امتحانات شهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠5 العادية

عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (3 points)

In the complex plane referred to a direct orthonormal system $(O ; \vec{u}, \vec{v})$, consider the points E, M and M' of respective affixes i, z and z', where $z' = iz + 1 + i$.

1) Find the algebraic form of z' when $z = \sqrt{2}e^{i\frac{\pi}{4}}$.
2) Determine the modulus and an argument of z if $z' = 1 + \sqrt{3} + 2i$.
3) Determine the value of z, for which the points M and M' are confounded.
4) a- Show that $z' - i = i(z - i)$.
   b- Deduce that when M moves on the circle (C) of center E and radius 3, then the point M' moves on the same circle.

II - (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider :

- the plane (P) of equation $2x + y - 3z - 1 = 0$;
- the plane (Q) of equation $x + 4y + 2z + 1 = 0$;
- the line (d) defined by : $\begin{cases} x = 2t + 1 \\ y = -t - 1 \\ z = t \end{cases}$ (t is a real parameter).

1) Prove that the line (d) is included in the plane (P).
2) Find an equation of the plane (S) that is determined by the point O and the line (d).
3) Consider the point $E\left(0 ; -\frac{1}{2} ; -\frac{1}{2}\right)$.
   Prove that E is the orthogonal projection of the point O on the line (d).
4) a- Show that the planes (P) and (Q) are perpendicular.
   b- Let (D) be the line of intersection of (P) and (Q).
   Calculate the distance from E to (D).

III - (5 points)

A certain store sells only jackets, coats and shirts.
During a week, **120** customers were served in this store.
90 of those customers bought each one jacket, while the other **30** customers bought each one coat.
40% of those who bought jackets bought each also a shirt, while **20%** of those who bought coats bought each also a shirt.

A customer is chosen at random from those **120** customers and is interviewed.

1) Consider the following events :

   J : « the interviewed customer has bought a jacket ».

   C : « the interviewed customer has bought a coat ».

   S : « the interviewed customer has bought a shirt ».

   a- Verify that the probability of the event $S \cap J$ is equal to $\frac{3}{10}$.

   b- Calculate the following probabilities :

   $P(S \cap C)$, $P(S)$, $P(C/S)$ and $P(C/\overline{S})$.

2) The prices of the clothes in this store are as shown in the following table :

Kind	Jacket	Coat	Shirt
Price in LL	150 000	200 000	60 000

   Let X designate the random variable that is equal to the amount paid by a customer.

   a- Give the four possible values of X.

   b- Determine the probability distribution of X .

   c- Calculate the mean (expected value) E(X) .

   d- Estimate the amount of sales collected by the store during that week.

IV- **(8 points)**

Consider the function f that is defined, on $I =]\,1\,;+\infty\,[$, by $f(x) = x + 1 - \dfrac{3e^x}{e^x - e}$

and let (C) be its representative curve in an orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,)$.

1) a- Prove that the line of equation $x = 1$ is an asymptote to (C) .

   b- Calculate $\lim\limits_{x \to +\infty} f(x)$ and show that the line (d) of equation $y = x - 2$ is an asymptote to (C).

   c- Determine the relative position of (C) and (d) .

2) Prove that $f'(x) > 0$ for all values of x in I, and set up the table of variations of f.

3) Prove that the equation f(x) = 0 has a unique root α and verify that $2.6 < \alpha < 2.7$.

4) Draw the curve (C) .

5) Designate by (D) the region that is bounded by (C) , the line (d) and the lines of equations $x = 3$ and $x = 4$.

   Calculate $\displaystyle\int_3^4 \frac{e^x}{e^x - e}\,dx$ and deduce the area of the region (D).

6) a- Prove that f, on the interval I, has an inverse function g.

   b- Prove that the equation f(x) = g(x) has no roots .