Interferometric Phase Image Estimation via Sparse Coding in the Complex Domain

José M. Bioucas Dias

Instituto Superior Técnico and Instituto de Telecomunicações
Technical University of Lisbon

PORTUGAL

bioucas@lx.it.pt, www.lx.it.pt/~bioucas

Joint work with *Hao Hongxing* (IST)
Gonçalo Valadão (IT)
Vladimir Katkovnik (TUT)

Outline

1. The phase estimation problem
2. Examples in InSAR and in MRI
3. Phase unwrapping
4. Interferometric phase estimation via sparse coding
5. Non-Gaussian and non-additive noise
6. Concluding remarks

Absolute Phase Estimation problem

Given a set of observations $e^{j\phi_p} \equiv (\cos\phi_p, \sin\phi_p)$,
for $p \in \mathcal{V} \equiv \{1, \ldots, n\}$, determine ϕ_p (up to a constant)

$e^{j\phi_p}$ is 2π-periodic $\Rightarrow$ nonlinear and ill-posed inverse problem

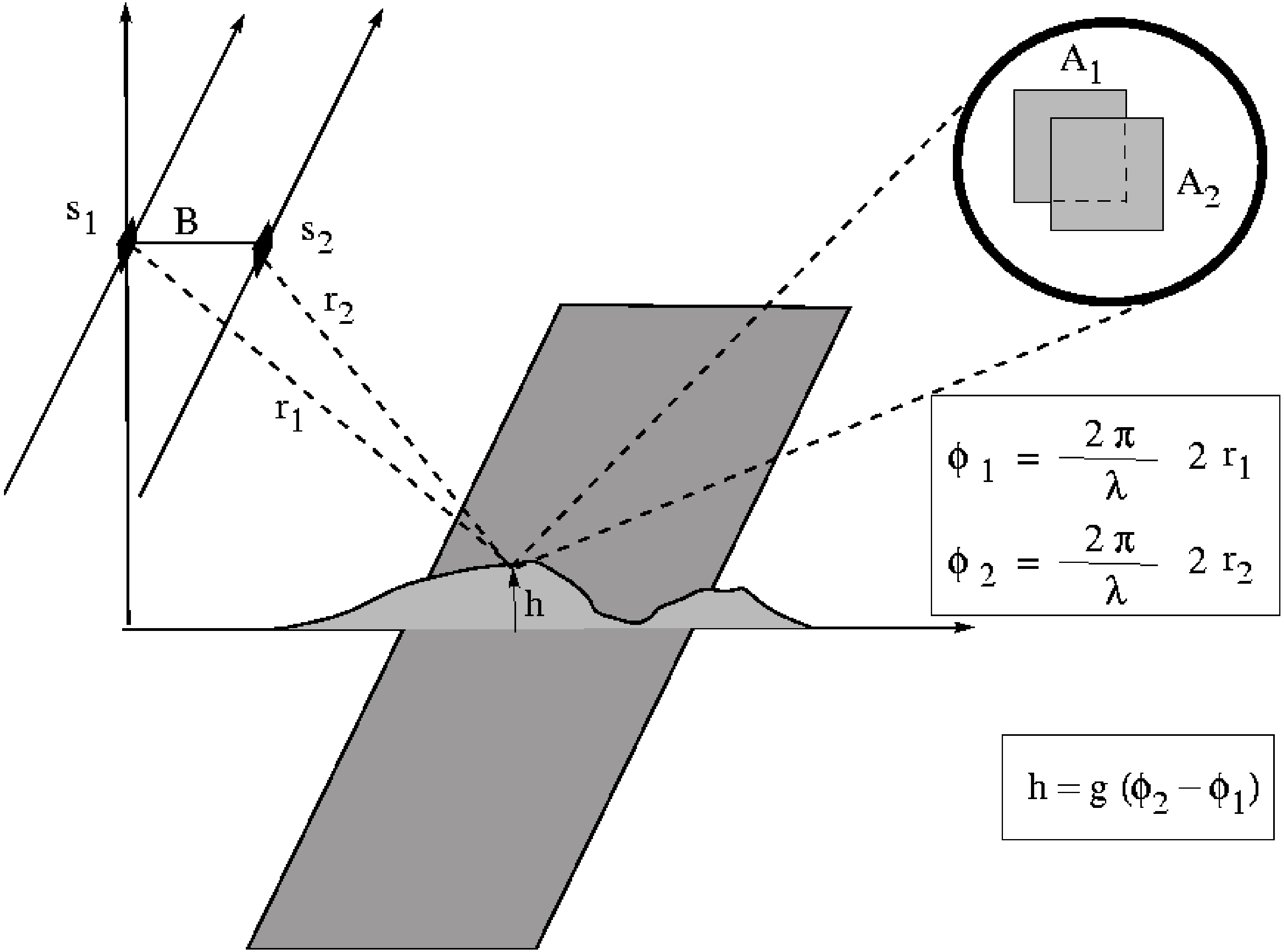
Continuous/discrete flavor: $\phi = \mathcal{W}[\phi] + 2k\pi \qquad \mathcal{W}: \mathbb{R} \to [\pi, \pi[$

Phase Unwrapping (PU)	Phase Denoising (PD)
↓	↓
Estimation of $k \in \mathbb{Z}$	Estimation of $\mathcal{W}(\phi) \in [\pi, \pi[$ (wrapped phase)

Applications

- Synthetic aperture radar/sonar
- Magnetic resonance imaging
- Doppler weather radar
- Doppler echocardiography
- Optical interferometry
- Diffraction tomography

Absolute Phase Estimation in InSAR (Interferometric SAR)



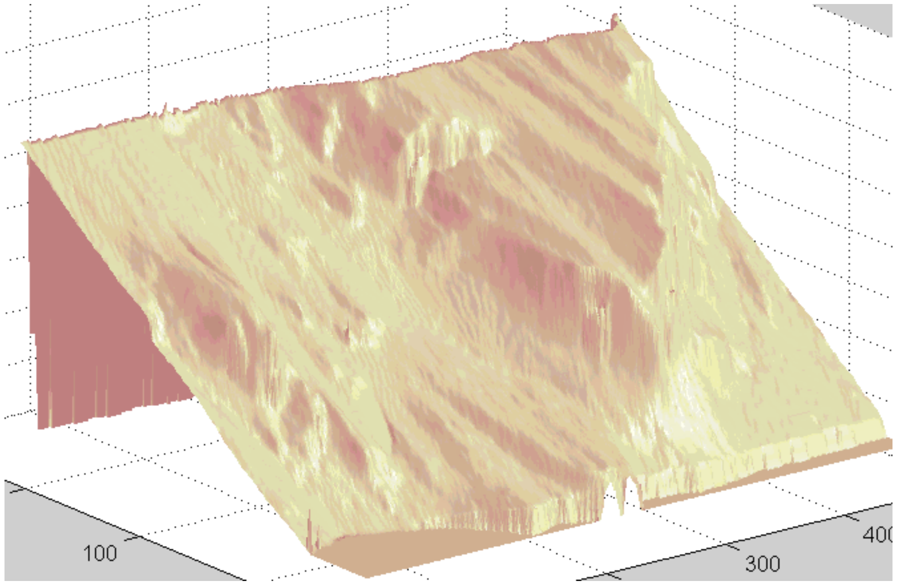
$$\phi_1 = \frac{2\pi}{\lambda} 2 r_1$$

$$\phi_2 = \frac{2\pi}{\lambda} 2 r_2$$

$$h = g(\phi_2 - \phi_1)$$

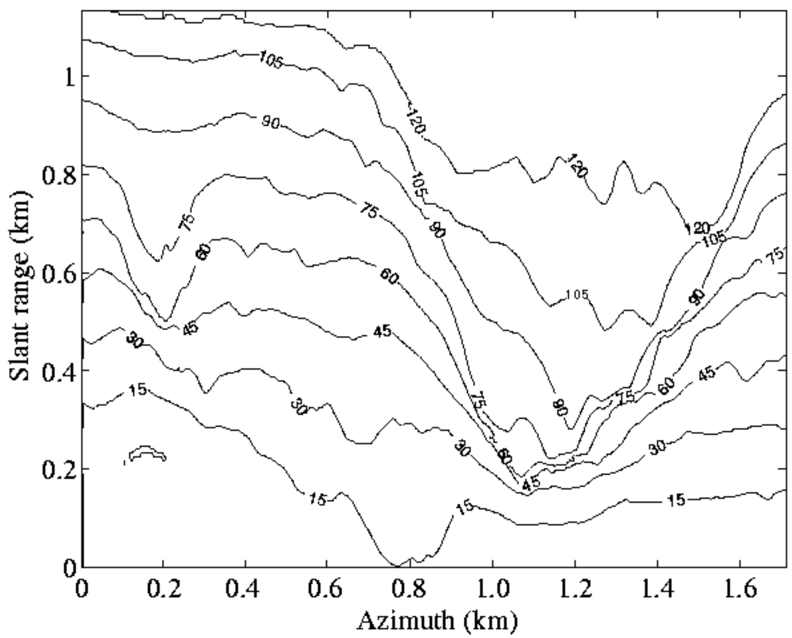
InSAR Problem: Estimate ϕ_2-ϕ_1 from signals read by s_1 and s_2

InSAR Example

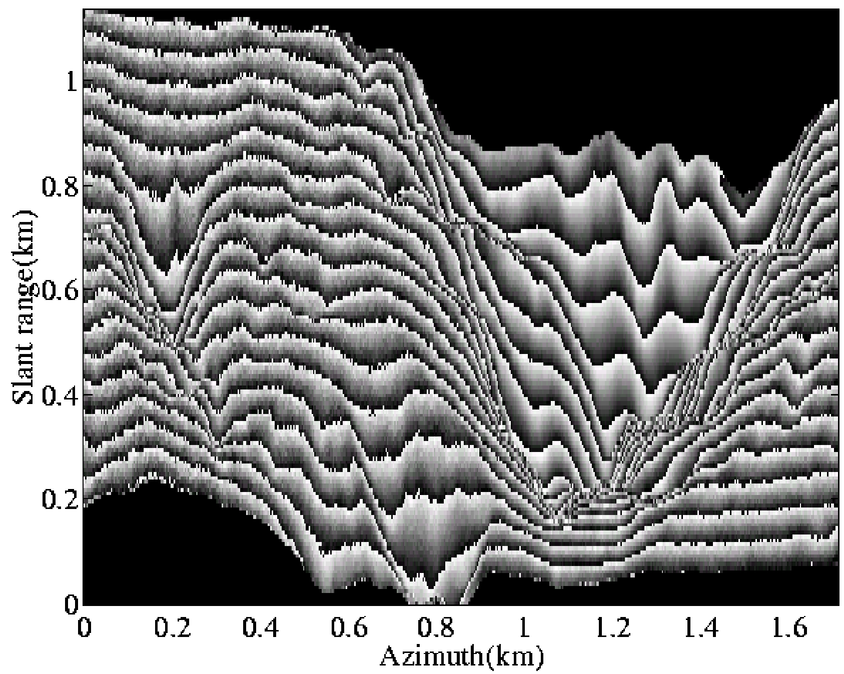


Mountainous terrain around Long's Peak, Colorado

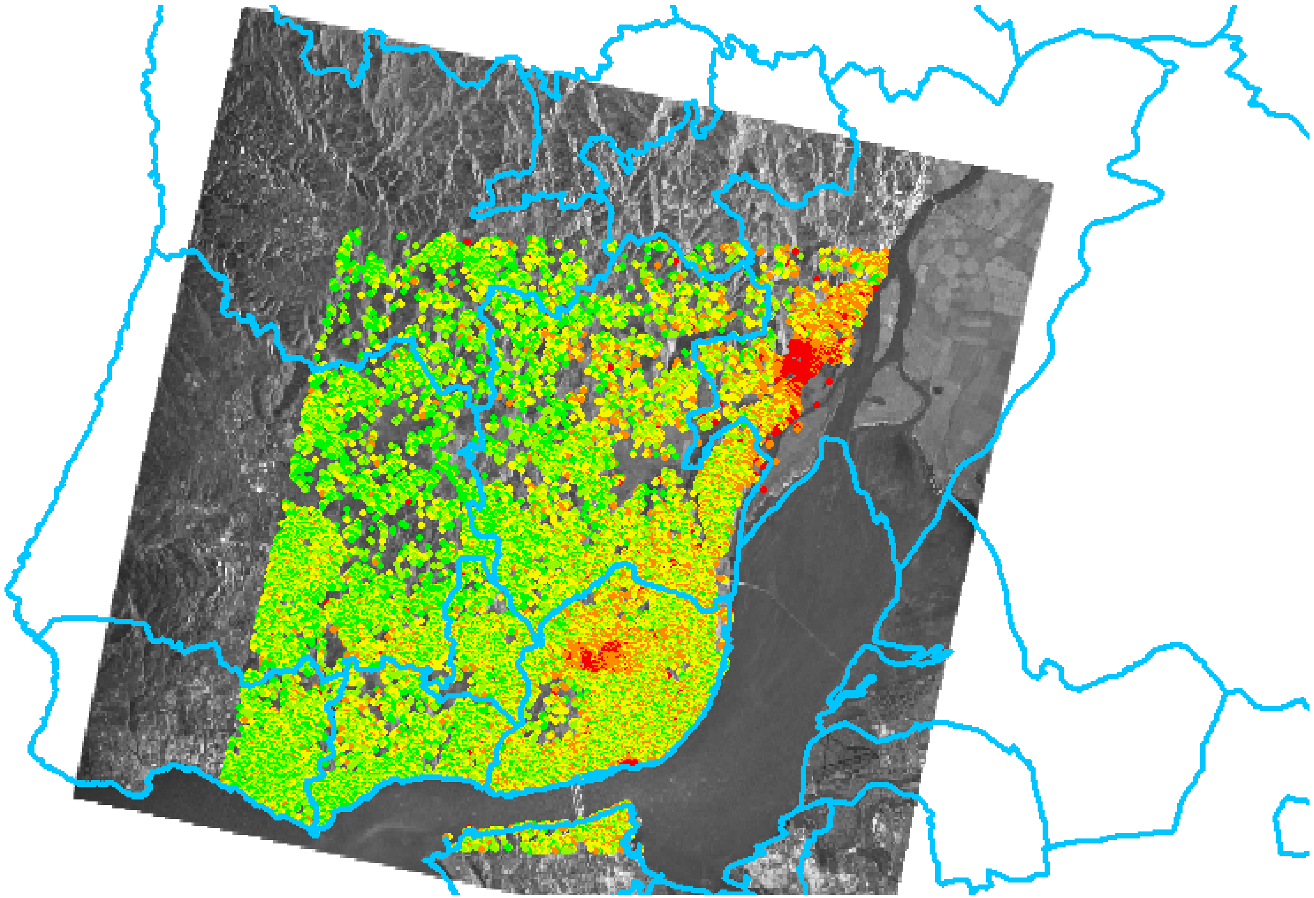
$\phi_1 - \phi_2$ Contour map (rad)



Interferogram $\mathcal{W}(\phi_1 - \phi_2)$



Differential Interferometry

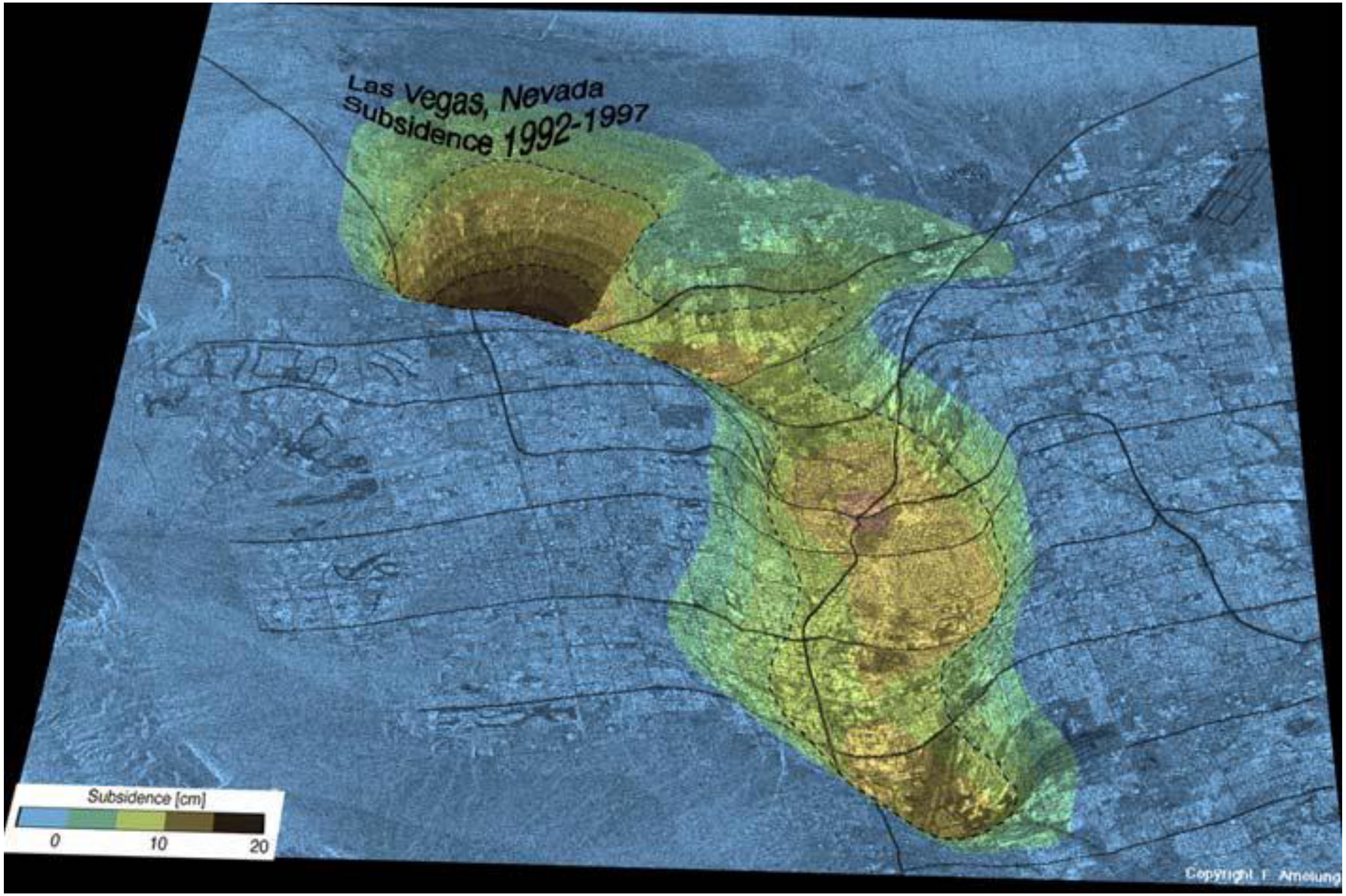


Height variation

7 mm/year

-17 mm/year

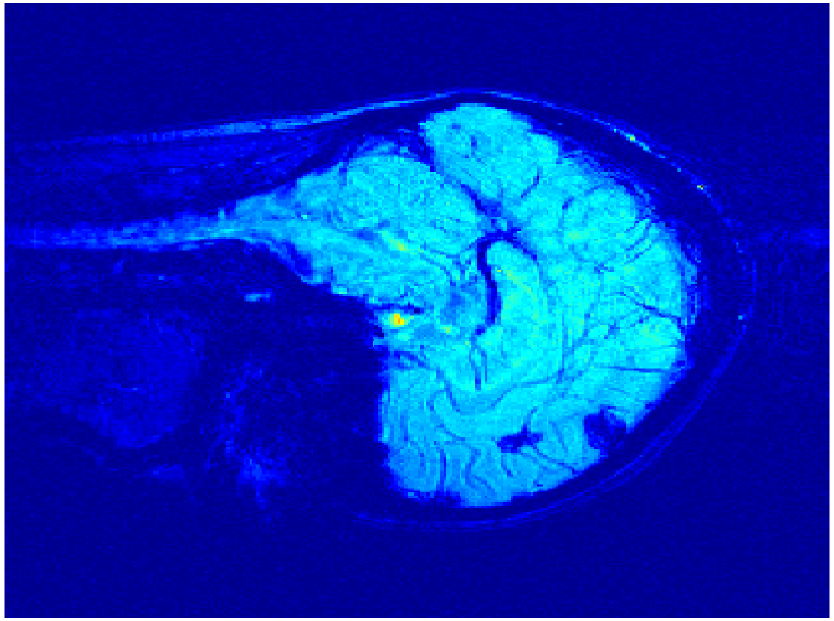
Differential Interferometry



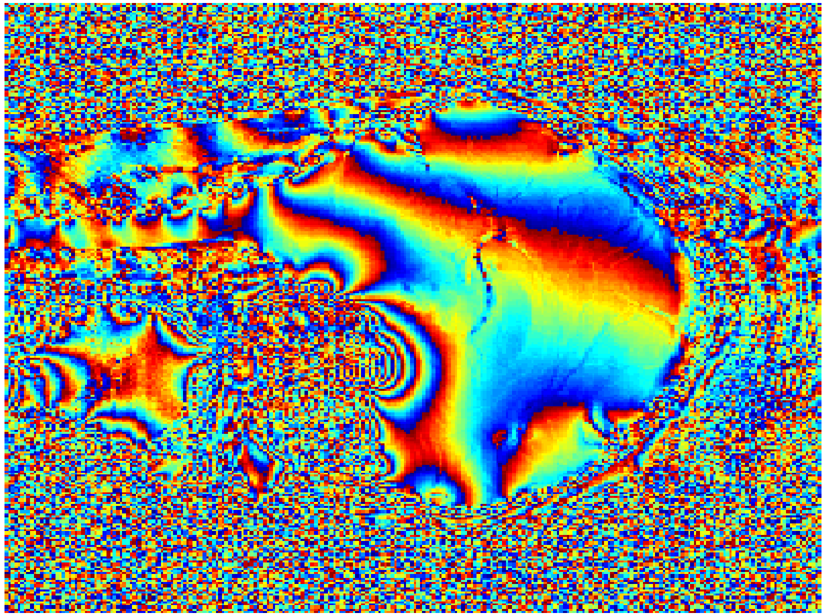
Differential InSAR derived subsidence in Las Vegas between 1992 and 1997 (from [Amelung et al., 1999]).

Magnetic Resonance Imaging - MRI

Intensity



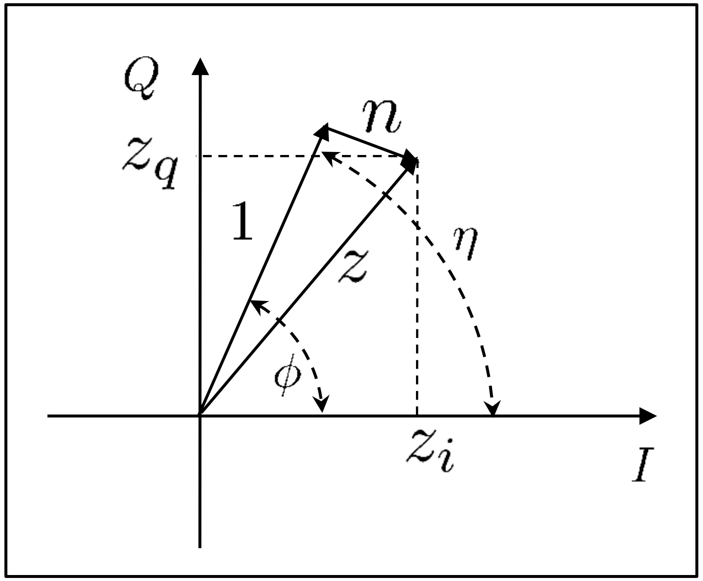
Interferometric phase



Interferomeric phase

- measure temperature
- visualize veins in tissues
- water-fat separation
- map the principal magnetic field

Forward Problem: Sensor Model



$$z_i = \cos\phi + n_i \qquad n = (n_i, n_q)$$

$$z_q = \sin\phi + n_q \qquad z = (z_i, z_q)$$

$$(n_i, n_q) \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_2)$$

$$\boxed{p(z|\phi) \propto c\, e^{\lambda \cos(\phi - \eta)}} \longrightarrow \widehat{\phi}_{ML} = \eta + 2k\pi$$

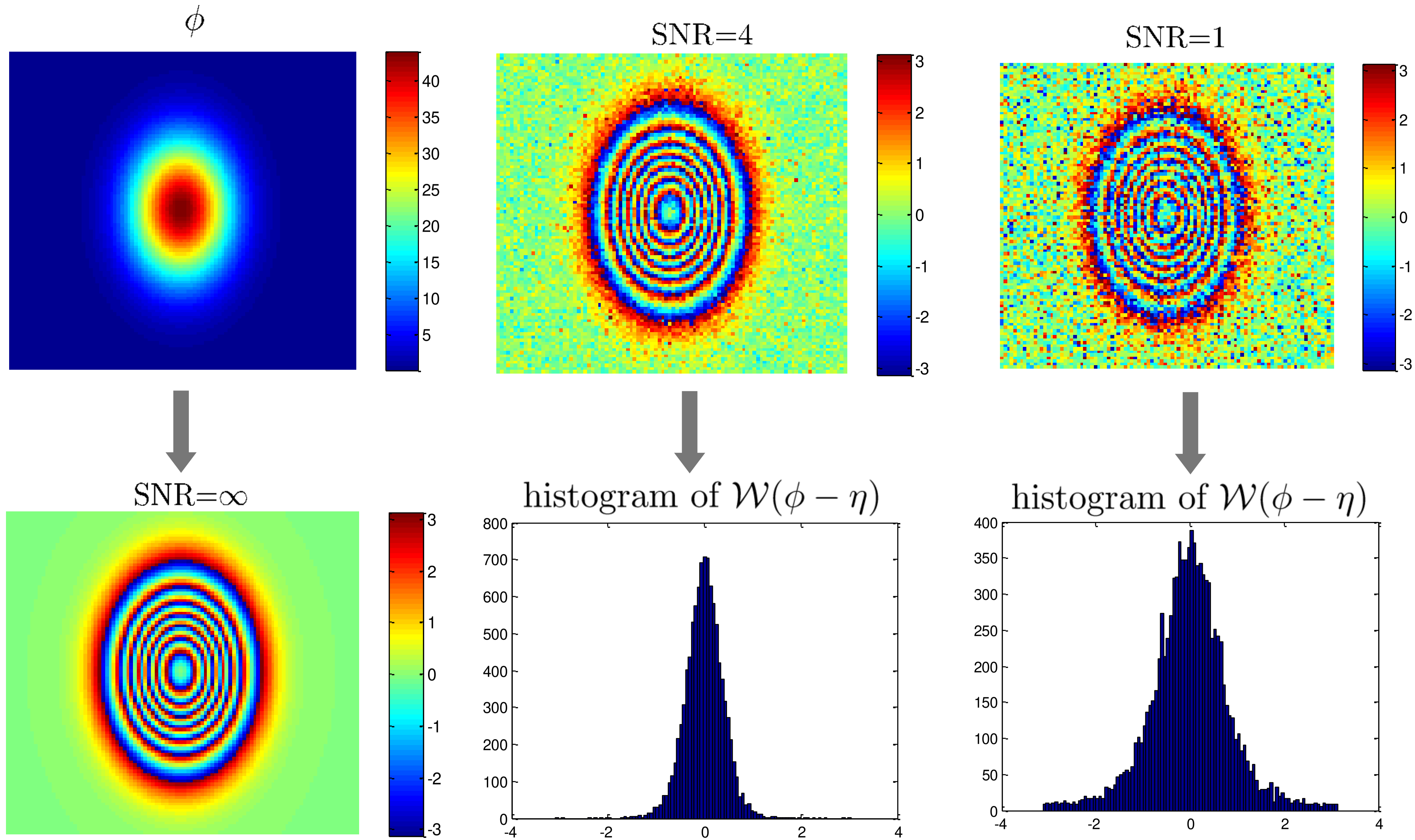
$$\eta = \arg(z)$$

$$\lambda = \frac{2|z|}{\sigma^2}$$

Simulated Interferograms

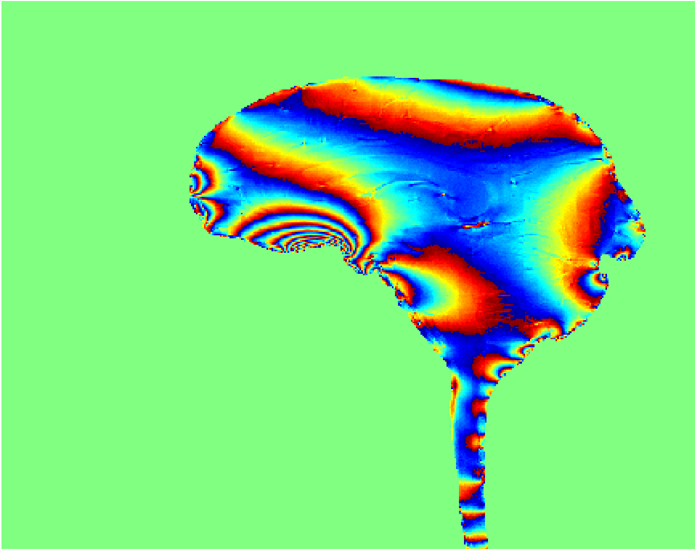
Images of $\eta = \arg(e^{j\phi} + n)$

$$\mathrm{SNR} \equiv \frac{1}{2\sigma^2}$$

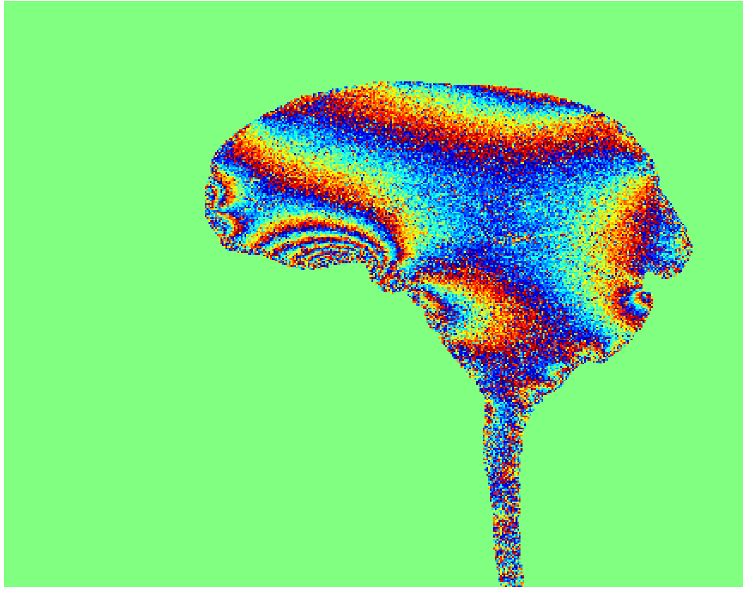


Real Interferograms

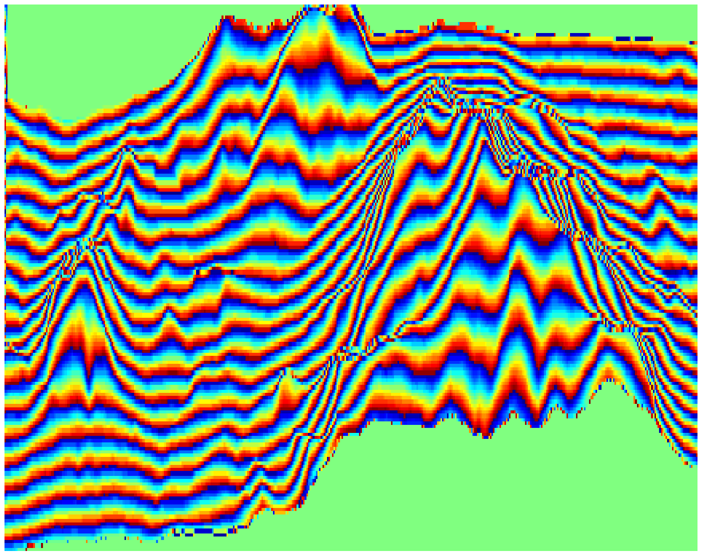
MRI



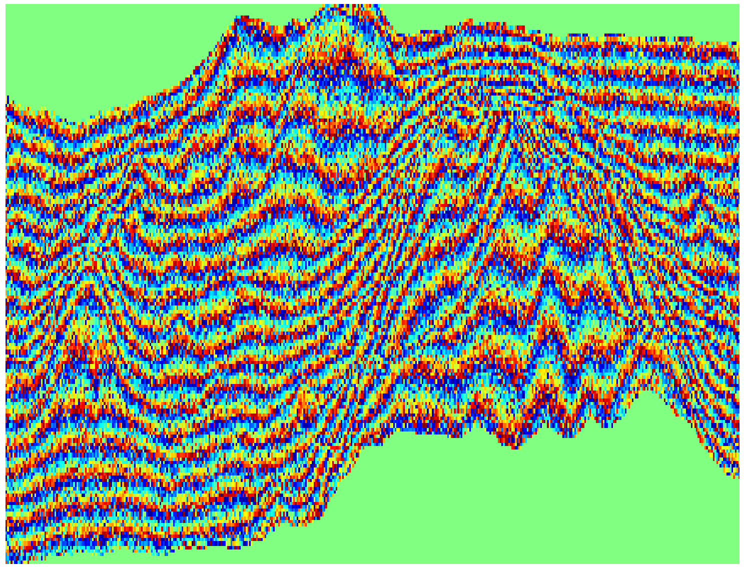
MRI



InSAR



InSAR



Bayesian Approach

data density: $$p(\mathbf{z}|\boldsymbol{\phi}) = \prod_{p\in\mathcal{V}} p(z_p|\phi_p)$$

prior (MRF): $$p(\boldsymbol{\phi}) = \frac{1}{Z} e^{-\sum_{\{p,q\}\in\mathcal{E}} E_{pq}(\phi_p - \phi_q)}$$

- $\mathcal{E} = \{\{p,q\} : p \sim q\}$ clique set
- $E_{pq}(\cdot)$ clique potential (pairwise interaction)

$E_{pq}(\cdot)$ convex

$\downarrow$

Enforces smoothness

$E_{pq}(\cdot)$ non-convex

$\downarrow$

Enforces piecewise smoothness (discontinuity preserving)

Maximum a Posteriori Estimation Criterion

- $\widehat{\phi} \in \arg\max_{\boldsymbol{\phi} \in \mathbb{R}^n} \; p(\boldsymbol{\phi}|\mathbf{z})$ $\qquad p(\boldsymbol{\phi}|\mathbf{z}) \propto p(\mathbf{z}|\boldsymbol{\phi})\, p(\boldsymbol{\phi})$ posterior density

  $= \arg\min_{\boldsymbol{\phi}} E(\boldsymbol{\phi})$ $\qquad E(\boldsymbol{\phi}) = -\log p(\boldsymbol{\phi}|\mathbf{z}) + c^{te}$

$$\boxed{E(\boldsymbol{\phi}) = \sum_{p \in \mathcal{V}} -\lambda_p \cos(\phi_p - \eta_p) + \sum_{\{p,q\} \in \mathcal{E}} E_{pq}(\phi_p - \phi_q)}$$

- Phase unwrapping ($\lambda_p \to \infty$):

$$\phi_p = \eta_p + 2k\pi \;\; \text{for} \;\; k_p \in \{0, 1, \ldots, K-1\}$$

$$\widehat{\phi} \in \arg\min_{\mathbf{k} \in \mathbb{Z}^n} \; E(\mathbf{k}) \qquad E(\mathbf{k}) = \sum_{\{p,q\} \in \mathcal{E}} V_{pq}(k_p - k_q)$$

$$V_{pq}(k_p - k_q) = E_{pq}(\eta_p - \eta_q + 2\pi(k_p - k_q))$$

Phase Unwrapping Algorithms

$$\boxed{E(\mathbf{k}) = \sum_{\{p,q\}\in\mathcal{E}} V_{pq}(k_p - k_q)}$$

- $E_{pq}(\cdot) = |\cdot|_{2\pi-\text{quantized}}$

  [Flynn, 97] (exact) $\rightarrow$ sequence of positive cycles on a graph

  [Costantini, 98] (exact) $\rightarrow$ min-cost flow on a graph $(|\mathcal{V}| = n, |\mathcal{E}| = 4n)$

- $E_{pq}(\cdot) = (\cdot)^2$

  [B & Leitao, 01] (exact) $\rightarrow$ sequence of positive cycles on a graph $(|\mathcal{V}| = n, |\mathcal{E}| = 4n)$
  [Frey et *al*., 01] (approx) $\rightarrow$ belief propagation on a 1st order MRF

- $E_{pq}(\cdot)$ convex

  [B & Valadao, 05,07,09] (exact) $\rightarrow$ Sequence of *K* min cuts ($KT(n, 6n)$)

- $E_{pq}(\cdot)$ non-convex

  [Ghiglia, 96] $\rightarrow$ LPN0 (continuous relaxation)

  [B & G. Valadao, 05, 07,09] $\rightarrow$ Sequence of min cuts ($KT(n, 6n)$)

PUMA (Phase Unwrapping MAx-flow)

[B & Valadao, 05,07,09]

$\boldsymbol{\phi}^{(0)} = \boldsymbol{\eta}$

while success == false

$\boxed{\boldsymbol{\delta}' := \arg \min_{\boldsymbol{\delta} \in \{0,1\}^{|\mathcal{V}|}} E(\boldsymbol{\phi} + 2\boldsymbol{\delta}\pi)}$

if $E(\boldsymbol{\phi} + 2\boldsymbol{\delta}'\pi) < E(\boldsymbol{\phi})$ **then** $\boldsymbol{\phi} := \boldsymbol{\phi} + 2\boldsymbol{\delta}'\pi$

else success = true

end

PUMA finds a sequence of steepest descent binary images

PUMA: Convex Priors

- A local minimum is a global minimum
- Takes at most *K* iterations
- E is submodular: $2V_{pq}(0) \leq V_{pq}(1) + V_{pq}(-1)$

  $\Rightarrow$ each binary optimization has the complexity of a min cut $T(n, 6n)$

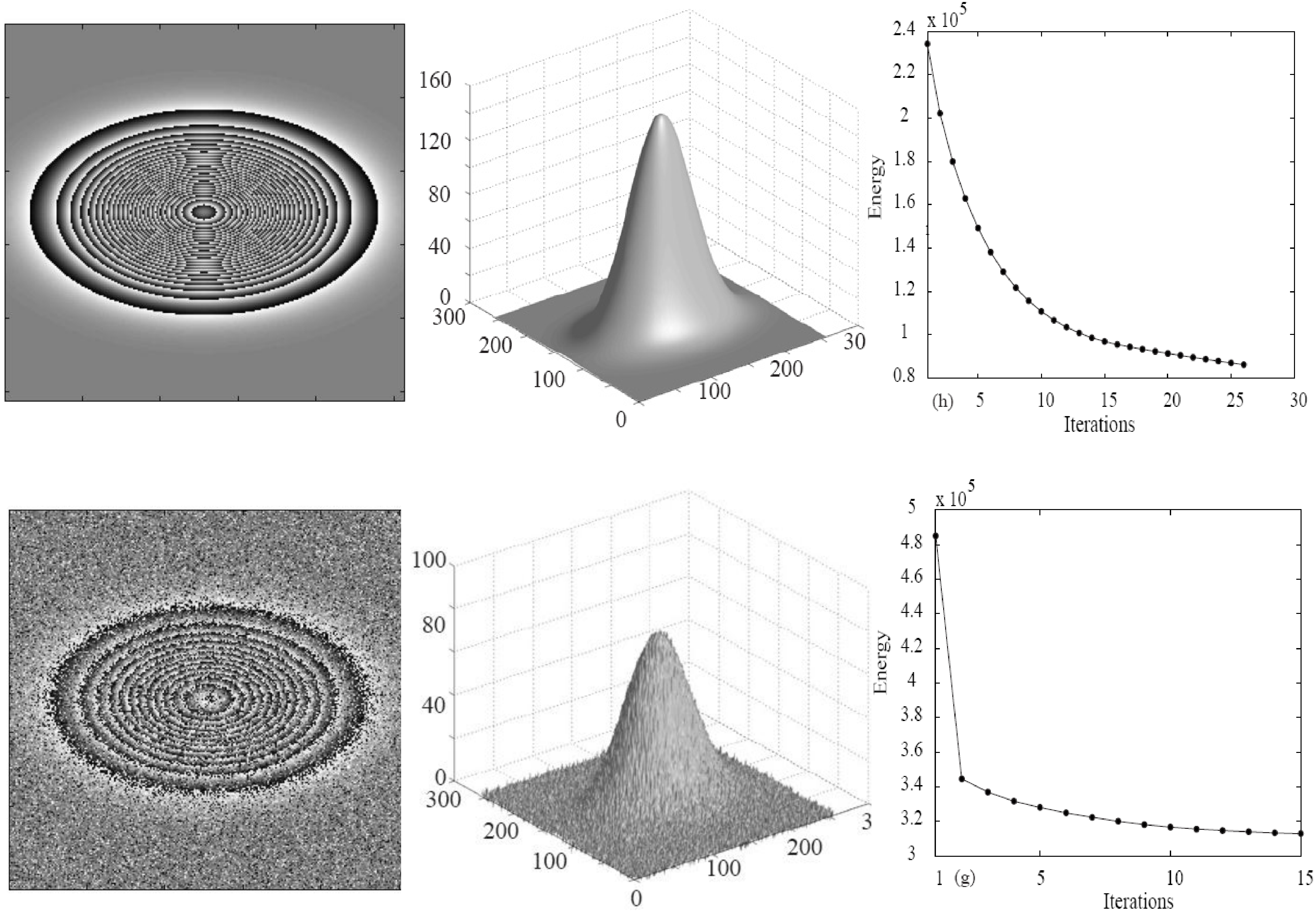
$$\boxed{E(\mathbf{k}) = \sum_{p \in \mathcal{V}} U_p(k_p) + \sum_{\{p,q\} \in \mathcal{E}} V_{pq}(k_p - k_q)}$$

- Related algorithms

  [Veksler, 99] (1-jump moves)
  [Murota, 03] (steepest descent algorithm for L-convex functions)
  [Ishikawa, 03] (MRFs with convex priors)
  [Kolmogorov & Shioura, 05,07], [Darbon, 05] (Include unary terms)
  [Ahuja, Hochbaum, Orlin, 03] (convex dual network flow problem)

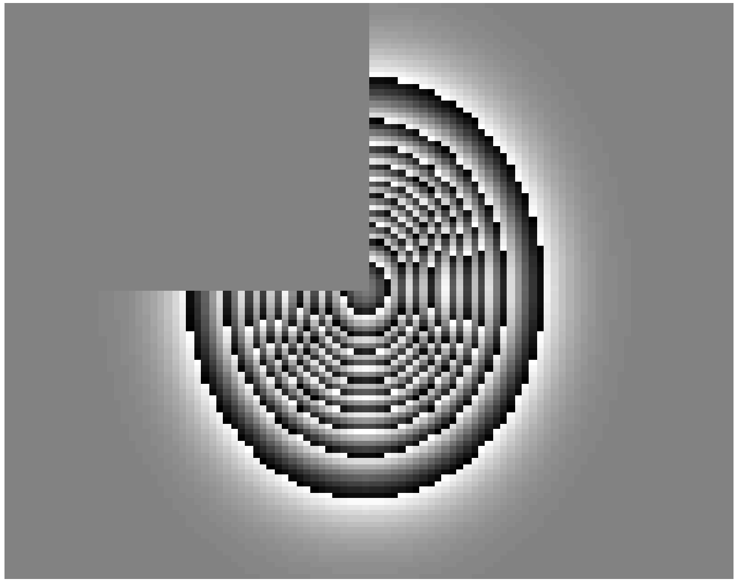
Results

$$E_{pq}(\cdot) = (\cdot)^2$$

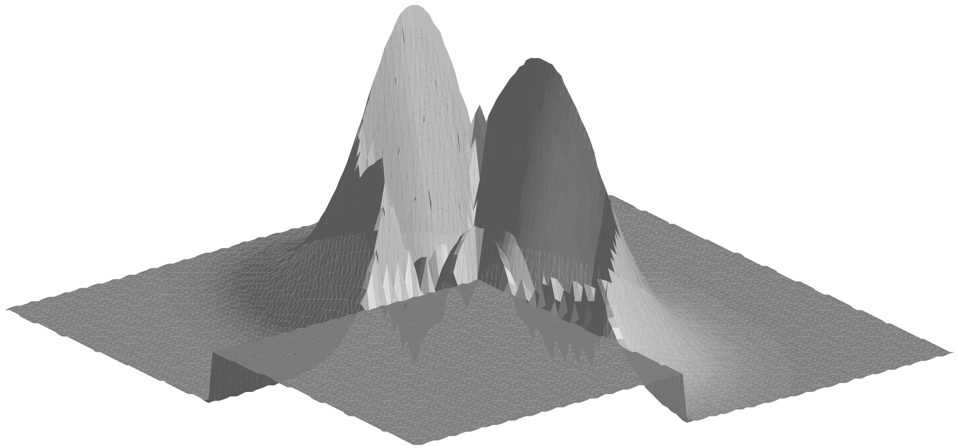


Results

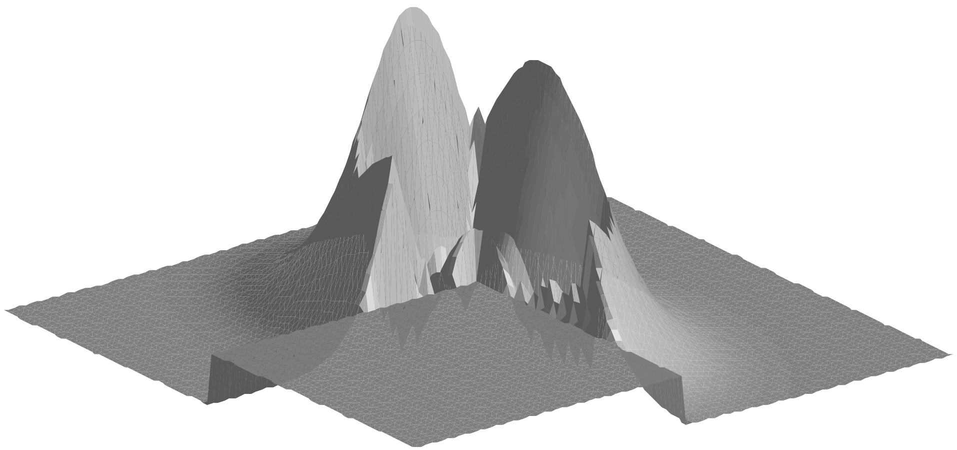
Convex priors do not preserve discontinuities



$$E_{pq}(x) = x^2$$

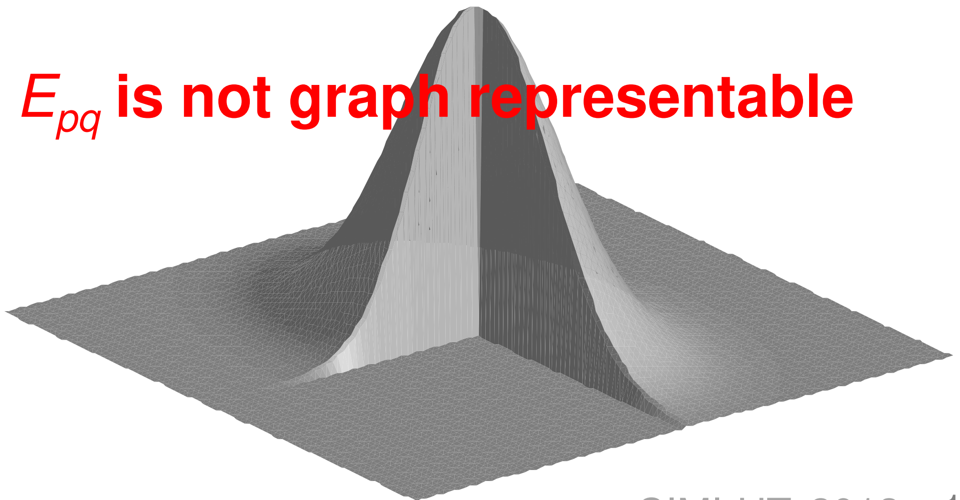


$$E_{pq}(x) = |x|$$



$$E_{pq}(x) = \begin{cases} x^2 & |x| \le \pi \\ \pi^2 |x/\pi|^{0.5} & |x| > \pi \end{cases}$$

E_{pq} is not graph representable

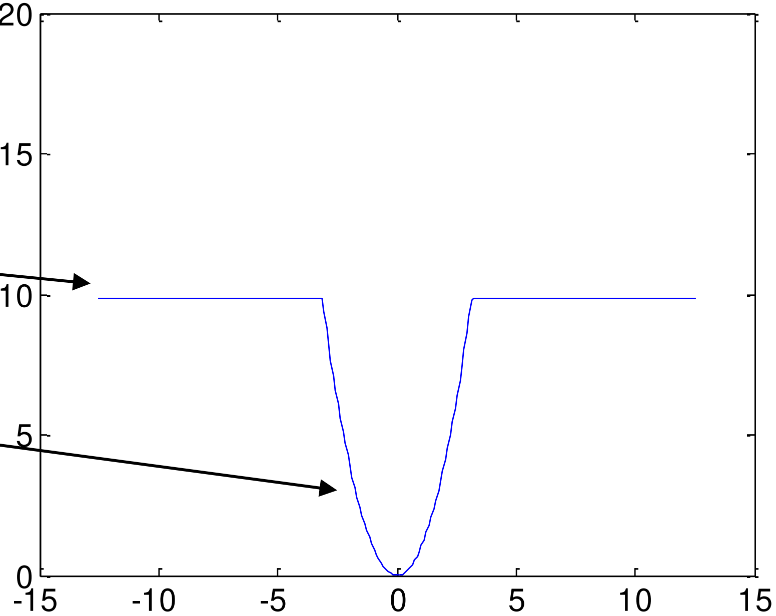


PUMA: Non-convex priors

Ex: $E(x) = \min(x^2, \pi^2)$

Models discontinuities

Models Gaussian noise



Shortcomings

- ❑ Local minima are no more global minima
- ❑ Energy contains nonsubmodular terms (NP-hard)

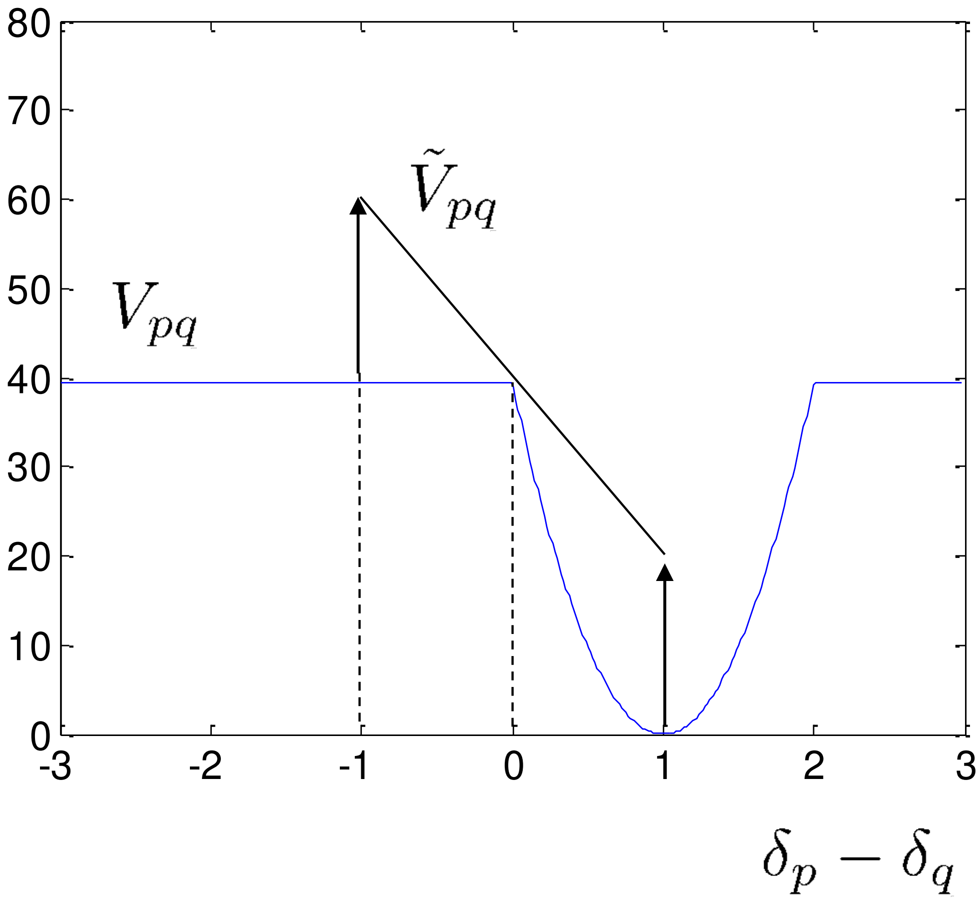
Proposed suboptimal solution: majorization minimization applied
To PUMA binary problem

Other suboptimal approaches

- ❑ Quadratic Pseudo Boolean Optimization (Probing [Boros et *al.*, 2006], Improving [Rother et *al.*, 2007])
- ❑ Sequencial Tree-Reweighted Message Passing (TRW-S) [Kolmogorov, 2006]
- ❑ Dual decomposition (DD) [Komodakis et al., 2011]

Majorizing Nonsubmodular Terms

Majorization Minimization (MM) [Lange & Fessler, 95]



$$\begin{cases} \tilde{V}(\mathbf{k}) = V(\mathbf{k}) \\ \tilde{V}(\mathbf{k}+\boldsymbol{\delta}) \geq V(\mathbf{k}+\boldsymbol{\delta}) \end{cases}$$

$$\boldsymbol{\delta}' = \arg\min_{\boldsymbol{\delta}} \tilde{V}(\mathbf{k}+\boldsymbol{\delta})$$

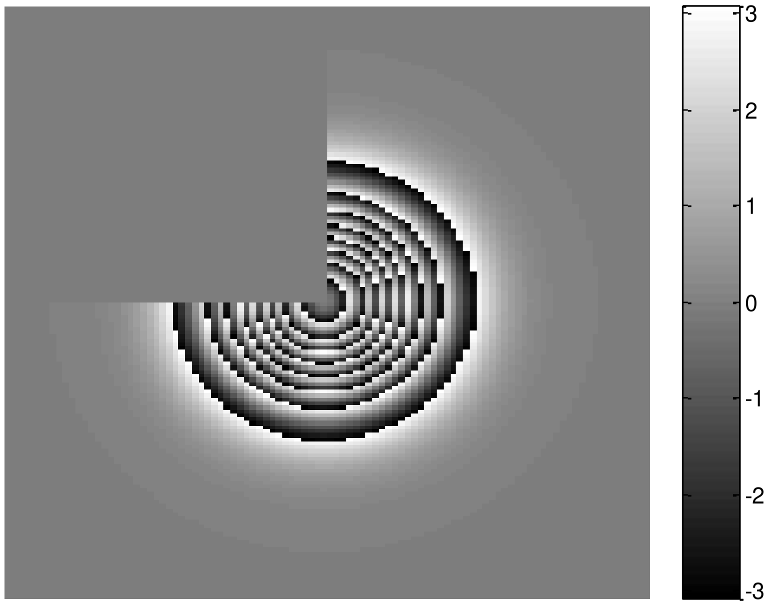
Non-increasing property

$$\boxed{V(\mathbf{k}+\boldsymbol{\delta}') \leq V(\mathbf{k})}$$

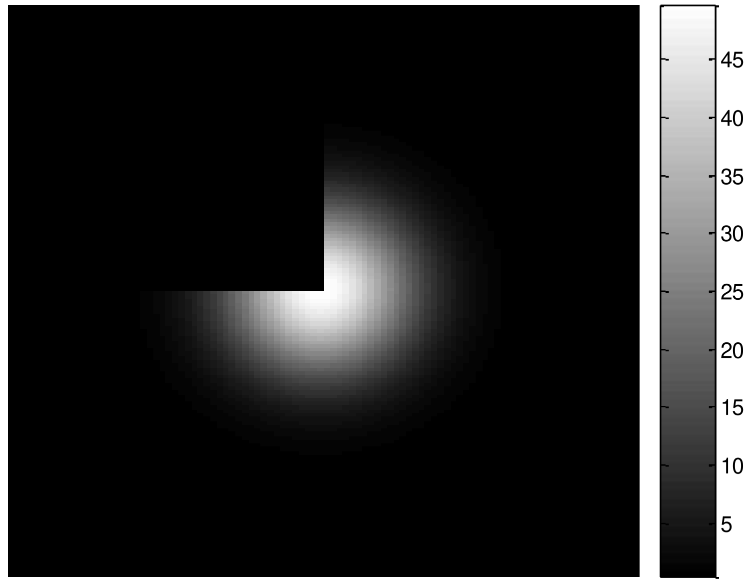
[Rother *et al.*, 05] $\rightarrow$ similar approach for alpha expansion moves

Results with PUMA (MM)

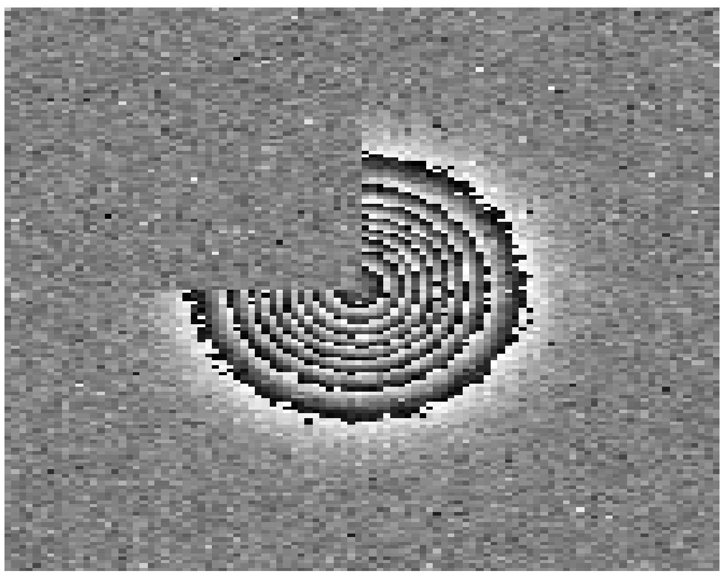
Interferogram η



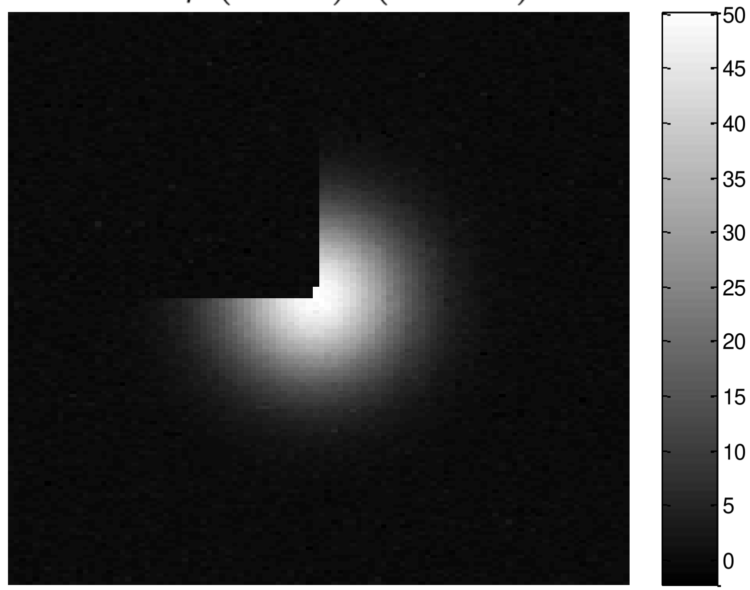
$\widehat{\phi}$(MM) (8 iter)



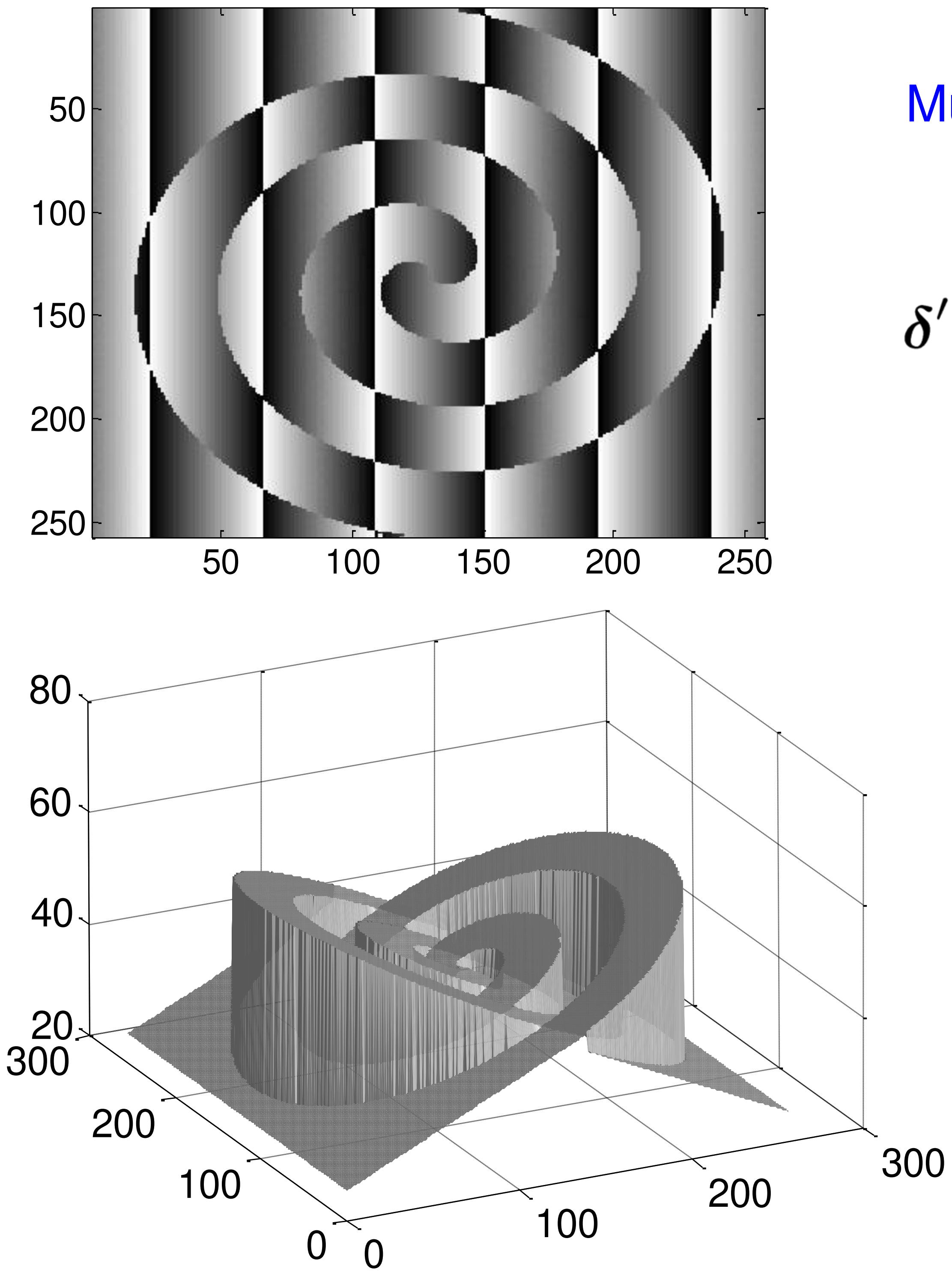
Interferogram η



$\widehat{\phi}$(MM) (8 iter)



Multi-jump version of PUMA (MM)
jumps d $\in$ [1 2 3 4]



$$\boldsymbol{\delta}' := \arg \min_{\boldsymbol{\delta} \in \{0,d\}^{|\mathcal{V}|}} E(\boldsymbol{\phi} + 2\boldsymbol{\delta}\pi)$$