# Managerial Incentive Problems: A Dynamic Perspective

Bengt Holmstrom RES, 1999

#### Motivation

- Tradeoff between allocating risk associated with incomplete observability and providing incentives for a proper supply of labor
- When time has effect on incentives (learning about ability), Fama proposed a theory which points out the importance of reputation. However, it's not correct in general.

#### Basic model

#### Output in each period:

$$y_t = \eta + a_t + \varepsilon_t,$$

Ability with mean m and precision h; labor input (efforts that you can put) a

$$w_t(y^{t-1}) = E[y_t | y^{t-1}] = E[\eta | y^{t-1}] + a_t(y^{t-1}).$$

Your wage in the current period is evaluated by the output of your last period

Your best choice of a is the solution of :

$$\max_{\{a_t(\cdot)\}} \sum_{t=1}^{\infty} \beta^{t-1} [Ew_t(y^{t-1}) - Eg(a_t(y^{t-1}))].$$

One important thing: "observing y will in equilibrium be equivalent to observing z", market expects you have invested a\* already

$$z_t \equiv \eta + \varepsilon_t = y_t - a_t^* (y^{t-1})$$

#### Basic model results

- Ability will finally be exactly known by market
- You can put more labor to increase your output, but it won't help in an equilibrium since market has expectations on your effort level. You are trapped in equilibrium.
- More uncertainties about ability, more returns to labor

## Stationary case, efficiency and reputation

Reputation works permanently only when ability cannot be fully resolved

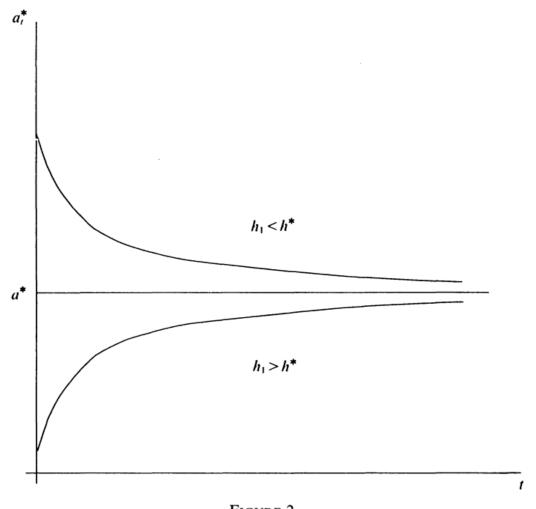
$$\eta_{t+1} = \eta_t + \delta_t$$

Efficiency means  $g'(\bar{a}) = 1$ 

Stationary labor supply:  $\frac{\beta(1-\mu^*)}{1-\mu^*\beta} = g'(a^*).$ 

Beta =1 → Fama's results, efficient. Otherwise not efficient

### Before a stationary state is reached



Young people overinvest in labor (卷,躺)

Noisy output, slow convergence, inefficiency

FIGURE 2

# More general conclusions and my interpretation

$$y_{t} = f(\eta_{t}) + a_{t} \qquad E[f'(\eta_{0} + \delta_{0} + \delta_{1})(f^{-1})'(f(\eta_{0} + \delta_{0}))] = E\left[\frac{f'(\eta_{0} + \delta_{0} + \delta_{1})}{f'(\eta_{0} + \delta_{0})}\right]$$

If f' is convex (or, return to ability is convex), oversupply of labor; f' is concave, undersupply of labor

Concave: return grows faster in low ability regime than in high ability regime, social media celebrity. Underinvestemt of labor (活少钱多)

Convex: return grows slower in low ability regime than in high ability regime, traditional stars. Overinvestment of labor