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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Assume A={1,2,3} and ρ(A) be its power set. Let ⊆ be the subset relation on the power ser. Draw the Hasse diagram of (ρ(A),⊆) (3)

2 Let R denote a relation on the set of ordered pairs of positive integers such that (x, y)R(u, v) iff xv = yu. Show that R is an equivalence relation (3)

3 Prove that in any group of six people, at least three must be mutual friends or at least three must be mutual strangers. (3)

4 Define GLB and LUB for a partially ordered set. Give an example (3)

PART B

Answer any two full questions, each carries 9 marks.

5 a) Suppose f(x)=x+2,g(x)= x-2 and h(x)=3x for x ϵ R,where R is the set of real numbers. Find g o f ,f o g, f o f, g o g, f o h, h o g,h o h and (f o h) o g (4)

b) Prove that every equivalence relation on a set generates a unique partition of the set with the blocks as R-equivalence classes (5)

6 a) Show that the set N of natural numbers is a semigroup under the operation x*y=max(x,y). Is it a monoid? (3)

b) Solve the recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$ (6)

7 a) Show that for any commutative monoid <M,*>, the set of idempotent elements of M forms a submonoid. (5)

b) Define subsemigroups and submonoids. (4)

PART C

Answer all questions, each carries 3 marks.

8 Show that, for an abelian group, $(a * b)^{-1} = a^{-1} * b^{-1}$ (3)

9 Show that every chain is a distributive lattice. (3)

10 Simplify the Boolean expression a’b’c+ab’c+a’b’c’ (3)

11 Let $G = \{1, a, a^2, a^3\}$ $(a^4 = 1)$ be a group and $H = \{1, a^2\}$ is a subgroup of G under multiplication. Find all cosets of H. (3)

PART D

Answer any two full questions, each carries 9 marks.

12 a) Show that the order of a subgroup of a finite group divides the order of the group. (6)

b) Define ring homomorphism. (3)

13 Show that $(I,\Theta,\odot)$ is acommutative ring with identity, where the operations Θ and $\odot$ are defined, for any $a,b \in I$, as $a \Theta b = a+b-1$ and $a \odot b = a+b-ab$. (9)

14 a) Let $(L,\leq)$ be a lattice and $a,b,c,d \in L$. Prove that if $a \leq c$ and $b \leq d$, then (5)

(i) $a \vee b \leq c \vee d$

(ii) $a \wedge b \leq c \wedge d$

b) Show that in a Boolean algebra,for any a,b,c (4)

$$(a \wedge b \wedge c) \vee (b \wedge c) = b \wedge c$$

PART E

Answer any four full questions, each carries 10 marks.

15 a) a) Construct truth table for $(\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$ (6)

b) Explain proof by Contrapositive with example. (4)

16 Prove the following implication (10)

$(x)(P(x) \vee Q(x)) \Longrightarrow (x) P(x) \wedge (\exists x) Q(x)$

17 a) Represent the following sentences in predicate logic using quantifiers (6)

(i) "x is the father of the mother of y"

(ii) "Everybody loves a lover"

b) Determine whether the conclusion C follows logically from the premises (4)

H_1: $\sim p \vee q$, H_2: $\sim(q \wedge \sim r)$, H_3: $\sim r$ C: $\sim p$

18 a) Without using truth table prove $p \rightarrow (q \rightarrow p) \Leftrightarrow \sim p \rightarrow (p \rightarrow q)$ (4)

b) Determine the validity of the following statements using rule CP. (6)

"my father praises me only if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore if my father praises me then I do not study well"

19 a) Show that $r \rightarrow s$ can be derived from the premises $p \rightarrow (q \rightarrow s)$, $\sim r \vee p$, q (4)

b) Prove, by Mathematical Induction, that $n(n + 1)(n + 2)(n + 3)$ is divisible by 24, for all natural numbers n (6)

20 a) "If there are meeting, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. There was no meeting". Show that these statements constitute a valid argument. (6)

b) Show that $2^n < n!$ For $n \geq 4$ (4)

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