Selling dynamics modeling using the waiting time distribution

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If we have divided items into different clusters. For each of such clusters, items within it have a similar selling trend. Let us consider the selling dynamics of some item within a given cluster.

1. General theory

Denote x as the number of pieces of the item that are sold. There are N pieces of the item in total.

Denote t as the time. Our aim is to find the time T when $x = N$. Initially, at $t = 0$, there is $x = 0$ pieces of the item sold.

Denote $\eta(x,t)$ as the probability that there are x pieces of the item **just** sold at t. Then we can write

$$\eta(x,t) = \sum_{x'=0}^{x} \sum_{t'=0}^{t} \eta(x',t')\psi(x,t;x',t'), \tag{1}$$

where $\psi(x,t;x',t')$ is the transition probability that the number of the pieces sold is increased from x' to x as time is changed from t' to t.

Denote $W(x,t)$ as the probability that there have been x pieces sold at time t. There is a connection between W and η that

$$W(x,t) = \sum_{t'=0}^{t} \eta(x,t')\Psi(x,t;t') + \delta_{t,0}\delta_{x,0}, \tag{2}$$

where $\Psi(x,t;t')$ is the probability that once there are x pieces sold at t', no piece is sold from t' to t. And finally, we can express Ψ in terms of the waiting time distribution $w(x,t;t')$ as

$$\Psi(x,t;t') = 1 - \sum_{t''=t'}^{t} w(x,t'';t'), \tag{3}$$

where $w(x,t'';t')$ is the waiting time probability that given x pieces has been sold at t', no more piece is sold until t''.

Equations (1)-(3) constitute a highly nonstationary nonlinear discrete Fokker-Planck equation of $W(x,t)$, and we convert the original problem into the so-called "first passage" problem of Brownian motion, i.e., we are going to find the first time that a Brownian particle hits the wall at $x = N$, starting from $x = 0$ at $t = 0$.

2. A simplified result (not necessarily holds in our case).

If the process is stationary in the sense that $\psi(x,t;x',t') = \psi(x-x',t-t')$ and $\Psi(x,t;t') = \Psi(t-t')$. Then we can see $w(x,t;t') = w(t-t') = \sum_{(x-x')=0}^{N} \psi(x-x',t-t')$. In this simple case, the problem is solvable, and we can find that if w is not too skewed, then in the continuous limit the process is just described by

$$\frac{\partial W(x,t)}{\partial t} = -v\frac{\partial W}{\partial x} + D\frac{\partial^2 W}{\partial x^2}, \tag{4}$$

where v and D are drift and diffusion coefficients, respectively.

3. Two important probabilities in our case.

If $\psi(x,t;x',t') = \psi(x-x',t-t')$ and $w(x,t;t')$ can possibly be further simplied (need numerical evidence to support), then we will have a simplified model. It might be worth taking a look at the features of these two probabilities, especially the (non-)staionarity, as well as the (non-)normality.