4/11/17

Permutations & Combinations

? How many variable names of 8 letters can be formed from the letters a, b, c, d, e, f, g, h, i without repeatation

A. This is a problem of permutation out of nine letters we have to select 8 letters without repeatation so the no: of permutations is given by ^nP_r. where $n = 9$ & $r = 8$.

$$^9P_8 = \frac{9!}{(9-8)!} = \frac{9!}{1!} = \underline{\underline{9!}}$$

? There are 10 persons called on an interview each one is capable to be selected for the job. How many permutations are there to select 4 from 10.

A. Out of 10 people we have to select 4.

$\therefore$ No: of permutations $= {}^{10}P_4 = \frac{10!}{6!} = \underline{\underline{5040}}$

? How many 6 digit numbers can be formed by using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8. If every number is to start with 30. with no digit repeated.

A. Out of 9 numbers 30 is fixed & hence

we can select from only 7

$\therefore n = 7$

For the selection of 6 digit number. Two numbers are already fixed and hence 4 numbers can only be selected $\therefore r = 4$.

$\therefore$ No:of permutations $= {}^7P_4 = \frac{7!}{3!} = 840$

? How many permutations can be made out of the letter of the word 'COMPUTER'. How many of these

i) begin with C

ii) begin with C & end with R

iii) C & R occupy the end places

iv) end with R.

A. There are 8 letters in the word COMPUTER and all are distinct. $\therefore n = 8$

$\therefore$ Total no:of permutations $= 8!$

i) begin with C

In this case the first position is filled by the letter C & we can only arrange the remaining 7 letters & hence the number of permutations $= 1 \times 7!$

ii) begin with C & end with R

Here the first place is filled with C & last place is filled with R. So we can make the arrangements only for the remaining 6 letters. ∴ No:of permutations = $1 \times 6! \times 1$

iii) C & R occupy the end places.

Here C & R occupy end places. Here the end places can be arranged either by C or R and the other places have to be arranged by the remaining 6 letters.

∴ No:of permutations = $6!\,2!$

? Determine the no:of permutations that can be made out of the letters of the word PROGRAMMING.

A. There are 11 letters in the word PROGRAMMING out of which G, M & R are two each

∴ No:of permutations = $\dfrac{11!}{2!\,2!\,2!}$ =

? There are four blue 3 red & 2 black penes in the box which are drawn one by one. Determine all the possible permutations

A. No:of Permutations $= \frac{9!}{4!\,3!\,2!}$

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? How many 4 digit numbers can be formed by using the digits. 2,4,6,8 when repeatation is allowed.

A. We have to make the 4 digit numbers were repeatation is allowed. So the no:of ways of filling the unit's place. = 4

No:of filling the 10's place = 4

No:of filling the 100th place = 4

" " " 1000's place = 4

∴ Total no:of permutations = $4\times4\times4\times4 = 256$.

? How many 2 digit even number can be formed. by using the digits. 1,3,4,6,8 when repeatation of digits are allowed.

A. In our given numbers three numbers are even & two numbers are odd. We are asked to form a 2 digit even number. ∴ The no:of filling the unit's place = 3. & that of ten's place is 5

∴ Total no:of permutation $= 5 \times 3 = 15$

? In how many ways can the letters a, b, c, d, e, f be arranged in a circle

A. This is a problem of circular permutation
Here $n = 6$.

∴ Total no:of permutations $= (6-1)! = 5!$

Combination

? How many 16 bit string are there containing exactly five zeros

A. This is a problem of combination because it is only told to have 5 zeros in the 16 bit string and were to put the zero is not given. ∴ There no order for the arrangement.

∴ No:of combinations $= 16C_5 = \dfrac{16!}{5! \cdot (16-5)!}$

$$= \frac{16!}{5!\ 11!} = 4368.$$

? From 10 programmers in how many ways can 5 be selected when

i) a particular programmer is included every time.

ii) a particular programmer is not included at all.

A. From the 10 programmers the combination to select 5 is given by $^{10}C_5$

i) A particular programmer is always selected means we can select only 4 programmers from the remaining 9 programmers

$\therefore$ No:of combination $= {}^9C_4 = \frac{9!}{(9-4)!\,4!} = \underline{\underline{126.}}$

ii) A particular programmer is not included means we have to select 5 programmers out of 9 programmers.

$\therefore$ No:of combinations $= {}^9C_5 = \frac{9!}{5!\,(9-5)!} = \underline{\underline{126}}$

? Show that if any 4 members from 1 to 6 are choosen then 2 of them will add to 7

A. We have given the numbers 1, 2, 3, 4, 5, 6. we have to choose any 4 numbers such that when we add 2 numbers of it we get a sum 7.

The 2 numbers whose sum is 7 are :

$A = \{2,5\}$ $B = \{3,4\}$ $C = \{1,6\}$

$\therefore$ whenever we select the 4 numbers anyone of the set A, B or C will be included in our selection.

Hence the proof.

? Show that atleast 2 people must have their birthday in the same month if 13 people are assembled in a room. $n = 13$ $m = 12$ PHP

? Show that if 9 colours are used to paint 100 houses atleast 12 houses will be of the same colour. $n = 100$ $m = 9$ extended pigeonhole principle

9/11/17

Principle of Inclusion - Exclusion

? Out of 1200 students at a college. 582 took Economics 627 took English 563 to Mathematics. 217 took both Economics & English. 307 took both Economics & maths. 250 took both English & maths. 222 took all the courses. How many of them took none of these

A. Denote economics by A English by B & Mathematics by C.

Cardinality of,

$$|A| = 582.$$
$$|B| = 627$$
$$|C| = 543$$
$$|A \cap B| = 217$$
$$|A \cap C| = 307.$$
$$|B \cap C| = 250.$$
$$|A \cap B \cap C| = 222$$

We want to find the no: of students who have not take any of the courses. It can be found out by subtracting the no: of students who have taken any of the courses from the total no: of students.

$|A \cup B \cup C|$ indicates the no:of students who have taken any of the courses.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 582 + 627 + 543 - 217 - 307 - 250 + 22.$$

$$= \underline{\underline{1200}}.$$

$\therefore$ The no:of students who have not taken the courses $= 1200 - 1200 = 0$ //

? Among 100 students, 32 study Maths, 20 study Physics, 45 Biology, 15 study Maths & Biology, 7 study Maths & Physics, 10 study Physics & Biology, 30 donot study any of the three subjects. Find the no:of students studying all the three subjects

A. Denote Maths by ~~M~~ A, Physics. B, Biology C.

$|A| = 32.$ $\quad |A \cap C| = 15.$

$|B| = 20.$ $\quad |A \cap B| = 7$

$|C| = 45.$ $\quad |A \cap C| = 10.$

$$|A \cup B \cup C| = 100 - 30$$
$$= \underline{\underline{70}}$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$|A \cap B \cap C| = |A \cup B \cup C| - |A| - |B| - |C| + |A \cap B| + |A \cap C| + |B \cap C|$$

$$= 70\cancel{100} - 32 - 20 - 45 + 15 + 7 + 10$$

$$= \underline{\underline{5}}$$

Module II.

Review of Permutation & Combination

Definition

An ordered selection of r elements of a set containing n distinct elements is called an r-permutation of n elements and is denoted by $P(n,r)$ or ^nP_r, where $r \leq n$.

An unordered selection of r elements of a set containing n distinct elements is called an r-combination of n elements and is denoted by $C(n,r)$ or ^nC_r or $\binom{n}{r}$.

Values of $P(n,r)$ & $C(n,r)$

1) The number of different permutations of n distinct objects taken r at a time, $r \leq n$ is given by $\boxed{P(n,r) \text{ or } {}^nP_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n-r+1)}$

2) The number of permutations of n things taken all at a time is $n!$.
(ie) $\boxed{P(n,n) = n!}$

3) The number of permutations of n objects of which n_1 identical objects, n_2 identical objects, ..., n_k identical objects, when all are taken at a time is given by

$$\boxed{\frac{n!}{n_1!\, n_2! \ldots n_k!}}$$

4) The circular permutation are the permutations in which the objects are placed in a circle.
The number of circular permulations of n different objects is $\boxed{(n-1)!}$.

5) The number of combinations of n different things taken r at a time is given by,

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}, \quad 1 \le r \le n}$$

6) The number of combinations of n things taken all at a time is 1. (ie) $\boxed{{}^nC_n = 1}$

7) The number of combinations of n things taken none at a time is 1 (ie) $\boxed{{}^nC_0 = 1}$

Pigeon Hole Principle

Statement

If n pigeon's are accomodated in m pigeon holes and $n > m$, then atleast one pigeon hole will contain two or more pigeons.

Proof:

Let the n pigeons be labelled $P_1, P_2, \ldots, P_n$ and m pigeon holes be labelled $H_1, H_2, \ldots, H_m$.

If $P_1, P_2, \ldots, P_m$ are accomodated in to $H_1, H_2, \ldots, H_m$ respectively, then we are left with $(n-m)$ pigeons $P_{m+1}, P_{m+2}, \ldots, P_n$.

If these left over pigeons are accomodated again to the m pigeon holes $H_1, H_2, \ldots, H_m$ in any random manner, then atleast one pigeon hole will contain two or more pigeons.

Hence proved.

Note:) Extended Pigeon Hole Principle

If n pigeons are accomodated in m pigeonholes and $m < n$, then one of the pigeon hole must contain atleast $\left[\frac{n-1}{m}\right]+1$ pigeons, where $[x]$ denotes the greatest integer less ~~that~~ than or equal to x, where x is a real number.

Principle of Inclusion-Exclusion

(1) If A and B are two finite sets on a Universe U, then $\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$, where $|A|$ denotes the cardinality of A.

(2) If A and B are finite disjoint sets, then

$$\boxed{|A \cup B| = |A| + |B|}$$

(3) If A, B, C are the three finite sets, then

$$\boxed{|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|}$$

∴ In general,

Let $P_1, P_2, \ldots, P_n$ are finite sets, then

$$\boxed{\begin{aligned}|P_1 \cup P_2 \cup \ldots \cup P_n| &= \sum_{i=1}^{n} |P_i| - \sum_{1 \le i < j \le n} |P_i \cap P_j| \\ &+ \sum_{1 \le i < j < k \le n} |P_i \cap P_j \cap P_k| + \cdots + \\ &(-1)^{n-1} |P_1 \cap P_2 \cap \ldots \cap P_n|\end{aligned}}$$

Recurrence Relations

S.J

A recurrence relation is a functional relation between the independent variable x, dependent variable $f(x)$ and the difference of various order of $f(x)$.

A recurrence relation is also called a difference equation.

eg:) The equation $f(x+3h)+3f(x+2h)+6f(x+h)+9f(x)=0$.

It can also be written as

$$a_{n+3}+3a_{n+2}+6a_{n+1}+9a_n=0. \text{ or}$$

$$y_{n+3}+3y_{n+2}+6y_{n+1}+9y_n=0.$$

For eg:) The Fibonacci sequence is defined by the recurrence relation $a_n = a_{n-2} + a_{n-1}$, $n \geq 2$ with initial conditions $a_0 = 1$, $a_1 = 1$.

Order of the Recurrence Relation

The order of the recurrence relation is defined to be the difference between the highest and lowest subscripts of $f(x)$ or a_n or y_n.

eg:) The eq^n. 13 $a_n + 20a_{n-1} = 0$.

Here, the highest order subscript is n and lowest order subscript is $n-1$

difference is $n-(n-1) = n-n+1 = 1$.

∴ The given eq^n is a first order recurrence relation.

The eqⁿ. $8f(x)+4f(x+1)+8f(x+2)=k(x)$

can be written as $8a_n+4a_{n+1}+8a_{n+2}=k(x)$

∴ Highest subscript value $= n+2$

Lowest subscript value $= n$.

difference $= n+2-n = 2$

∴ The given eqⁿ. is second order recurrence relation.

Degree of the Recurrence Relation

The degree of the recurrence relation is defined to be the highest power of $f(x)$ or a_n or y_n.

1) The eqⁿ. $y_{n+3}^{3}+2y_{n+2}^{2}+2y_{n+1}=0$. has degree 3, since highest power of y_{n+3} is 3.

The eqⁿ. $a_n^4+3a_{n-1}^3+6a_{n-2}^2+4a_{n-3}=0$ has degree 4, since highest power of a_n is 4

The eqⁿ. $y_{n+3}+2y_{n+2}+4y_{n+1}+2y_n=k(x)$ has degree 1 and have order $n+3-n=3$

D) Linear Recurrence relation with constant coefficients. —

A recurrence relation is called linear, if its degree is one.

general form of linear recurrence relation with constant coefficients is

$$C_0 y_{n+r} + C_1 y_{n+r-1} + C_2 y_{n+r-2} + \cdots + C_n y_n = R(n)$$

where $C_0, C_1, C_2, \ldots, C_n$ are constants and $R(n)$ is a function of independent variable n.

Particular solution

ⓐ Homogeneous Linear Difference Equations

We can find the particular solution of the difference equation, when the equation is of homogeneous linear type by putting the values of the initial conditions in the homogeneous solution.

(1)

Solution of linear homogeneous recurrence relation with constant coefficients :-

Consider,

$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \cdots + C_k a_{n-k} = 0$ —— (1), $C_k \neq 0$

which is the general form of l.h. recu relation with const. coefficients of order k.

put $a_n = r^n$ $(r \neq 0)$ in equ. (1). Then,

$C_0 r^n + C_1 r^{n-1} + C_2 r^{n-2} + \cdots + C_k r^{n-k} = 0$

$\Rightarrow r^{n-k}\left[C_0 r^k + C_1 r^{k-1} + C_2 r^{k-2} + \cdots + C_k\right] = 0$

$\Rightarrow C_0 r^k + C_1 r^{k-1} + C_2 r^{k-2} + \cdots + C_k = 0$, since $r \neq 0$ —— (2)

This equ. (2) is called characteristic equ. and the roots of the char. equ. are called char. roots.

To find $a_n^{(P)}$

To find $a_n^{(P)}$ (i.e. particular solution) we make use of the 'method of undetermined coefficients'. The following table gives certain forms of $f(n)$ and corresponding choice for $a_n^{(P)}$.

(cc)

Notes

① If $f(n)$ is a linear combination of terms in the 1st column, then $a_n^{(P)}$ is assumed as linear combination of the corresponding terms in the 2nd column.

2) If $f(n) = r^n$ or $(A+Bn)r^n$, where 'r' is a non repeated characteristic root, then $a_n^{(P)}$ is assumed as Anr^n or $n(A+Bn)r^n$.

3) If $f(n) = r^n$, where 'r' is a twice repeated char. root then $a_n^{(P)}$ is taken as An^2r^n and so on

Problems of Recurrence Relation ③ 5J ①

Solution for homogeneous Recurrence Relation with Constant Coefficients.

Consider,

$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \dots + C_k a_{n-k} = 0 \rightarrow ①$, $C_k \neq 0$.

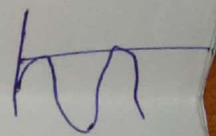
Steps for finding solutions:-

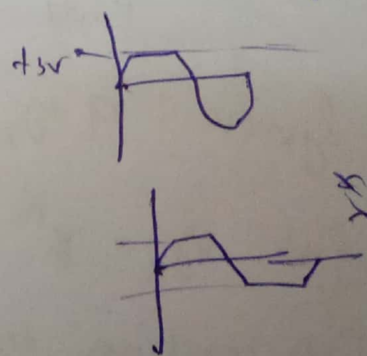
1] Put the unknown variable $a_n = r^n (r \neq 0)$ in equation ①.

2] Find the characteristic equation

3] Find the characteristic roots.

4] Write the solution in the general form according to the cases.





Q) Solve $a_n - 6a_{n-1} + 8a_{n-2} = 0$.

A) Given, $a_n - 6a_{n-1} + 8a_{n-2} = 0 \rightarrow ①$

Put $a_n = r^n$ in ①, $(r \neq 0)$

$\therefore$ ① becomes,

$r^n - 6r^{n-1} + 8r^{n-2} = 0$ [Subscript will become the power of r in each term]

Taking the lowest power of r outside, we have

$r^{n-2}[r^2 - 6r + 8] = 0$.

$\rightarrow r^2 - 6r + 8 = 0$, [Since $r \neq 0 \Rightarrow r^{n-2} \neq 0 \Rightarrow \frac{0}{r^{n-2}} = 0$]

This is the characteristic equation

$$r = \frac{6 \pm \sqrt{36-32}}{2}$$

$$= \frac{6 \pm \sqrt{4}}{2} = \frac{6+2}{2}, \frac{6-2}{2}$$

$$= \frac{8}{2}, \frac{4}{2}.$$

$$= 4, 2.$$

∴ The characteristic roots, $r_1 = 4$ and $r_2 = 2$.

The general solution is,

∴ $a_n^{(h)} = C_1 4^n + C_2 2^n$ [Since r_1 & r_2 are distinct roots].

Q) Solve $9y_{k+2} - 6y_{k+1} + y_k = 0$

A) Given $9y_{k+2} - 6y_{k+1} + y_k = 0 \longrightarrow (1)$

Put $y_k = r^k$, where $r \neq 0$, in (1) ~~[According~~ $\to (2)$ ~~to the variable in the given equation,~~

~~[According to the variable~~

~~Put eqn. (2)]~~

∴ (1) becomes,

$$9r^{k+2} - 6r^{k+1} + r^k = 0.$$

$$r^k [9r^2 - 6r + 1] = 0$$

∴ $9r^2 - 6r + 1 = 0$ is the characteristic equation.

$$r = \frac{6 \pm \sqrt{36 - 36}}{18}$$

$$= \frac{6 \pm \sqrt{0}}{18} = \frac{6}{18} = \frac{1}{3}, \frac{1}{3}$$ [Since eq^n is quadratic, it must have 2 roots].

$\therefore r_1 = 1/3$ & $r_2 = 1/3$

Since the roots are equal, the general solution is

$$y_k^{(h)} = [C_1 + C_2 k](1/3)^k.$$

Q) Solve the recurrence relation,

$a_n = 2(a_{n-1} - a_{n-2})$.

A) 1st write in the general ~~...~~ equation form,

(ie) $a_n = 2a_{n-1} - 2a_{n-2}$

$\Rightarrow a_n - 2a_{n-1} + 2a_{n-2} = 0. \longrightarrow (1)$

Put $a_n = r^n$, $(r \neq 0)$ in (1).

(1) becomes,

$r^n - 2r^{n-1} + 2r^{n-2} = 0$

$r^{n-2}[r^2 - 2r + 2] = 0.$

$\therefore r^2 - 2r + 2 = 0$ is the characteristic equation.

$$r = \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$$

S Jose.
Gun Con

$$= 1 \pm i.$$

$\therefore r_1 = 1+i$ and $r_2 = 1-i$ are complex roots.

$\therefore$ The general solution is,

$$a_n^{(h)} = C_1 (1+i)^n + C_2 (1-i)^n.$$

Particular solution in homogeneous recurrence relation

Step 1) Find the general solution of homogeneous recurrence relation as above.

Step 2) Apply the conditions given the problem to find out the constant values in the general solution.

Step 3) Substitute the constants in the general solution and then the solution is called particular solution.

Q) Solve $a_{n+2} + 4a_{n+1} + 4a_n = 0$, $n \geq 0$, $a_0 = 1$, $a_1 = 2$.

A) $a_{n+2} + 4a_{n+1} + 4a_n = 0 \rightarrow (1)$

Put $a_n = r^n$ in (1), $(r \neq 0)$

becomes,

$r^{n+2} + 4r^{n+1} + 4r^{n} = 0.$

$r^{n}\left[r^{2} + 4r + 4\right] = 0$

$\therefore$ $r^2 + 4r + 4 = 0$ is the characteristic equation.

$\therefore$ $r = \dfrac{-4 \pm \sqrt{16-16}}{2} = \dfrac{-4}{2} = -2, -2$ are equal roots.

$\therefore$ General sol^n. is given by,

$a_n^{(h)} = (C_1 + C_2 n)(-2)^n \longrightarrow$ (2)

Given $a_0 = 1 \Longrightarrow$ when $n = 0$, $a_n = 1$.
$a_1 = 2 \Longrightarrow$ when $n = 1$, $a_n = 2$. $\Big\} \rightarrow$ (3)

$\therefore$ Applying (3) in (2), we have

$\underline{n=0}$ $a_0 = (C_1 + C_2 \times 0)(-2)^0$

$\longrightarrow 1 = C_1 \times 1 \longrightarrow \boxed{C_1 = 1}$

$\underline{n=1}$

$a_1 = (C_1 + C_2 \times 1)(-2)^1$

$\Longrightarrow 2 = (C_1 + C_2) - 2$

$\Longrightarrow 2 = -2C_1 - 2C_2.$

$\Longrightarrow C_1 + C_2 = -1$ [Dividing throughout by -2]

$\Longrightarrow 1 + C_2 = -1$ [since $C_1 = 1$]

$\Longrightarrow C_2 = -1 - 1 = -2$

$\Longrightarrow \boxed{C_2 = -2}$.

Substituting the value of c_1 & c_2 in ②, we have

$a_n^{(h)} = (1-2n)(-2)^n$ is the particular solution.

Q) Solve $a_r - 7a_{r-1} + 10a_{r-2} = 0$ with $a_0 = 0, a_1 = 6$.

A) $a_r - 7a_{r-1} + 10a_{r-2} = 0 \longrightarrow$ ①

Put $a_r = K^r$, $K \neq 0$, in ①

$$K^r - 7K^{r-1} + 10K^{r-2} = 0$$

$$K^{r-2}\left[K^2 - 7K + 10\right] = 0$$

$\therefore K^2 - 7K + 10 = 0$ is the characteristic equation.

$$\therefore K = \frac{7 \pm \sqrt{49-40}}{2} = \frac{7 \pm \sqrt{9}}{2}$$

$$= \frac{7 \pm 3}{2} = \frac{7+3}{2}, \frac{7-3}{2}$$

$$= \frac{4}{2}, \frac{10}{2} = 2, 5$$ are distinct roots.

$\therefore$ The general solution is given by,

$$a_r^{(h)} = c_1 2^r + c_2 5^r \longrightarrow ②.$$

given, $a_0 = 0$ & $a_1 = 6 \longrightarrow$ ③

sub. ③ in ②, we have

$r=0$

$$a_0 = c_1 2^0 + c_2 5^0$$

$$\Rightarrow 0 = c_1 + c_2$$

$$\Rightarrow c_1 = -c_2.$$

$r=1$

$$a_1 = c_1 2^1 + c_2 5^1$$

$$\Rightarrow 6 = 2c_1 + 5c_2.$$

Sub. $C_1 = -C_2$ in $2C_1 + 5C_2 = 6$, we have

$$-2C_2 + 5C_2 = 6$$

$$3C_2 = 6$$

$$\boxed{C_2 = 2}$$

Sub. $C_2 = 2$ in $C_1 = -C_2$

$$\Rightarrow \boxed{C_1 = -2}$$

$\therefore$ Sub. C_1 & C_2 in (2), we have

$a_r^{(h)} = -2 \cdot 2^r + 2 \cdot 5^r$ is the particular solution.

Solution of non-homogeneous recurrence relation with constant coefficients.

eq^n. is of the form,

$$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \dots + C_n a_{n-k} = f(n) \rightarrow (1)$$ where

$f(n) \neq 0$, $C_k \neq 0$

General sol^n. of eq^n. (1) is

$$a_n = a_n^{(h)} + a_n^{(p)},$$

where $a_n^{(h)}$ is the general solution of homogeneous recurrence relation and $a_n^{(p)}$ is the particular solution of non-homogeneous recurrence relation.

Method of finding $a_n^{(h)}$ is followed as above.

To find $a_n^{(P)}$ (Particular solution of non-homogeneous

Step 1) write the general form ~~of~~ from the choice of $a_n^{(P)}$ from the below table corresponding to given $f(n)$ in ①.

$f(n)$	Choice of $a_n^{(P)}$
z, z is constant	A
z^r, z is constant	$A \cdot z^r$
$P(r)$, a polynomial of degree n	$A_0 r^n + A_1 r^{n-1} + \dots + A_n$
$z^r \cdot P(r)$, $P(r)$ polynomial of degree n & z constant	$(A_0 r^n + A_1 r^{n-1} + \dots + A_n) z^r$

2) $F(n) = r^n$, non repeated.

$F(n) = 2^r$.

3) $F(n) = r^n$, ~~r~~ r repeated

$A\, n^2 r^n$

General soln

$a_n = a_n^{(h)} + a_n^{(P)}$

→ To Find a_n

$a_n - 2a_{n-1} = 0$.

Remark:)

① If $f(n)$ is a linear combination of terms in the 1st column, ~~(e)~~ [eg: $2^n + 3$], then $a_n^{(P)}$ is assumed as linear combination of the corresponding terms in the 2nd column.

for eg:- Suppose $f(n) = 2^n + 3$, then the corresponding choice of ~~(an)~~ $a_n^{(P)}$ is $A \cdot 2^n + A$.

If $f(n) = r^n$ or $(A+Bn)r^n$, where 'r' is a non-repeated characteristic roots, then $a_n^{(P)}$ is assumed as Anr^n or $n(A+Bn)r^n$.

③ If $f(n) = r^n$, where 'r' is a twice repeated characteristic roots, then $a_n^{(P)}$ is taken as An^2r^n.

If $f(n) = r^n$, where 'r' is repeated thrice, then $a_n^{(P)}$ is taken as An^3r^n and so on.

Q) Solve $a_n - 2a_{n-1} = 3^n$, $a_1 = 5$.

A) $a_n - 2a_{n-1} = 3^n \longrightarrow (1)$

The general solution for the non-homogeneous recurrence relation is

$$a_n = a_n^{(h)} + a_n^{(P)} \longrightarrow (2)$$

~~Consider the~~

To find $a_n^{(h)}$

Consider the homogeneous equation from (1),

(ie) $a_n - 2a_{n-1} = 0 \longrightarrow (3)$

Put $a_n = r^n$ in (3) [Proceed as in homogeneous form]

~~Sub this~~

$\therefore$ (3) becomes, $r^n - 2r^{n-1} = 0$

$r^{n-1}[r-2] = 0$

$r-2=0$ is the characteristic equation

$\therefore r=2$ is the characteristic root.

$\therefore a_n^{(h)} = c_1 2^n \rightarrow (4)$

To find $a_n^{(p)}$

~~from~~ Since R.H.S of (1) is $f(n) = 3^n$. and since 3 is not a characteristic root, we can get the corresponding particular solution from the table.

$\therefore$ The choice of $a_n^{(p)} = A \cdot 3^n \rightarrow (5)$

sub. the choice in (1), we have

$A \cdot 3^n - 2A\,3^{n-1} = 3^n$. [Power of 3 depends on subscripts]

$\cancel{A\,3^{n-1}[3-2] = 3^n}$

$\Rightarrow 2A\,3^{n-1} = 3^n$

$\Rightarrow 2A = \dfrac{3^n}{3^{n-1}} \Rightarrow 2A = 3^n \cdot 3^{-(n-1)}$

$\Rightarrow 2A = 3^{n-n+1}$

$\Rightarrow 2A = 3$

$\Rightarrow A = 3/2$

ICH1 CH2 1ns TB

Taking 3^n outside,

$3^n\left[A - 2A \cdot \frac{1}{3}\right] = 3^n$

$\therefore \cancel{A - 6A = 1}$

$$\Rightarrow A - \frac{2A}{3} = 1$$

$$\Rightarrow 3A - 2A = 3$$

$$\Rightarrow \boxed{A = 3}$$

Sub. $A=3$ in eqn. 5,

$$a_n^{(P)} = 3 \cdot 3^n = 3^{n+1} \longrightarrow 6$$

Sub. 4 & 6 in 2, we have

$$a_n = c_1 2^n + 3^{n+1} \longrightarrow 7.$$

Given $a_1 = 5$.

(ie) $n=1$

$$a_1 = c_1 2^1 + 3^{1+1}$$

$$\Rightarrow 5 = 2c_1 + 9$$

$$\Rightarrow 2c_1 = -9+5 = -4$$

$$\Rightarrow 2c_1 = -4$$

$$\Rightarrow c_1 = -2.$$

Sub. $c_1 = -2$ in 7, we have

$$a_n = -2 \cdot 2^n + 3^{n+1}$$

(ie) $a_n = 3^{n+1} - 2^{n+1}$

Q) Solve, $a_n = 2a_{n-1} + 2^n$, $a_0 = 2$.

A) The general form is

$$a_n - 2a_{n-1} = 2^n \quad \longrightarrow ①.$$

General solution is,

$$a_n = a_n^{(h)} + a_n^{(P)} \longrightarrow ②.$$

To find $a_n^{(h)}$

The homogeneous equation of ① is

$$a_n - 2a_{n-1} = 0. \longrightarrow ③$$

Put $a_n = r^n$, $(r \neq 0)$ in ③,

③ becomes

$$r^n - 2r^{n-1} = 0$$

$$r^{n-1}[r-2] = 0$$

$\Longrightarrow$ $r-2=0$ is the characteristic equation.

$\therefore$ $r=2$ is the characteristic root.

$$\therefore \quad a_n^{(h)} = c_1 2^n. \longrightarrow ④$$

To find $a_n^{(P)}$

Since R.H.S of ① is 2^n, but 2 is the characteristic root of ① and hence the corresponding choice of $a_n^{(P)}$ will be $An2^n$. [refer Remark 2]

$$\therefore \quad a_n^{(P)} = An2^n \longrightarrow ⑤$$

~~Sub ⑤ in~~

sub. 5 in 1, we have

$$An2^n - 2A(n-1)2^{n-1} = 2^n \Rightarrow An2^n - A(n-1)2^n = 2^n.$$

Taking 2^n outside,

$$\cancel{2^n\left[An - 2An \cdot \tfrac{1}{2}\right] = 2^n}$$

$$\cancel{\Rightarrow nA - nA = 1}$$

$$2^n\left[An - A(n-1)\right] = 2^n.$$

$$An - An + A = 1$$

$$A = 1$$

$\therefore$ 5 becomes $\underline{a_n^{(p)} = n2^n.} \rightarrow 6$

$\therefore$ sub. 4 & 6 in 2,

$$\underline{a_n = c_1 2^n + n2^n.} \rightarrow 7$$

Given $a_0 = 2$,

$\underline{n=0}$

$$a_0 = c_1 2^0 + 0 \times 2^0$$

$$2 = c_1$$

Sub. $c_1 = 2$ in 7, we have

$$a_n = 2 \cdot 2^n + n2^n.$$

$$\underline{\underline{a_n = 2^{n+1} + n2^n = 2^n(2+n)}}$$

Q) Solve $a_{n+2} - 6a_{n+1} + 9a_n = 3\cdot 2^n + 7\cdot 3^n, n \geq 0.$

A) Given, $a_{n+2} - 6a_{n+1} + 9a_n = 3\cdot 2^n + 7\cdot 3^n \rightarrow (1)$

The general solution of (1) is

$$a_n = a_n^{(h)} + a_n^{(p)} \rightarrow (2)$$

To find $a_n^{(h)}$

The homogeneous equation of (1) is,

$$a_{n+2} - 6a_{n+1} + 9a_n = 0 \rightarrow (3)$$

Put $a_n = r^n \; (r \neq 0)$ in (3)

(3) becomes,

$$r^{n+2} - 6r^{n+1} + 9r^n = 0$$

$$r^n\left[r^2 - 6r + 9\right] = 0$$

$r^2 - 6r + 9 = 0$ is the characteristic equation.

$$r = \frac{6 \pm \sqrt{36-36}}{2} = \frac{6}{2} = 3, 3.$$

$$\therefore a_n^{(h)} = (C_1 + C_2 n)3^n \rightarrow (4)$$

To find $a_n^{(p)}$

R.H.S of (1) is $3\cdot 2^n + 7\cdot 3^n$.

Corresponding to $3\cdot 2^n$, we can assume $a_n^{(p)}$ as $A_0\cdot 2^n$. (table)

Corresponding to $7\cdot 3^n$, we can assume $a_n^{(p)}$ as $A_1 n^2 3^n$ [refer Remark 3]

The choice of $a_n^{(p)} = A_0 2^n + A_1 n^2 3^n$ — ⑤

sub ⑤ in ①,

~~$a_{n+2} - 6a_{n+1} + 9a_n = A_0 2^n + A_1 n^2 3^n$~~

$$A_0 2^{n+2} + A_1 (n+2)^2 3^{n+2} - 6\left[A_0 2^{n+1} + A_1 (n+1)^2 3^{n+1}\right] + 9\left[A_0 2^n + A_1 n^2 3^n\right] = 3\cdot 2^n + 7\cdot 3^n$$

(i) $A_0 2^{n+2} + A_1 (n+2)^2 3^{n+2} - 6A_0 2^{n+1} - 6A_1 (n+1)^2 3^{n+1} + 9A_0 2^n + 9A_1 n^2 3^n = 3\cdot 2^n + 7\cdot 3^n$

(ii) $2^n\left[A_0 2^2 - 6A_0\cdot 2 + 9A_0\right] + 3^n\left[A_1 (n+2)^2\cdot 3^2 - 6A_1 (n+1)^2 3 + 9A_1 n^2\right] = 3\cdot 2^n + 7\cdot 3^n$

equating the coefficients of 2^n,

$4A_0 - 12A_0 + 9A_0 = 3$ ⟶ ⑥

equating the coefficients of 3^n,

$9A_1 (n+2)^2 - 18A_1 (n+1)^2 + 9A_1 n^2 = 7$ ⟶ ⑦

$\underline{A_0 = 3}$ &

$9A_1 (n^2+4n+4) - 18A_1 (n^2+2n+1) + 9A_1 n^2 = 7$

$9A_1 n^2 + 36A_1 n + 36A_1 - 18A_1 n^2 - 36A_1 n - 18A_1 + 9A_1 n^2 = 7$

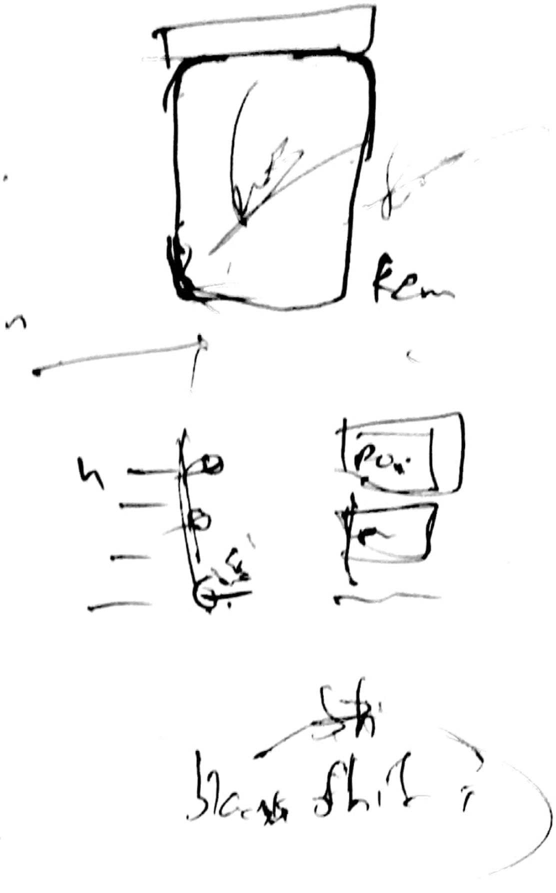
$18A_1 = 7$

$A_1 = 7/18$

sub. in ⑤, we have $a_n^{(p)} = 3\cdot 2^n + \frac{7}{18} n^2 3^n$ ⟶ ⑧

Sub. (1) & (2) in (*),

$$a_n = (C_1 + C_2 n)\,2^n + 3\cdot 2^n + \frac{7}{18}\, 3^n \cdot n.$$



) PT $(A-B)-C = A-(B\cup C)$

LHS = $(A-B)-C$

$x \in [(A-B)-C]$

$x \in A \quad x \notin B \quad x \notin C$

$\Rightarrow x \in A \quad x \notin B$ or C.

$x \in A \quad x \notin B\cup C$

$(A-B)-C \subset A-(B\cup C)$

RHS: $x \in [A-(B\cup C)]$

$x \in A \quad x \notin (B\cup C)$