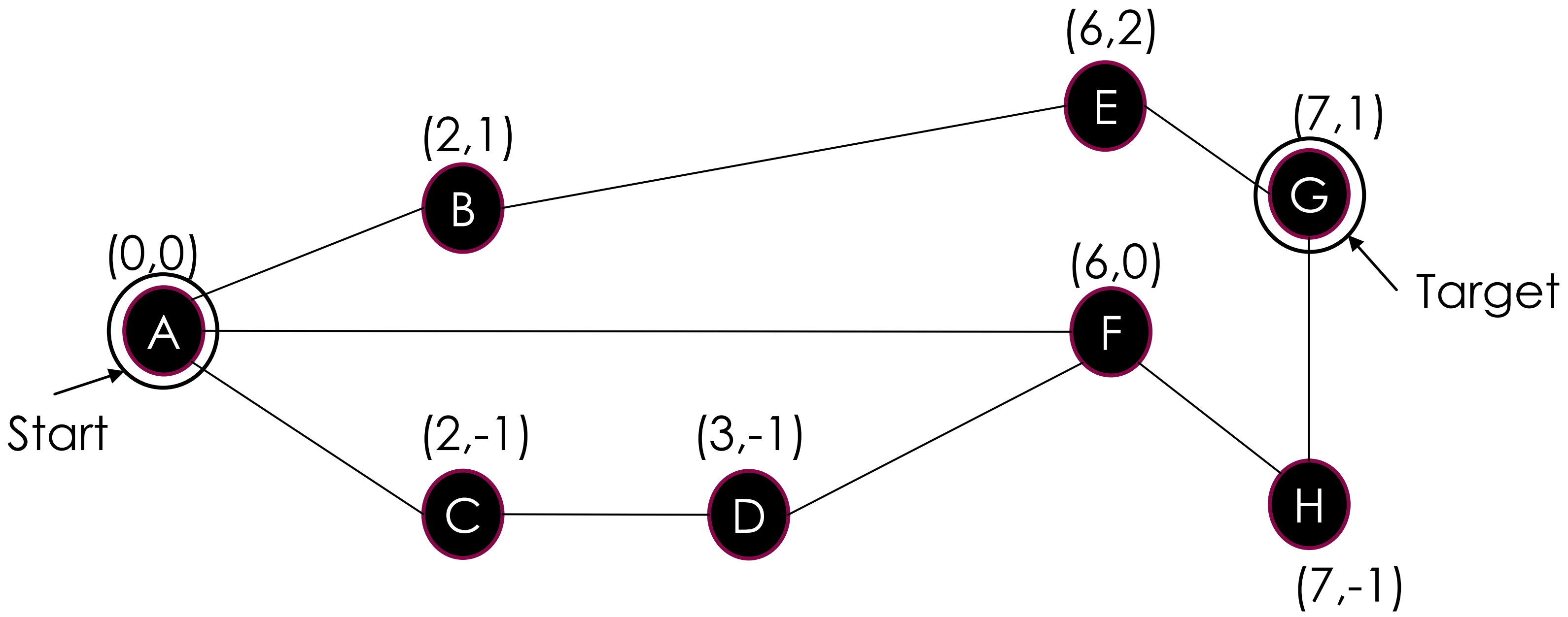
2020 AI Review

Question 1

Use A* algorithm and design a reasonable heuristic function ***h(x)*** to find a path from the Start to the Target in the following graph.

1) Assume your A* cost function is ***f(x) = g(x) + h(x)***. Draw the search tree and give the value of ***f(x)*** for each node.

2) List the Priority Queue and Explored Set for each step.



A* Algorithm

```
function A-STAR-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE : /* Cost f(n) = g(n

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

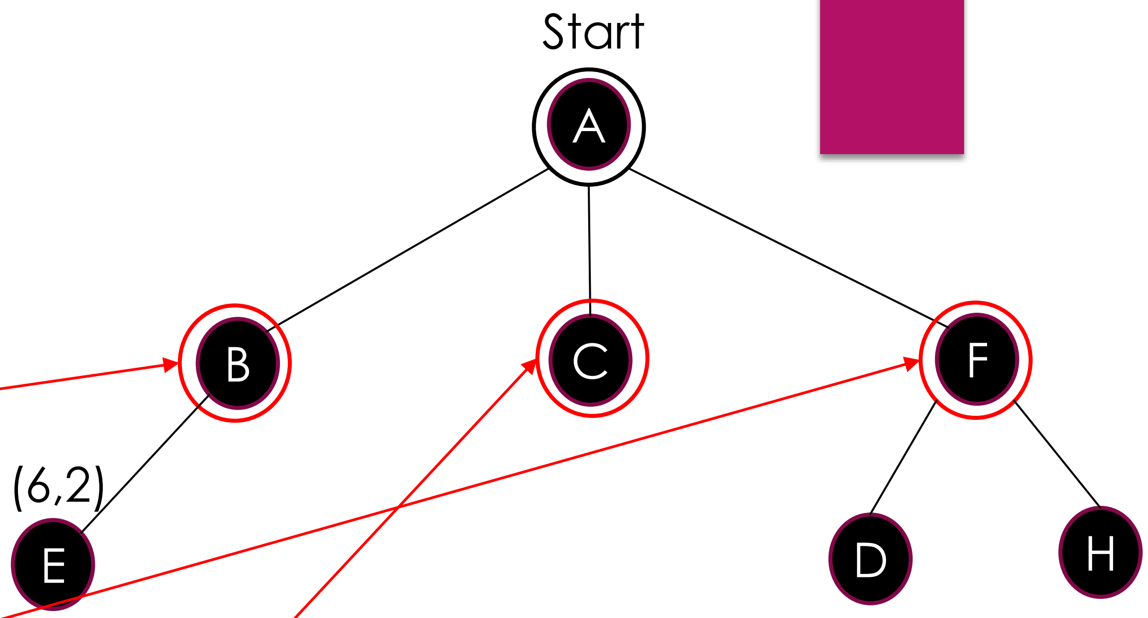
Question 1 Analysis

Initial
Priority Queue: A(7.071)
Explored Set: {}

Step1
Priority Queue: B(2.236+5),C(2.236+5.383),F(6+1.414)
Explored Set: {A(7.071)}

Step2
Priority Queue: F(6+1.414), C(2.236+5.383), E(6.359+1.414)
Explored Set: {A(7.071), B(2.236+5)}

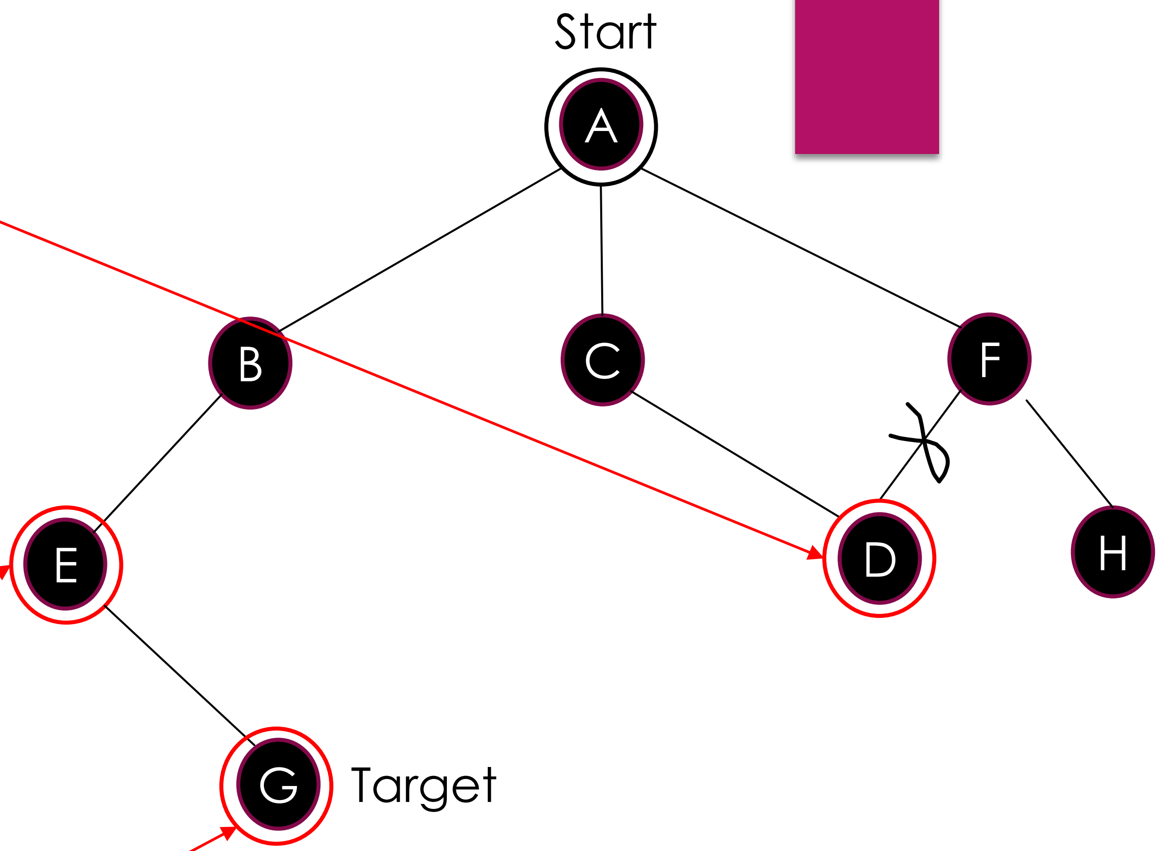
Step3
Priority Queue: C(2.236+5.383),
E(6.359+1.414),D(9.162+4.472),H(7.414+2)
Explored Set: {A(7.071), B(2.236+5), F(6+1.414)}



Step4
Priority Queue:
D(3.236+4.472),E(6.359+1.414),~~D(9.162+4.472)~~,H(7.414+2)
Explored Set: {A(7.071), B(2.236+5), F(6+1.414)
C(2.236+5.383)}

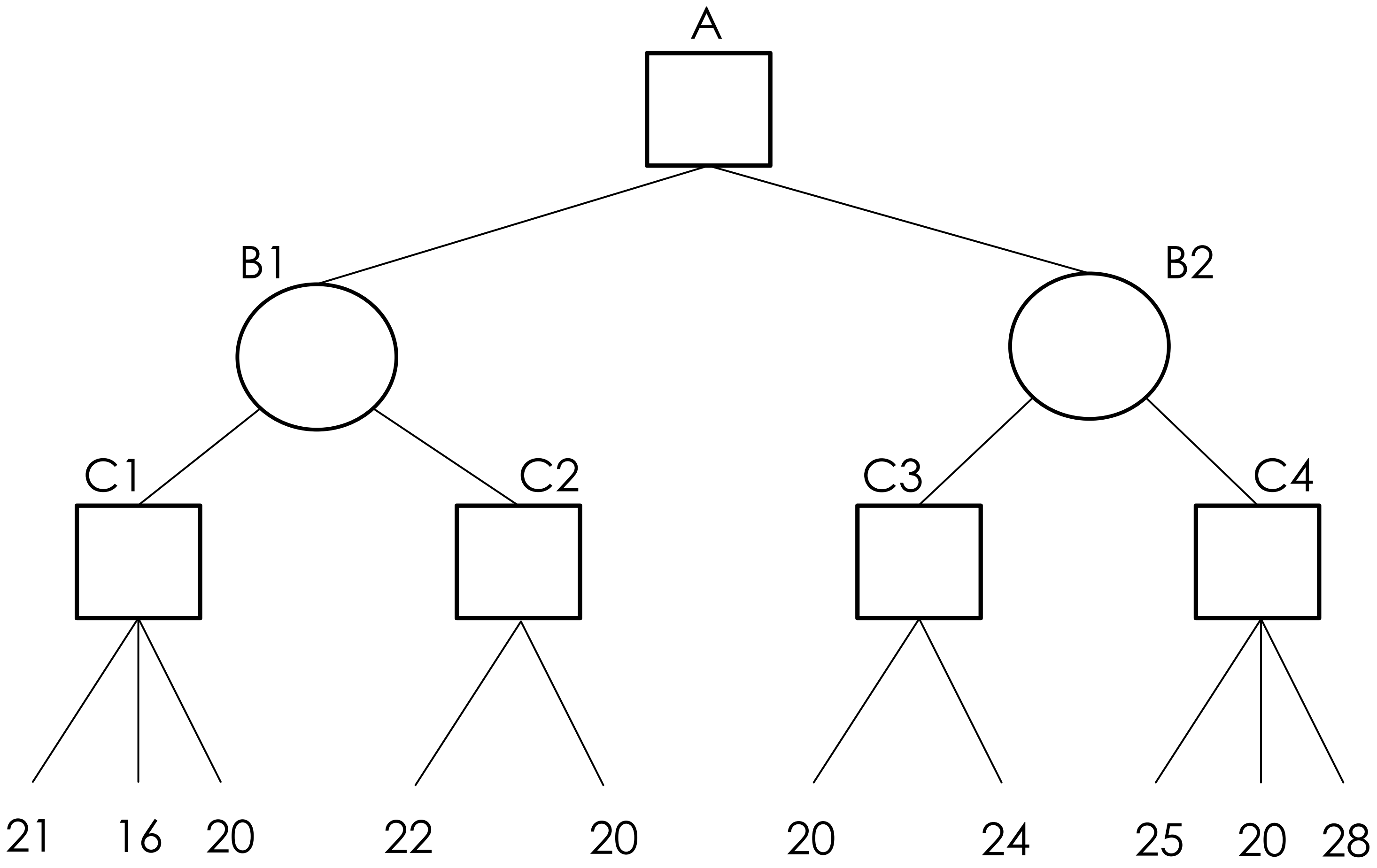
Step5
Priority Queue: E(6.359+1.414),H(7.414+2)
Explored Set: {A(7.071), B(2.236+5), F(6+1.414)
C(2.236+5.383), D(3.236+4.472)}

Step6
Priority Queue: G(7.773),H(7.414+2)
Explored Set: {A(7.071), B(2.236+5), F(6+1.414)
C(2.236+5.383), D(3.236+4.472), E(6.359+1.414)}



Question 2

For the following game tree, in which the numbers at the leaf nodes indicate their utility values, apply Alpha-Beta pruning to prune unnecessary branches. Please directly label the nodes with (Alpha, Beta) values, and put a "X" on branches that should be pruned.



Alpha-Beta Pruning

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE($state, -\infty, +\infty$)
  return the *action* in ACTIONS(*state*) with value v

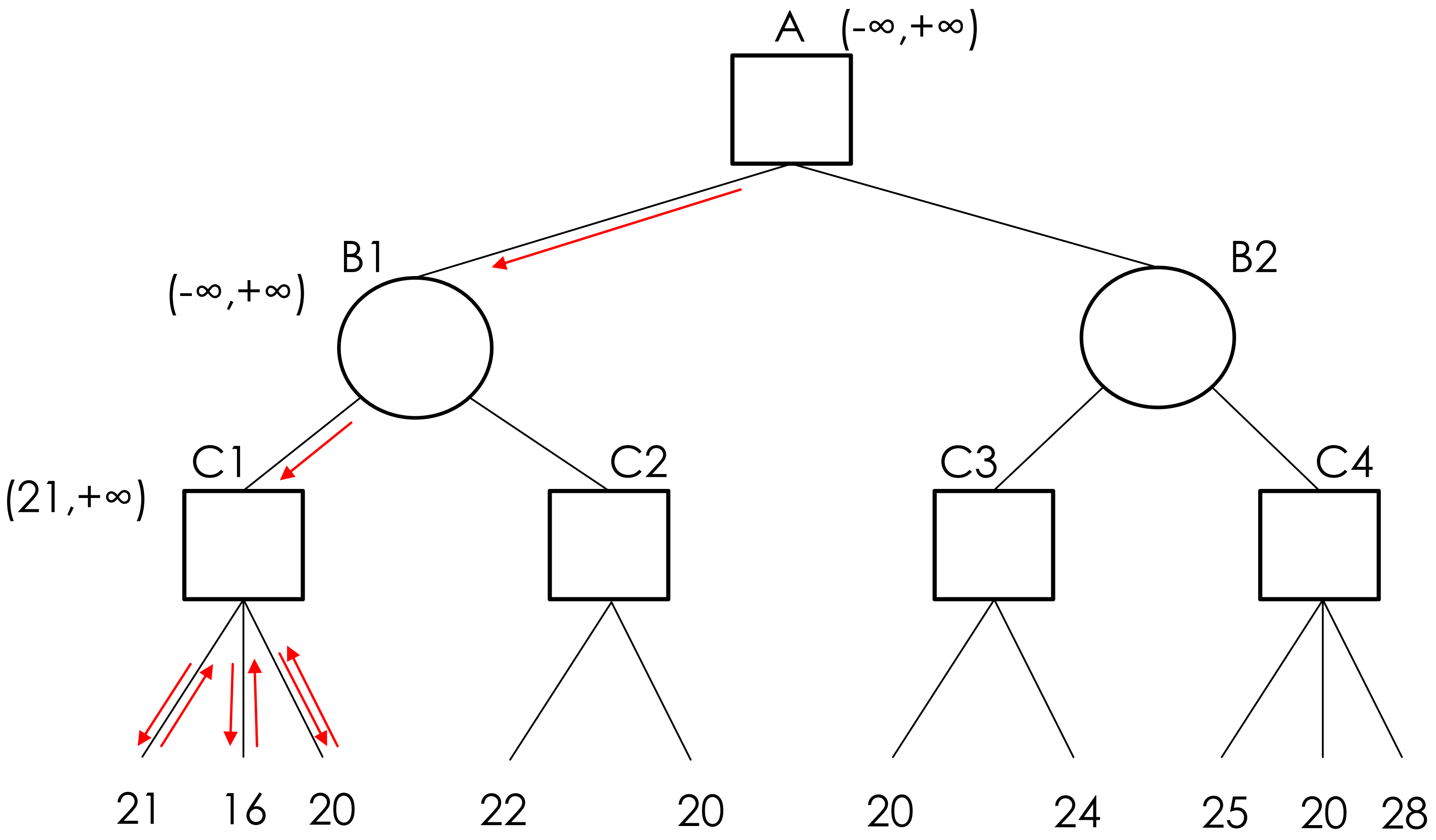
---

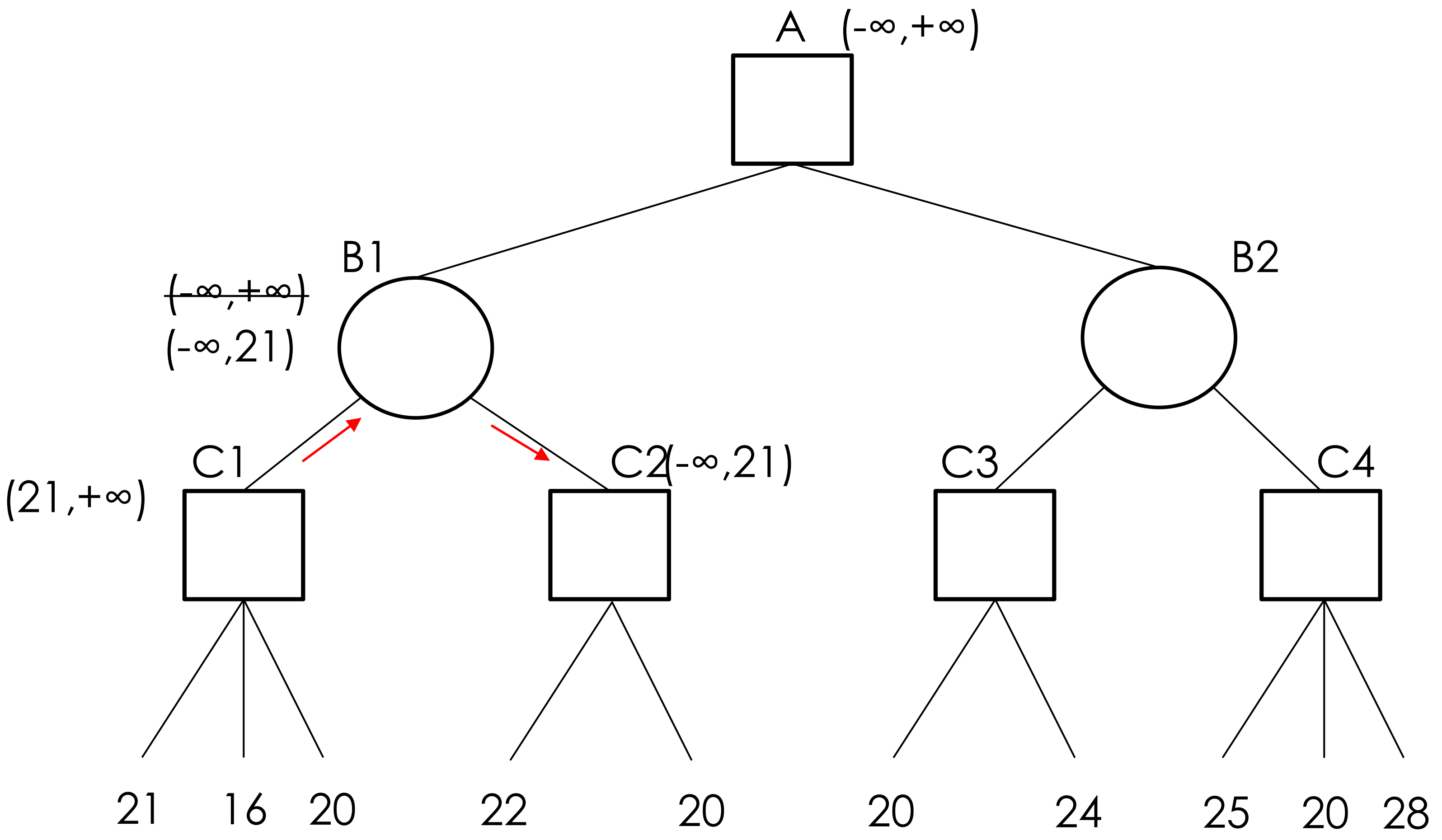
function MAX-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

---

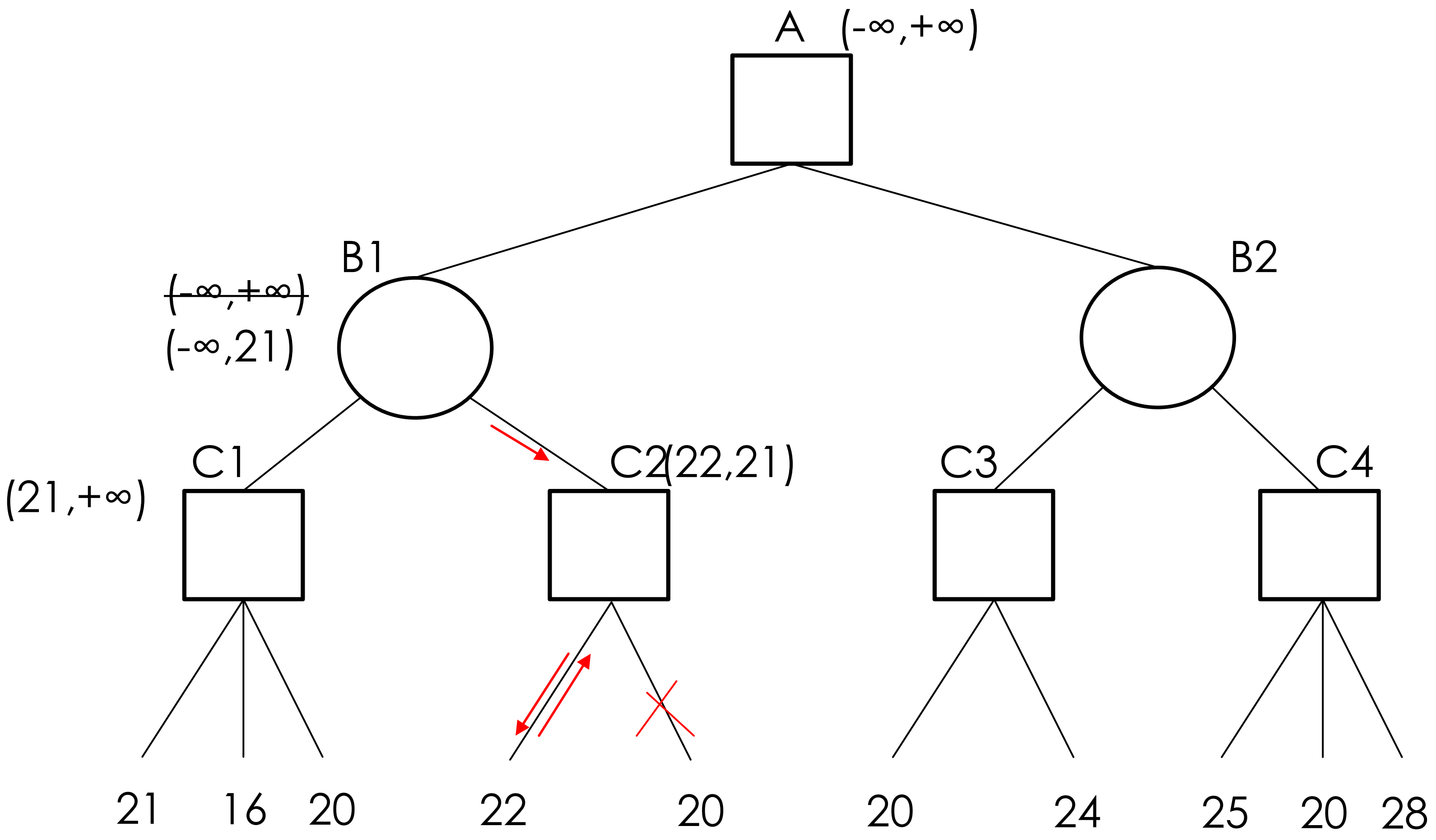
function MIN-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Question 2 Analysis

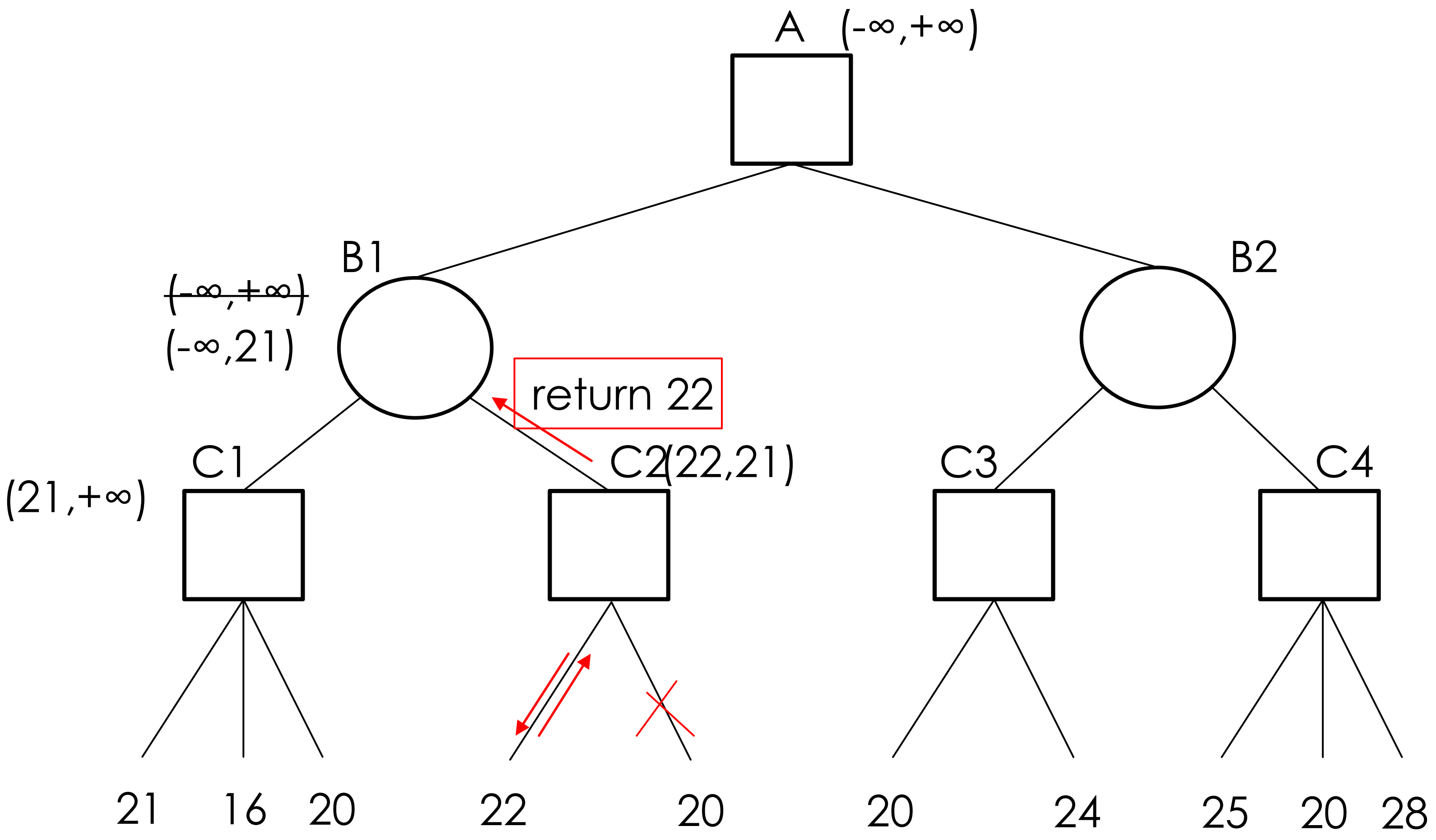


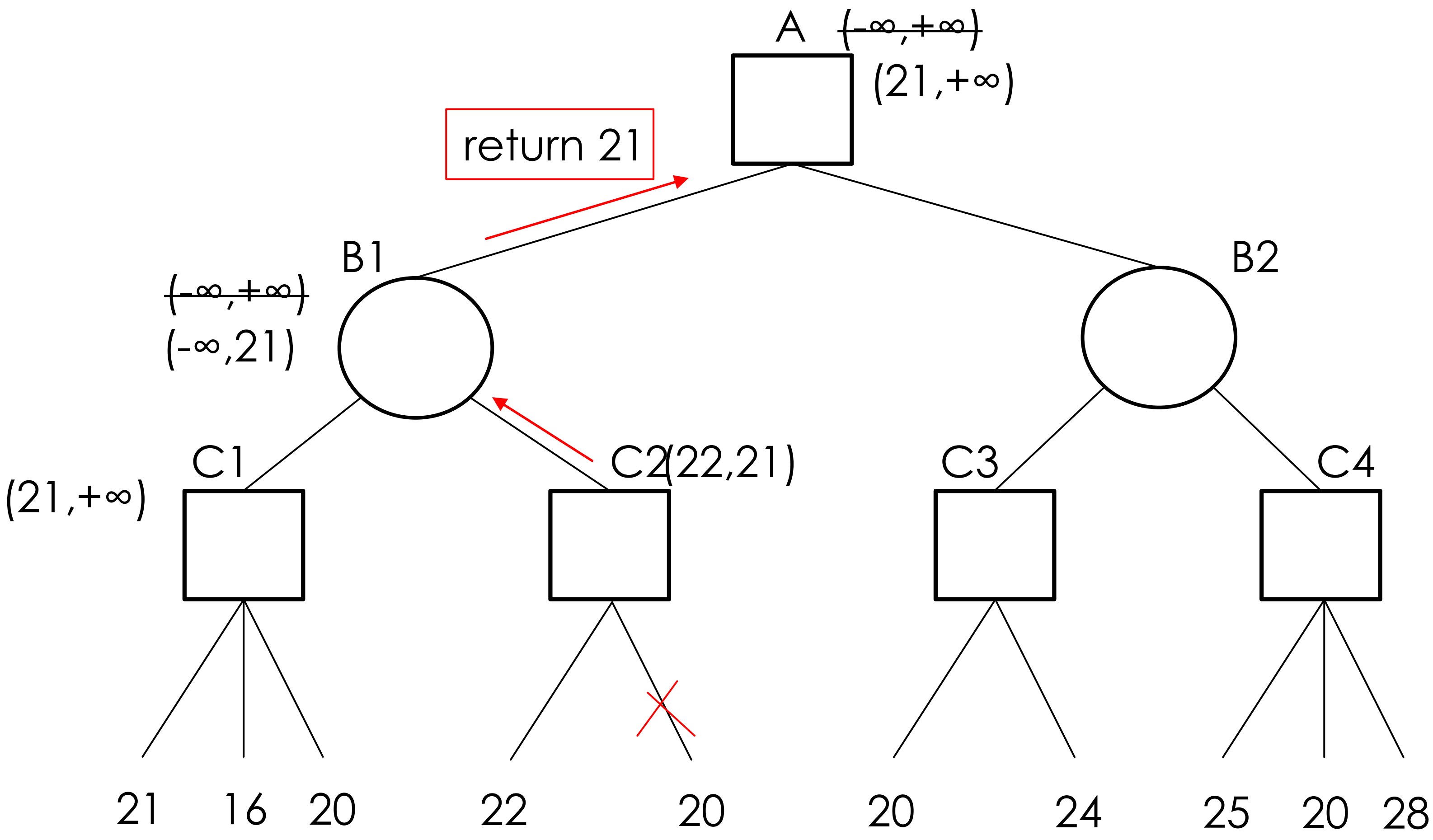


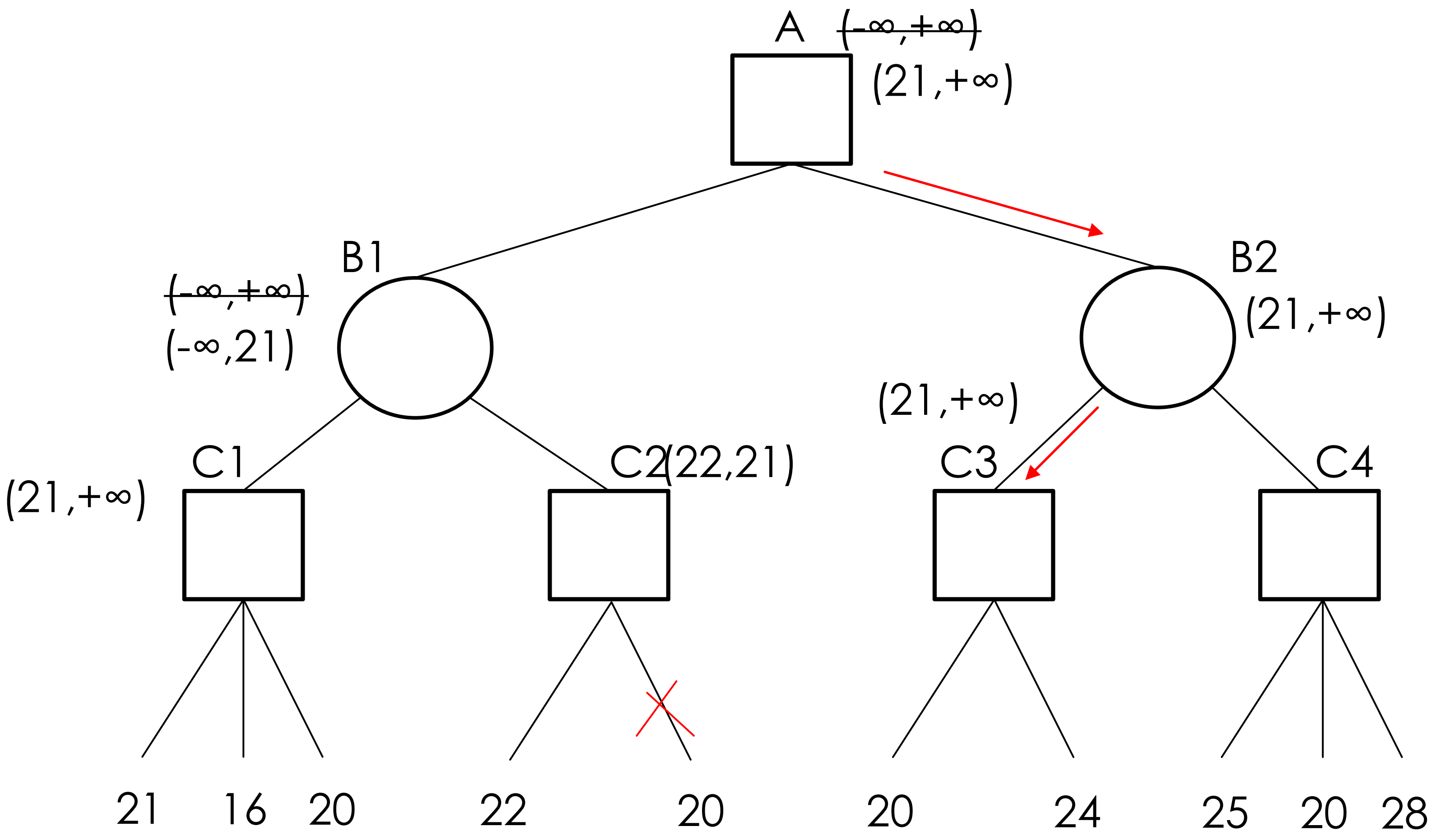
A
(-∞,+∞)
B1
~~(-∞,+∞)~~
(-∞,21)
B2
C1
(21,+∞)
C2
(-∞,21)
C3
C4
21
16
20
22
20
20
24
25
20
28

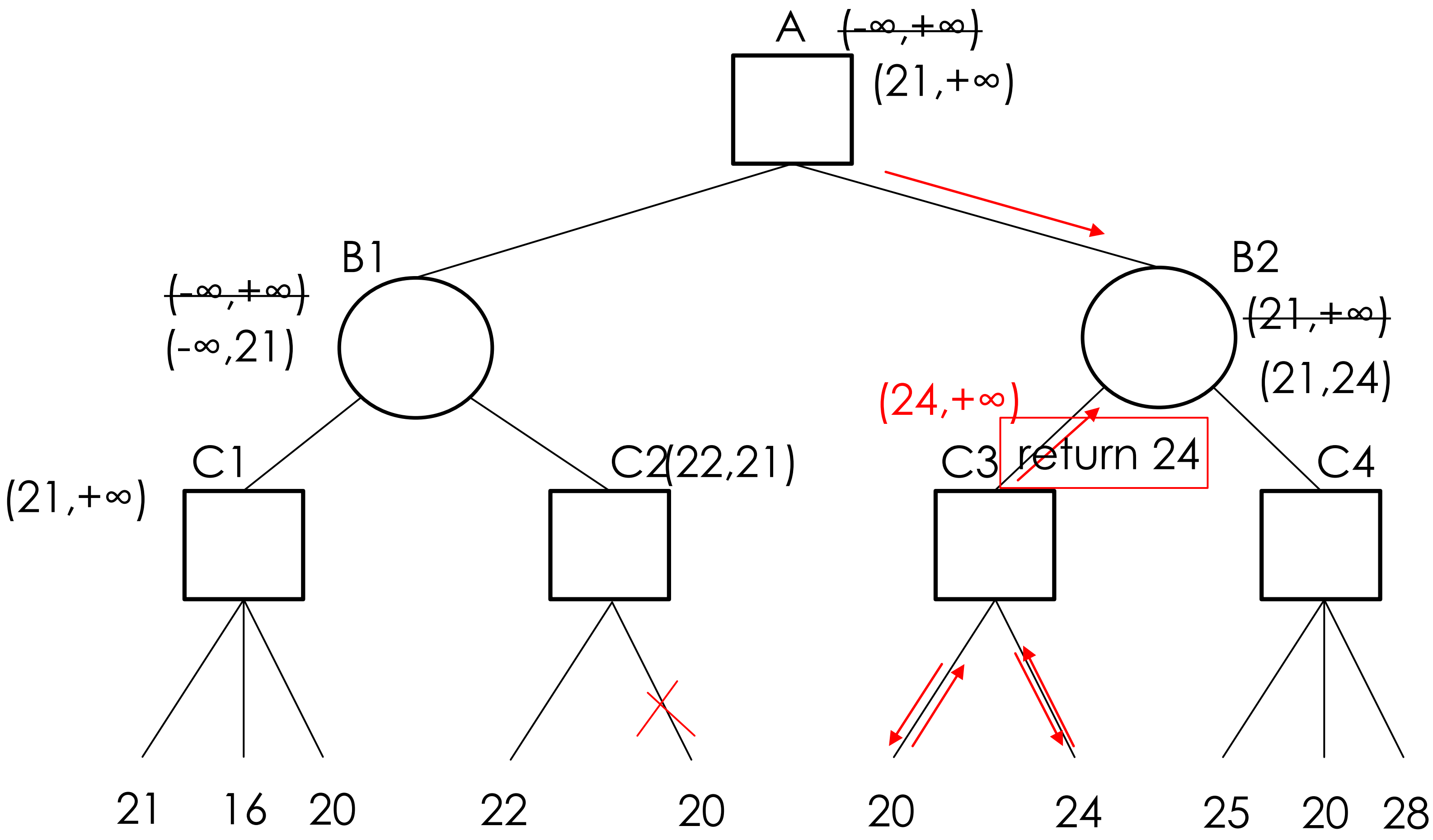


A (-∞,+∞)
B1
~~(-∞,+∞)~~
(-∞,21)
B2
C1
(21,+∞)
C2(22,21)
C3
C4
21 16 20
22 20
20 24
25 20 28

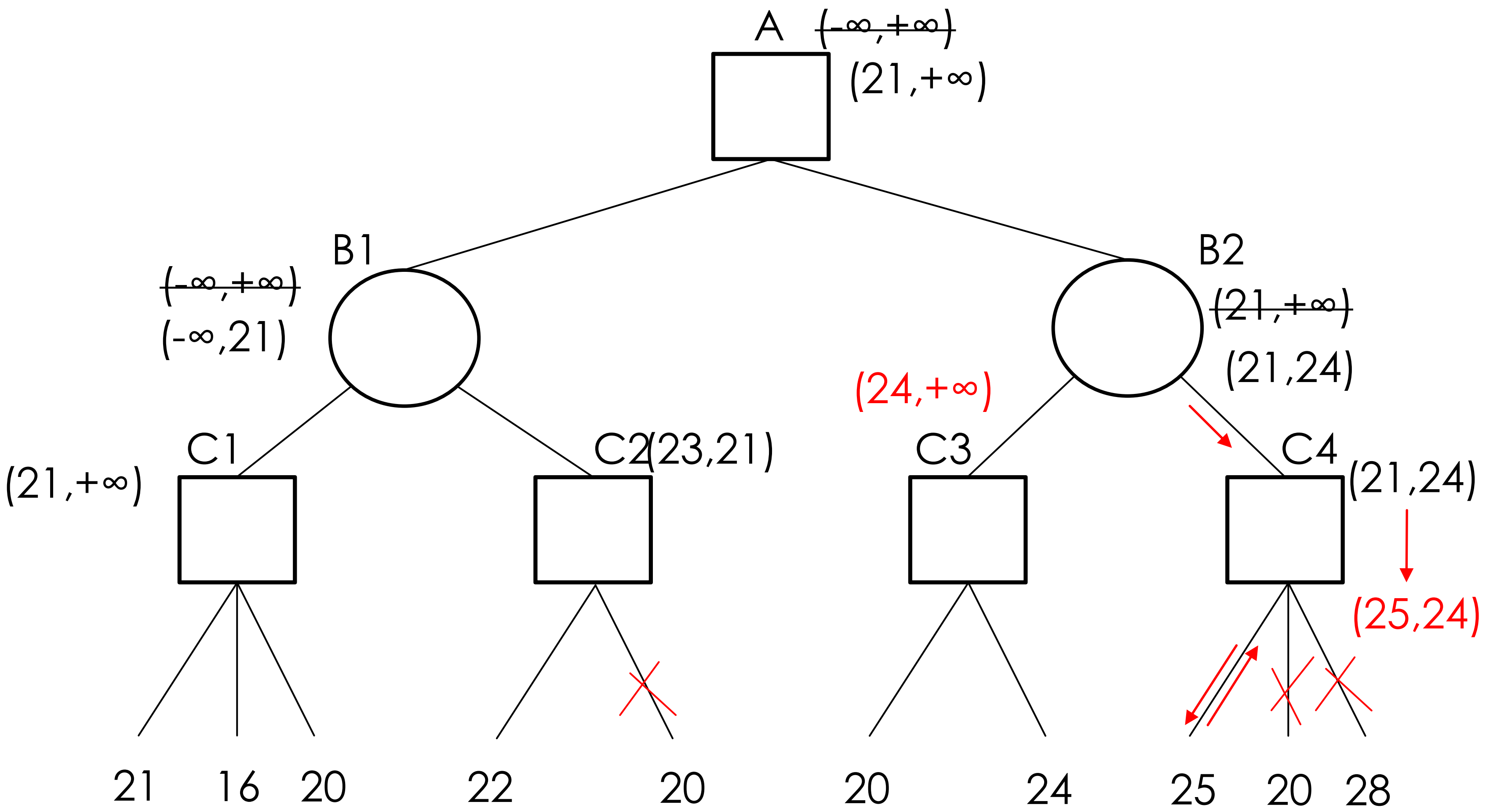

A (-∞,+∞)
B1
~~(-∞,+∞)~~
(-∞,21)
B2
return 22
C1
(21,+∞)
C2(22,21)
C3
C4
21
16
20
22
20
20
24
25
20
28


A
~~(-∞,+∞)~~
(21,+∞)
return 21
B1
~~(-∞,+∞)~~
(-∞,21)
B2
C1
(21,+∞)
C2(22,21)
C3
C4
21
16
20
22
20
20
24
25
20
28

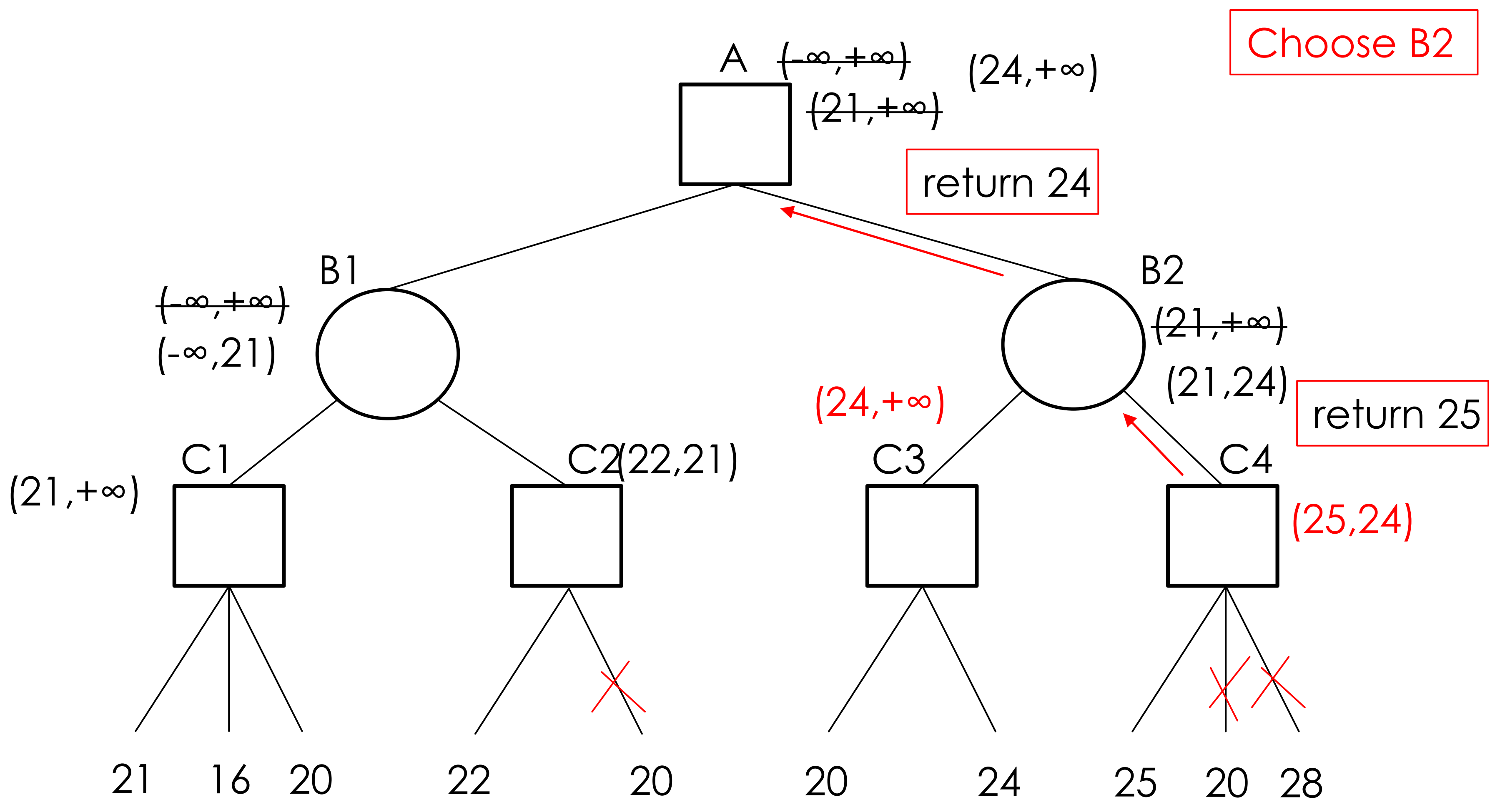

A
~~(-∞,+∞)~~
(21,+∞)
B1
~~(-∞,+∞)~~
(-∞,21)
B2
(21,+∞)
(21,+∞)
C1
(21,+∞)
C2
(22,21)
C3
C4
21 16 20
22 20
20 24
25 20 28



A
~~(-∞,+∞)~~
(21,+∞)
B1
~~(-∞,+∞)~~
(-∞,21)
B2
~~(21,+∞)~~
(21,24)
(24,+∞)
return 24
C1
(21,+∞)
C2
(22,21)
C3
C4
21
16
20
22
20
20
24
25
20
28

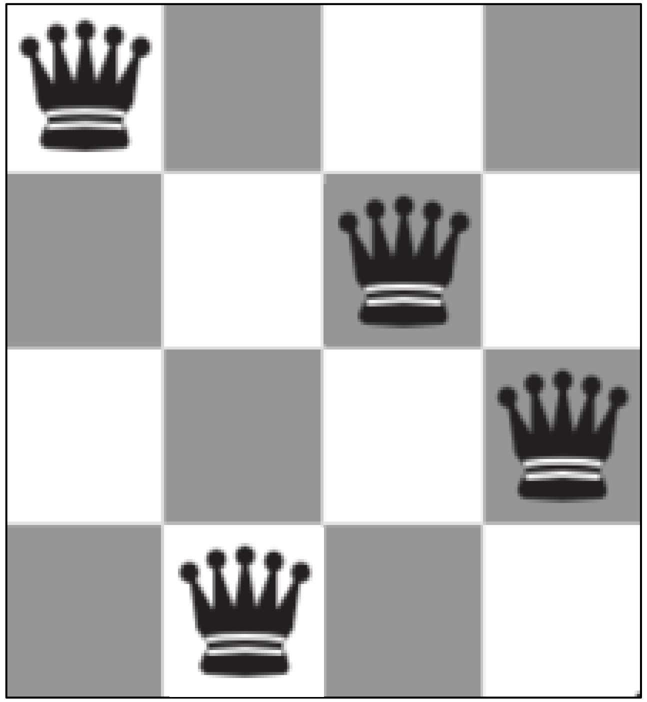


A
~~(-∞,+∞)~~
(21,+∞)
B1
~~(-∞,+∞)~~
(-∞,21)
B2
~~(21,+∞)~~
(21,24)
(24,+∞)
C1
(21,+∞)
C2
(23,21)
C3
C4
(21,24)
(25,24)
21
16
20
22
20
20
24
25
20
28


Choose B2
A
~~(-∞,+∞)~~
~~(21,+∞)~~
(24,+∞)
return 24
B1
~~(-∞,+∞)~~
(-∞,21)
B2
~~(21,+∞)~~
(21,24)
return 25
(24,+∞)
C1
(21,+∞)
C2
(22,21)
C3
C4
(25,24)
21
16
20
22
20
20
24
25
20
28

Question 3

Here is the initial status of a 4-queens problem. Using min-conflicts algorithm to solve this problem. Draw every step and tell why? (Finding one solution with the minimal number of steps can get full marks)



Min-Conflicts Algorithm

```
function MIN-CONFLICTS(csp, max_steps) returns a solution or failure
   inputs: csp, a constraint satisfaction problem
           max_steps, the number of steps allowed before giving up

   current ← an initial complete assignment for csp
   for i = 1 to max_steps do
      if current is a solution for csp then return current
      var ← a randomly chosen conflicted variable from csp.VARIABLES
      value ← the value v for var that minimizes CONFLICTS(var, v, current, csp)
      set var = value in current
   return failure
```

Question 3 Answer

Step1:choose queen no.2

Q		1	
		Q	
		3	Q
	Q	2	

Q		Q	
			Q
	Q		

Step2:choose queen no.0

Q		Q	
0			
3			Q
1	Q		

		Q	
Q			
			Q
	Q		

Step3：no conflict queen, exit

Question 4

- You want to go to Germany as a tourist and visit the 9 cities Berlin, München, Frankfurt, Nürnberg, Hamburg, Stuttgart, Dresden, Eisenach, and Weimar (each at most once).Only Berlin, München, and Frankfurt have international airports, so you can **arrive in Germany and leave Germany ONLY from one of these three cities.**In Germany, you want to travel by train. All the 9 cities are directly connected by train. Each train connection has a **travel time (duration) and a rating**. The worst rating is 1 star, which means that the train ride is uninteresting and in an uncomfortable train. The best rating is 5 stars and it stands for comfortable train trips that go through a nice scenery.

How can you make your train trips **both as short and as pleasant as possible, AND feasible**?

Write something for each of the points below. If an element is not needed, say so!

- (1)What is a solution space in this context?
- (2)How would you represent such solutions internally for the optimization algorithms? In other words, how would you encode them and which search space would you use?
- (3)Define the objective function(s) and whether they are maximized or minimized. (a textual description or short pseudo-code is enough)
- (4)Define the search operations that you would employ. It is sufficient to shortly describe what they do.

Question 4 Answer

- (1)The solution space is the permutation of 9 cities in which the first city and the last city should be one among Berlin, München and Frankfurt. Permutation means one city only appear once.
- (2)A solution can be represented as a permutation of numbers from 1 to 9, each number denotes a city. The first number and last number in one solution should be one among 1, 2, 3.
- (3)There are two objective functions:

  total travel time (distance), to be minimized

  total travel rating, to be maximized
- (4)Possible search operations:

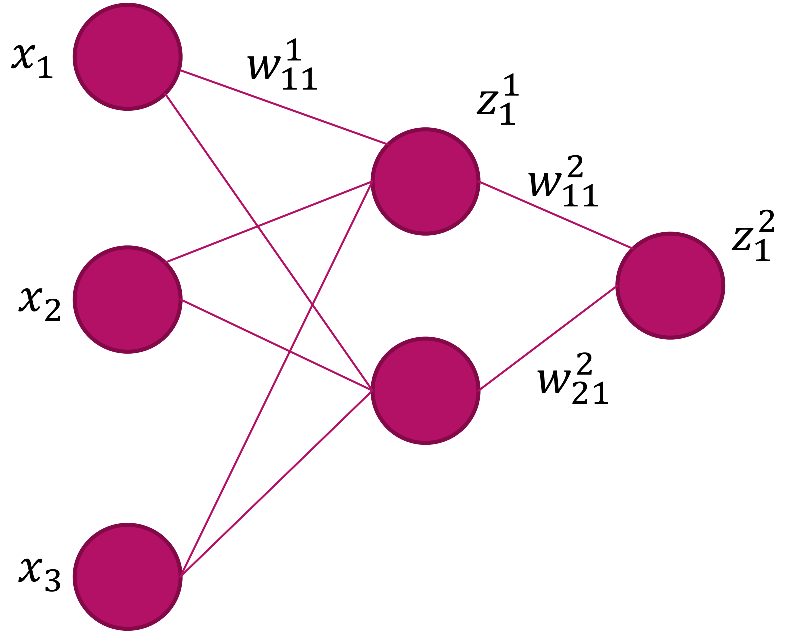
  Initialization: initialize lists of 1-9 and then randomly shuffle.

  Mutation: randomly choose two elements in one solution and exchange them.

  Repair: If the first element in one solution is not 1, 2, or 3, exchange the first element with the first appearing 1, 2, or 3 in the list. If the last element in one solution is not 1, 2, or 3, exchange the last element with the last appearing 1, 2 or 3 in the list.

Question 5

- Assume x = $[1\ 2\ 3]^T$, y = 3. $w^1 = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix}$, $w^2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Activity function for hidden layer and output layer is Relu: $f(x) = \max(0, x)$.
- (1)Please calculate the z_1^2.
- (2)We want to minimize $E(z_1^2) = (y - z_1^2)^2/2$, using gradient descent. Let the step size be $\alpha = 0.001$, which w^1 and w^2 will be at after ONE iteration of gradient descent?



Question 5 Answer

- (1)$z_1^1 = z_2^1$ = f(1+4+9) = 14

z_1^2 = f(14+28) = 42

- (2)

$$f(x) = \max(0, x)$$

$$f'(x) = \begin{cases} if\ x > 0, f'(x) = 1 \\ else\ f'(x) = 0 \end{cases}$$

$$\frac{\partial E}{\partial w_{ij}^{(2)}} = \frac{\partial E}{\partial z_j^{(2)}} \frac{\partial (z_j^{(2)})}{\partial a_j^{(2)}} \frac{\partial (a_j^{(2)})}{\partial w_{ij}^{(2)}} = -(y - z_1^2)*1*(z_i^{(1)})$$

$$\frac{\partial E}{\partial w_{ij}^{(1)}} = \frac{\partial E}{\partial z_1^{(2)}} \frac{\partial (z_1^{(2)})}{\partial a_1^{(2)}} \frac{\partial (a_1^{(2)})}{\partial z_j^{(1)}} \frac{\partial (z_j^{(1)})}{\partial (a_j^{(1)})} \frac{\partial (a_j^{(1)})}{\partial w_{ij}^{(1)}} = -(y - z_1^2)*1*\ w_{j1}^{(2)} * 1 * x_i$$

$$w^2 = w^2 - 0.001* \frac{\partial E}{\partial w_{ij}^{(2)}} = \begin{bmatrix}1\\2\end{bmatrix} - 0.001*(-1)(3-42) * \begin{bmatrix}14\\14\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}0.546\\0.546\end{bmatrix} = \begin{bmatrix}0.454\\1.454\end{bmatrix}$$

$$w^1 = w^1 - 0.001* \frac{\partial E}{\partial w^{(1)}} = = \begin{bmatrix}1 & 1\\2 & 2\\3 & 3\end{bmatrix} - 0.001 * (-1) * (3-42) * \begin{bmatrix}1\\2\\3\end{bmatrix} * [1\ 2] = \begin{bmatrix}1 & 1\\2 & 2\\3 & 3\end{bmatrix} - \begin{bmatrix}0.039 & 0.078\\0.078 & 0.156\\0.117 & 0.234\end{bmatrix} = \begin{bmatrix}0.961 & 0.922\\1.922 & 1.844\\2.883 & 2.766\end{bmatrix}$$

Question 6

- Consider a support vector machine (SVM) with decision boundary $w^T x + b = 0$ for a 3D feature space. The weight vector is $w = [3\ 2\ 1]^T$ and $b = 2$.

(1) Which of the following points will be classified *incorrectly* by this SVM in training process?

(a) $x_1 = [0\ 0\ 0]^T$, $y_1 = -1$

(b) $x_2 = [1\ 1\ 1]^T$, $y_2 = +1$

(c) $x_3 = [-1.88\ \ -2.99\ \ \ 11]^T$, $y_3 = +1$

(2) If the above w and b were obtained with the above three points, what is the range of the slack variables ξ_i?

Hint: the constraints are $y_i(w^T x_i + b) \geq 1 - \xi_i (\xi_i \geq 0)$.

Question 6 Answer

- (1)

(a) y_1 *(3*0+2*0+1*0+2)= -2 < 1 *incorrect*

(b) y_2*(3*1+2*1+1*1+2) = 8 ≥ 1 *correct*

(c) y_3*(3*(-1.88)+2*(-2.99)+1*11+2) = 1.38 ≥ 1 *correct*

(2) $\xi_1 \geq 3$ $\xi_2 \geq 0$ $\xi_3 \geq 0$