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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks.

Marks

- 1 a) Find the points where Cauchy-Riemann equations are satisfied for the function $f(z) = xy^2 + i x^2 y$. Where does $f'(z)$ exist? Is the function $f(z)$ analytic at those points? (7)
- b) If $v = e^x (x \sin y + y \cos y)$, find an analytic function $f(z) = u + iv$. (8)
- 2 a) Show that $u = x^2 - y^2 - y$ is harmonic. Also find the corresponding conjugate harmonic function. (7)
- b) (i) Find a bilinear transformation which maps $(-i, 0, i)$ onto $(0, -1, \infty)$. (8)
- (ii) Test the continuity at $z = 0$, if $f(z) = \frac{\operatorname{Im} z}{|z|}, z \neq 0$
 $= 0, z = 0$
- 3 a) Find the image of the lines $x=1, y=2$ and $x>0, y<0$ under the mapping $W = z^2$ (8)
- b) Find the image of the semi-infinite strip $x > 0, 0 < y < 2$ under the transformation $w = iz + 1$. Draw the regions. (7)

PART B

Answer any two full questions, each carries 15 marks.

- 4 a) Evaluate $\oint_C \operatorname{Re} z^2 dz$ over the boundary C of the square with vertices $0, i, 1+i, 1$ clockwise (8)
- b) Evaluate $\int \frac{4-3z}{z(z-1)} dz$ over the circle $|z| = \frac{3}{2}$ (4)
- c) Evaluate $\int \frac{3z^2+7z+1}{z+1} dz$ over the circle $|z+i|=1$ (3)
- 5 a) Expand $\frac{z}{(z-1)(z-2)}$ in (1) $0 < |z-2| < 1$, (2) $|z-1| > 1$ (8)
- b) Evaluate $\int_0^{2\pi} \frac{1}{2+\cos \theta} d\theta$ (7)
- 6 a) Using Residue theorem evaluate $\int \frac{z^2}{(z-1)^2(z+2)} dz$ over the circle $|z|=3$ (7)
- b) Find the Taylor series of $\frac{\sin z}{z-\pi}$ about the point $z = \pi$ (4)

- c) Evaluate $\int \frac{\sin z}{z^6} dz$ over the circle $|z|=2$ using Cauchy's Residue theorem. (4)

PART C

Answer any two full questions, each carries 20 marks.

- 7 a) Solve by Gauss-Elimination method $x + y + z = 6$, $x + 2y - 3z = -4$, $-x - 4y + 9z = 18$. (7)
- b) Find the values of 'a' and 'b' for which the system of equations $x + y + 2z = 2$, $2x - y + 3z = 10$, $5x - y + az = b$ has: (7)
- (i) no solution (ii) unique solution (iii) infinite number of solutions.
- c) Verify whether the vectors $(1, 2, 1, 2)$, $(3, 1, -2, 1)$, $(4, -3, -1, 3)$ and $(2, 4, 2, 4)$ are linearly independent in \mathbb{R}^4 . (6)
- 8 a) Write down the matrix associated with the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$. By finding eigen values, determine nature of the quadratic form. (7)
- b) Diagonalise the matrix $A = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}$ (7)
- c) If A is a symmetric matrix, verify whether AA^T and A^TA are symmetric? (6)
- 9 a) Find the eigen vectors of $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$ (8)
- b) Find the null space of $AX=0$ if $A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \\ 4 & 4 & -1 & 9 \end{bmatrix}$ (6)
- c) Verify whether $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$ is orthogonal. (6)

What can you say about determinant of an orthogonal matrix? Prove or disprove the result.
