



# *Computational modelling of synapses and learning in brains*

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# Why study learning?



**KordingLab**  
@KordingLab



In computational neuroscience can we realistically understand computation (say how we see a hand) or can we at best hope to understand the learning that gives rise to the computation?

4:30 PM · Oct 14, 2021 · Twitter Web App

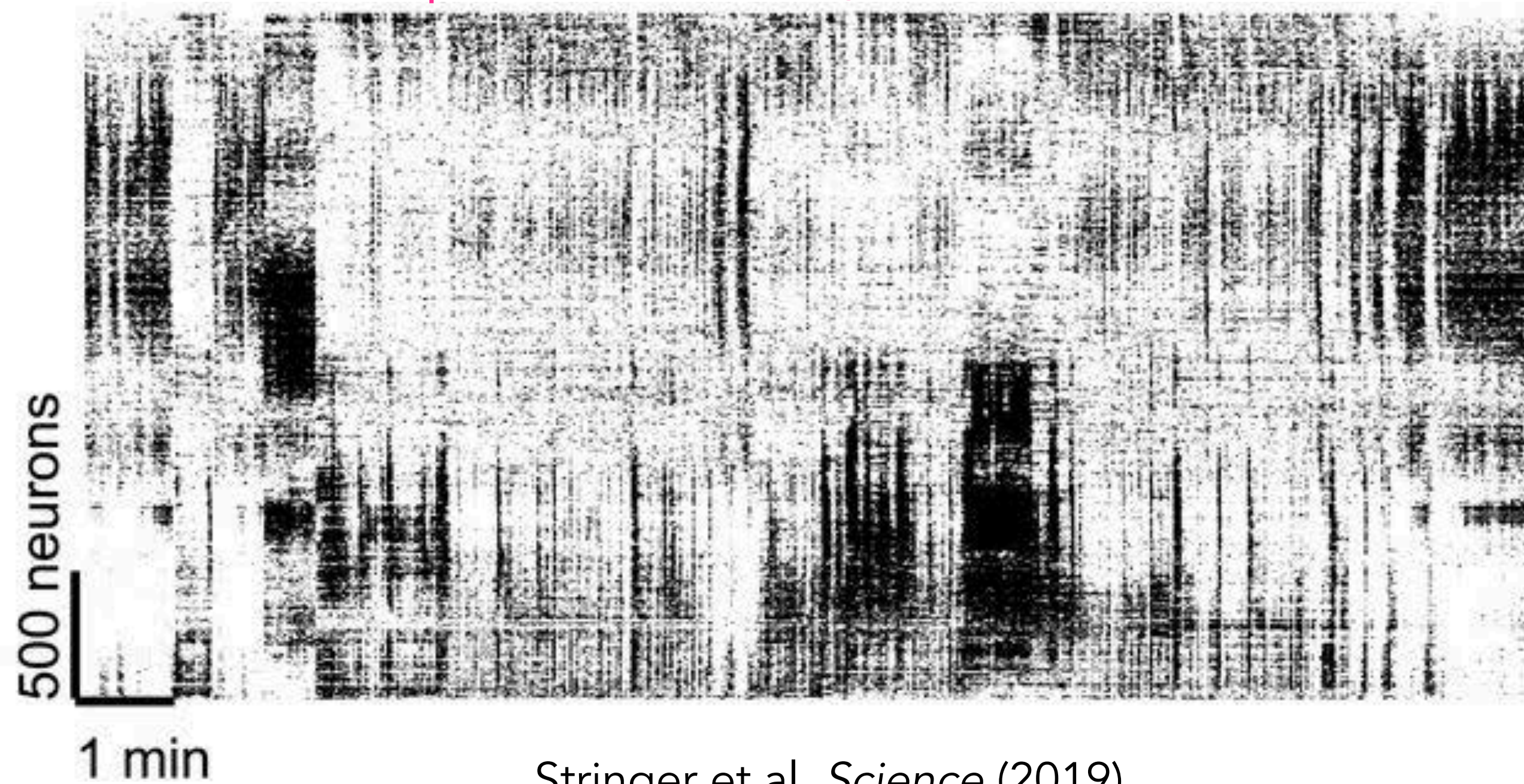
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*What does it mean to understand a neural network?*  
Lillicrap & Kording, arXiv (2019)



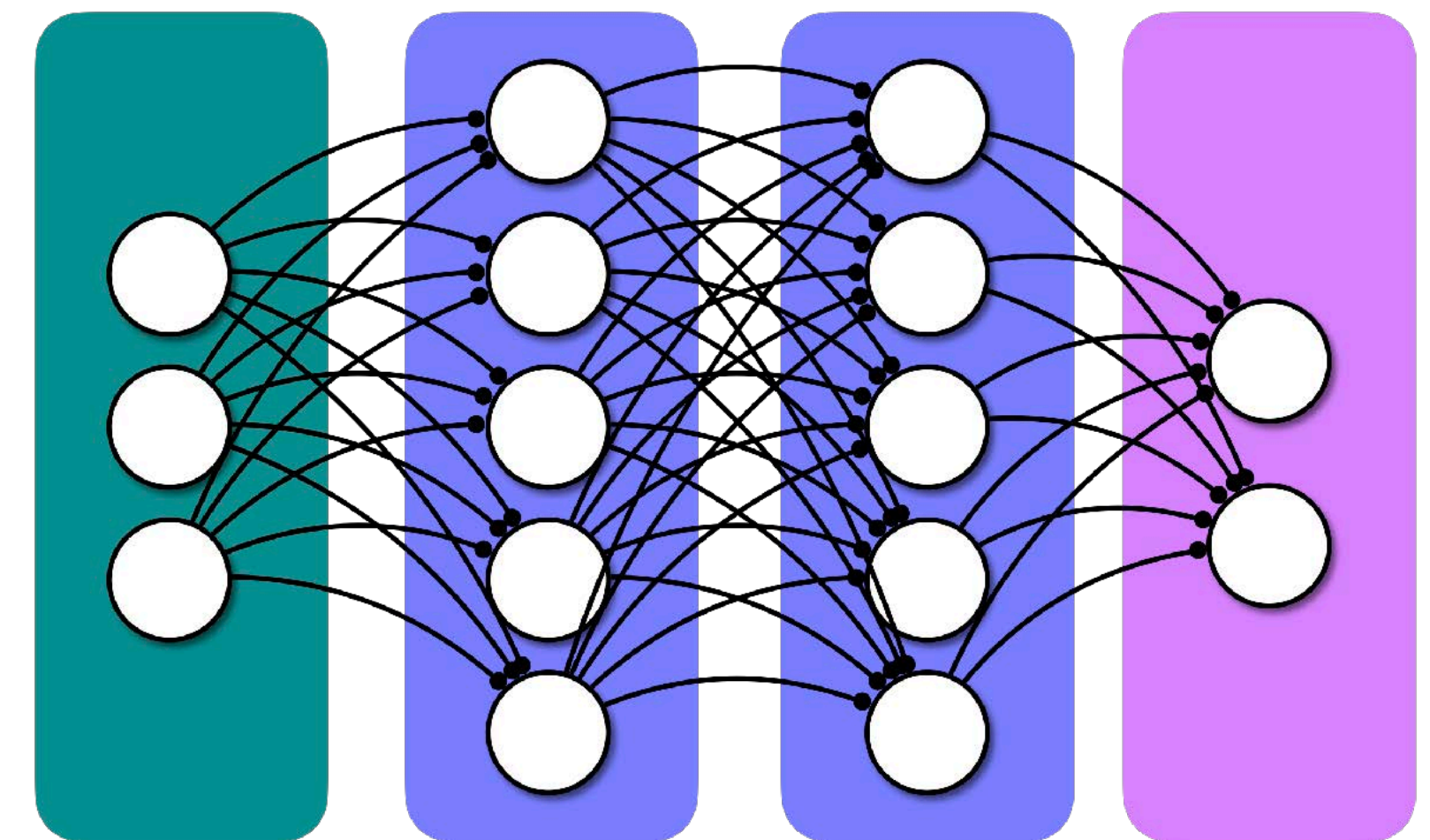
# Why study learning?

Population activity in the brain



Stringer et al, *Science* (2019)

Deep neural network





# What we will cover

## 1. Synapses

- What is a synapse and how does it work?
- Basic computational models of synaptic transmission

## 2. Synaptic plasticity

- Classic rate-based models of plasticity, Hebbian learning.
- Spike-timing dependent plasticity.

## 3. Associative memories and attractor networks

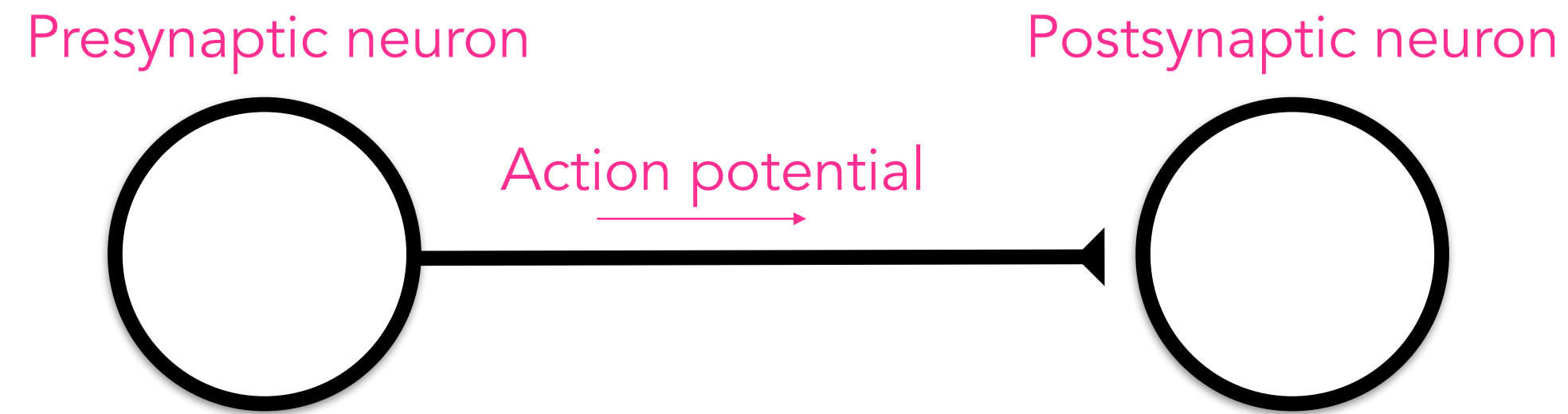
- The hippocampus
- Hopfield networks

## 4. Links to artificial neural networks

- The backpropagation rule
- Mapping deep learning to neurobiology

# 1. Synapses

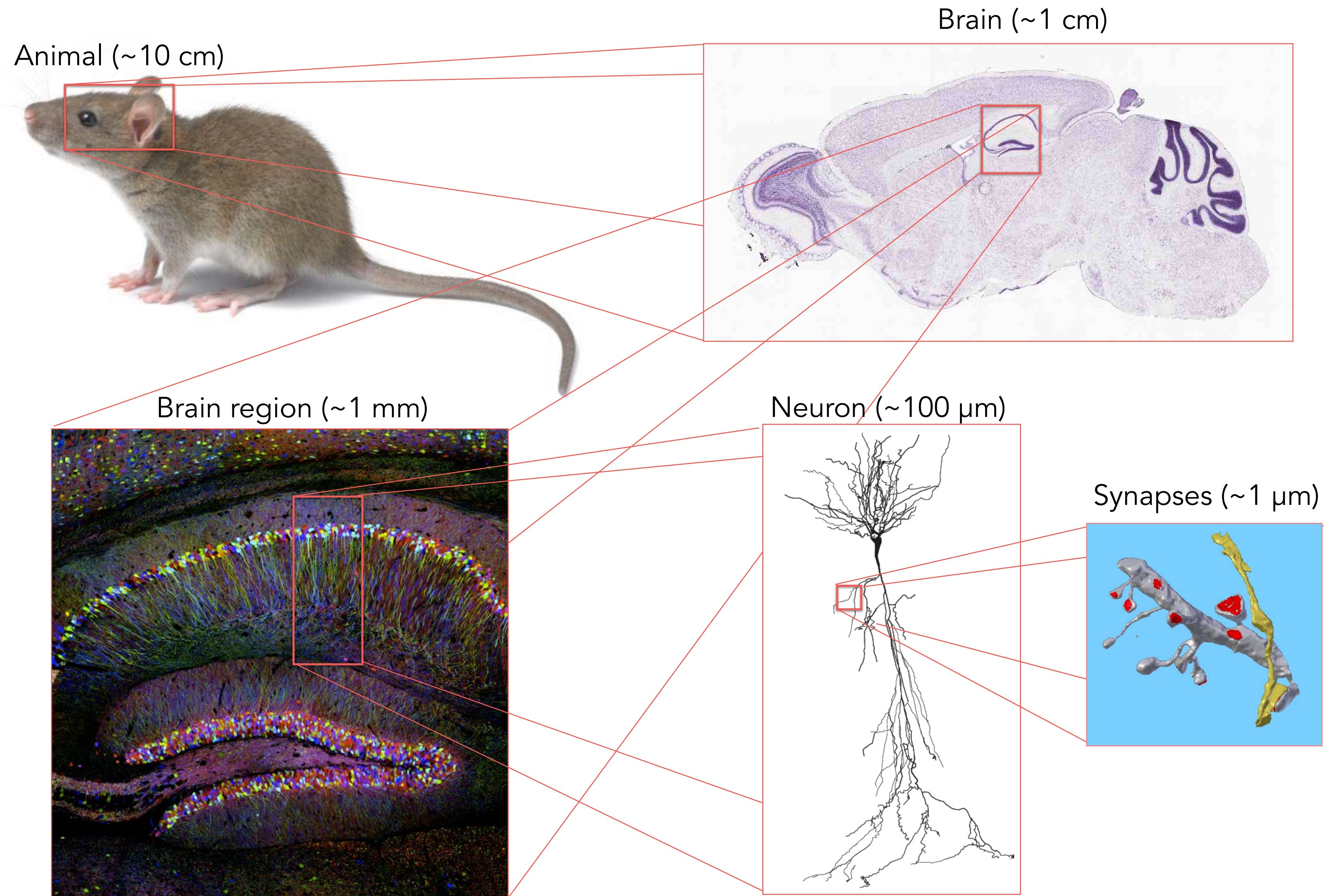
# What is a synapse?



- Synapses are the connections between neurons.
- They convert the action potential from one neuron's axon into a 'post-synaptic-potential' in the dendrite of another neuron.



# Zooming in on synapses





# How do synapses work?

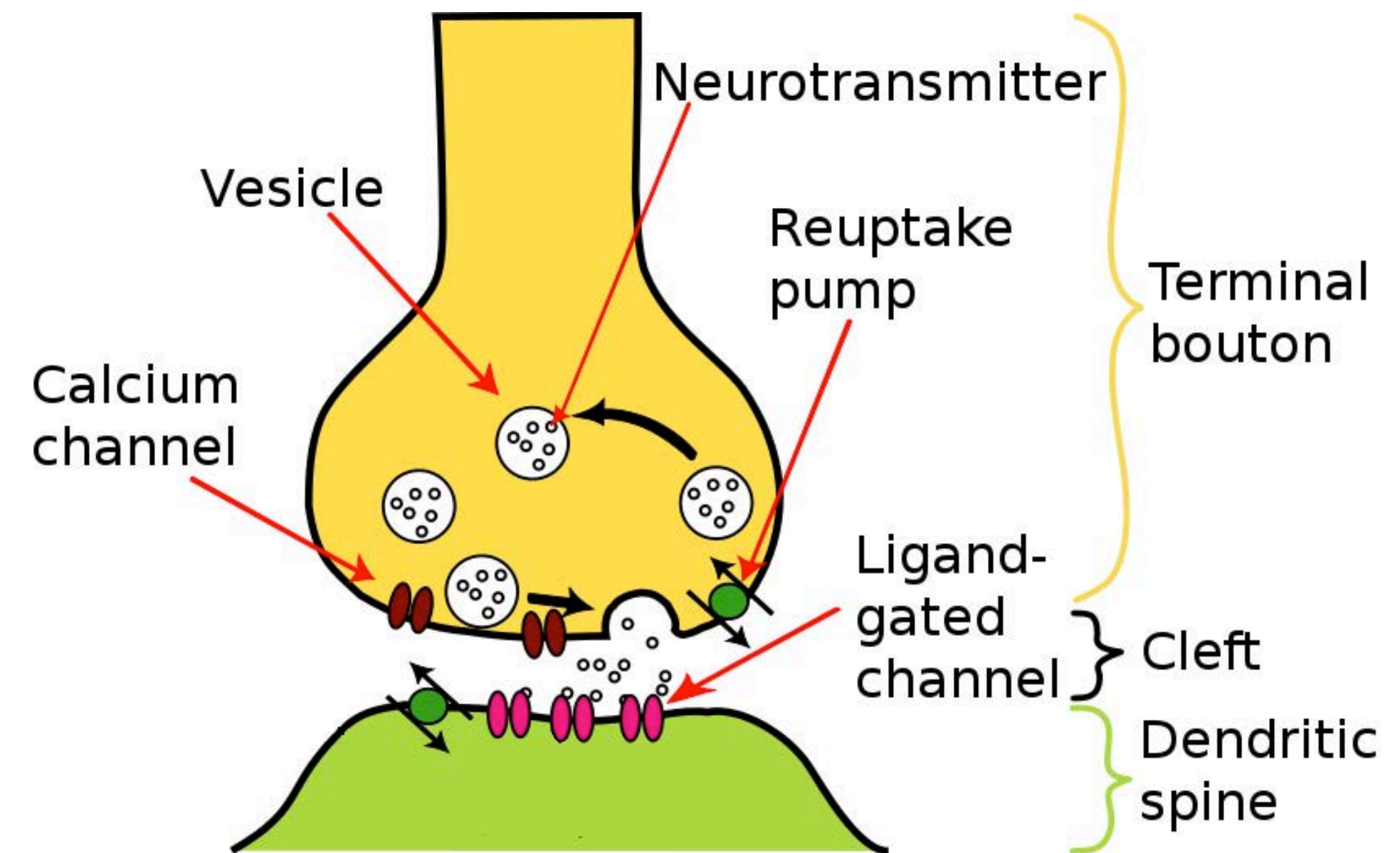
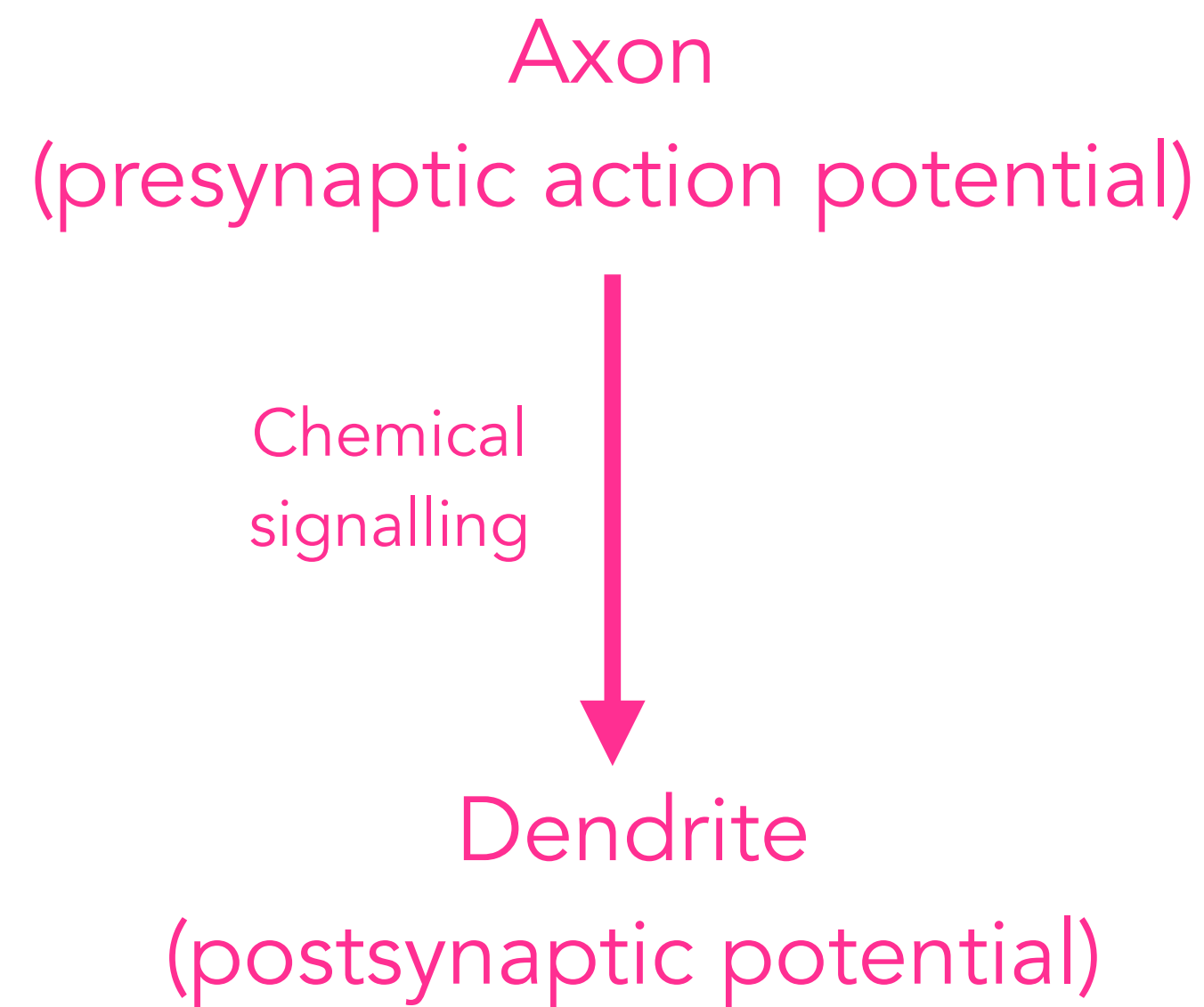
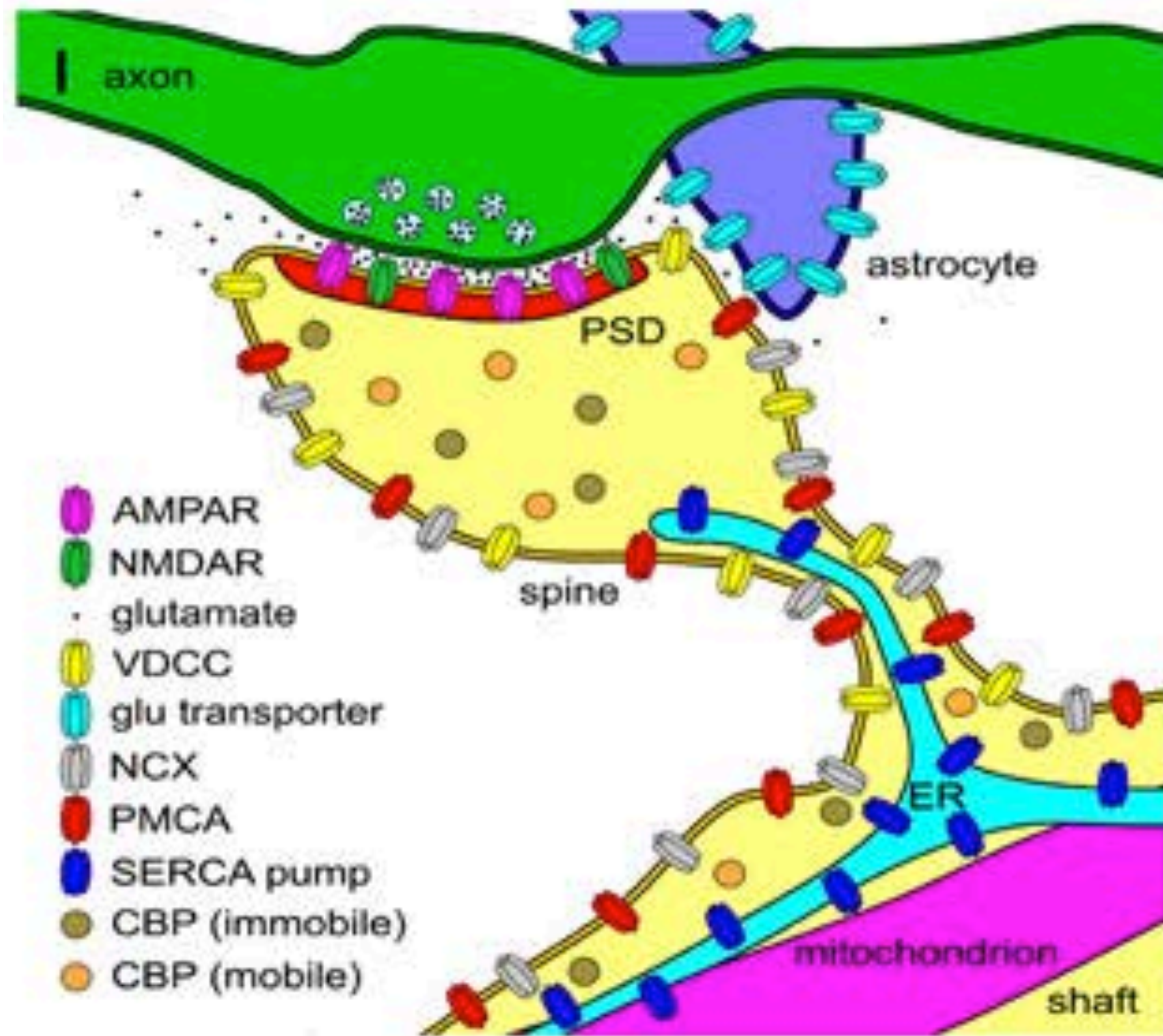


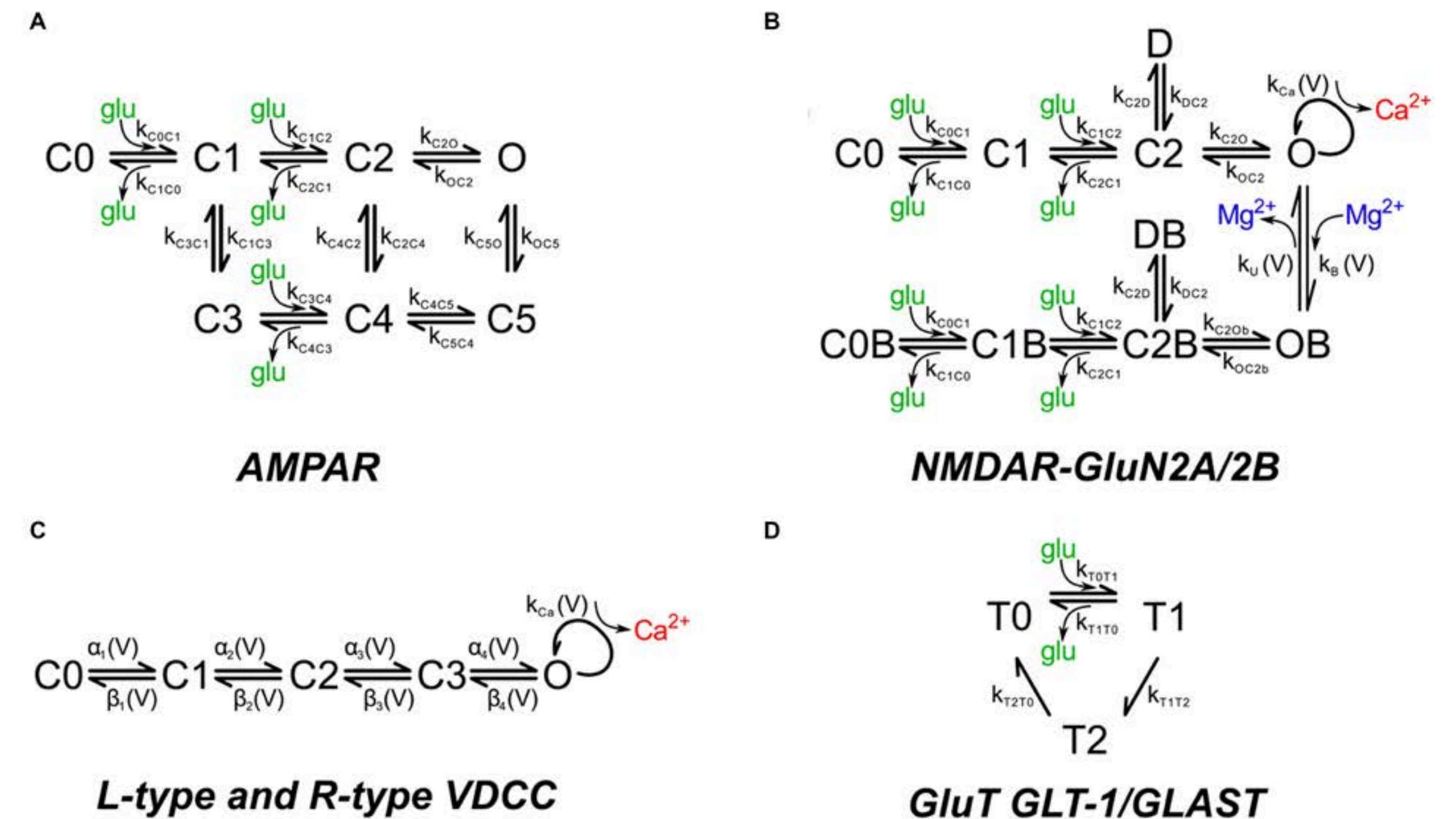
Image from Wikipedia (modified by C Houghton)



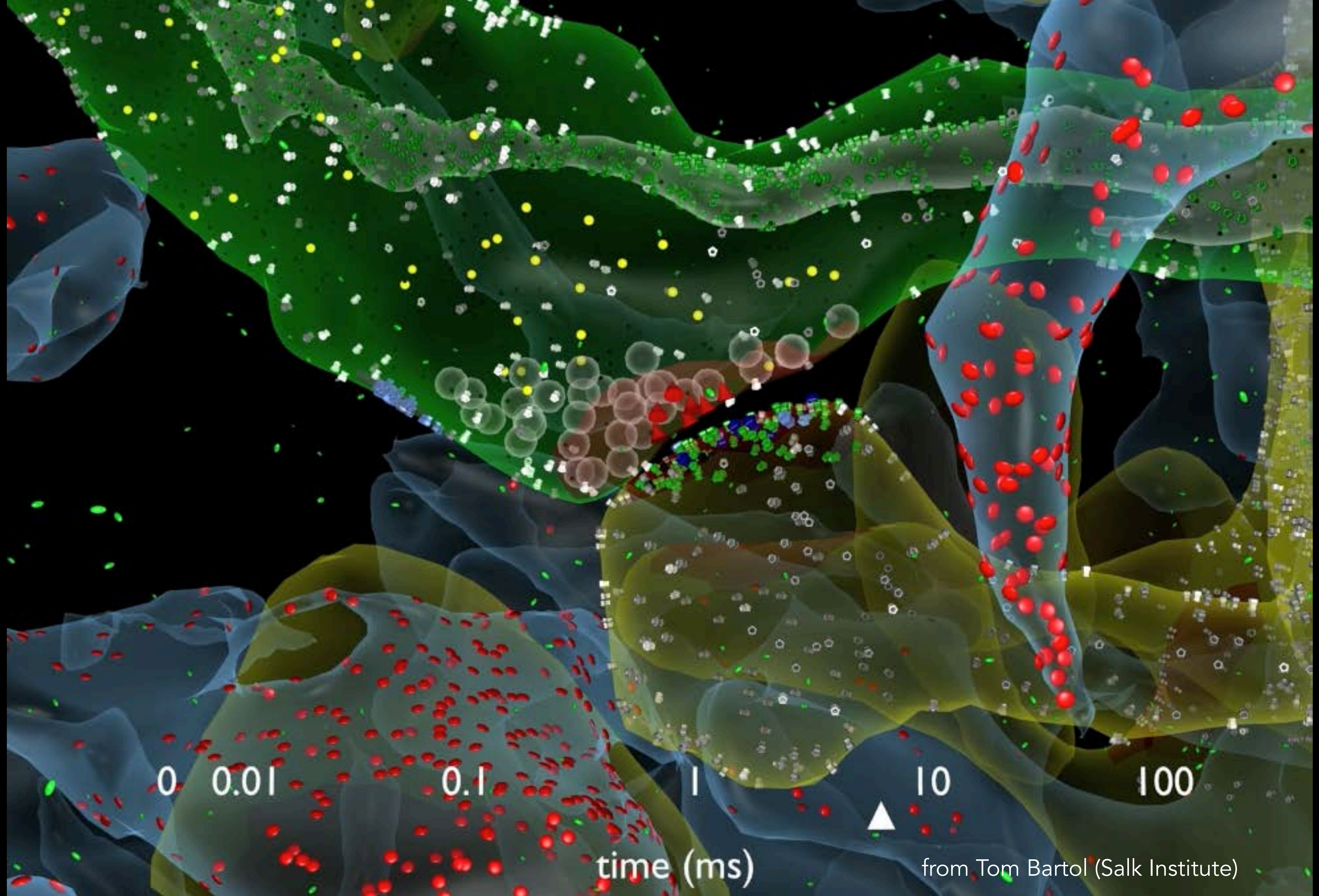
# MCell simulation of synaptic release



Bartol et al, *Frontiers Syn Neuro* (2015)









# How can we computationally model a synapse?

- We *could* simulate the dynamics of each molecule involved in the signalling process (like the MCell simulation).
- But since that is very computationally *expensive*, we might instead go for a reduced mass-action chemical-kinetics model.
- However a lot of people still find even that too expensive and parameter-heavy, so instead use even simpler phenomenological models that black-box the synapse as a simple input-output system.



How detailed should a model be?



# Details vs realism

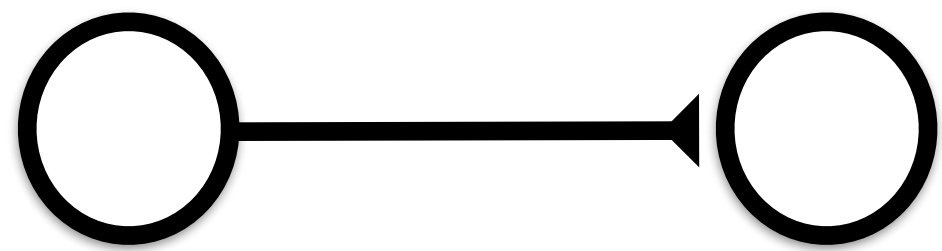


c.f. Brette, *PLoS Comput Biol*, 2015

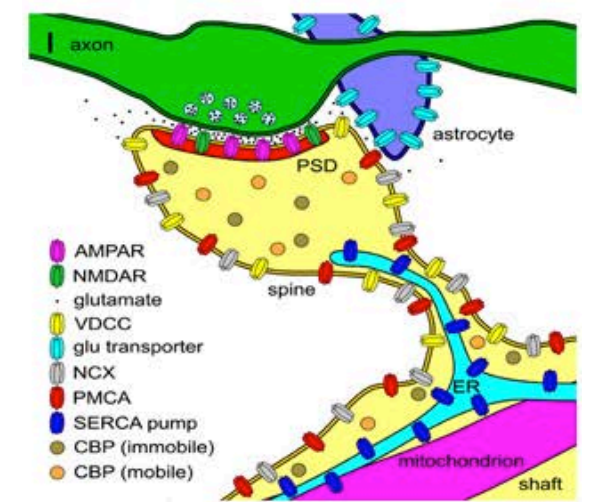


# Models

Abstract



Realistic



## Abstract models

Simple

Hard to relate to biology

Few parameters

Fast simulation

Mathematical analysis

Generic

vs

vs

vs

vs

vs

vs

## Realistic models

Detailed

Contains stuff you could measure

Lots of parameters

Slow simulation

Intractable

Specific



# Simple synapse models

The most common way to phenomenologically model a synapse is as a time-dependent conductor in series with a battery.

$$I_s(t) = \bar{g}_s s(t) (E_s - V)$$

The value of  $E_s$  determines whether the synapse is excitatory or inhibitory:

for excitatory synapses  $E_s$  usually = 0 mV

for inhibitory synapses  $E_s$  usually =  $V_{rest}$

But how should we model  $s(t)$ ?

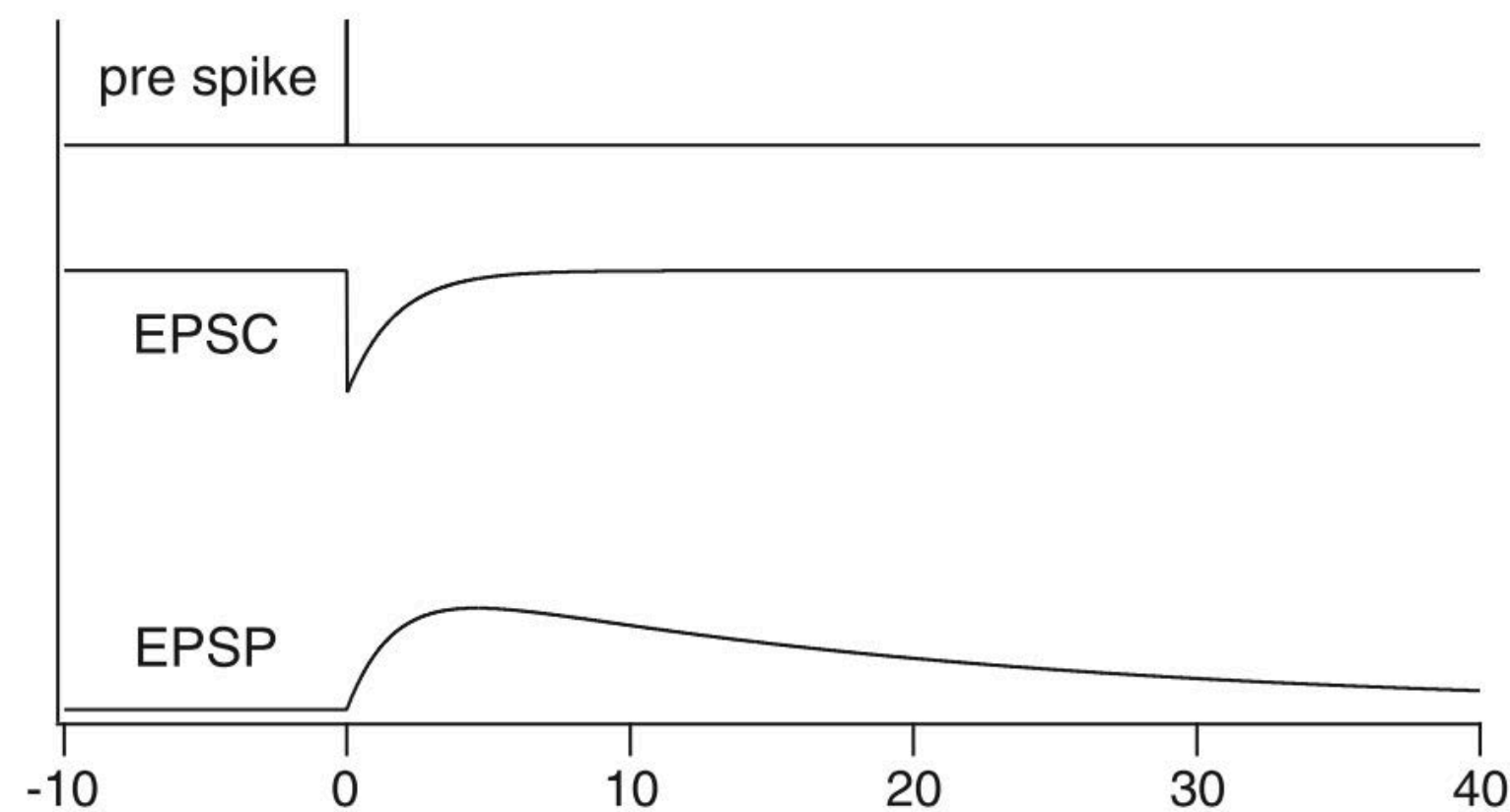


# Simple synapse models

## Single exponential

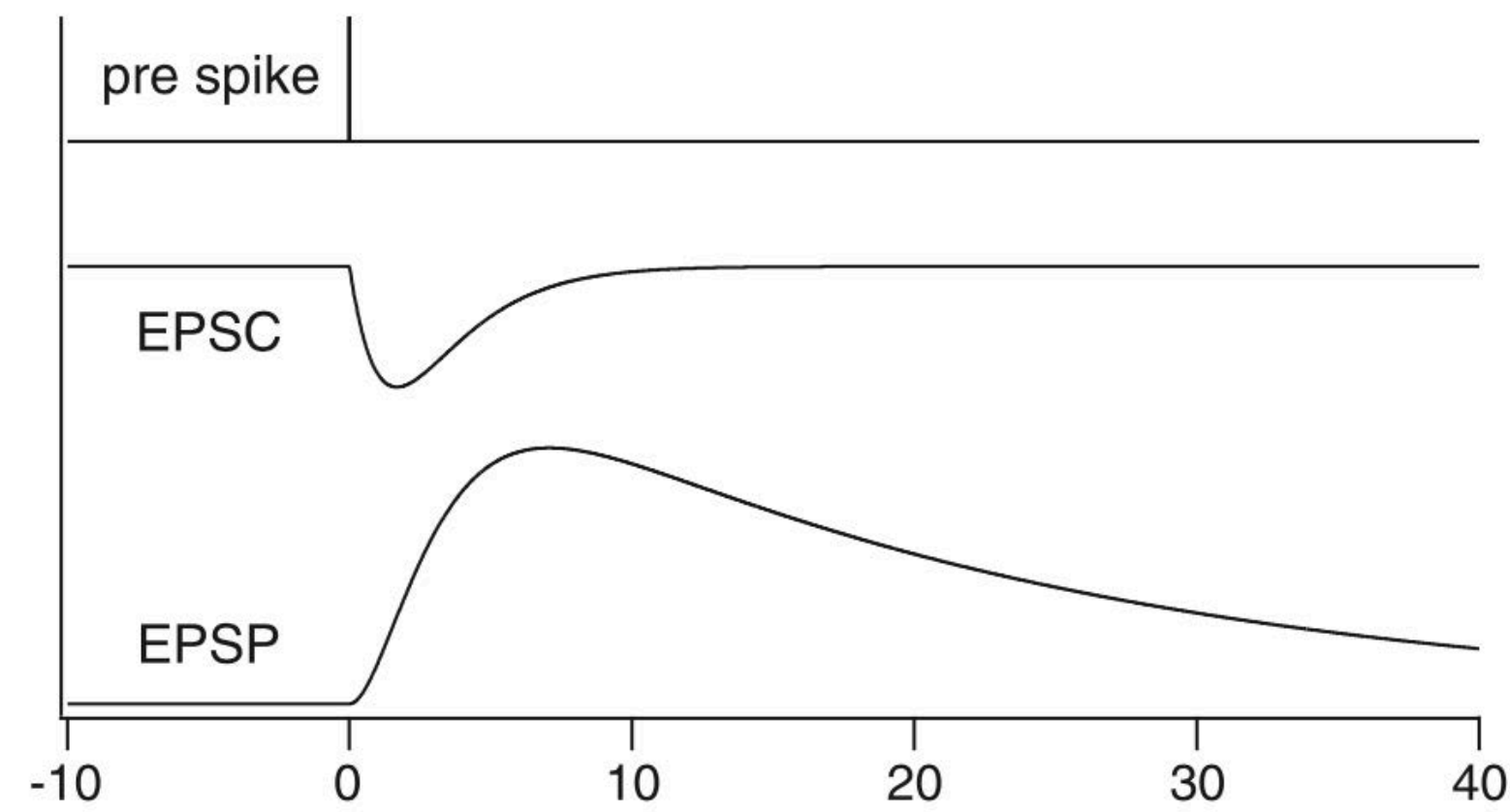
$$s(t) \rightarrow s(t) + 1$$

$$s(t) = e^{-t/\tau_s}$$



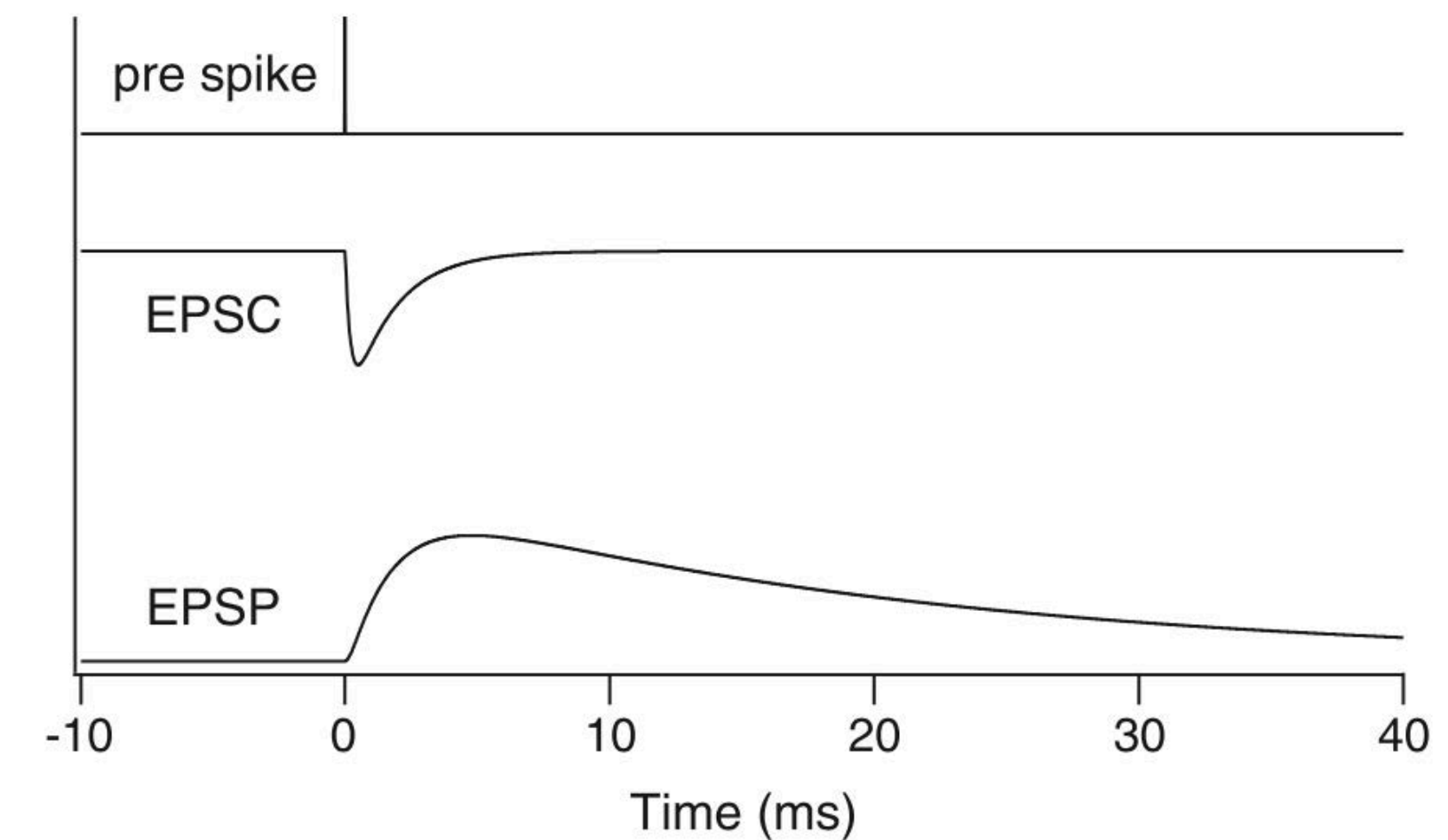
## Alpha function

$$s(t) = te^{-t/\tau_s}$$



## Difference of two exponentials

$$s(t) = e^{-t/\tau_{decay}} - e^{-t/\tau_{rise}}$$



Figures from Roth & van Rossum

[http://homepages.inf.ed.ac.uk/mvanross/reprints/roth\\_mvr\\_chap.pdf](http://homepages.inf.ed.ac.uk/mvanross/reprints/roth_mvr_chap.pdf)



# Summary on synapses

- Synapses are the **connections** between neurons.
- They **convert** the pre-synaptic **action potential** to a (excitatory or inhibitory) **post-synaptic potential** via a chemical intermediate stage.
- They are **complicated** molecular machines.
- We can model them at multiple levels of granularity, as **appropriate for the task at hand**.

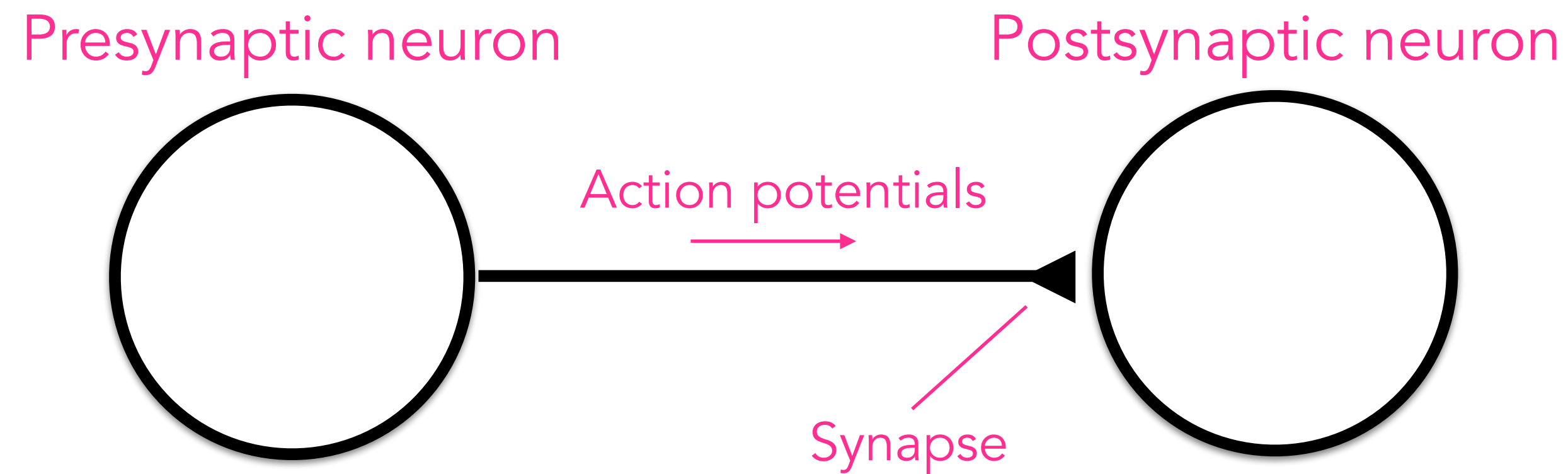


## 2. Synaptic plasticity



# Long-term synaptic plasticity

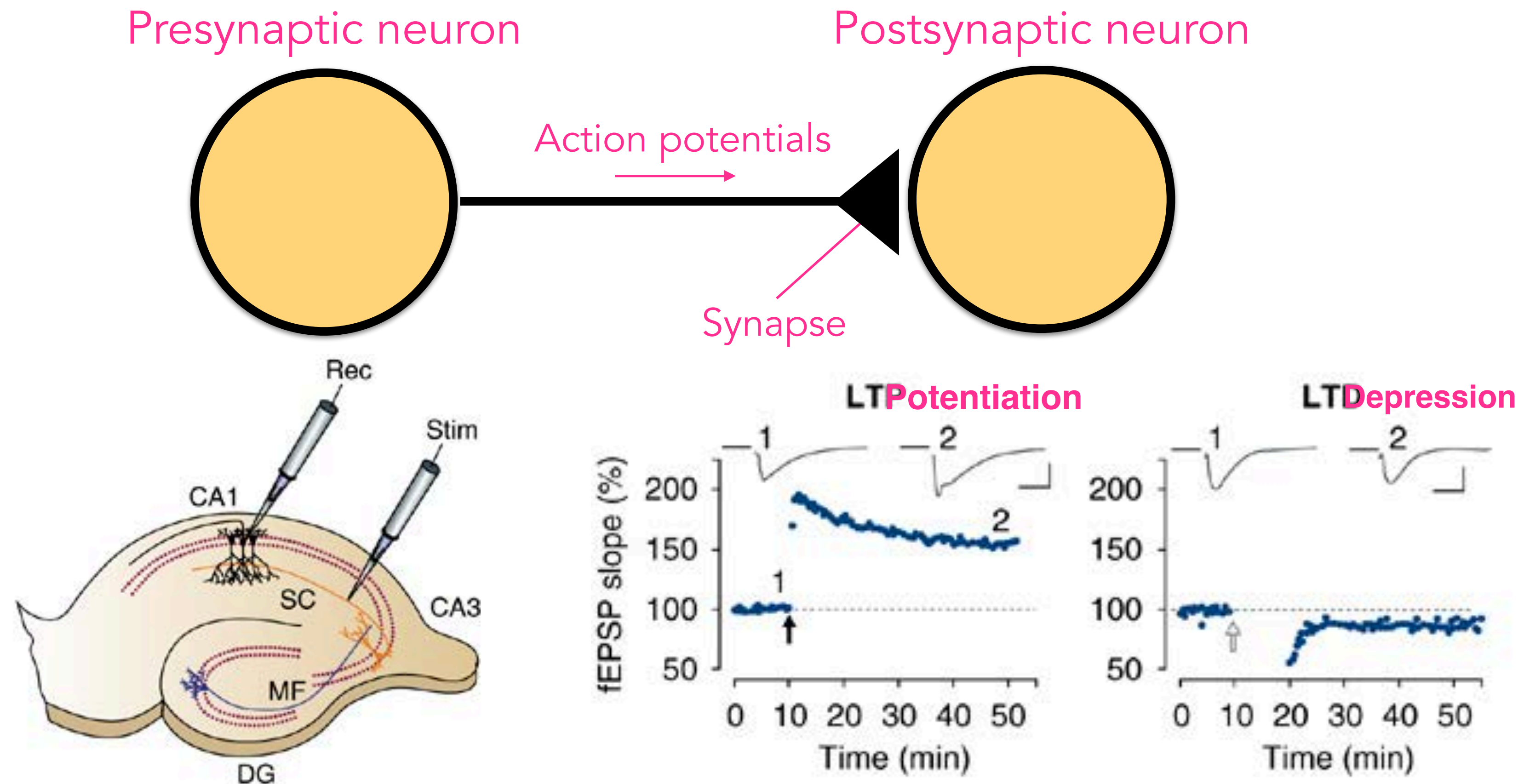
Long-term synaptic plasticity is a (activity-dependent) semi-permanent change in the strength of the connection from one neuron to another.





# Long-term synaptic plasticity

Long-term synaptic plasticity is a (activity-dependent) semi-permanent change in the strength of the connection from one neuron to another.





# Learning and memory via synaptic plasticity

- Synaptic plasticity is generally believed to underlie long-term memory.
- Synapses change their strength according to certain 'rules of plasticity'.
- Synaptic plasticity is linked to learning and memory as follows:
  - Neural activity during learning triggers synaptic strength changes.
  - Synaptic strength changes alters the propensity for neurons to fire.
  - Next time the same neural circuit receives an input, it responds in a different fashion than it otherwise would have. **That's memory.**



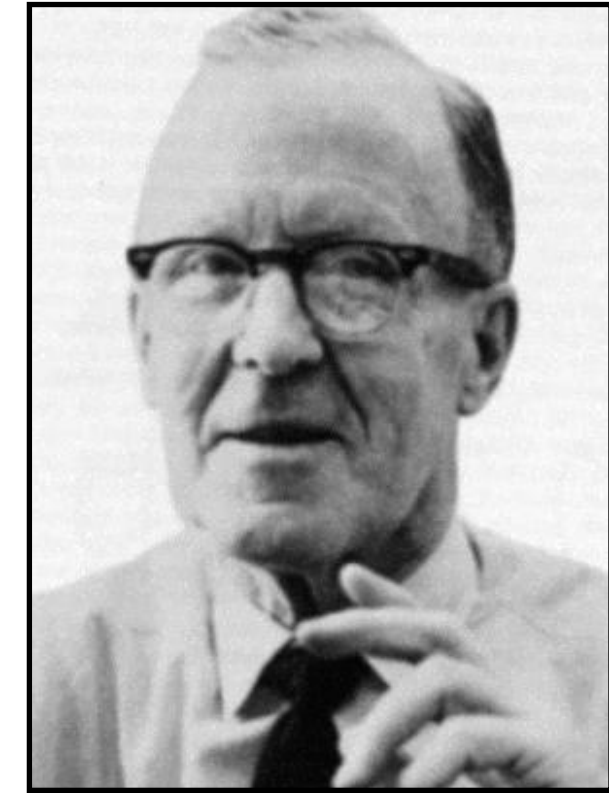
What are these 'rules of synaptic plasticity'?

Some mathematical function of the pre- and post-synaptic neurons' activities... and maybe other stuff?

$$f(pre, post, weight, error, \dots) = ?$$



# Hebbian plasticity



Donald Hebb

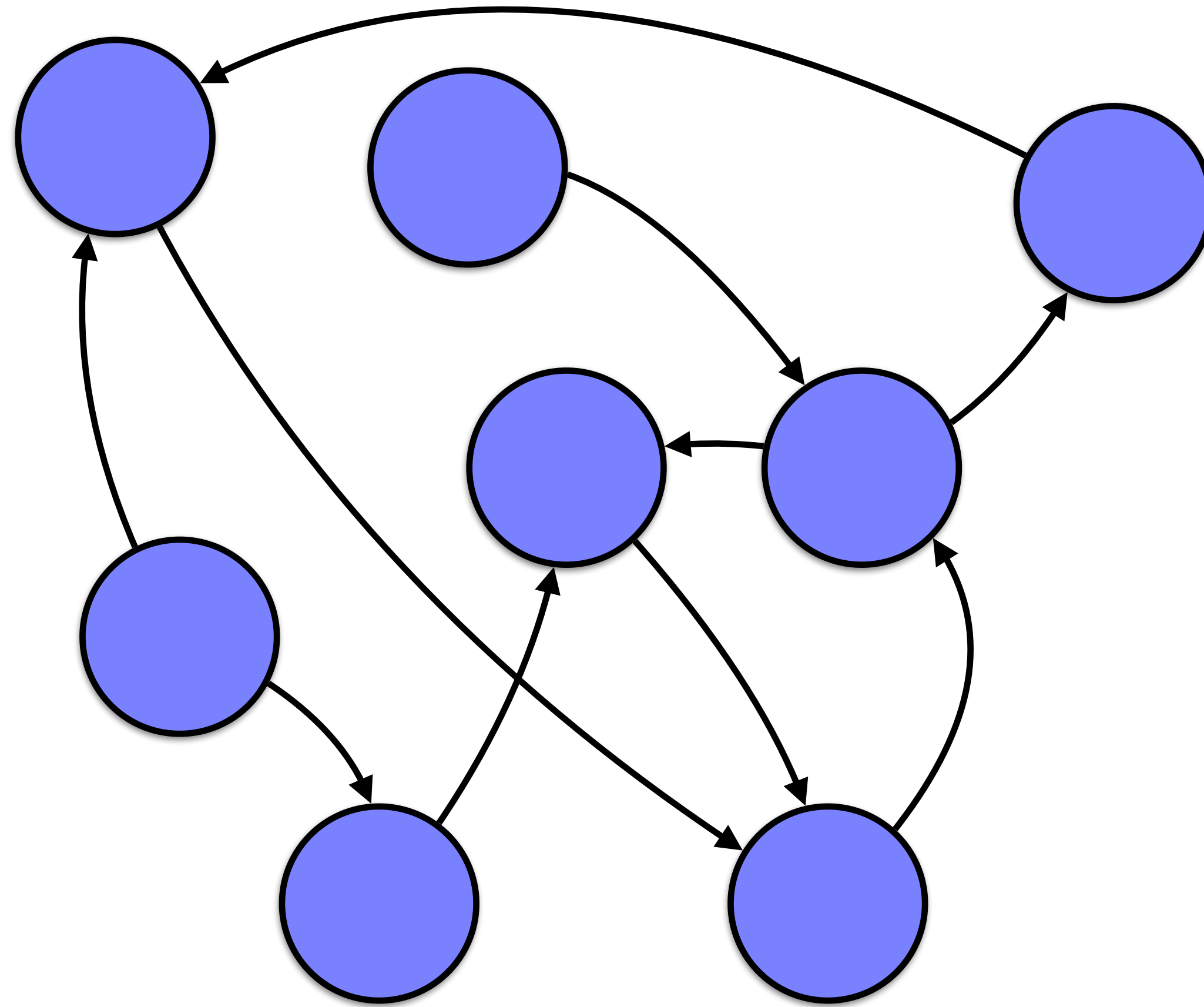
*"When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased."*

— Donald Hebb (1949)

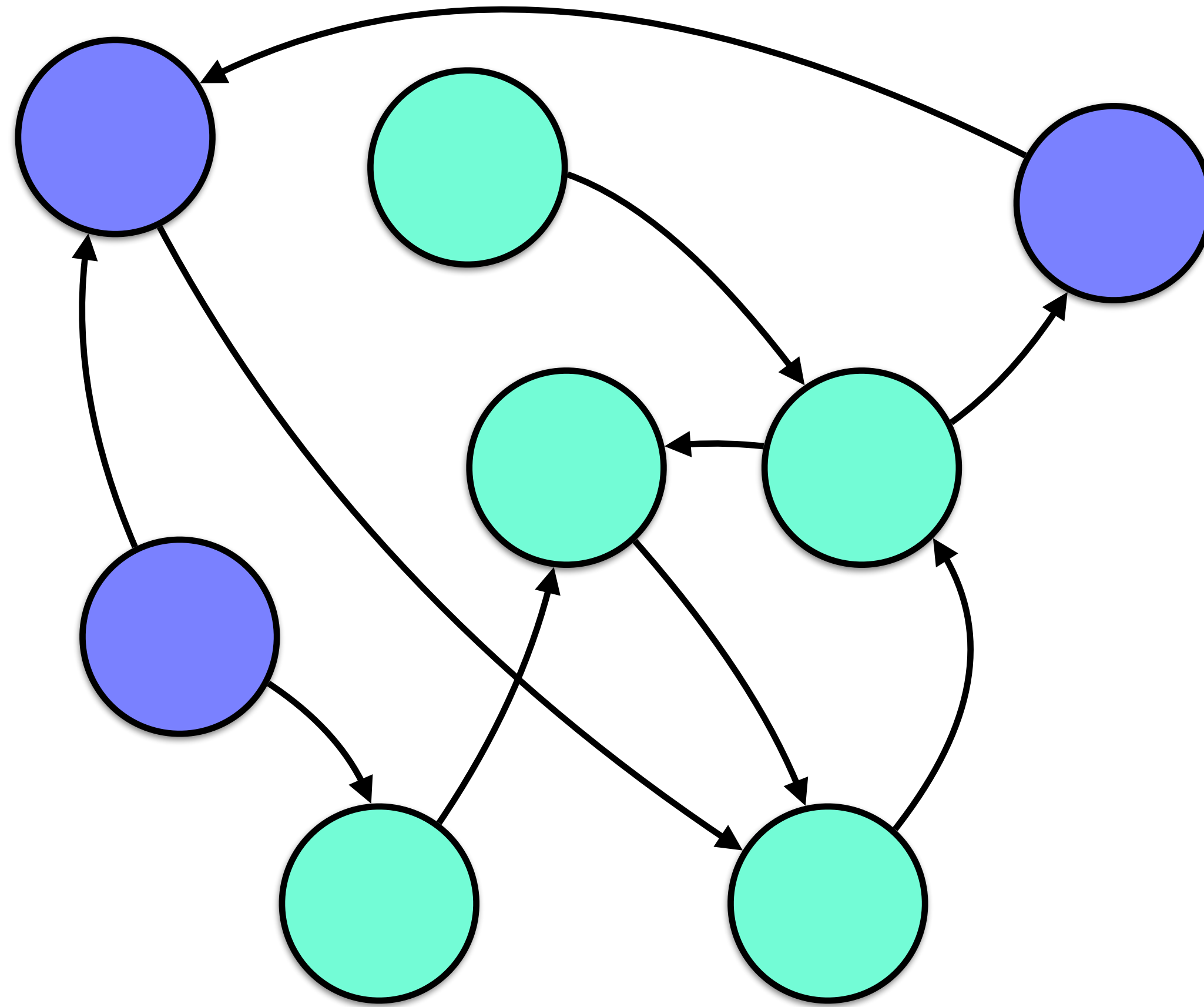
a.k.a. "neurons that fire together wire together." (Carla Shatz)



# Hebbian plasticity

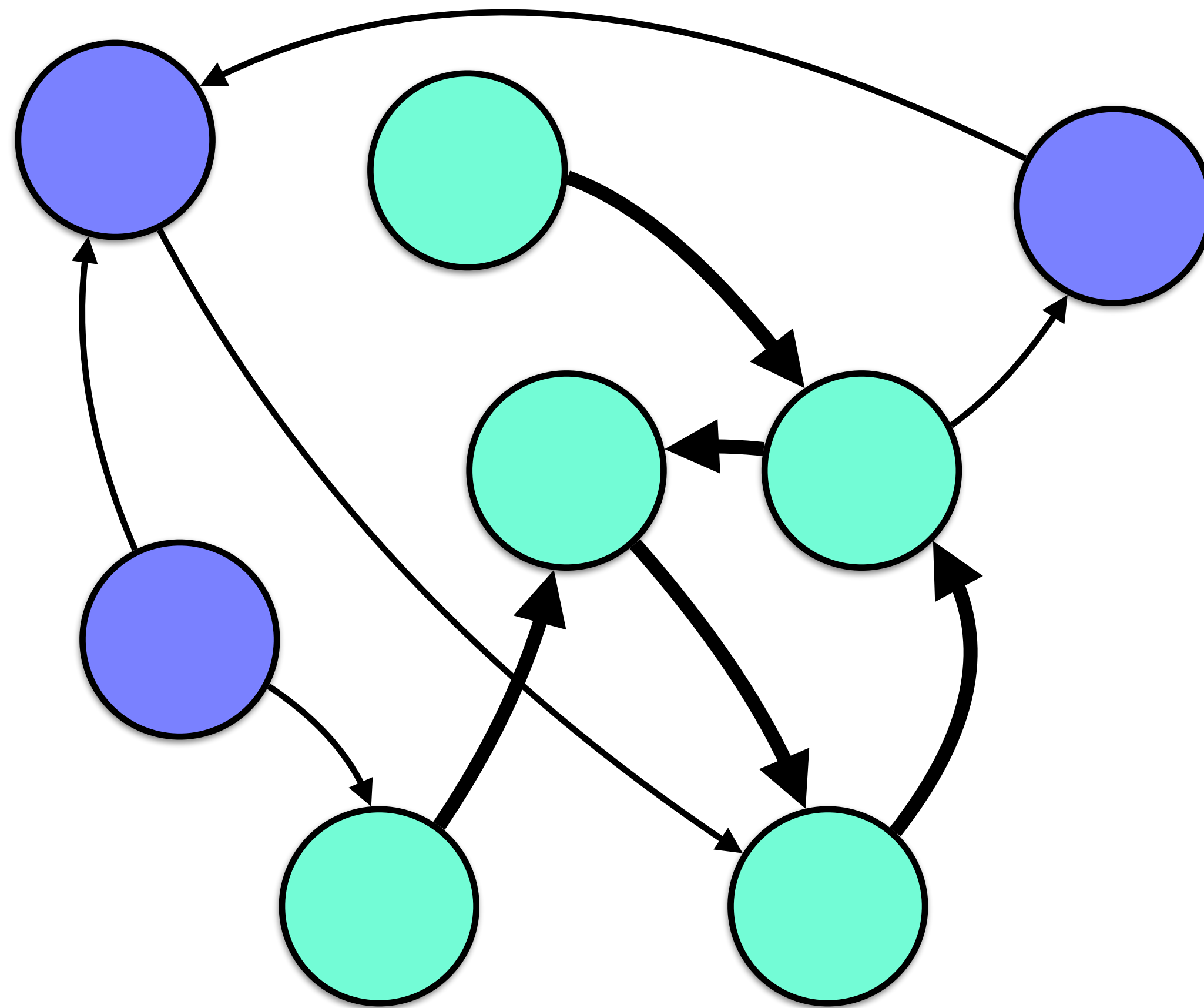


# Hebbian plasticity

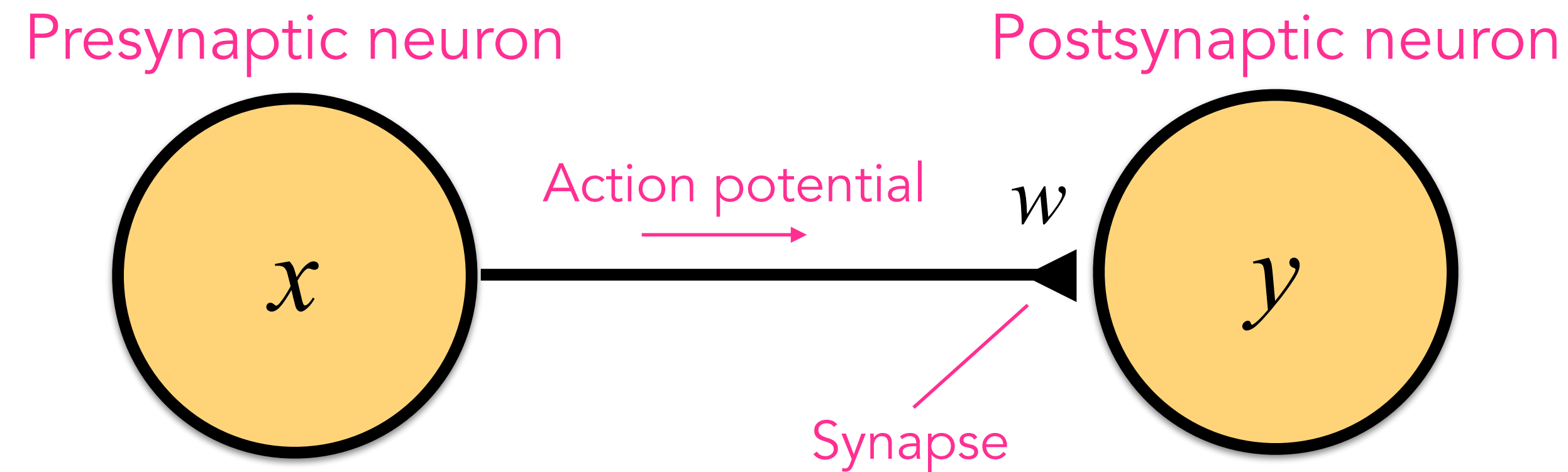




# Hebbian plasticity



# Modelling Hebbian plasticity



$$\Delta w = f(x, y) = ???$$

A Hebbian rule:  $\Delta w = \eta xy$

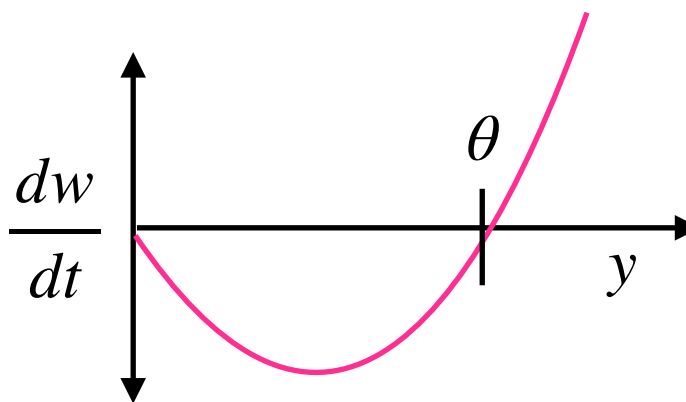
Note that **dynamics are unstable**:  $w$  and therefore  $y$  grow without bound.



# How to stabilise plasticity?

Idea 1: BCM (Bienenstock, Cooper & Munro, 1982)

Modify the basic Hebbian rule by including a postsynaptic threshold for plasticity:

$$\frac{dw}{dt} = \eta_w xy(y - \theta)$$


The graph shows the change in weight  $\frac{dw}{dt}$  on the vertical axis versus the postsynaptic activity  $y$  on the horizontal axis. A pink curve starts at the origin (0,0), dips into the negative region (depression) for low values of  $y$ , reaches a minimum, and then crosses the horizontal axis at a point labeled  $\theta$ . For values of  $y$  greater than  $\theta$ , the curve is positive (potentiation) and increases quadratically.

The key idea for stability is that the threshold is also plastic:

$$\frac{d\theta}{dt} = \eta_\theta (y^2 - \theta_\infty)$$

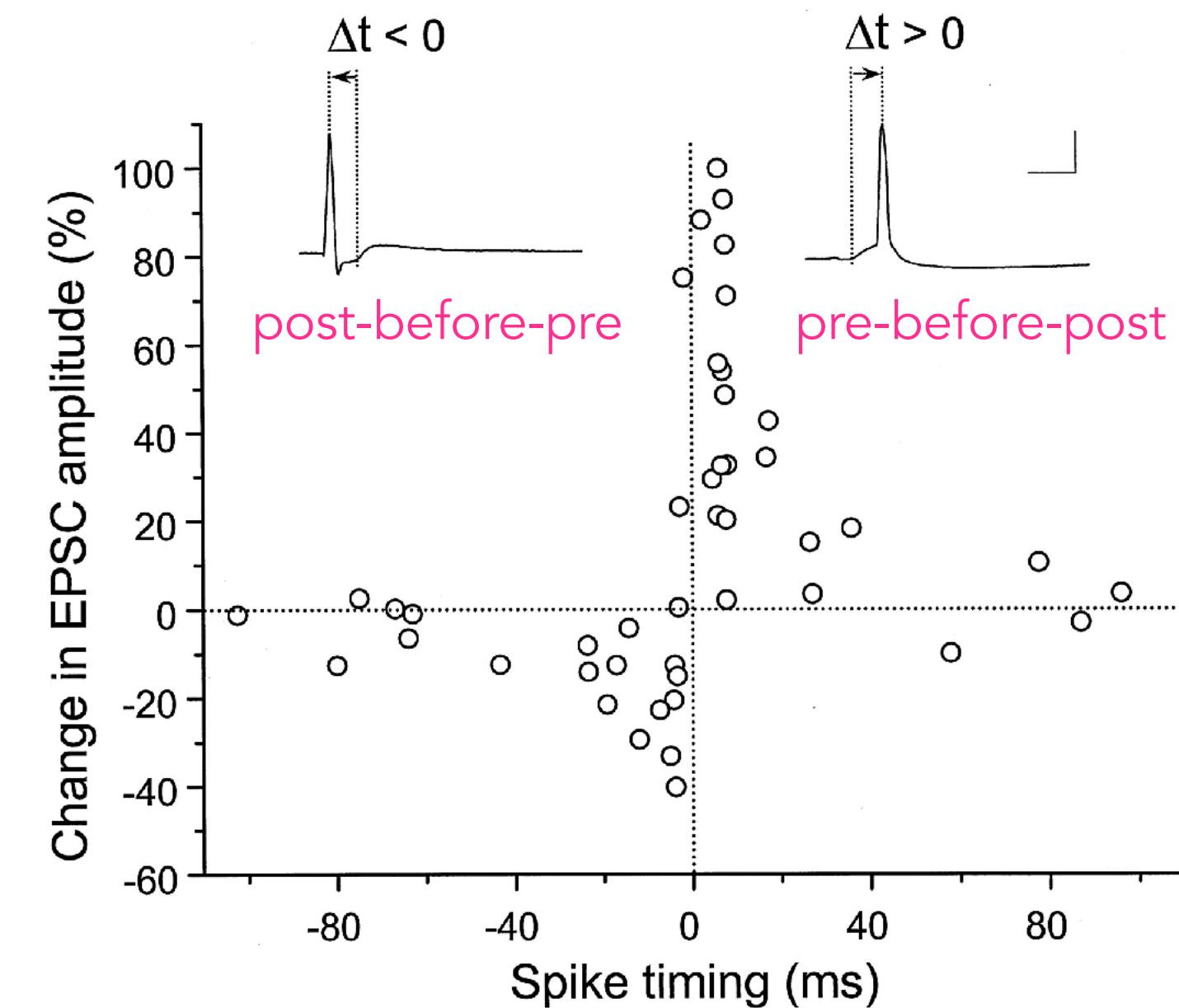
Idea 2: Homeostatic plasticity/synaptic scaling (Turrigiano et al, 1998)

Make the weights scale multiplicatively on a slow timescale, to keep postsynaptic activity at some target level:

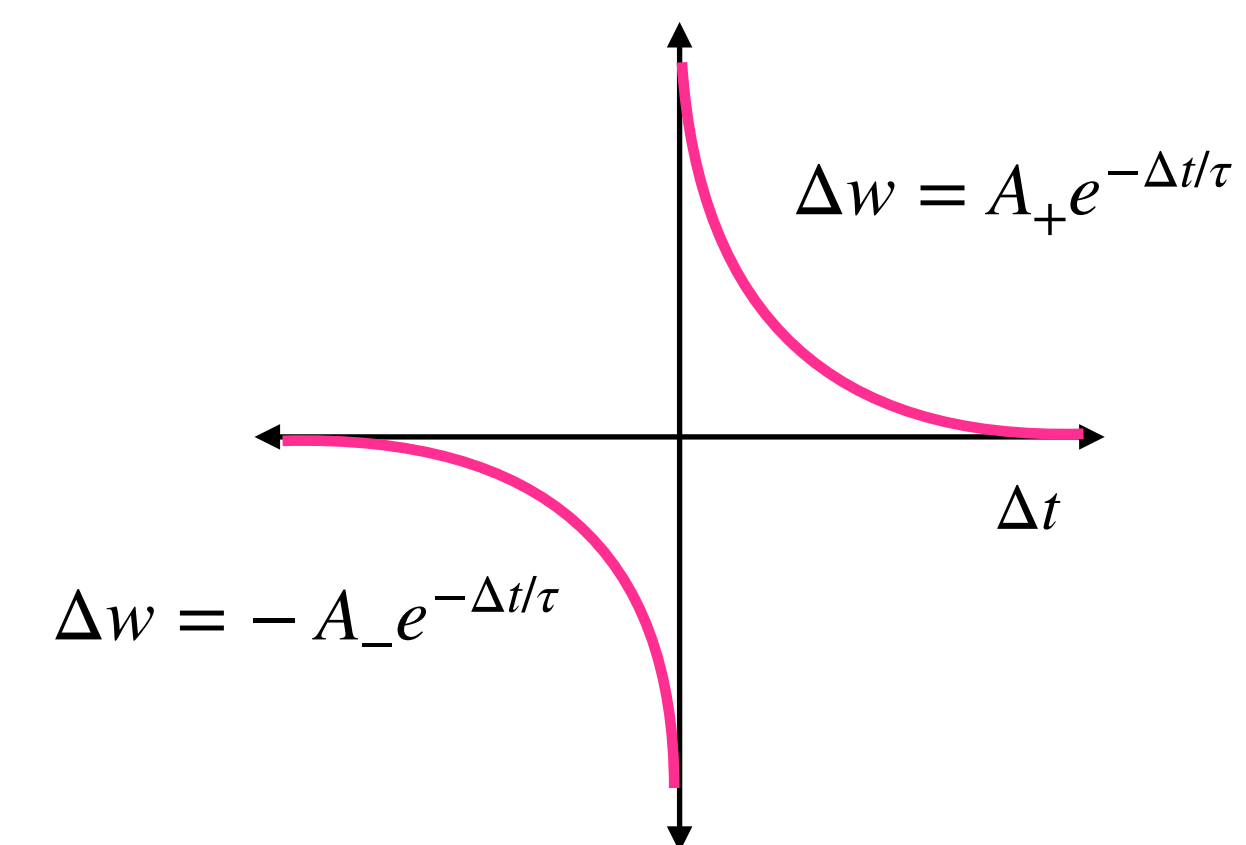
$$\frac{dw}{dt} = \eta_{ss} w (y_{target} - y)$$

# Spike-timing-dependent plasticity

- STDP (discovered in late 1990s) encapsulates the idea of causality implied by Hebb:
  - if presynaptic spike A happened just *before* postsynaptic spike B, A *could have caused* B.
  - on the other hand, if presynaptic spike A happened just *after* postsynaptic spike B, A *could not have caused* B.
- Classic STDP: Pre-before-post causes LTP, post-before-pre causes LTD.
- STDP's existence implies that synapses can detect millisecond-level differences in spike timing when deciding whether to strengthen or weaken.
- When first discovered it was seen as the possible "atom of plasticity".
- "Things turned out to be just as simple as we first thought"  
— *No biologist, ever*

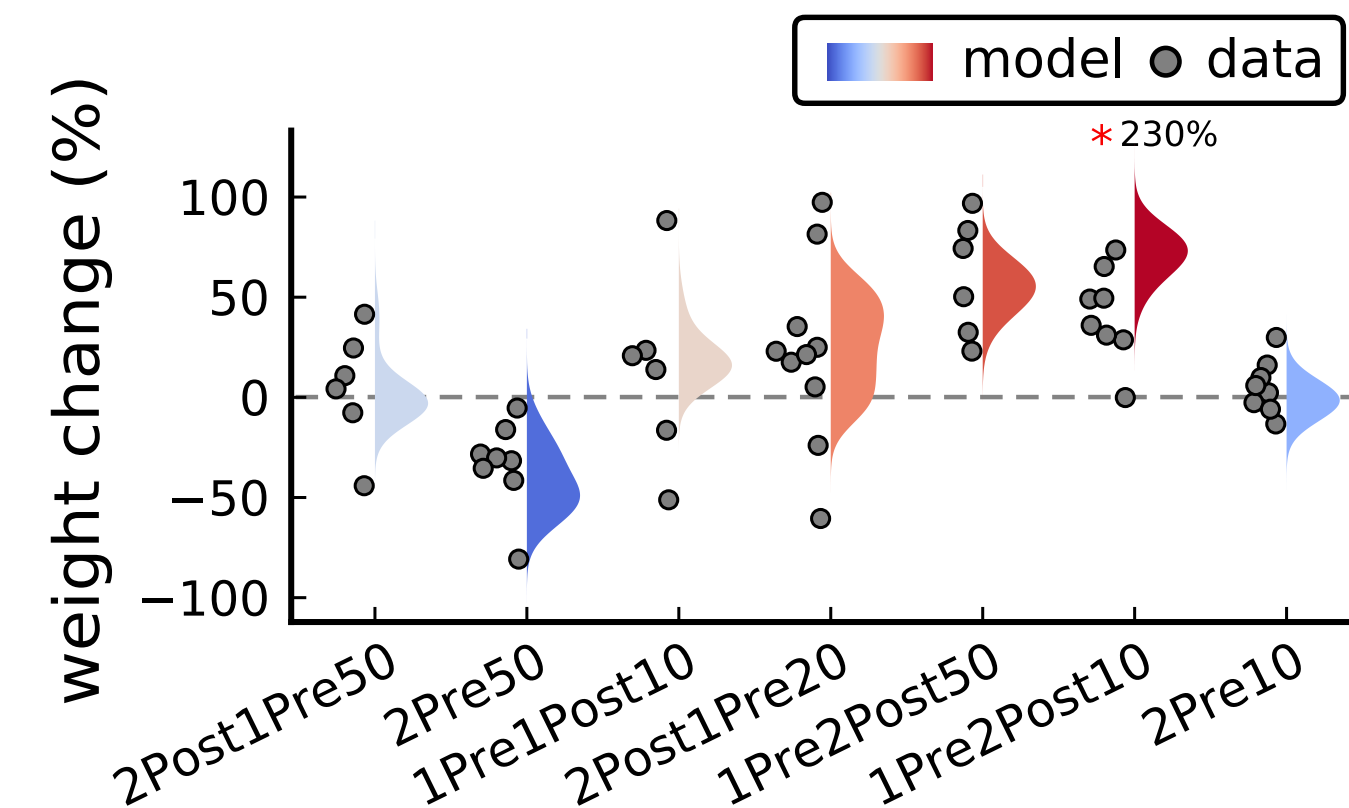
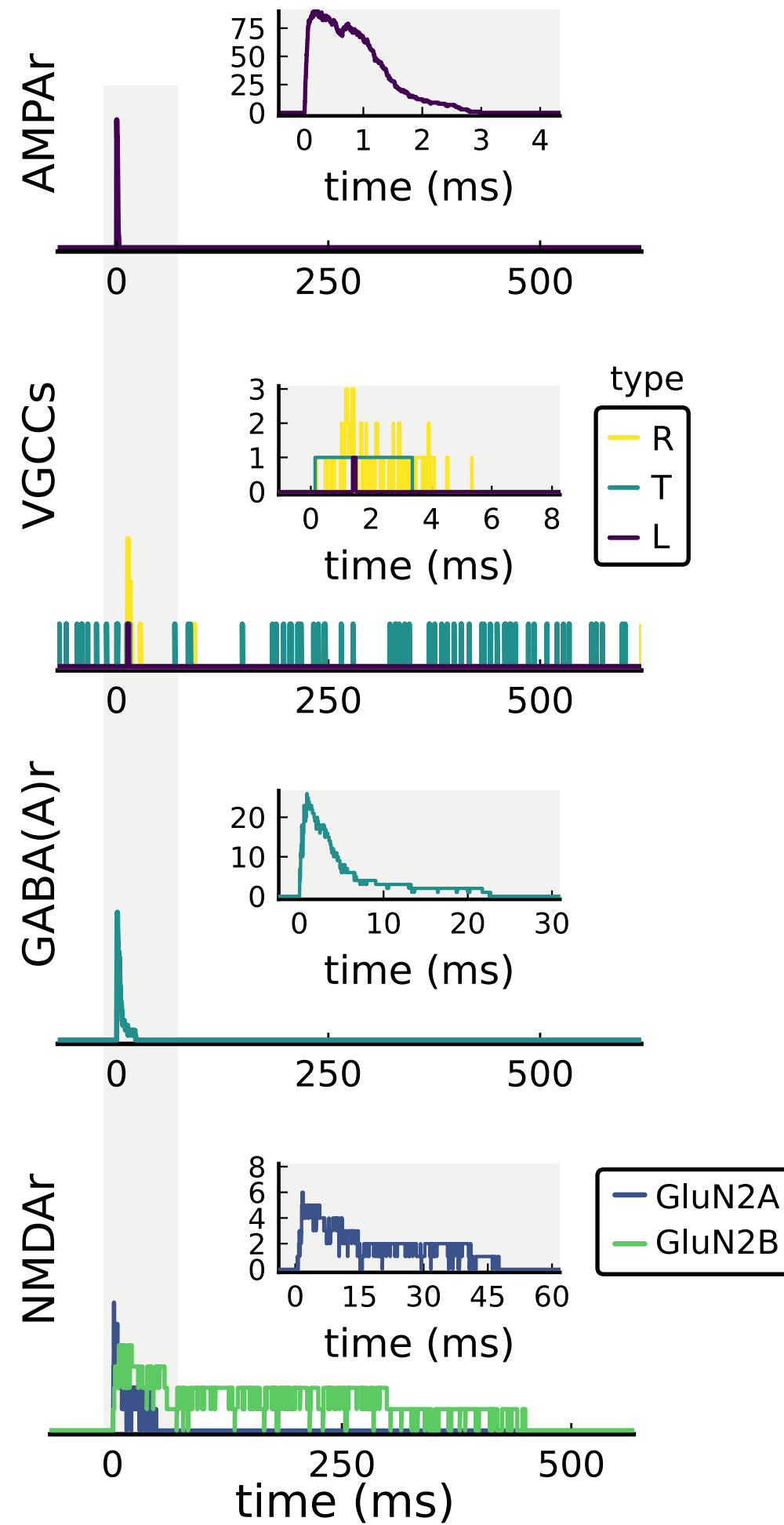
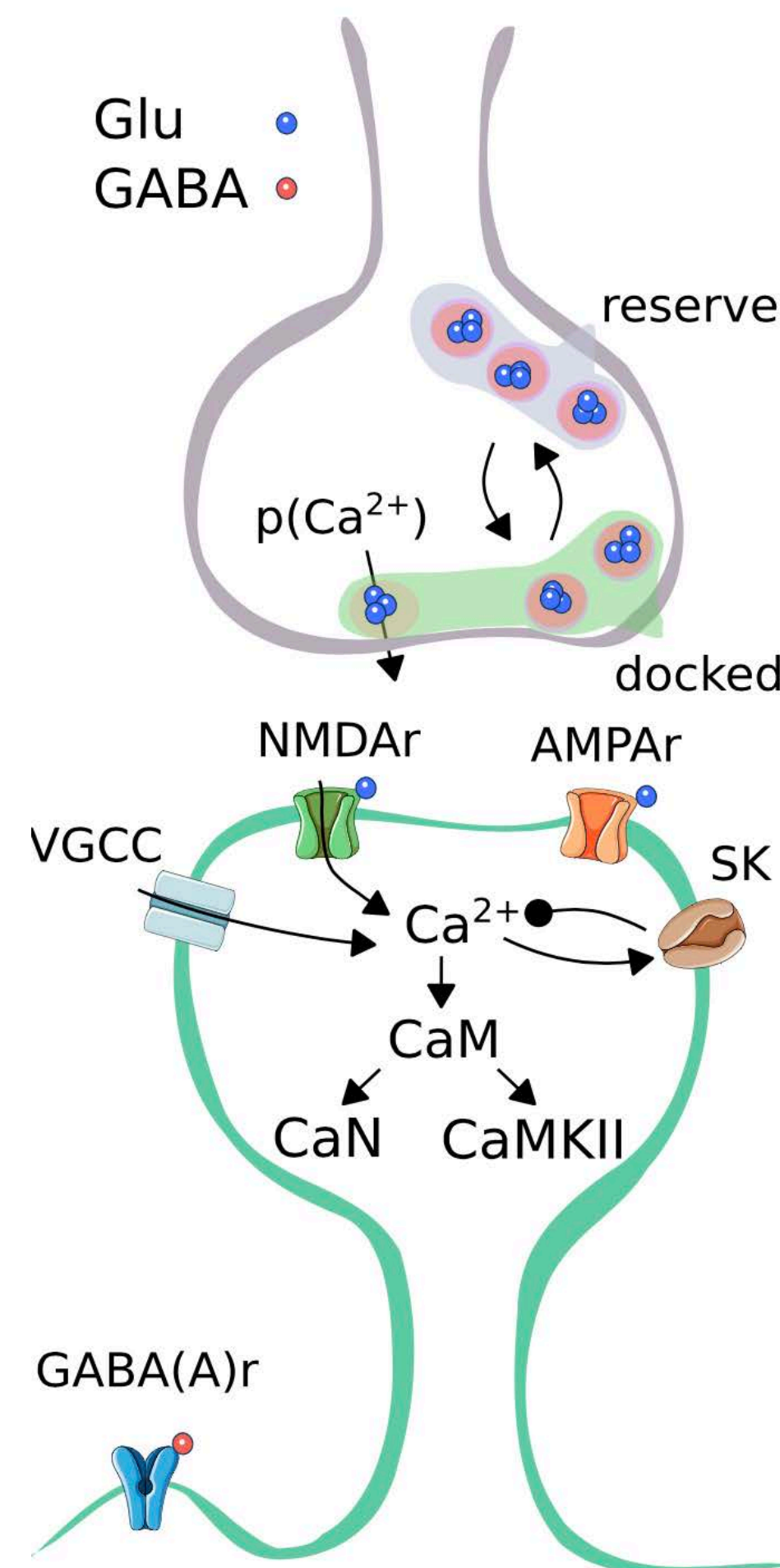


Bi & Poo, *J Neurosci* (1998)

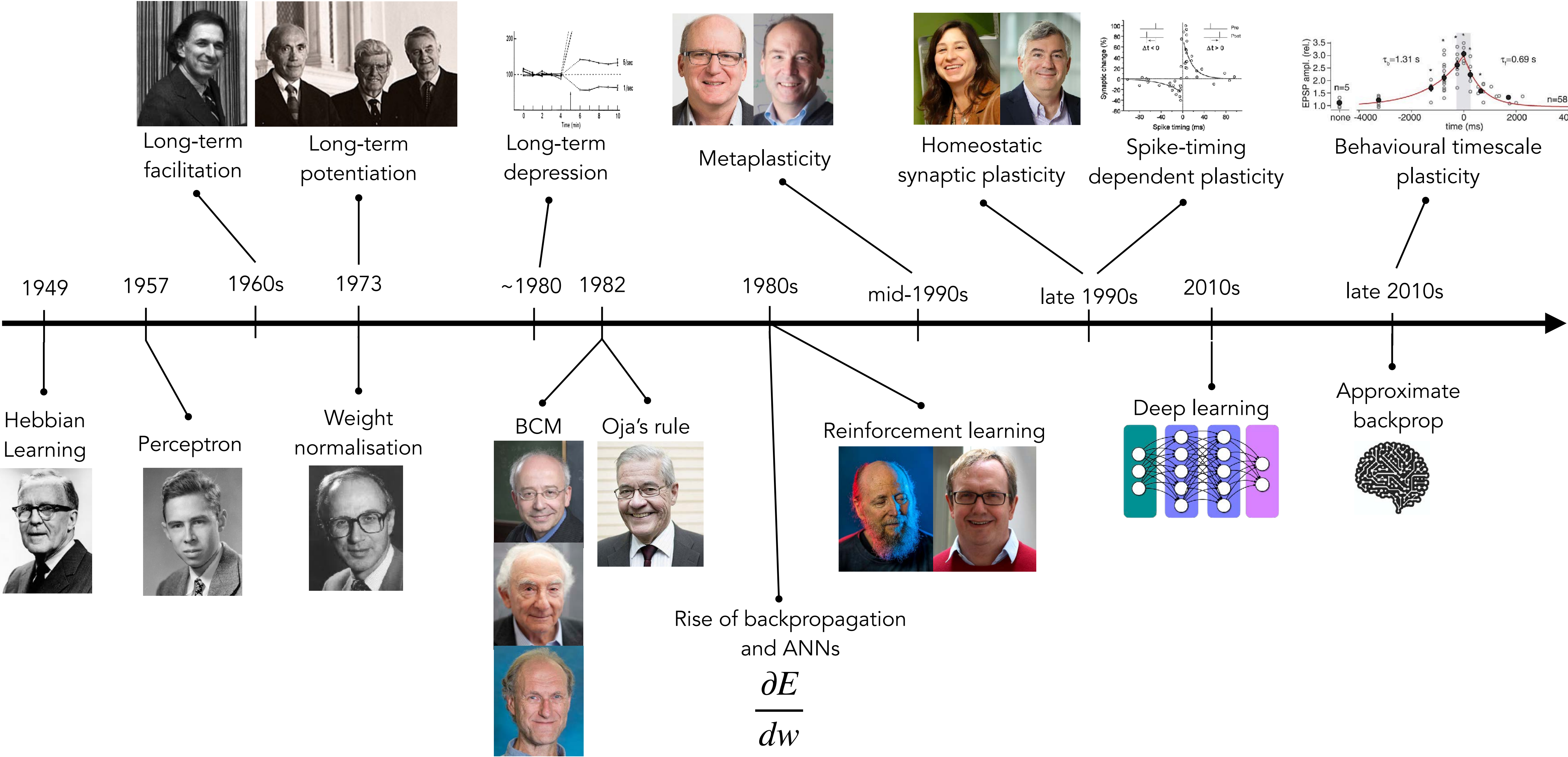




# Adding more biology



# A brief history of learning rules





# Where we are

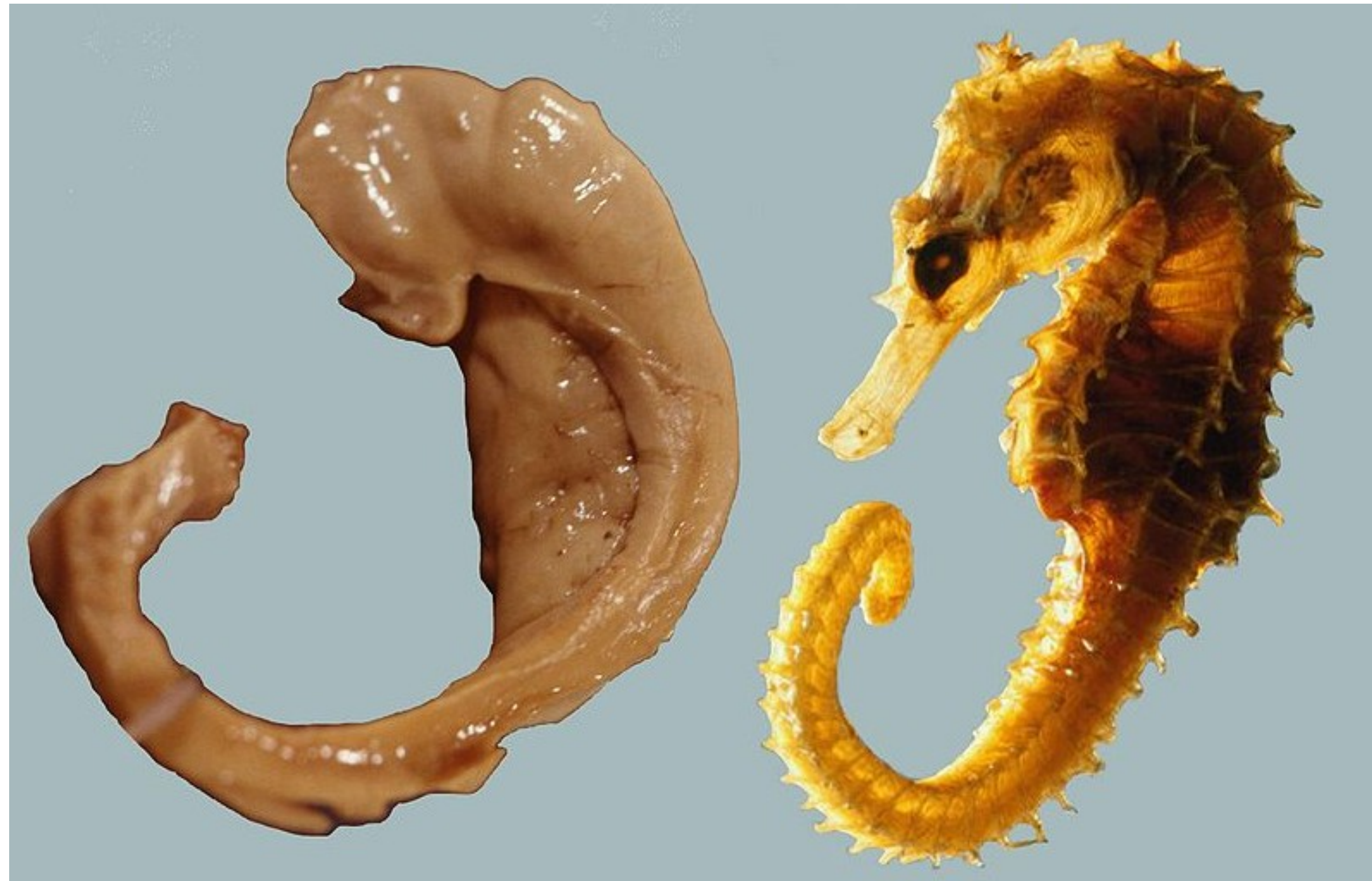


Learning type	Unsupervised	Supervised	Reinforcement
Example rules	Hebbian learning, STDP, BTSP	Backpropagation	Temporal Difference, Q-learning
Can achieve human-level performance on complex tasks?	Not really 😞	Yes, many and improving 🔥	Yes in specific cases 😎
Known mapping to neurobiology?	Pretty good	Several deep incompatibilities	Less clear, mostly high-level

# 3. Attractor networks



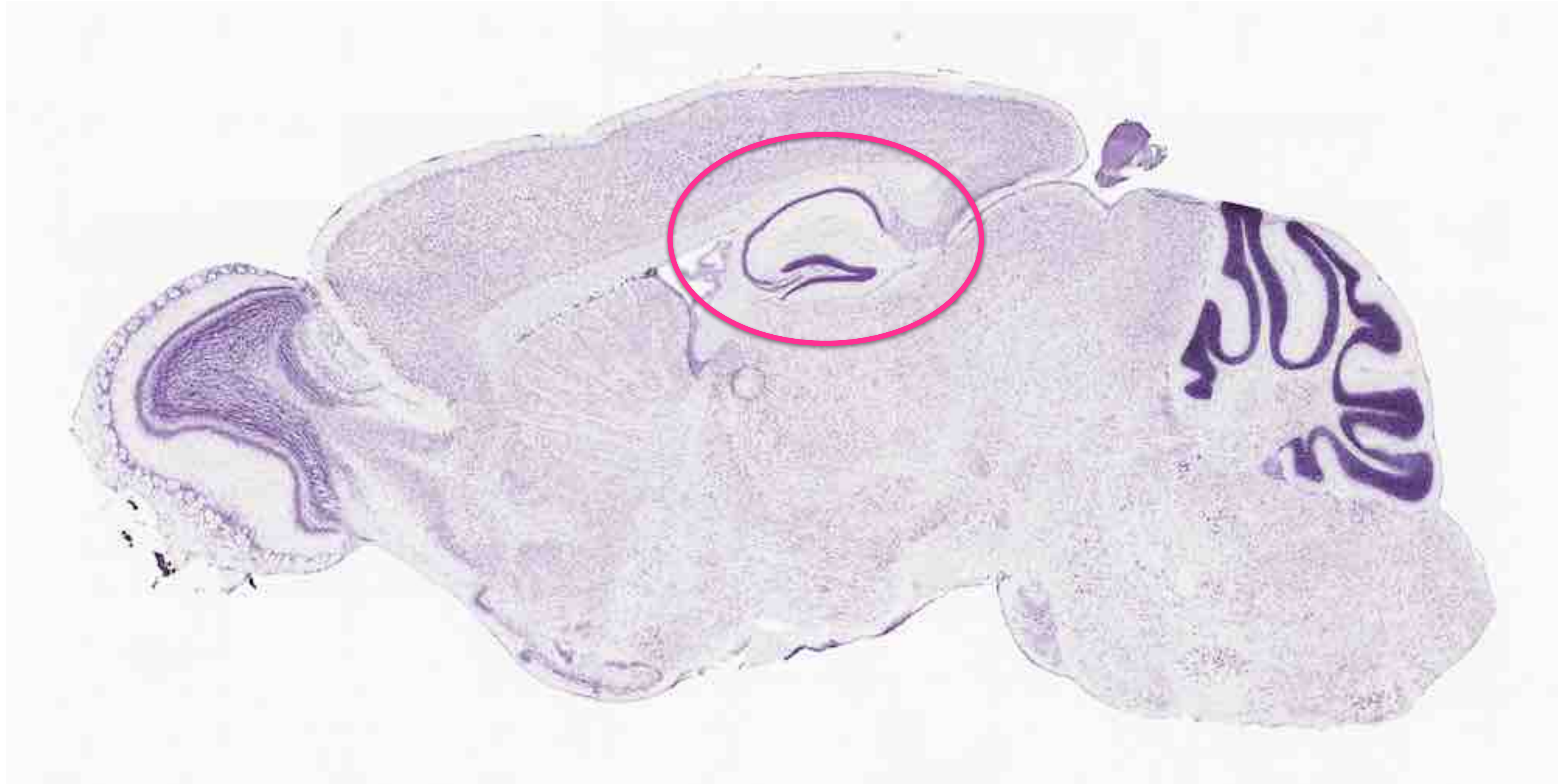
Hippocampus, from the greek words for "horse" and "sea-monster"



[https://en.wikipedia.org/wiki/Hippocampus#/media/File:Hippocampus\\_and\\_seahorse\\_cropped.JPG](https://en.wikipedia.org/wiki/Hippocampus#/media/File:Hippocampus_and_seahorse_cropped.JPG)



# Anatomy of the hippocampus



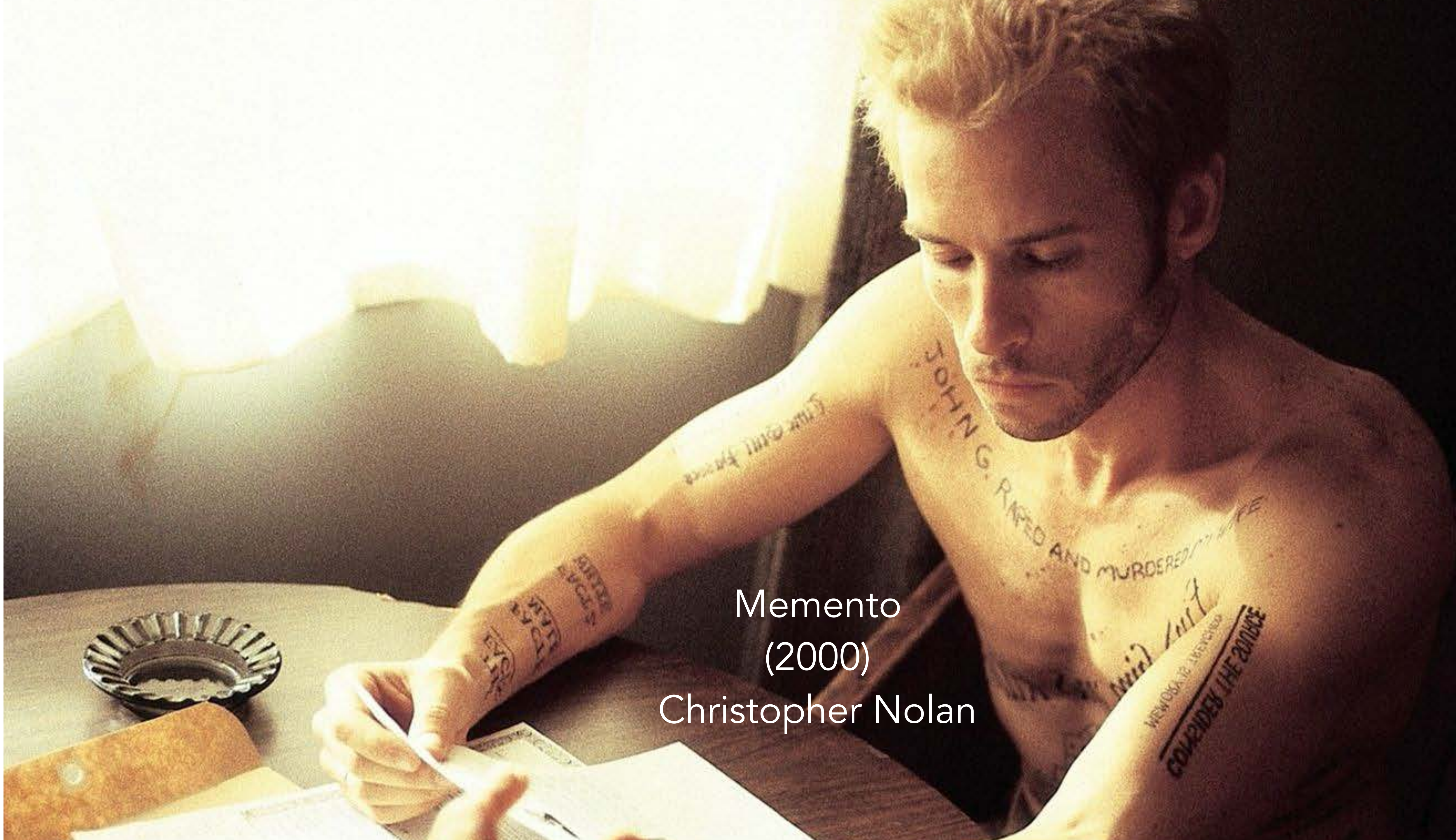
<https://mouse.brain-map.org>



# Hippocampus and memory

- Patient HM (who had his hippocampus surgically removed) could not form new long-term memories, and also had time-limited retrograde amnesia.
- The hippocampus is specifically needed for encoding new episodic memories, but is not necessary for other memories (e.g. procedural).
- Memory encoding requires synaptic plasticity in the hippocampus.





Memento  
(2000)  
Christopher Nolan



# Hopfield networks



John  
Hopfield

- A basic model of associative memory recall, as in the CA3 region of the hippocampus.
- A Hopfield network is a recurrent network of “neurons” with synaptic connections set so to store memories as attractors (proposed in 1982).
- The network state evolves dynamically, typically toward some “attractor” state.
- A simple Hebb-like synaptic plasticity rule can imprint attractors in the network weights.
- Incredibly influential model in the history of computational neuroscience (attracted a bunch of physicists to the field).

# Hopfield networks

- Network state evolves as:

$$\text{if } \sum_{j \neq i}^N [w_{ij}x_j(t) - \theta] > 0 \quad \text{then } x_i \rightarrow +1$$
$$\text{otherwise } x_i \rightarrow -1$$

- There are two common flavours: synchronous or asynchronous updates.

- The weights follow a Hebbian-like rule:

$$w_{ij} = \frac{1}{N_{patterns}} \sum_a^{N_{patterns}} x_i^a x_j^a$$

- Usually the weights are symmetric:  $w_{ij} = w_{ji}$   
and the connectivity is all-to-all.

- The network dynamics evolve to minimise an "energy":

$$E = -\frac{1}{2} \sum_{ij} w_{ij} x_i x_j$$



# Hopfield network dynamics

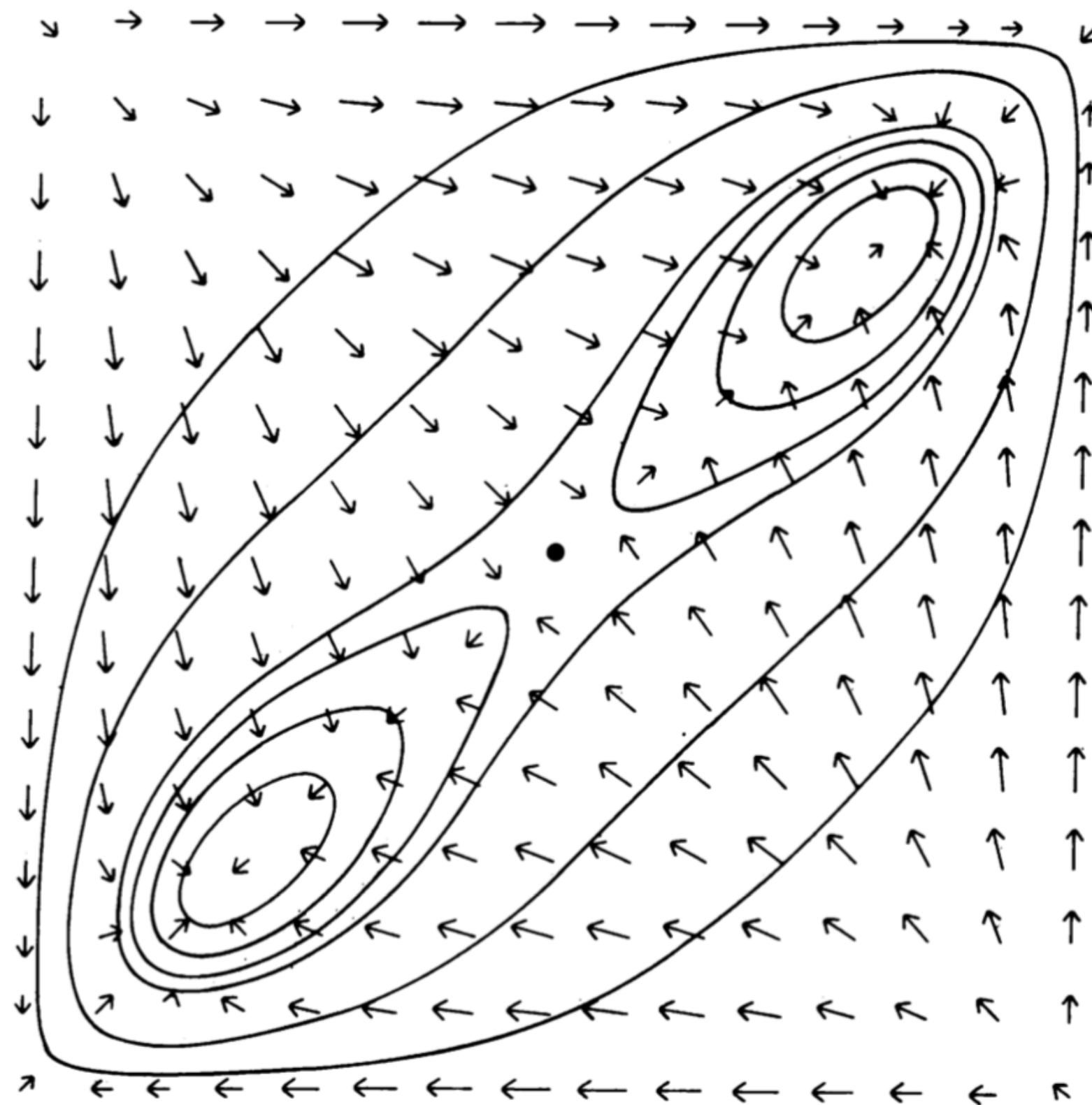


FIG. 3. An energy contour map for a two-neuron, two-stable-state system. The ordinate and abscissa are the outputs of the two neurons. Stable states are located near the lower left and upper right corners, and unstable extrema at the other two corners. The arrows show the motion of the state from Eq. 5. This motion is not in general perpendicular to the energy contours. The system parameters are  $T_{12} = T_{21} = 1$ ,  $\lambda = 1.4$ , and  $g(u) = (2/\pi)\tan^{-1}(\pi\lambda u/2)$ . Energy contours are 0.449, 0.156, 0.017, -0.003, -0.023, and -0.041.

- Each of local minima in the energy landscape is known as an "attractor".
- The network can do pattern completion: retrieving the full pattern from a partial cue.
- The capacity of the network, or maximum number of attractors, is  $\sim 0.14N$ .



# Content-addressable memory in Hopfield networks



Hertz, Krogh & Palmer, *Introduction to the Theory of Neural Computation* (1991)



## 4. Links between learning in biological and artificial neural networks

# The back-propagation learning rule

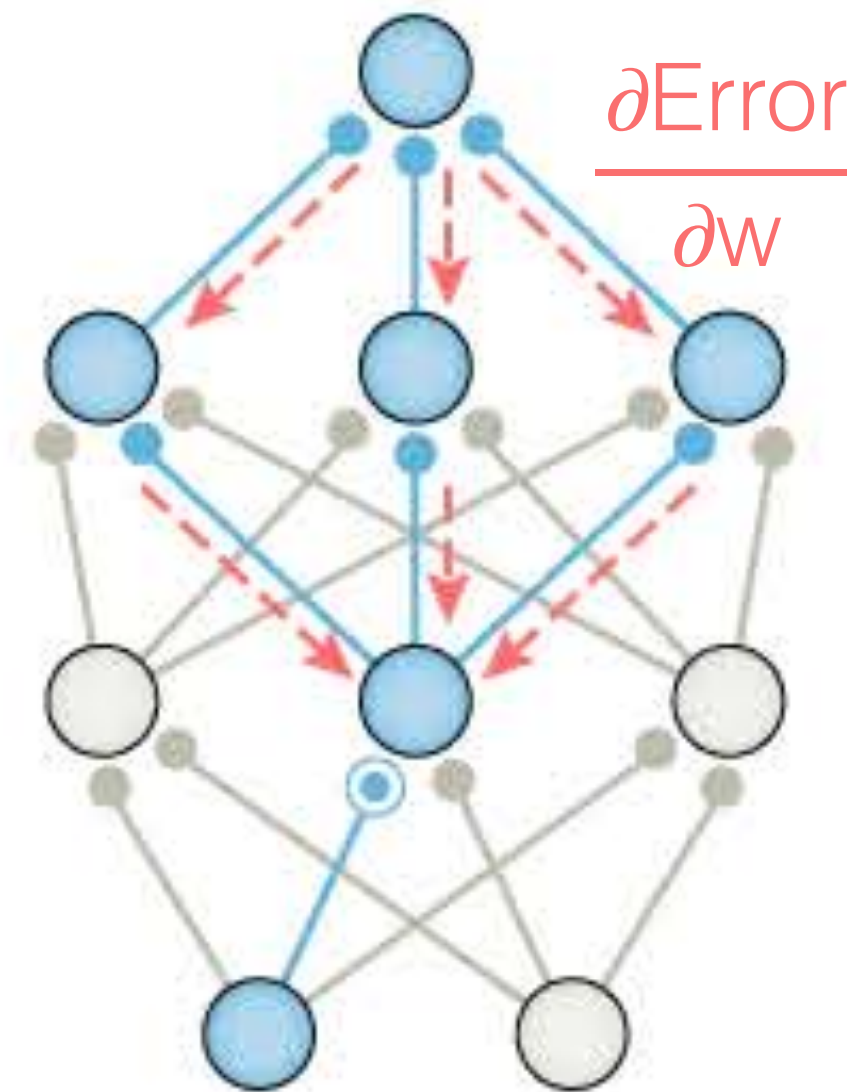
- Unlike Hebb's rule, in machine learning **artificial neural networks are trained via supervised learning**: performance errors are fed back into the system to adjust connection weights, so reducing future errors.
- The main method for doing this is called **back-propagation** (basically the calculus chain rule).



# The back-propagation learning rule

Feedforward  
network

Output

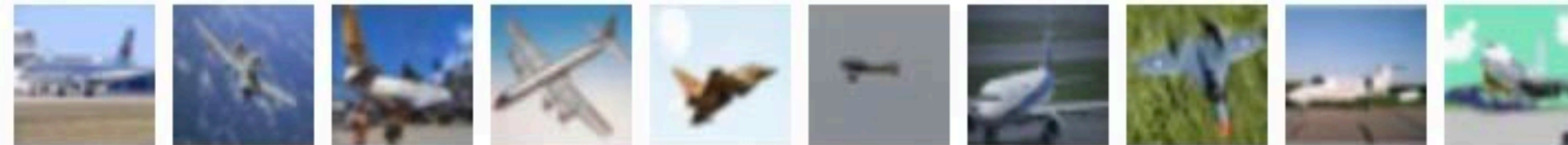


Input



# Deep learning is everywhere

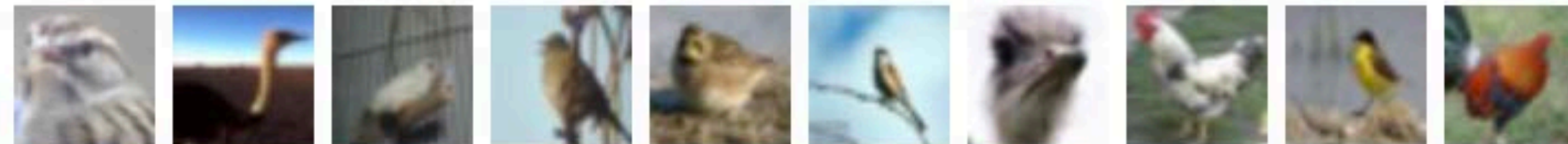
**airplane**



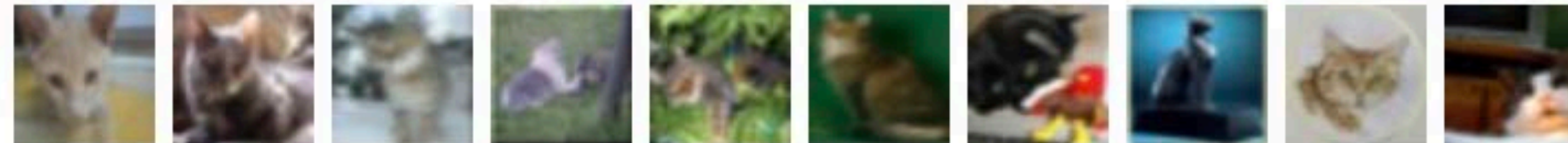
**automobile**



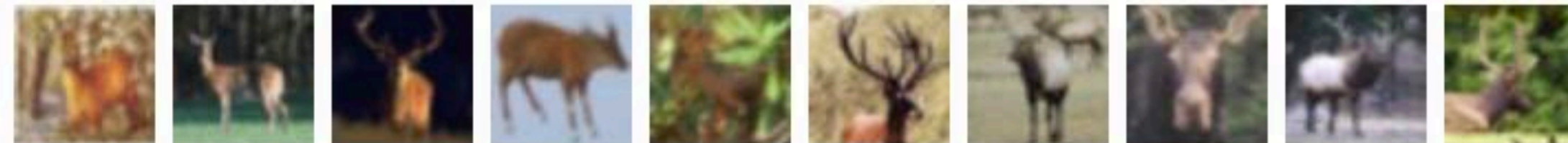
**bird**



**cat**



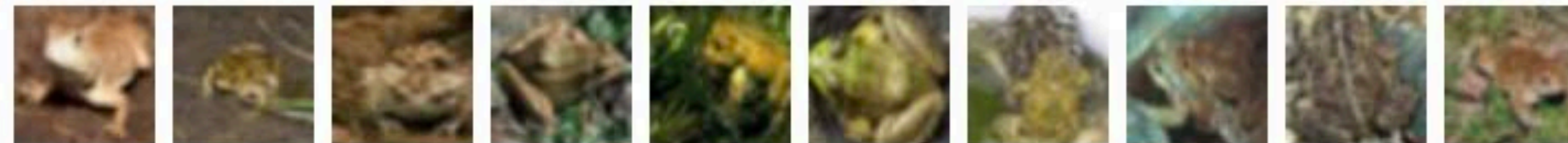
**deer**



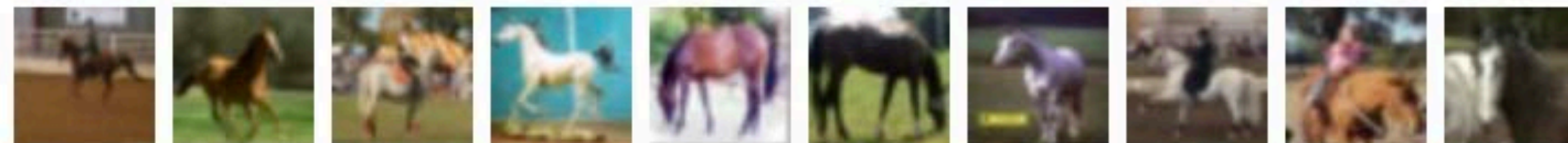
**dog**



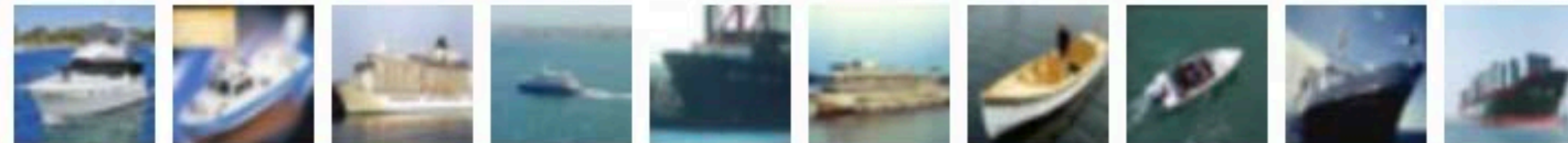
**frog**



**horse**



**ship**

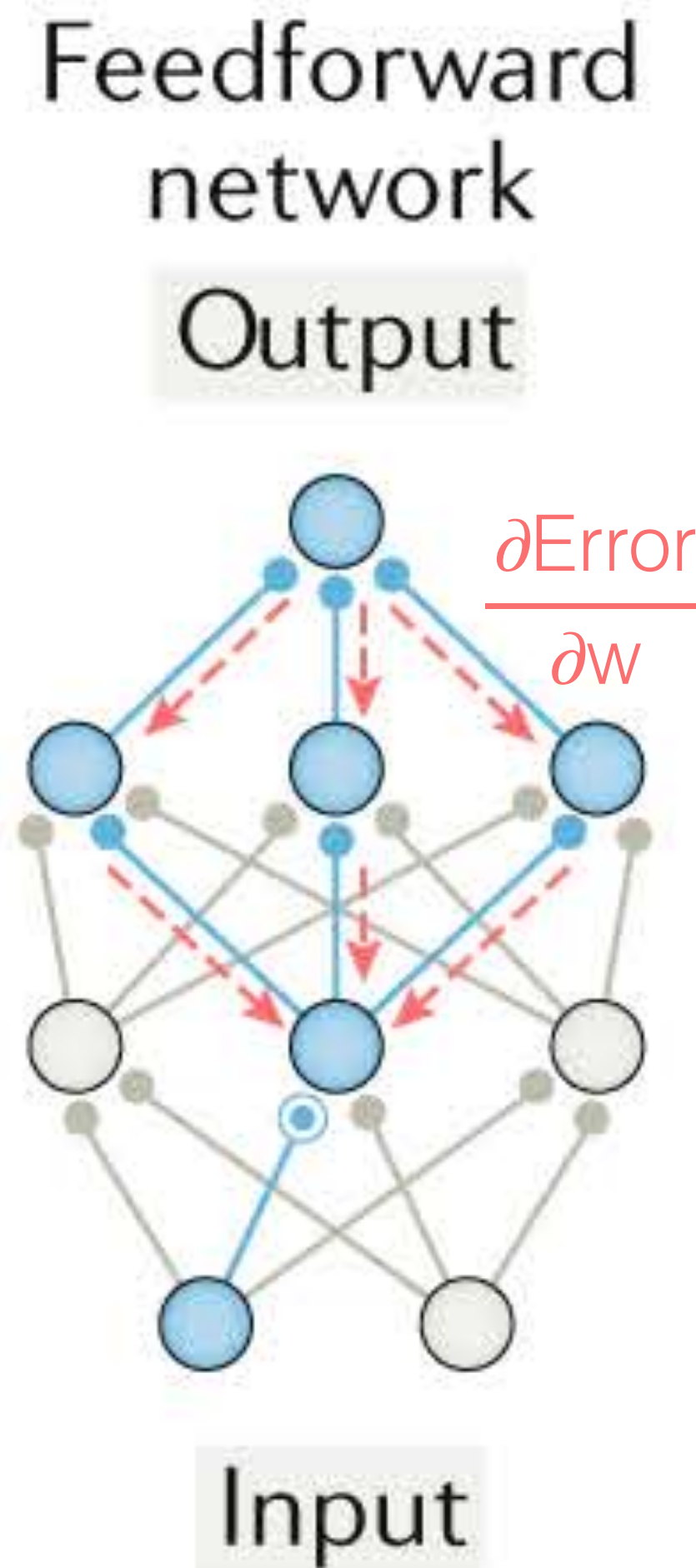


**truck**





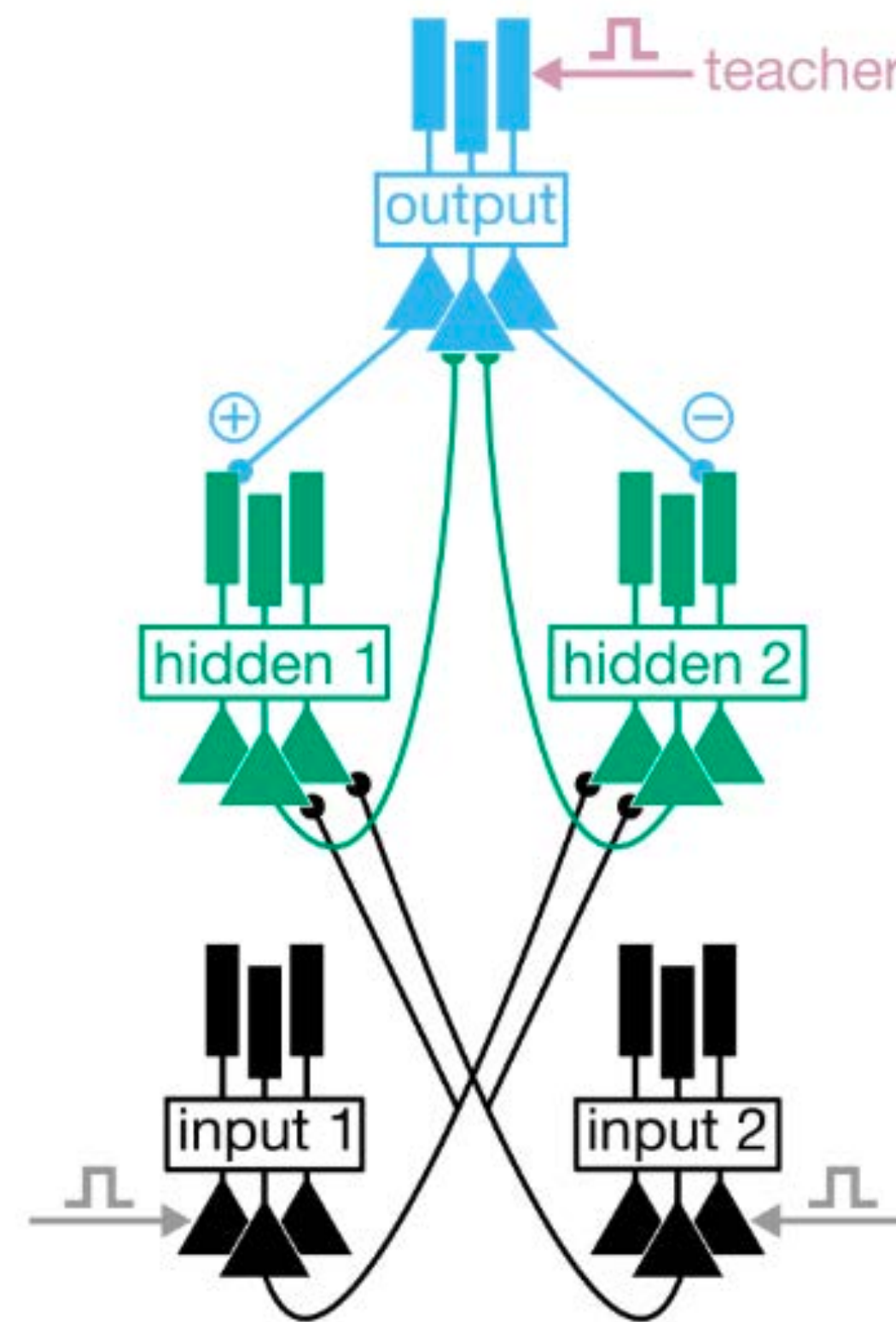
# Issues with backprop as a model of brain learning



- Plasticity rules at each synapse needs non-local information.
- Requires symmetry in backwards and forwards weights.
- Error signals need to contain sign information (+/- error).
- Feedforward and feedback 'passes' need to be separated in time.

# Example ideas for biologically plausible backprop

**Burst-prop:** feedback signals to distal dendrites, driving bursts that gate plasticity.



Payeur et al, *Nature Neurosci* (2021)



# Deep learning and brains summary

- Backpropagation of error gradients is a ubiquitous algorithm for learning in artificial neural networks.
- **But** the basic version is not biologically realistic.
- Current research is trying to dream up biologically plausible **alternatives that approximate backprop**.
- However the idea that the brain even learns by supervised gradient descent is controversial! Maybe ***unsupervised*** or ***reinforcement learning*** rules are all it takes.

# Overall session summary

1. Synapses transmit signals between neurons. Changes in their strength underlie long-term memory.
2. Classic plasticity rules: Hebb, STDP.
3. The hippocampus as a Hopfield attractor network.
4. Does the brain do deep learning?? Maybe.



Thanks