Assignment - 02

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Q.1: Is the set of odd numbers with the binary addition i.e. <O,+> an abelian group?

Answer: False.

Explanation: The set of odd numbers O is not under addition. For example, $a = 1$, $b = 3$ both odd numbers. Then $a+b = 4$, which is even. Since, closure fails $\langle O, + \rangle$ is not even a group. So, it cannot be an abelian group.

Q.2: Let, G be a group of order pq. where p and q are distinct primes. prove that G is abelian.

Answer: False.

Explanation: The symmetric group S_3, which has order $2 \times 3 = 6$. S_3 is not abelian as it containing non-commutativity elements $(1\,2)$ and $(1\,2\,3)$ permutations.

Q.3: Prove that, if G is a group of order p^2, where p is prime, then G is abelian if and only if it has $p+1$ subgroups of order p.

Answer: False.

Explanation: Every group of order p^2 is abelian. However the number of subgroups of order p depends on the structure: If G is cyclic, it has exactly one subgroup of

order p: if G is elementary abelian it has $p+1$ subgroups of order p.
"If and only if" conditions fails because a cyclic group of order p^2 is abelian but doesn't have $p+1$ subgroups of order p.

Q:4: Let G be a finite group and H be a proper subgroup of G. prove that the union of all conjugates of H cannot be equal to G.

Answer: True.

Explanation: This is a standard result in group theory. The union of all conjugates of a proper subgroup H

is a proper subset of G. This can be shown using the formula for the number of conjugates and the fact that the intersection of the conjugates has inden at least 2, leading to a size contradiction if the union were equal to G.

Q.5: Let G be a group and N be a normal subgroup of G. If G/N is cyclic and N is cyclic, prove that G is abelian.

Answer: False.

Explanation: Let, N be the alternating subgroup A_3, which is cyclic of order

3. Then G/N is cyclic order 2. However S_3 is non-abelian, showing that the conditions do not gurantee that G is abelian.

Q.6: Prove that, in any group G, the set of elements of finite order forms a subgroup of G.

Answer: False

Explanation: In the finite dehedral group D_∞, the elements of oreder are the reflections, but the product of two distinct reflections is a translation, which has finite order. Thus the set of elements of finite order is not closed under multiplication is not a subgroup.

Q.7: Let G be a finite group and p be the smallest prime dividing $|G|$. prove that any subgroup of ~~it~~ index p in G is normal.

Answer: True.

Explanation: If H is a subgroup of index p in G and p is the smallest prime dividing $|G|$ then H is normal. This can be proven using the action of G on the closets of H and considering the homomorphism into the symmetric group S_p.

Q.8: Let G be a group and $a, b \in G$. prove that, $a^4 = b^2$ and $ab = ba$, then $(ab)^6 = e$.

Answer: False.

Explanation: Let, $G = \langle a \rangle$, where a is order 4. Set, $b = e$ (the identity). Then $a^4 = e = b^2$ and $ab = ba$. But, $(ab)^6 = a^6 . a^2 = e . a^2 = a^2 \neq e$. Thus, the statements fails.