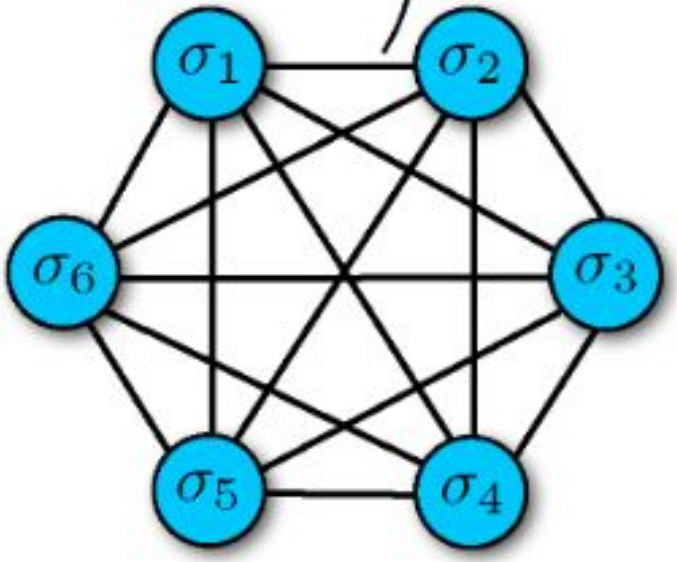
The possible capacity and accuracy of a triple-well Hopfield model

“Foyled” - By Charlotte, Pranav, Rosalind & James

$$J_{12} = \sum_{\mu=1}^{3} \xi_1^\mu \xi_2^\mu$$

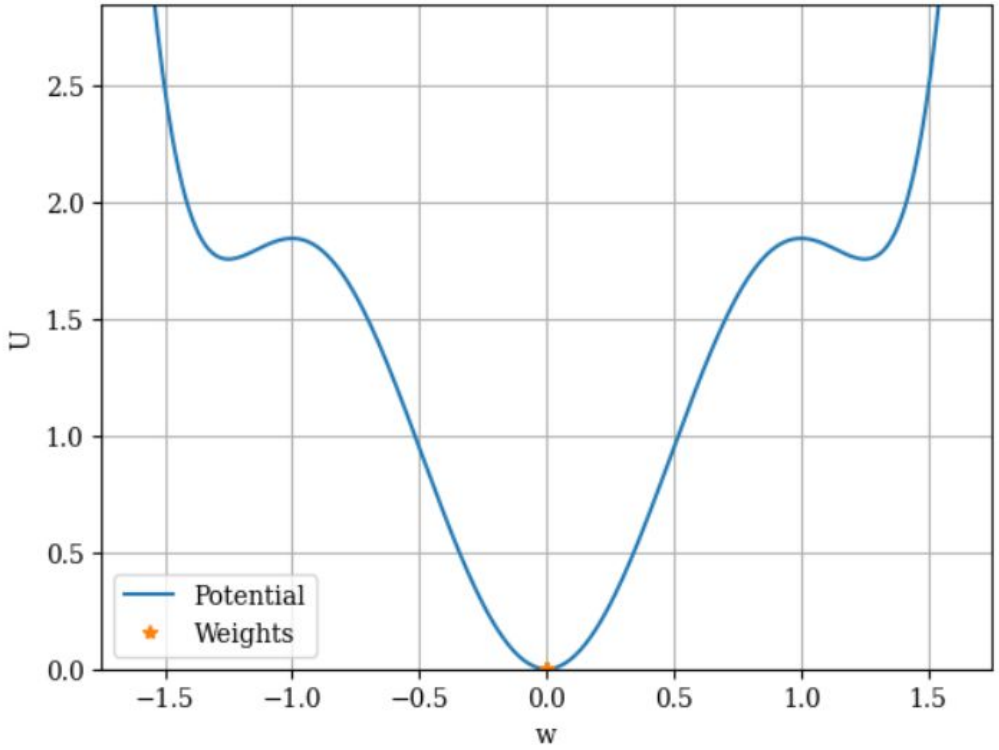


Hopfield Network

"Magic Number"

~ 0.138N

Triple well Hopfield Model



Objective:

To try and get an idea of the capacity of this “triple well model” to store patterns.

What is its accuracy like?

How does this capacity change over varying sizes of network?

Our prediction?

This model will store fewer memories/patterns than the traditional Hopfield model. This is because of the inherent restrictions in weights prescribed by the “weight potential” landscape.

A paper (Amit & Fusi, 1994) which talks about a model with a distribution of “synaptic values” suggests that its capacity is ~ log(n). Another paper (Storkey, 1998) refers to “forgetful” models which have a capacity of up to 0.05n.

What did we do?

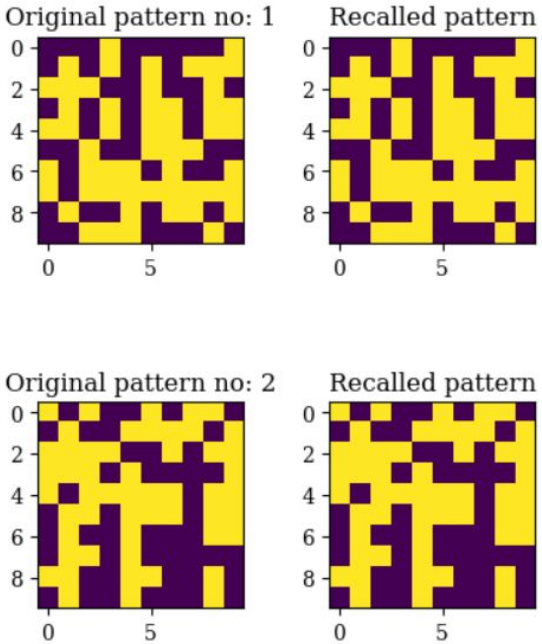
We estimated the accuracy of the model to store different numbers of patterns for the following numbers of neurons:

$$n = 25, 100, 225$$

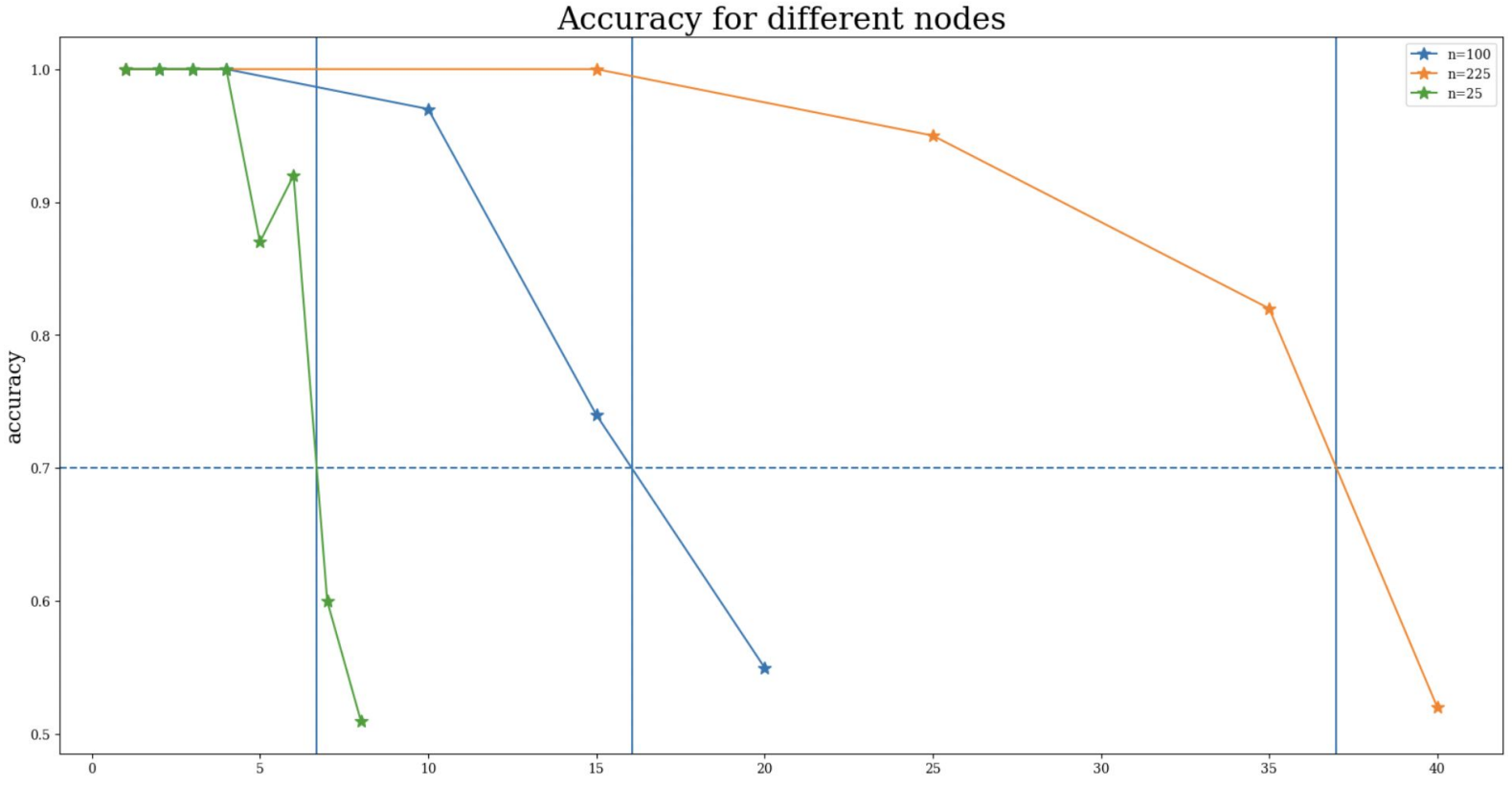
We calculated the accuracy of the model with respect to the patterns stored by using an estimate of the Pearson correlation coefficient (inspired by Feng & Brunel, 2023 and Gerstner et al., 2014) which is:

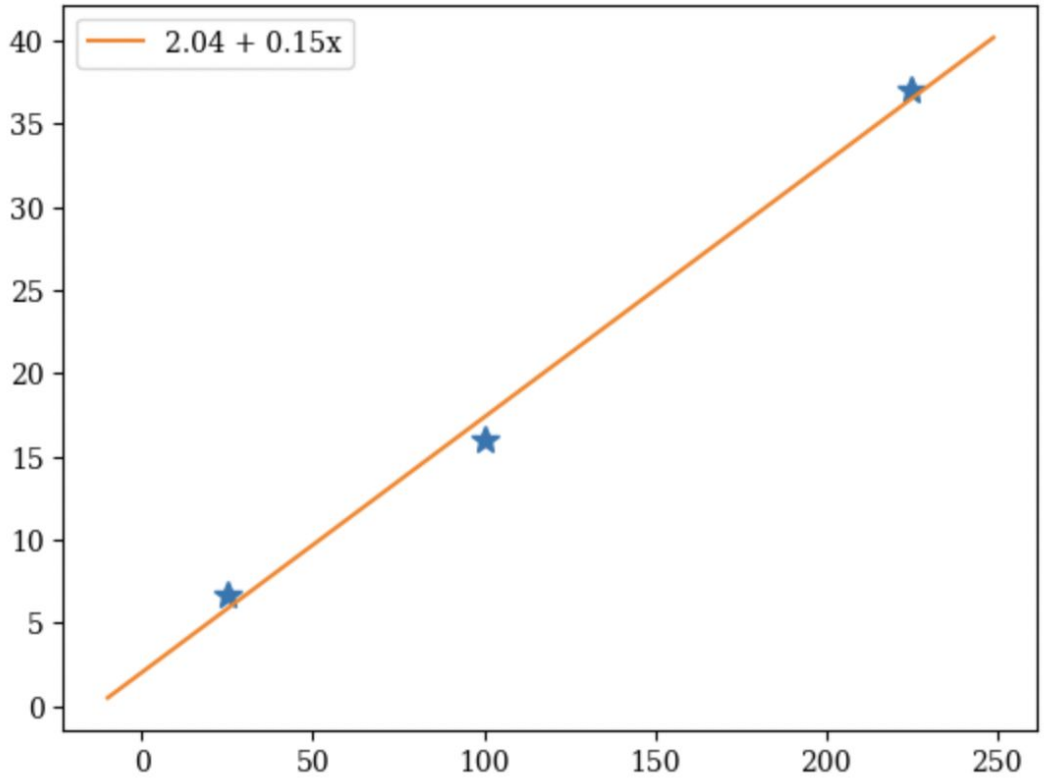
$$\frac{1}{N}\sum x_i y_i$$

And then calculated the average across patterns.



Plots


Accuracy for different nodes
accuracy
1.0
0.9
0.8
0.7
0.6
0.5
0
5
10
15
20
25
30
35
40
n=100
n=225
n=25



Well, well, well!

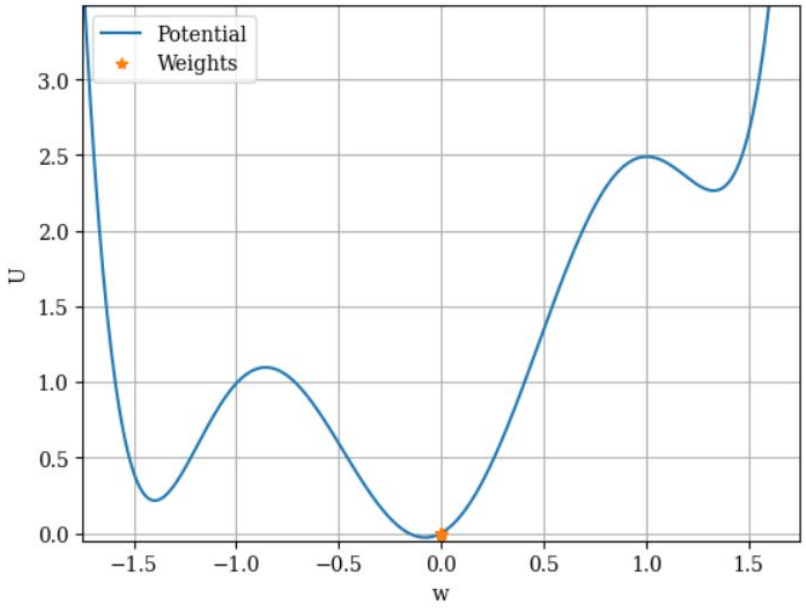
Moving forward…

Run it several times and plot errors bars, confidence intervals, etc.

We had difficulties with the timestep, so ran the simulations manually. With more time we could set it up to manage the timestep automatically. This would make the above step easier…

We would explore the impact of varying learning rate and noise on the network capacity.

We would also have liked to fully explore the characteristics of an asymmetric weight potential.



References

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Thank you!