

Illinois Space Society IREC Avionics Team
TARS Control System State-Space Model

March 6, 2021

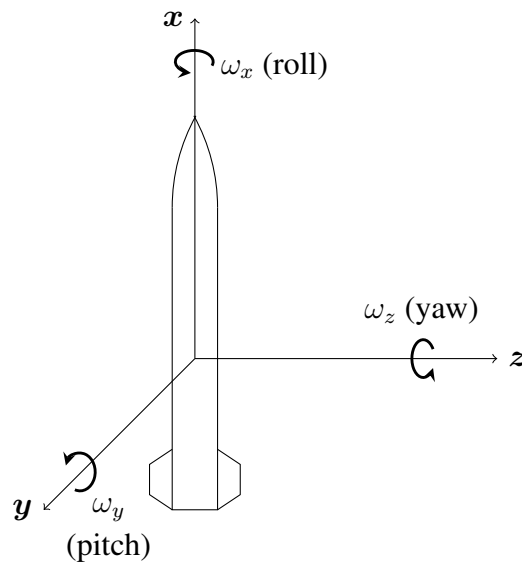
1. INTRODUCTION

This article serves as a means of documenting the progress made by the IREC Avionics team in the development of a state-space model describing the dynamics of a sounding rocket that is to compete in the Spaceport America Cup (formerly IREC) rocketry competition. The article is the culmination of the team's understanding in state-space control methods at the time of writing. We have limited expertise in the subject matter as we are a student team, therefore, care should be taken when referencing any material from this article.

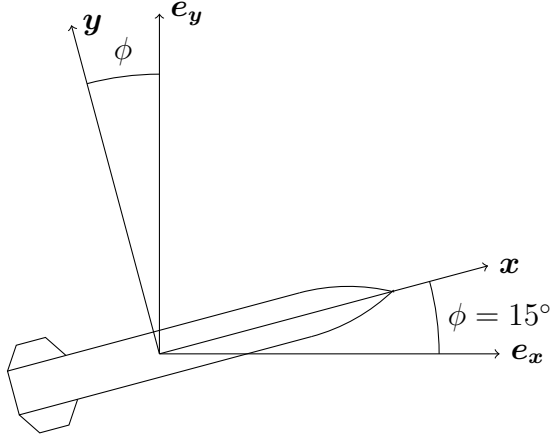
2. PRELIMINARIES

2.1 Rocket Frame of Reference

In order to maintain congruency with roll-pitch-yaw conventions commonly employed in aerospace dynamics, the following reference frame will be used to express the vehicle's motion. The origin of this frame shall be the center of mass (COM) of the vehicle, with the x axis aligned to the axis of the rocket.



2.2 Tait-Brian Angles (aka Euler Angles)



2.3 Quaternions

$$q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}, \quad \dot{q} = \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

Euler angle to quaternion conversion:

$$q(\phi, \theta, \psi) = \begin{bmatrix} q_0 = \cos(\phi/2) \cos(\theta/2) \cos(\psi/2) + \sin(\phi/2) \sin(\theta/2) \sin(\psi/2) \\ q_1 = \sin(\phi/2) \cos(\theta/2) \cos(\psi/2) - \cos(\phi/2) \sin(\theta/2) \sin(\psi/2) \\ q_2 = \cos(\phi/2) \sin(\theta/2) \cos(\psi/2) + \sin(\phi/2) \cos(\theta/2) \sin(\psi/2) \\ q_3 = \cos(\phi/2) \cos(\theta/2) \sin(\psi/2) - \sin(\phi/2) \sin(\theta/2) \cos(\psi/2) \end{bmatrix}$$

Quaternion to euler angle conversion:

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arctan\left(\frac{2(q_0 q_1 + q_2 q_3)}{1 - 2(q_1^2 + q_2^2)}\right) \\ \arcsin(2(q_0 q_2 - q_3 q_1)) \\ \arctan\left(\frac{2(q_0 q_3 + q_1 q_2)}{1 - 2(q_2^2 + q_3^2)}\right) \end{bmatrix}$$

$$\dot{q} = \frac{1}{2} \vec{\omega} \cdot q = \frac{1}{2} \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$