

CHAPTER-2

Review of Literature

This chapter provides a literature review and important developments in phase unwrapping. Design considerations of phase unwrapping through the MAP-MRF framework, the importance of filter in phase unwrapping process and the developments in graph cut are outlined. Phase unwrapping utilizes graph cuts for minimizing MRF's. Also, some algorithms which solve the phase unwrapping problem quicker are studied.

2.1 REVIEW OF LITERATURE

The problem of unwrapping can be achieved in many ways. One of the widely used techniques is adapting the Bayesian approach and that too especially MAP-MRF approach for unwrapping problem. The earlier works related to phase unwrapping using MAP-MRF framework are studied.

Abdallah W B et al [16] proposed phase unwrapping method based on a Markov Random Field model. The phase image is modelled using MRF where the corresponding energy function is defined. The filtered phase image is estimated by minimizing the proposed energy function. In this paper, the genetic algorithm is used as minimization step.

Suksmono A B et al [17] proposed a PU method based on 5th order non-causal complex valued Markov Random Field model. Non-residue areas are used for estimating pixels of residual areas. Using a CMRF lattice complex-valued neural-network, the error energy is minimized by LMS steepest descent algorithm.

Abdallah W B et al [18] proposed a phase unwrapping method based on Markovian Random Field model. At first, the phase image is segmented into homogeneous regions by applying the Structural Feature Set algorithm. Then segmented phase images are modeled using MRF model and conjugate gradient algorithm is used to minimize an energy function.

Hou Q et al [19] proposed a robust algorithm for the phase unwrapping. The proposed algorithm is for borehole application using MAP-MRF approach. The

Bayesian estimator was applied to recover the desired phase, as the optimal field solution of the maximum a posteriori (MAP) estimation criterion.

Chen R et al [20] proposed an integrated phase denoising and unwrapping algorithm modeled by Markov Random Fields (MRFs). At first, MRF is used to model the relationship between the elements and then the energy function is defined. Finally, the energy function is minimized to obtain the estimate of the true phase value.

Abdallah W B et al [21] proposed phase filtering and unwrapping method based on a Markov random field (MRF) model. The phase image is modelled using a joint MRF with a corresponding energy function related simultaneously to noise filtering and phase unwrapping (PU). The unwrapped phase image is estimated by minimizing the proposed energy function using the genetic algorithm.

Dong Y et al [22] presents a novel phase unwrapping method using a region-based Markov Random Field (MRF) model. Firstly, the phase image is segmented into regions within which the phase is not wrapped. Then, the phase image is unwrapped between different regions using an improved Highest Confidence First algorithm.

Ferraiuolo G et al [23] proposed an algorithm which uses the MAP-MRF framework for phase filtering. The image prior is modeled as a suitable Markov Random field, and the filtered phase field is the configured with maximum a posteriori probability. Assuming the image to be residue free and generally smooth, a two-component MRF model is adopted, where the first component penalizes residues, while the second one penalizes discontinuities. Constrained simulated annealing is then used to find the optimal solution.

Gudmundsson S et al [24] proposed a phase unwrapping algorithm based on MRF regularization aided by GPS data to unwrap differential interferograms. At first, virtual unwrapped interferogram's are created by ordinary kriging of GPS-measured range change and then optimized further by using MRF regularization. A simulated annealing optimization algorithm is used to find an optimal solution of the MRF regularization.

Denis L et al [25] propose an algorithm which jointly filters phase and amplitude data in a Markovian framework. The regularization term is expressed by the minimization of the total variation. In this paper, two applications based on the proposed technique are described. First one, a 3-D reconstruction in urban areas using the proposed algorithm and the second one is an adaptation of this framework to the fusion of SAR and optical data.

Ferraiuolo G et al [26] present a Phase unwrapping technique based on a MAP-MRF approach. Prior knowledge is modelled by MRF or Gibbs Model and energy function is defined by imposing two constraints, i.e. on the absence of residues and the smoothness of the phase field and finally, MAP estimation is carried out by simulated annealing.

The ideas of filtering the wrapped phase or unwrapped phase are widely used for attaining better results and to reduce the noise. Broadly, the denoising can be handled during the unwrapping or in two different steps of denoising and unwrapping. The earlier works related to the filtering in unwrapping algorithms are considered and its effect on output results are focused.

Loeffler C et al [27] proposed a simple three-stage unwrapping algorithm. The system is composed of two first-order linear filters and a median filter. This three-stage system is a generalization of the median filter and intended to remove both pulse and unit steps from a signal.

Lee J S [28] presented two multiplicative noise-smoothing algorithms. These two algorithms are computationally efficient to suppress the speckle noise due to the coherent interference of radar echoes from target scatters.

Bo G et al [29] proposed a pre-filtering unwrapping method. The algorithm uses local statistics method to design an adaptive filter. At first, the local spatial content of the phase image is exploited and then, a non-linear adaptive filtering function, based on the local estimation of noise and signal standard deviation, is adopted. The combination of adaptive filter and phase unwrapping algorithm reduces distributed noise and low coherence areas.

Capanni A et al [30] presents a noise reduction filter for the phase unwrapping of data from phase-shifting speckle pattern interferometry. This filter is effective in reducing the noise in wrapped phase maps without affecting the edges.

Huang M J et al [31] proposed a local histogram data-orientated filtering algorithm to remove noise from the deformation phase map obtained by a phase stepping electronic speckle pattern interferometry. The output result is free from phase inconsistency so simple unwrapping rule, like Macy's method is applied for unwrapping.

Servín M et al [32] proposed a denoising along with unwrapping method based on a Regularized Phase-Tracking system. PU is attained in two steps. Firstly, two phase-shifted fringe patterns are obtained and then demodulate them by using the Regularized Phase-Tracking technique.

Yun H Y [33] developed an accurate and robust phase-estimation method which assumes that the phase variation with a gradient within a fitting window. A cost function based on least-squares criterion is formulated and Powell's iteration method is applied to it to derive the phase and its gradient. Finally, an algorithm that improves the initial guess of the iteration is developed to handle phase of large noise and steep variations.

Kemao Q [34] developed two algorithms, one on filtering and the other on a similarity measure. In this paper, implementations of these algorithms are addressed and provided an overview of windowed Fourier transform.

Kraemer R [35] proposed the autocorrelation function method based on an Extended Kalman filter. The Kalman filter provides local slope information and this information is optimally fused with the information directly obtained from a real and imaginary part of the interferogram to provide unwrapped phase.

Trouvé E et al [36] developed a faster algorithm dedicated to the processing of interferogram's and a measure of confidence in the estimate is proposed. This algorithm is able to estimate fringe local width, orientation and this information is applied to restore noisy phase data.

Trouvé E et al [37] proposed two conventional unwrapping algorithms, the first is a local and the other is a global approach based on local frequency estimates. The local approach is based on path-following techniques and able to eliminate residues due to the noise. A global approach is based on weighted least-squares methods and local frequency estimates.

Aebischer H A et al [38] proposed a sine or cosine average filter which works as an automatic adaptive filter for filtering speckle-interferometric phase fringe patterns.

Palacios F et al [39] developed an algorithm based on a combination of sine or cosine average filtering with masking filtering techniques for filtering of the phase of the fringe pattern. This algorithm is able to preserve the borders of phase fringes of the object and removes the isolated discontinuities and false phase fringes.

Rosso V et al [40] developed a simple and systematic method to highlight the properties of filters for their application in temporal phase shifting interferometry. The advantages and the failures of elementary filters were enhanced.

Tang C et al [41] proposed an algorithm based on sine or cosine average filter and least-squares fitting algorithm. The proposed algorithm is able to reduce the noise from electrical speckle pattern interferometry phase fringe patterns.

Kemao Q et al [42] proposed an algorithm based on windowed filtering technique. First, the noise is removed by windowed Fourier transform approach and then sequential line scanning method was used for phase unwrapping. Proposed method is able to unwrap noisy phase wraps.

Kemao Q et al [43] studied the effects of windowed Fourier filtering on edges. Finally, this paper concludes that the phase near an edge remains unchanged and no special consideration is needed for the edges if Windowed Fourier Filtering is used for Phase unwrapping operation.

Kemao Q et al [44] analyzed different filtering methods of exponential wrapped phase maps. Finally, it is concluded that the principles of all the considered filtering methods are equivalent and elegantly unifies all these methods for filtering unwrapped phase maps.

Lorenzo-Ginori J V et al [45] studied the effects of Non-Linear vector filtering techniques to de-noise the wrapped phase. Results indicate a significant noise reduction, especially when NL filters based on angular distances are used for filtering.

Martinez-Espla J J et al [46] proposed a multi-purpose algorithm which unwraps and filter the phase simultaneously. This algorithm filters the phase using the particle filter and matrix-pencil local slope estimator. Once the phase is filtered then it unwraps the phase by using the optimized region-growing. The MP estimator provides better resolution, the particle filter enables simultaneous unwrapping and filtering, and the implemented region-growing technique ensures an optimum solution.

Villa J et al [47] proposed a local filtering technique for denoising fringe patterns. The proposed algorithm is a Bayesian estimation technique where the prior information is provided by MRF. The regularization term imposes smoothness constraints only along the fringe's tangent direction and contains fringe orientation information.

Villa J et al [48] proposed an algorithm to de-noise the wrapped phase. To attain this, complex-valued Markov random fields with orientation information of the filtering direction along iso-phase lines are used. The proposed method has an advantage of clearly separating the noise and phase by anisotropic filter even in high-frequency zones.

Li Y H et al [49] proposed filtering method based on Bayesian estimation method. The cost function of this method uses the discrete form of MRF's. This algorithm offers three advantages, easy implementation of filtering, phase discontinuities are well preserved and little computational effort.

Bioucas-Dias J et al [50] proposed a simultaneously filtering and unwrapping algorithm. At first, the phase is filtered using the LPA technique. Once the phase is denoised then the noise-free phase is unwrapped using PUMA.

Kemao Q et al [51] proposed an algorithm which post-processes the results of windowed Fourier filtered and quality guided phase unwrapping algorithm by using the combination of congruence operation and least squares fitting.

The requirement of unwrapping the phase at a faster rate and consumes less memory is of more demanding. Active research is being undertaken to develop reliable and high-speed unwrapping algorithms. Some of the available algorithms are as below.

Chen K et al [52] proposed a new quality-guided phase unwrapping algorithm. In the proposed algorithm, the entire wrapped image is divided into high quality and low-quality pixels by detecting edge pixels on the object image. In order to improve the computational efficiency, these two types of pixels are unwrapped by different unwrapping algorithms. The proposed approach consumes less than 2% of computational resources for an image size 925×925 and achieves the same level of 3D measurement robustness compared with the quality-guided flood-fill algorithm.

Zhong H et al [53] proposed a blocked minimum discontinuity phase unwrapping algorithm. In this approach, the wrapped phase image is divided into small blocks. Once the small blocks are available, then one after the other block is unwrapped and joined together. For an image size of 700×1400 , the proposed algorithm takes only 13.5% time than Flynn proposed algorithm [54].

Yu H et al [55] proposed fast PU method for large-scale interferogram's. This algorithm firstly partitions the interferogram into tiles using the tiling strategy and individual tiles are unwrapped by using the minimum-spanning tree technique. For an interferogram of $18,000 \times 20,000$, the proposed algorithm is able to unwrap within 2,578 seconds and shows superior performance than SNAPHU algorithms.

Chen K et al [56] proposed fast and reliable unwrapping phase shifted based three-dimensional fringe pattern profilometry. Firstly the shadow areas and the phase discontinuities are identified in an interferogram and isolated. The remaining areas are unwrapped by group-merging phase unwrapping method. The proposed approach is characterized by less than 2% unwrapping time than conventional quality-guided phase unwrapping method for an image size of 925×925 .

LiX et al [57] proposed a fast algorithm that only does the one-dimensional searching and therefore reduces the complexity significantly of the CRT algorithm [58]. The total number of searches is reduced from 540 to 15.

Ferraioli G et al [59] proposed a fast algorithm to unwrap the interferometric phase in the multichannel configuration. The combination of prior provided by Total Variation algorithm and optimization provided by graph cut is used in this algorithm. For a simulated image of size 128X128, the proposed algorithm is able to find the solution within 24 seconds, whereas the multichannel phase unwrapping method proposed in [60] requires 300 seconds.

An L et al [61] proposed an algorithm which implements the minimum spanning tree PU algorithm in a new way. This algorithm reduces the complexity from $O(n^2)$ to $O(n \log_2 n)$. For an MRI wrapped phase image of size 256 X 256, the proposed method provides a solution within 4.5 seconds but the conventional implementation spent 63 minutes to complete.

Eineder M et al [62] developed a new Minimum Cost Flow implementation, optimized with respect to memory and speed for In-SAR phase unwrapping. For speed up the existing algorithm, the new cost function is used for unwrapping. An optimized version of the algorithm allows unwrapping even for large interferogram of size 5000 x 11000 within 35 minutes.

Costantini M et al [63] proposed a phase unwrapping method that makes use of the discrete space. Firstly, the proposed algorithm locates the areas where the estimated discrete vector field is corrected. And the further correction is performed using the prior information. By using the FFT's faster runtimes are achieved. The computational time required to unwrap the interferogram of 2616 x 4650 is about 25 minutes.

Schofield M A et al [64] presents an algorithm that solves the phase unwrapping problem, using a combination of Fourier techniques. The execution time of the proposed technique is same as executing the 8 FFT's.

Jeught S V et al [65] presents a high-speed phase unwrapping algorithm by combining Fourier-based phase unwrapping techniques with a custom parallel implementation of the two-dimensional discrete cosine transform and by optimizing its execution on the graphics processing unit of a standard low-cost graphics card. For an image of 640 x 480, the total processing time of the phase unwrapping algorithm is less than 5 milliseconds.

Asundi A et al [66] proposed an algorithm based on grey-scale mask and the flood-fill for phase unwrapping. The proposed algorithm unwraps phase from an area with higher reliability to one with lower reliability. The speed of the algorithm proposed is much faster than conventional routines.

Zhu Y J et al [67] proposed a DCT-based phase-unwrapping algorithm. The proposed algorithms avoid big errors in unwrapping and results are obtained by Schofield's 4-FFT algorithm and Volkov's deterministic 2-FFT algorithm. The proposed algorithm is able to achieve a higher computation speed and reliable solution even though the experimental phase map is destroyed by sharp variance or discontinuities.

Arevalillo-Herráez M et al [68] proposed an algorithm uniquely combines the benefits of both the quality-guided and the residue-based for phase unwrapping. The proposed measure is based on the probability that a 2×2 grid becomes a residue or estimation error. It achieves faster run times by adding a slope to wrapped phase signals.

Zhao Z et al [69] proposed a method relies on a derivative Zernike polynomial fitting technique. The Zernike polynomials are estimated using the least squares method. The proposed algorithm is a very faster as it is equivalent to the calculation of these Zernike polynomials. It combines the phase unwrapping and the wave-front fitting process.

Drugman T et al [70] proposed an algorithm which exploits the link between the value of the unwrapped phase at a Nyquist frequency and the number of zeros of the z-transform. This number was rapidly determined using modified SchurCohn's technique. The proposed method consists of an iterative procedure and assures to provide a perfect accuracy at a relatively low computational cost.

Chen K et al [71] proposed a quality-guided spatial phase unwrapping algorithm. In the proposed method, the phase map is divided into high-quality areas corresponding to smooth phase variance and unwrapped by a simple algorithm and low-quality ones to rough phase changes.

Chen K et al [72] proposed a quality-guided phase unwrapping algorithm based on a quality of a pixel. At first, pixels are classified into high and low-quality

pixels by posing a threshold level. Once the segregation is completed, simple fast phase unwrapping is used to unwrap the high-quality pixels.

Lu J et al [73] proposed a fast phase unwrapping algorithm based on region partition according to a quality map of wrapped phase. At first, the wrapped phase image is divided into several regions using partition thresholds and then each region is unwrapped by using a simple path-following phase algorithm. Finally, all regions are merged according to their priorities. The proposed method is much faster, more accurate, and robust to noise than the Goldstein and flood algorithm in unwrapping complex phase image.

Herráez M A et al [74] proposed an algorithm based on reliability function and at first, the pixels with higher reliability values follows a non-continuous path to perform the unwrapping operation.

Herráez M A et al [75] proposed a fast phase unwrapping algorithm based on the principle of unwrapping the areas of similar phase values. Two inherent parameters allow tuning of the algorithm to images of different quality and nature. Execution times for an image of size 512×512 pixels, phase distribution are in the order of a half second.

Abdul-Rahman H et al [76] proposed a three-dimensional phase unwrapping algorithm based on the quality map. The technique is to unwrap the most reliable voxels first and the least reliable voxels later. The technique follows a discrete unwrapping path to perform the unwrapping process. The proposed algorithm is robust and fast compared with other 3D phase unwrapping algorithms.

Graph cuts are widely used as an optimization algorithm in the Bayesian framework. The earlier works related to phase unwrapping using graph cuts as an optimization step is studied.

Dong J et al [77] proposed a water and fat separation methods applied to MRI data. In this paper, the general phase wrap and chemical shift problem is formulated using a single species fitting and it is solved using graph cuts with conditional jump moves. This method is noted as simultaneous phase unwrapping and removal of chemical shift.

Xie W et al [78] proposed a new algorithm for phase unwrapping based on graph cuts. The proposed algorithm takes a patch-based approach which has the advantages of simplicity and robustness to phase noise. The energy function is designed to combine both spatial and temporal constraints.

Ferraioli G et al [79] proposed a phase unwrapping algorithm for Multichannel data. The proposed algorithm follows a Bayesian approach where the prior is provided by total variation and graph-cut based optimization algorithms. In this paper, two graph-cut based optimization algorithms have been used, the one is Ishikawa algorithm and the other is alpha-expansion algorithm.

Bioucas-Dias J M et al [15] proposed an algorithm of two variants using a MAP-MRF framework based on clique potential. The first variant is for convex potentials and finds the exact minimum and the other is an approximate algorithm for non-convex potentials.

Bioucas-Dias J M et al [80] proposed a combinatorial phase unwrapping algorithm based on diversity and graph cuts. The proposed algorithm is a MAP-MRF approach where a first-order Markov random field (MRF) prior is assumed and graph cuts are used as an optimization algorithm.

Bioucas-Dias J M et al [81] proposed a graph cut technique based on diversity approach, where different phase images of the same scene acquired with different frequencies are considered. Once diversity and denoising are formulated as integer optimization problems then finally graph cuts techniques are used as optimization technique.

Shabou A et al [82] presented an approach for phase unwrapping and digital elevation model (DEM) generation using multichannel In-SAR data. The proposed algorithm is able to unwrap and regularize the observed data. The proposed algorithm exploits both the amplitude and phase of the available complex data for accurate DEM reconstructions. In this approach, a Markovian approach together with a graph-cut-based optimization algorithm has been considered.

Bioucas-Dias J M et al [83] proposed a new max-flow/min-cut approach for phase unwrapping convex potentials ($0 < p < 1$). The algorithm is for convex potentials where at first the energy is expressed in terms of the binary potentials and

then it is minimized using the graph cuts. This approach is named as PUMF (for phase unwrapping max-flow).

Shabou A et al [84] proposed a Markovian Approach for DEM estimation from Multiple InSAR data with atmospheric contributions. This work provides a method that aims to solve the Digital Elevation Model (DEM) estimation problem through a Bayesian formulation with the Markovian energy minimization framework. The DEM is generated from a set of multi-frequency or multi baseline interferogram's using a multichannel phase unwrapping algorithm combined with an estimation method of the atmospheric artifacts.

Shabou A et al [85] proposed a Markovian approach for interferometric synthetic aperture radar (In-SAR) phase reconstruction. The proposed approach provides a way to estimate local hyperparameters to adjust the prior model and preserve discontinuities in profiles. The proposed work aids discrete optimization algorithms based on the graph-cut technique.

Shabou A et al [86] proposed a phase unwrapping processing chain for 3D reconstruction using very high-resolution multi-baseline data. The proposed method achieves accurate and reliable DEM of the observed urban areas which in turn used for 3D mapping of urban areas. This proposed method make use amplitude data within the unwrapping chain which helps in preserving sharp discontinuities, typical of urban areas.

Denis L [87] proposed a method for 3D reconstruction using both interferometric SAR and optical data. This algorithm defines the regularized elevation in the framework of Markov random fields (MRF) and derives a smoothness prior that both preserves sharp boundaries (based on total variation minimization) and is driven by the structures present in the optical image.

Hongxing H et al [88] et al proposed an algorithm which filters the PUMA unwrapping results. A pointwise local second-order polynomial approximation method is considered to suppress the noise. In the denoising step, adaptive local window sizes are selected to compromise the fitting error and the suppression of noise.

Dong J et al [89] proposed a dual decomposition acceleration method to speed up a three-dimensional graph cut-based phase unwrapping algorithm. At first, the phase unwrapping problem is formulated as a global discrete energy minimization problem, whereas the technique of dual decomposition is used to increase the computational efficiency. High acceleration in computation time is achieved.

Wu X et al [90] proposed a seismic phase unwrapping based on graph cuts for constructing a relative geologic time volume. The unwrapping method is aided by seismic horizon and unconformity constraints. Horizon constraints are introduced for guiding the phase unwrapping to ensure a constant unwrapped phase on a constraining horizon to avoid unwrapping errors.

Valadao G et al [91] proposed a discontinuity-preserving Bayesian approach to absolute phase estimation. The proposed algorithm is a two-step approach, where phase unwrapping is employed to remove 2π periodic ambiguity and then a multi-precision optimization algorithm, based on graph flows, to estimate the original phase and to eliminate noise.

Valadao G et al [92] proposed a combinational Phase unwrapping and denoising algorithm based on a MAP-MRF algorithm. The unwrapping algorithm detects the discontinuity locations which are used in denoising step. Multi-precision combinatorial optimization algorithm based on graph cuts de-noised the unwrapping results.

Valadao G et al [93] proposed a phase imaging algorithm: phase unwrapping with diversity and denoising with multi-resolution. Frequency diversity technique is put forward through a graph-cuts algorithm that minimizes an MRF and then in denoising step an iterative multi-resolution MAP-MRF energy minimization graph cuts is used.

Graph cuts algorithms are used as an optimization technique in many other algorithms of computer vision due to their efficiency in computing globally optimal solutions and in minimizing the energy functions. A graph cut optimizes MRF by constructing a graph and computing the mincut. The earlier works related to graph cuts are studied.

Hiroshi Ishikawa et al [94] proposed a graph cut technique which reduces any higher-order Markov random field with binary labels into a first-order. The proposed technique combines the reduction with the fusion-move and QPBO algorithms to optimize higher-order multi-label problems. The proposed algorithm is a ready alternative for Belief Propagation algorithms in both optimization performance and speed.

Strandmark P et al [95] proposed an algorithm which processes faster on multi-core computers and has the capability to handle larger graphs. The proposed algorithm solves the max-flow /mincut problem in parallel by splitting the graph into multiple subgraphs for parallel computation in both shared and distributed memory. The proposed algorithm achieves a global optimum solution by splitting a graph and by iteratively solving subproblems in parallel.

Using both splitting and merging approaches Yu M et al [96] proposed a dynamic parallel and distributed graph-cuts algorithm. The proposed algorithm guaranteed convergence to the globally optimal solutions within a predefined number of iterations. High Performance is achieved using more sophisticated splitting and merging strategies.

Sleator D D et al [97] proposed an augmenting-path based graph cut algorithm. Authors proposed a data structure that solves the dynamic trees problem. The proposed data structure is to maintain a collection of vertex-disjoint trees with two operations that are link and cut. Two versions of data structures are proposed, one with a time bound of $O(\log n)$ per operation while the other is a bit complicated of a worst-case per-operation time bound of $O(\log n)$.

Boykov Y et al [12] proposed a fast graph cut algorithm for grids like topology based on augmented paths. It maintains two trees of residual arcs, S rooted from s and T rooted in t . Each tree has active and internal vertices. The outer loop of the algorithm consists of three stages: growth, augmentation, and adoption. In most examples, the proposed algorithm worked 2-5 times faster than any of the other methods, including the push-relabel algorithms.

Goldberg A V et al [13] proposed a faster version of Boykov-Kolmogorov (BK) graph cut by augmenting on shortest paths. The main bottleneck of BK

algorithm is the computation time. Most of the time is consumed during the graph construction. Incremental Breadth First Search (IBFS) algorithm address this issue by building the auxiliary network for computing augmenting paths in an incremental manner by updating the existing network after each augmentation.

Goldberg A V et al [98] proposed an augment path max flow algorithm based on the pre-flow concept. The proposed algorithm maintains a pre-flow in the original network and pushes local excess flow towards the sink. This method achieves an $O(n^3)$ time bound on the 'n'-vertex graph and dynamic version of the algorithm achieves a running time $O(nm \log(n^2/m))$ on a 'n'-vertex and 'm'-edge graph. The parallel implementation running in $O(n^2 \log n)$ time using 'n' processors and $O(m)$ space is obtained.

Cherkassky B V et al presented [99] a push-relabel method for the maximum flow problem. Faster run-times are observed due to the usage of heuristics. The highest-level selection strategy achieves better results when combined with both global and gap relabeling heuristics. The maximum flow problem can be solved without generating flows explicitly by using maximum 's'-excess problem.

Hochbaum D S introduced the pseudo flow algorithm for solving the maximum flow problem by employing pseudo flows [100]. The pseudo flow algorithm solves the minimum cut problem by first finding out the maximum blocking cut. The simplest version of pseudo flow algorithm has been proposed and it reaches an optimal solution quickly. Parametric variants of pseudo flow and pseudo-flow-simplex are also proposed and all the proposed algorithms complexities are $O(mn \log n)$ on a graph with 'n' nodes and 'm' arcs.

Goldberg A V [101] proposed a variant of the push-relabel method that is the partial augment-relabel algorithm. The proposed algorithm is robust and outperforms highest-label push-relabel algorithm. The partial augment-relabel algorithm is a push-relabel algorithm that maintains a pre-flow and a distance labeling. The proposed algorithm looks for augmenting paths in the same way as augment-relabel. Depending upon the existence of the arc the algorithm extends or shrinks the path.

Goldberg A V [102] proposed two processor efficient maximum distance discharge algorithms for the maximum flow problem. The first implementation runs

in $n^2 \log \frac{2m}{n+p} \sqrt{\frac{m}{p}}$ time and the other runs in $n^2 \log n \sqrt{\frac{m}{p}}$ time where ‘n’ and ‘m’ area number of vertices and arcs.

Bader D A et al [103] implemented cache-aware of Goldberg’s parallel algorithm for modern shared-memory parallel computers. The proposed algorithm is first to parallel implementation and analysis of the gap relabeling heuristic that runs from 2.1 to 4.3 times faster for sparse graphs.

Delong A et al [104] proposed a region push-relabel algorithm. The proposed algorithm follows Goldberg and Tarjan’s ‘discharge’ variant of push-relabel [94]. The proposed algorithm selects an active region and tries to push all excess flow to neighbours $R \subseteq V - \{s, t\}$ outside R while only modifying labels inside R . The overall algorithm complexity is $O(n^2m)$.

Goldberg A V et al [105] proposed two algorithms based on the pre-flow concept. The first algorithm maintains a pre-flow in the original network and pushes local flow excess towards the sink. The idea is to select an active vertex $v \in V$, and repeatedly call push (v, w) and relabel (v) functions until all excess flow $e(v)$ has been pushed to v ’s neighbours. The first algorithm achieves an $O(n^3)$ time bound on an n -vertex graph. This faster algorithm runs in $O(nm \log(n^2/m))$ time on ‘n’-vertex and ‘m’-edge graph.

Vineet V et al [106] present an implementation of the push-relabel algorithm for graph cuts on the GPU. At first, divide the task among the multiprocessors of the GPU and exploit its shared memory for high performance. Periodic global relabeling improves the running time. Faster run times, for instance, 60 graph cuts per second on 1024×1024 images and over 150 graph cuts per second on 640×480 images on a Nvidia 8800 GTX are achieved by the proposed implementation.

Liu J et al [107] proposed a novel adaptive bottom-up approach to parallelize the BK algorithm. At first, partitioning uniformly the graph into a number of regularly-shaped disjoint subgraphs and process them in parallel, then incrementally merge the subgraphs in an adaptive way to obtain the global optimum. The proposed algorithm achieves near-linear speedup for common vision problems and it is cache-

friendly, keeps balanced workloads, causes little overhead and is adaptable to the number of available cores.

Kohli P et al [108] presented a fast new fully dynamic algorithm for the mincut/max-flow problem. MAP solutions of dynamically changing MRFs can be efficiently computed by reusing flow. Specifically, the proposed algorithm describes how the residual graph can be transformed to reflect the changes in the original graph using graph re-parameterization. The time taken by it is roughly proportional to the total amount of change in the edge weights of the graph.

Juan O et al [109] proposed a new Active Cuts approach for solving the max-flow problem. The proposed algorithm has arbitrary initial cuts, converging intermediate unique aspects. Active Cuts significantly improves practical efficiency (2-6 times) and applicability of graph cuts to problems in image analysis. Before converging towards the global minima, Active Cuts provides the intermediate cuts which can be used to accelerate or improve graph cuts real-time performance. The proposed method can also be combined with many previous methods for accelerating graph cuts.

Goldberg A V et al [110] proposed Excesses Incremental Breadth-First Search (Excesses IBFS) algorithm for maximum flow problems. Excesses IBFS algorithm generalizes IBFS algorithm while maintaining the same polynomial running time guarantee of $O(mn^2)$ as IBFS. Excesses IBFS naturally extends to this dynamic setting and is competitive in practice with other dynamic methods. Excess IBFS shows that it has the same worst-case time bounds as IBFS: $O(mn \log(n^2/m))$ with dynamic trees and $O(mn^2)$ without. Excesses IBFS offers an even faster, still theoretically justified and dynamic alternative to all existing methods.

Fix A et al [111] proposed a graph-cut for Higher-order Markov Random Fields. The proposed algorithm achieves better performance than [94,112] both theoretically and experimentally. The proposed method transforms a group of the higher-order terms into a first-order one. Running on the dataset used in [94,112] the proposed algorithm optimally label significantly more variables (96% versus 80%) and converge more rapidly to a lower energy.

Ishikawa H [112] extended his previous graph cut technique which reduces any higher-order Markov random field with binary labels into a first-order [81]. In this proposed paper a new generalization of the reduction, discussions on the polynomial-time minimize the ability of the reduced energy and a necessary number of auxiliary variables, and an investigation of relations between the new algorithm and other known methods have been made.

Jamriska O et al [14] proposed a compact data structure with cache-efficient memory layout for the representation of graph instances that are based on regular N-D grids with topologically identical neighborhood systems. The proposed algorithm shows an unconditional improvement in terms of speed and memory consumption and works only on grid-like topology.

Lombaert H et al [113] proposed a multilevel banded heuristic for computation of graph cuts that is motivated by the well-known narrow band algorithm in level set computation. The proposed algorithm drastically reduces both the running time and the memory consumption of graph cuts while producing nearly the same segmentation result as the conventional graph cuts.

Juan O et al [114] proposed an augmentation graph cut which improves the theoretical complexity of BK algorithm [12] to weakly polynomial by reducing the number of augmentations required to converge to the global minimum. The proposed algorithm significantly outperforms BK on denser grids and in cases with clutter (many local minima) and noise.

Arora C et al [115] present a Graph Cut algorithm which runs faster on all 2D samples and is at least two to three times faster on 70% of 2D and 3D samples in comparison to the algorithm reported in [12].

2.2 ORIGIN OF WORK

- a. As size of the interferogram increases, most of the PU algorithms are very slow while unwrapping phase of a large-scale In-SAR data (18,000×20,000).
- b. However, these algorithms can be demanding and the memory bandwidth is being the main bottleneck.
- c. Most of the PU Algorithms have adopted legacy techniques and exhibits poor performance.

2.3 OBJECTIVES OF THE WORK

- a. To design an algorithm that provides improvements in terms of speed and memory consumption.
- b. Designing an algorithm which should blindly detect the discontinuities.
- c. To design an algorithm which should simultaneously remove the noise and unwrap phase correctly.

2.4 THESIS ORGANIZATION

Chapter1 primarily dealt with the introduction of the thesis. Remote sensing through Radar Imaging systems and different radar imaging systems are listed. A brief introduction about Synthetic Aperture Radar and its principles, processing, applications, techniques are also mentioned. Specific technique of SAR that is Interferometry SAR is explained in detail. The basic principle of In-SAR, its acquisition, its processing, different types of In-SAR and its applications are outlined. The next sections are focused on the subject of Phase Unwrapping. At first, the explanation of coherent processing technique and the devices which utilize this principle are provided. The most common issue found in all of these devices are phase unwrapping. The basic definition of the phase unwrapping and its preliminary solution provided by ITOH are presented. Different methods are available to unwrap the phase are also explained in detail. Finally, some of the devices which use the coherent processing techniques are also studied. The rest of the chapter focuses on the estimation theory and its role in finding the optimal estimator. This chapter concludes with the discussion on the "MAP-MRF" framework and the usage of Graph Cuts in a MAP-MRF approach. Some of the graph-cut algorithms which are utilized in this work are also discussed. The motivation of the work and thesis organization is also included in this chapter.

Chapter2 presents a detailed literature review of algorithms which followed the MAP-MRF approach to find the PU solution. Filtering algorithms which are applied before or after unwrapping step (MAP-MRF approach) are also surveyed. Earlier works conducted on speeding up of the Phase Unwrapping algorithms are also outlined. Recent developments in graph cut techniques and recent works on unwrapping phase via graph cuts techniques are also studied.

Chapter3 concentrates on the PU algorithm based on MAP-MRF approach, one of the algorithm based on MAP-MRF is PUMA. But the PUMA algorithm faces issues while unwrap images of large size. In order to reduce the complexity of the PUMA algorithm, in this work, proposed a new phase unwrapping algorithm and named it as "**I-PUMA**". For this new algorithm, MAP-MRF approach is followed. According to this new approach, first the energy is mapped onto the graph cut and then it is minimized by using the Incremental Breadth-First Search (IBFS) algorithm. The proposed algorithm is explained in detail and tested on the simulated and real image (Interferometry SAR data, ENVISAT Data). Some of the important factors like discontinuity (mask), noise, PSNR and elapsed time are also analyzed. Results show that the proposed algorithm unwrap the solution faster than the existing phase unwrapping via graph cut (PUMA) algorithm.

Chapter4 focused on the point that the usage of cache-efficient techniques can improve the performance of the phase unwrapping algorithms. An algorithm is designed which uses the cache efficient techniques for unwrapping the phase. The proposed algorithm in this chapter follows the approach of the MAP-MRF and utilized the cache efficient techniques during the optimization step. GridCut is a multi-core high-performance max-flow solver for grid-like graphs. The implementation of GridCut as an optimization step in MAP-MRF approach is proposed. The proposed algorithm is described in detail and tested on simulated and real interferograms. Results show that the proposed algorithm "**G-PUMA**" unwraps phase at a faster rate (40% to 80%) by consuming less memory of about 10 to 40% than PUMA.

Chapter5 starts with the discussion on the idea of using filtering and phase unwrapping steps at once. This chapter starts with the discussion about different filtering methods, its usage in In-SAR, its advantages and disadvantages. Herein, proposed a two-step approach that is denoising the interferogram and then unwraps it. LPA-ICI algorithm with varying windows size changes adaptively according to the smoothness of the profile is elaborated. By applying the algorithms as in chapter 3 and 4 to the results of LPA-ICI technique, two new algorithms are proposed in this chapter. The newly proposed algorithms in this chapter are outlined and tested on simulated and real interferogram (Interferometry SAR data). Faster execution times

even for larger SAR images are attained with the designed algorithms. Comparative analysis of proposed algorithms with PEARLS is also included in this chapter.

Chapter6 presents the alternative way of filtering that is first unwrapping and then de-noising the interferogram. The chapter starts with the outlining about the advantage of using the post-filtering method than pre-filtering method. Post Filtering algorithm based on Second order polynomial approximation technique is presented and its features are outlined. Proposed an algorithm based on G-PUMA and SPA to attain the finer and faster results. For faster unwrapping, a combination of G-PUMA and SPA techniques are adopted and detailed discussions regarding the proposed algorithm is presented. The effectiveness of the new “**G-PUMA-SPA**” algorithm is tested on continuous phase surfaces and discontinuous phase surfaces. Comparative analysis of the proposed algorithm with the existing algorithm PUMA-SPA is also presented.

Chapter7 concluded with a brief summary of entire work, achieved results, and final conclusions. Further works on the mainstream of research are also described.