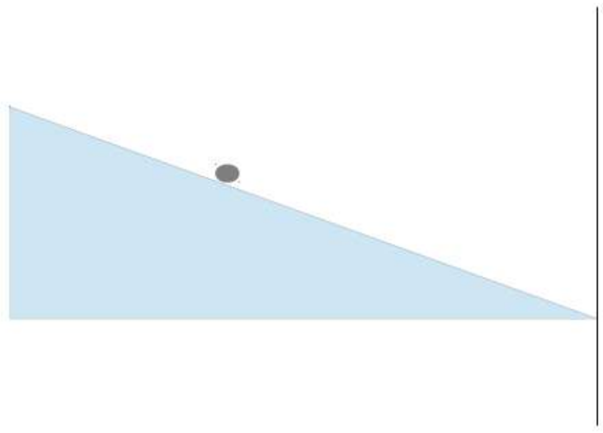
3. A particle with mass m slides from the top of a frictionless hemisphere with radius R. Find the place where the particle loses contact with the surface of the ball. What is its speed at this instant?

4. Solve the above problem if the initial speed of the particle (in the direction tangential to the sphere) is v_0.

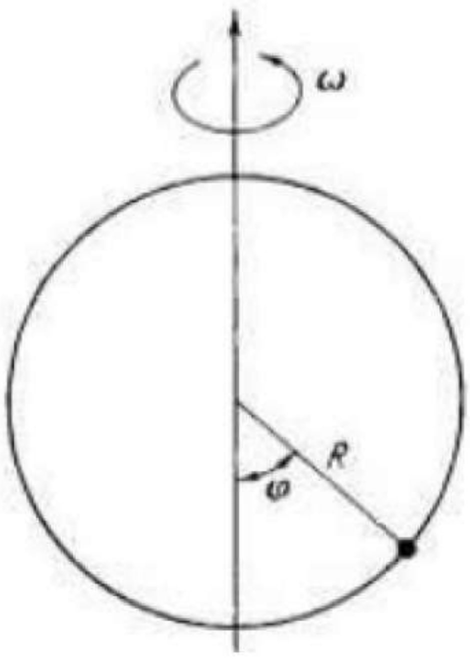
5. A box is filled with a liquid and is placed on a horizontal surface. Find the angle that the surface of the liquid forms with the horizontal if we pull the box with acceleration a.

6. A plane, inclined at an angle α to the horizontal, rotates with constant angular speed ω about a vertical axis (see the figure). Where on the inclined plane should we place a particle, so that it remains at rest? The plane is frictionless.



7. A small bead can slide without friction on a circular hoop that is in a vertical plane and has a radius R. Find points on the hoop, such that if we place the bead there it will remain at rest.

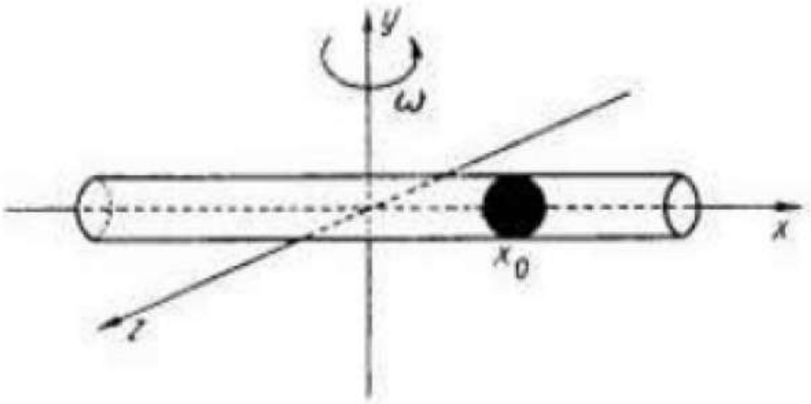
Acceleration due to gravity is g.



8. Will the oscillation plane of a Foucault pendulum, that is placed on the equator, rotate?

9. A particle with mass m is inside a pipe that rotates with constant angular velocity ω about an axis perpendicular to the pipe. The kinetic coefficient of friction is equal to μ_k. Write down (do not solve!) the equation of motion for this particle in the non-inertial frame of reference of the rotating pipe.

   There is no gravitational force in this problem.



11. A uniform cylinder of mass m, radius R and height h is floating vertically in a liquid, so that it is half-immersed in the liquid. Find the density of the liquid and minimum work needed to pull the cylinder completely above the liquid's surface.

12. Find work done by the force $\mathbf{F}_1(x, y) = -x\hat{n}_x - y\hat{n}_y$ and by the force $\mathbf{F}_2(x, y) = (2xy + y)\hat{n}_x + (x^2 + 1)\hat{n}_y$ if a particle is being moved from $(-1, 0)$ to $(0, 1)$ along

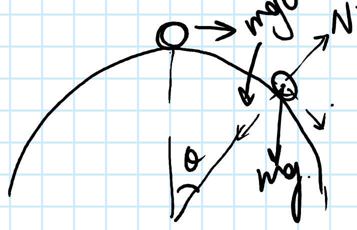
    (a) the straight line connecting these points,

    (b) the (shorter) arc of the circle $x^2 + y^2 = 1$,

    (c) the axes of the Cartesian coordinate system: first from $(-1, 0)$ to $(0, 0)$ along the x axis, then from $(0, 0)$ to $(0, 1)$ along the y axis.

    Notes: (1) there will be no time (and no need) for all details, so, e.g, for integrals like $\int \cos^2 t\, dt$ just quote the value of the integral; (2) Please pay attention to the fact that for $\mathbf{F}_1$ all paths you considered have yielded the same result.

④



$\Delta E_p = -R(1-\cos\theta)mg$.

$\Delta E_k = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

$0 = E = \Delta E_p + \Delta E_k = -R(1-\cos\theta)mg + \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \Rightarrow v^2 = v_0^2 + 2R(1-\cos\theta)g$

$N = mg\cos\theta - a_n m > 0$.

when $N=0$, the ball fly off the surface.

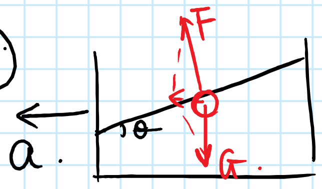
$a_n = \frac{v^2}{R} = g\cos\theta_0$ $\quad v_0^2 + 2R(1-\cos\theta_0)g = gR\cos\theta_0$

$3Rg\cos\theta_0 = 2Rg + v_0^2$.

$\cos\theta_0 = \frac{2}{3} + \frac{v_0^2}{3Rg}$.

If $\frac{2}{3} + \frac{v_0^2}{3Rg} > 0$, i.e. $v_0^2 > Rg$, the ball will fly off immediately.

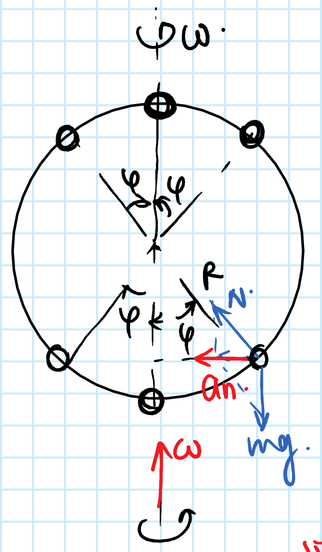
⑤



For water drop on surface, surrounding water can only provide force perpendicular to the surface. ⇔ a ball on frictionless surface.

$\sum F = G\tan\theta = a\cdot m$. $\quad a = g\tan\theta$.

⑦



remain at rest (w.r.t. the circular hoop).

w.r.t lab FOR: circular motion.

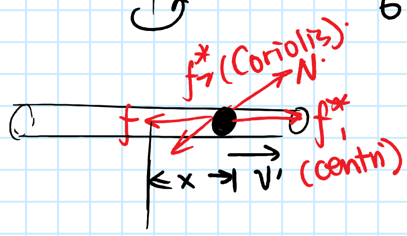
$a_n = \omega^2 R\sin\varphi$.

$\sum F = mg\tan\varphi = a_n m = \omega^2 Rm\sin\varphi$.

$\cos\varphi = \frac{\omega^2 R}{g}$

6 Positions in total.

⑨



fictitious forces:

d'Alembert force. $-m\vec{a_0} = 0$.

Euler force $\quad -m\frac{d\vec{\omega}}{dt}\times\vec{r'} = 0$

Coriolis force $\quad -2m(\vec{\omega}\times\vec{v'})$ $\quad f_3^* = 2m\omega\dot{x}$

centrifugal force $\quad -m\vec{\omega}\times(\vec{\omega}\times\vec{r'})$

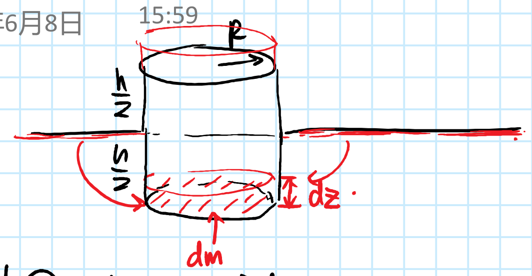
$f_1^* = m\omega^2 x$.

$N = f_3^* = 2m\omega\dot{x}$

$f = \mu_k N = 2\mu_k m\omega\dot{x}$

$$\boxed{\sum F = f_1^* - f = m\omega^2 x - 2\mu_k m\omega\dot{x} = \ddot{x}m}$$

(1)



· liquid density ρ_l.

$\frac{h}{2}\cdot \pi R^2 \rho_l = m$

$\rho_l = \frac{2m}{\pi h R^2}$

Sol ①. $dm_F \cdot \pi R^2 dz\, \rho_l$

$dW_l = dm\, z\, g$

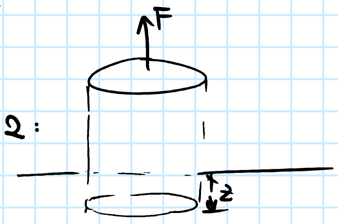
$W_l = \int_0^{\frac{h}{2}} dW_l = \int_{\frac{h}{2}}^{0} \pi R^2 \rho_l z g\, dz$

$= -\pi R^2 \cdot \frac{2m}{\pi h R^2} g \cdot \frac{h^2}{8} = -\frac{1}{4} mgh$

$W_m = mg\cdot \frac{1}{2}h$

$W_{total} = W_l + W_m = \frac{1}{4} mgh$

Sol 2:



$\frac{2m}{\cancel{\pi R^2} h} \cdot \cancel{\pi R^2}$

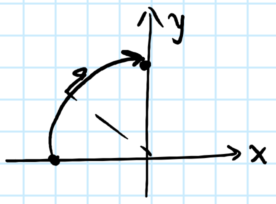
$F = mg - \rho_l \pi R^2 z g$

$W = \int_{\frac{h}{2}}^{0} F(-dz) = \int_{\frac{h}{2}}^{0} -\left(mg - \frac{2mg}{h} z\right) dz$

$= mg \cdot \frac{h}{2} + \frac{2mg}{h} \cdot \left[0 - \left(\frac{h}{2}\right)^2\right]$

$= \frac{1}{4} mgh$

(2) b). $x^2+y^2=1 \Rightarrow \begin{cases} x=\cos\theta \\ y=\sin\theta \end{cases}$



$\theta: \pi \to \frac{\pi}{2}$

$\vec{F}_1(x,y) = -\cos\theta \hat{n}_x - \sin\theta \hat{n}_y$

$\vec{F}_2(x,y) = (2\cos\theta\sin\theta + \sin\theta)\hat{n}_x + (\cos^2\theta + 1)\hat{n}_y$

$d\vec{l} = dx\,\hat{n}_x + dy\,\hat{n}_y = -\sin\theta\, d\theta\, \hat{n}_x + \cos\theta\, d\theta\, \hat{n}_y$

$W_1 = \int_c \vec{F}_1 \cdot d\vec{l} = \int_{\pi}^{\frac{\pi}{2}} (-\cos\theta \hat{n}_x - \sin\theta \hat{n}_y)\cdot(-\sin\theta\, d\theta\, \hat{n}_x + \cos\theta\, d\theta\, \hat{n}_y)$

$= \int_{\pi}^{\frac{\pi}{2}} (\sin\theta\cos\theta - \sin\theta\cos\theta)\, d\theta = 0$

$W_2 = \int_c \vec{F}_2 \cdot d\vec{l} = \int_{\pi}^{\frac{\pi}{2}} \left((2\cos\theta\sin\theta + \sin\theta)\hat{n}_x + (\cos^2\theta+1)\hat{n}_y\right)\cdot(-\sin\theta\, d\theta\, \hat{n}_x + \cos\theta\, d\theta\, \hat{n}_y)$

$= \int_{\pi}^{\frac{\pi}{2}} \left(-2\cos\theta\sin^2\theta - \sin^2\theta + \cos^3\theta + \cos\theta\right) d\theta$

$= 1 + \frac{1}{4}\pi$