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1.(a) $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$

$\Rightarrow x \equiv 7 \pmod{15}$ [since $7 \equiv 1 \pmod 3$, $7 \equiv 2 \pmod 5$]

Solve, $7 + 15k \equiv 3 \pmod 7$

$\Rightarrow 15k \equiv -4 \equiv 3 \pmod 7$

[since $15 \equiv 1 \pmod 7$],

$k \equiv 3$.

Thus, $x = 7 + 15.3$

$= 52$.

Modulus $= 3.5.7 = 105$

$\therefore x \equiv 52 \pmod{105}$ (Ans)

(b) Start with

$x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$

put $x = 5 + 11t$.

$11t \equiv 9 \pmod{29}$. The inverse of 11 mod 29 is 8, so, $8 \cdot 9 \equiv 14$. Hence $x = 5 + 11 \cdot 14 = 159$ and

$x \equiv 159 \pmod{319}$ $[319 = 11 \cdot 29]$

Now, combine with, $x \equiv 15 \pmod{31}$:

~~159 + 319u~~

$159 + 319u \equiv 15 \pmod{31}$

$\Rightarrow 319u \equiv -144 \equiv 11 \pmod{31}$

$(319 \equiv 9 \pmod{31})$ and inverse of 9 mod 31 is 7, so, $u \equiv 7 \cdot 11 \equiv 15$

Thus, $x = 159 + 319 \cdot 15 = 4944$

Modulus $= 319 \cdot 31 = 9889$

$x \equiv 4944 \pmod{9889}$ (Ans:)

(c) $x \equiv 5 \pmod{6}$, $x \equiv 4 \pmod{11}$

put, $x = 5 + 6t$, then, $6t \equiv -1 \equiv 10 \pmod{11}$

inverse of 6 mod 11 is 2, So,

$t \equiv 2.10 \equiv 9$ Hence, $x = 5 + 6.9 = 59$

and $x \equiv 59 \pmod{66}$

Combine with, $x \equiv 3 \pmod{17}$: $59 + 66u \equiv 3 \pmod{17}$

$\Rightarrow 66u \equiv -56 \equiv 12 \pmod{17}$

$66 \equiv 15 \pmod{17}$, inverse of 15 is 8,

So, $u \equiv 8.12 \equiv 11$.

Thus, $x = 59 + 66.11 = 785$

modulus $= 66.17 = 1122$

$x \equiv 785 \pmod{1122}$

(Ans.)