Stokes and Purdon [1] made some outdated but interesting points about the limitations on the use of Granger causality which attracted replies from renowned researchers on this area. The following is the Matlab code to simulate data according to Example 1 in the paper (also in a PPT’s slide):

Fs = 120; % sampling frequency is 120 Hz

T = 2\*Fs; % 2 senconds simulation

r = [0.9 0.7 0.8];

f = [40 10 50];

dt = 1/Fs;

theta = 2\*pi\*f\*dt;

N = 3; % number of nodes

p = 3; % true order of the MVAR model

A = zeros(N,N,p);

A(:,:,1) = diag(2\*r.\*cos(theta)) + [0 0 0; -0.356 0 0; 0 -0.3098 0];

A(:,:,2) = diag(-r.^2) + [0 0 0; 0.7136 0 0; 0 0.5 0];

A(:,:,3) = [0 0 0; -0.356 0 0; 0 -0.3098 0];

ss = 1; % space noise's standard deviation

x = ss\*randn(N,T);

for t = p+1:T

for k = 1:p

x(:,t) = x(:,t) + A(:,:,k)\*x(:,t-k);

end

end

B = [1 0.5 0.2; 0.5 1 0.5; 0.2 0.5 1]; % mixing matrix

so = 1; % observation noise's standard deviation

y = B\*x + so\*randn(N,T);

figure;

subplot 211; plot(x'); title('x: hidden dynamics'); axis tight

subplot 212; plot(y'); title('y: observed dynamics'); axis tight

where, the simulated example’s time series are represented by the variable x. Additionally, we added the observation equation *;* , where is the mixing matrix with coefficients .

1. Estimate the MVAR model using the correct order p=3 and an overestimated model order p=30 for both the time series in x and y, separately. Compare the results.
2. Calculate Granger causality in the frequency domain for both x and y and compare the results.
3. Calculate coherence, and imaginary coherence for both x and y time series. Compare the results with Granger causality, and discuss differences.
4. In your opinion, what are the advantage and disadvantages of the application of Granger causality using MVAR in this example, and for a general dataset (fMRI, EEG/MEG, etc.)?

**HINT**: use code provided by Faes et. al. [2]. Some matlab codes for solving this exercise are also provided with the course materials.

**References**:

[1] Stokes, P.A. and Purdon, P.L., 2017. A study of problems encountered in Granger causality analysis from a neuroscience perspective. *Proceedings of the national academy of sciences*, *114*(34), pp.E7063-E7072.

[2] Faes, L., Stramaglia, S. and Marinazzo, D., 2017. On the interpretability and computational reliability of frequency-domain Granger causality. *F1000Research*, *6*. <https://iris.unipa.it/retrieve/handle/10447/271763/526256/84-Faes-f1000Research_2017-00.pdf>