Project Title: Phase Unwrapping: A Min-Cut/Max-Flow Based Approach

Project Acronym: PHASE

Project Description

Summary

Recent results on energy minimization via Graph Cuts (see, e.g. [1], [2], [3], [4]) have opened the door to a series of advances in low-level image problems such as segmentation, restoration, classification, stereo, and multi-camera scene reconstruction.

By exploiting this new energy minimization results, this project aims at conceptual and algorithmic developments in the field of Phase Unwrapping (PU), namely, the following:

Conceptual

1. Design exact minimizers for L^p-norm based PU, for $p \geq 1$
2. Design sub-optimal minimizers for L^p-norm based PU, for $0 < p < 1$
3. Design new robust clique potentials tailored to PU, based on the Bayesian paradigm
4. Design PU schemes taking advantage of diversity (e.g., space or frequency), i.e., of more than one sources of observed wrapped phase.

Algorithmic

1. Develop, in C/C++, efficient Min-Cut/Max-Flow algorithms taking advantage of the graph structure inherent to the problems listed above
2. Develop efficient algorithms implementing the minimizers listed above
3. Develop "real world" application-oriented prototype demonstrators for these algorithms.

Furthermore, the project is also designed to partially support an ongoing collaboration between the project leader and Dr. Mario Ries from "Institut des Neorosciences de Bourdeaux", in the area of PU applied to Magnetic Resonance Imaging (MRI). Although, PU range of applications is wide, the project will be focused on MRI and Interferometric Synthetic Aperture Radar (InSAR).

Background and State of The Art

Phase is an important property of many classes of signals. For instance, interferometric SAR uses two or more antennas to measure the phase between the antennas and the terrain; the topography is then inferred from the difference between those phases [5]. In magnetic resonance imaging, phase is used, namely, to determine magnetic field deviation maps, which are used to correct echo-planar image geometric distortions [6]. In optical interferometry, phase measurements are used to detect objects shape, deformation, and vibration [7].

In all the examples above, in spite of phase being a crucial information, the acquisition system can only measure phase modulo-2π, the so-called principal phase value, or wrapped phase. Formally, we have

$$\phi = \psi + 2k\pi, \tag{1}$$

where ϕ is the true phase value (the so-called absolute value), ψ is the measured (wrapped) modulo-2π phase value, and $k \in \mathbb{Z}$ an integer number of wavelengths [8].

Phase unwrapping is the process of recovering the absolute phase ϕ from the wrapped phase ψ. This is, however, an ill-posed problem, if no further information is added. In fact, an assumption taken by most phase unwrapping algorithms is that the absolute value of phase differences between neighboring pixels is less than π, the so-called Itoh condition. If this assumption is not violated, the absolute phase can be easily determined, up to a constant. Itoh condition might be violated if the true phase surface is discontinuous, or if only a noisy version of the wrapped phase is available. In either cases, PU becomes a very difficult problem, to which much attention has been devoted [8], [9].

Phase unwrapping approaches belong to one of these following classes: path following [10], minimum L^p norm [11], Bayesian [12], and parametric modeling [13].

Path following algorithms apply line integration schemes over the wrapped phase image, and basically rely on the assumption that Itoh condition holds along the integration path. Wherever that condition is not met, different integration paths may lead to different unwrapped phase values. Techniques employed to handle these inconsistencies include the so-called *residues branch cuts* [10] and *quality maps* [8].

Minimum norm methods exploit the fact that the differences between absolute phases of neighbor pixels, are equal to the wrapped differences between correspondent wrapped phases, if Itoh condition is met. Thus, these methods try to find a phase solution ϕ for which L^p norm of the difference between absolute phase differences and wrapped phase differences (so a second order difference) is minimized. This is, therefore, a global minimization in the sense that all the observed phases are used to compute the unwrapped phase in a given site. With $p = 2$, we have a least squares method [14]. The exact solution with $p = 2$ is developed in [9] using network programming techniques. An approximation to the least squares solution can be obtained by relaxing the discrete domain $\mathbb{Z}^{MN}$ to $\mathbb{R}^{MN}$ and applying FFT or DCT based techniques[1]. A drawback of the L^2 norm is that this criterion tends to smooth discontinuities, unless they are provided as binary weights. L^1 norm performs better than L^2 norm in what discontinuity preserving is concerned. Such a criterion has been solved by Flynn [15] and Costantini [16], using network programming. With $0 \leq p < 1$ the ability of preserving discontinuities is further increased at stake, however, of highly complex algorithms [17].

The Bayesian approach relies on a data-observation mechanism model, as well as a prior knowledge of the phase to be modelled. For instance in [18], a non-linear optimal filtering is applied, while in [19] an InSAR observation model is considered, and is taken into account not only the image phase, but also the *backscattering coefficient* and *correlation factor* images, which are jointly recovered from InSAR image pairs.

Finally, parametric algorithms constrain the unwrapped phase to a parametric surface. Low order polynomial surfaces are used in [13]. Very often in real applications just one polynomial is not enough to describe accurately the complete surface. In such cases the image is partitioned and different parametric models are applied to each partition [13].

Project Line of Action

As stated in the Summary, the project aims at conceptual and algorithmic goals. Regarding the conceptual goals, the ultimate objective is to design minimizers for the L^p-norm of the complete set of phase differences between neighboring pixels, with the additional constraint of being 2π-congruent with wrapped phases. This is an integer optimization problem, which we intend to solve by a series of binary elementary optimizations in the vein of the work [9] for the $\mathbb{Z}\pi$M algorithm. The approach to be followed casts, however, the optimization problem as a min-cut/max-flow computation on a certain graph, building on energy minimization results presented in [3]. These concepts are valid for $p \geq 1$.

For $0 < p < 1$, the above referred integer optimization is NP-Hard. As for this range of values the discontinuities handling capability of the minimum L^p-norm phase unwrapping methods is, generally, rather enhanced, we will investigate the design of sub-optimal optimization schemes, still based on min-cut/max-flow computations, for which we have preliminary and surprising indications of good performance.

The above ideas are then to be extended to PU applications with observation diversity. Whether using, e.g., multiple SAR frequencies or acquisition geometries, on imaging a certain ground area with InSAR, or whether using, e.g., various echo times on imaging a certain body with MRI, the availability of diversity observations can dramatically improve

[1]Where M and N are the number of lines and columns respectively.

the phase unwrapping performance. The idea behind these techniques is, mainly, the extension of the ambiguity interval $[0\ 2\pi[$ (in which the wrapped phase values live), drawing on the Chinese remainder theorem.

References

[1] O. Veksler. *Efficient Graph-Based Energy Minimization Methods In Computer Vision.* PhD thesis, Cornell University, 1999.

[2] Y. Boykov, O. Veksler, and R. Zabih. Fast approximate energy minimization via graph cuts. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 23(11):1222–1239, 2001.

[3] V. Kolmogorov and R. Zabih. What energy functions can be minimized via graph cuts? *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 26(2):147–159, February 2004.

[4] Y. Boykov and V. Kolmogorov. An experimental comparison of min-cut/max-flow algorithms for energy minimization in vision. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 26(9):1124–1137, 2004.

[5] P. Rosen, S. Hensley, I. Joughin, F. LI, S. Madsen, E. Rodriguez, and R. Goldstein. Synthetic aperture radar interferometry. *Proceedings of the IEEE*, 88(3):333–382, March 2000.

[6] P. Jezzard and R. Balaban. Correction for geometric distortion in echo-planar images from B_0 field variations. *Magnetic Resonance in Medicine*, 34:65–73, 1995.

[7] S. Pandit, N. Jordache, and G. Joshi. Data-dependent systems methodology for noise-insensitive phase unwrapping in laser interferometric surface characterization. *Journal of the Optical Society of America*, 11(10):2584–2592, 1994.

[8] D. Ghiglia and M. Pritt. *Two-Dimentional Phase Unwrapping. Theory, Algorithms, and Software.* John Wiley & Sons, New York, 1998.

[9] J. Dias and J. Leitão. The $Z\pi M$ algorithm: A method for interferometric image reconstruction in SAR/SAS. *IEEE Transactions on Image Processing*, 11(4):408–422, 2002.

[10] R. Goldstein, H. Zebker, and C. Werner. Satellite radar interferometry: Two-dimensional phase unwrapping. In *Symposium on the Ionospheric Effects on Communication and Related Systems*, volume 23, pages 713–720. Radio Science, 1988.

[11] D. Ghiglia and L. Romero. Minimum L^p norm two-dimensional phase unwrapping. *Journal of the Optical Society of America*, 13(10):1999–2013, 1996.

[12] G. Nico, G. Palubinskas, and M. Datcu. Bayesian approach to phase unwrapping: theoretical study. *IEEE Transactions on Signal Processing*, 48(9):2545–2556, Sept. 2000.

[13] B. Friedlander and J. Francos. Model based phase unwrapping of 2-d signals. *IEEE Transactions on Signal Processing*, 44(12):2999–3007, 1996.

[14] D. Fried. Least-squares fitting a wave-front distortion estimate to an array of phase-difference measurements. *Journal of the Optical Society of America*, 67(3):370–375, 1977.

[15] T. Flynn. Two-dimensional phase unwrapping with minimum weighted discontinuity. *Journal of the Optical Society of America A*, 14(10):2692–2701, 1997.

[16] M. Costantini. A novel phase unwrapping method based on network programing. *IEEE Transactions on Geoscience and Remote Sensing*, 36(3):813–821, May 1998.

[17] C. Chen and H. Zebker. Two-dimensional phase unwrapping with use of statistical models for cost functions in nonlinear optimization. *Journal of the Optical Society of America A*, 18:338–351, 2001.

[18] J. Leitao and M. Figueiredo. Absolute phase image reconstruction: A stochastic nonlinear filtering approach. *IEEE Trans. Image Processing*, 7(6), June 1997.

[19] J. Dias and J. Leitão. Simultaneous phase unwrapping and speckle smoothing in SAR images: A stochastic nonlinear filtering approach. In *EUSAR'98 European Conference on Synthetic Aperture Radar*, pages 373–377, Friedrichshafen, May 1998.