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Module - VI

Eigen Values and Eigen Vectors

- Let A be a square matrix of order n. Then |A-AI|=0. 15 called the characteristic equation of A.
- The moots of the characteristic eqn is called the Eigen values on characteristic values on Latent noots.
- The set of eigen values is called spectrum of A.
- \Rightarrow The set of all solps to Ax = Ax is called the eigen apace of A and .95 denoted by $E(\lambda)$.
- → The eigen values ob A and A' are some (A'-toranspose)
- → If h 9s an eigen value of A than him 9s an eigen value
- ⇒ If he an Eigen value of A then kh is an Eigen value of kA and 1-k 95 an Eigervalue of A-KI
- => If I is an Eigen value of a non-sing was material then Kis an eigenvalue of A-1 and 1A1 1800000 95 the cigen " value of adjoint of A
 - => Ergen value of torrangular matorices and aragonal mater ces are its discourse now diagonal materia
- => The sum of eggen values of a maloux esthe sum of ets diagonal clements
- ⇒ The product of Ergen values
- ⇒ The eigen values of a symmetric matrix are real and of a skew aymmetric matrix are purely imaginary on **ತಲಾ**ಂ .

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for find er gen vectors.

Casa
$$\mathbf{e} \subseteq J$$
, $A = 3$.

$$\begin{bmatrix} A - \lambda x \end{bmatrix} = \begin{bmatrix} i - 3 & 2 \\ 2 & i - 3 \end{bmatrix} \\
 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \\
 = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} & R_1 \rightarrow R_{1/2} \\
 = \begin{bmatrix} 0 & 0 \end{bmatrix} & R_2 \rightarrow R_2 - 2R_1 \\
 = AX = AX$$

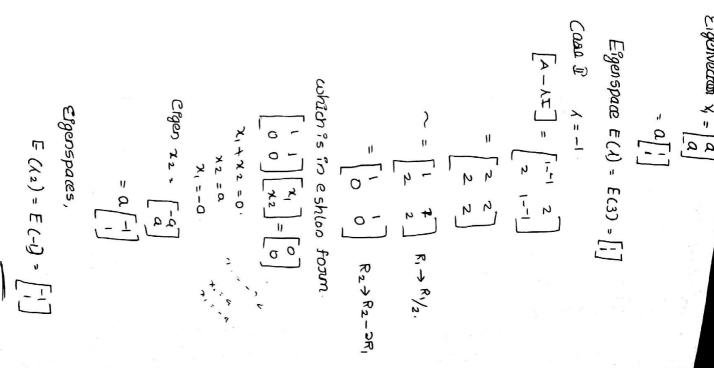
$$\Rightarrow \begin{bmatrix} i & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$X_1 - 2x = 0 \qquad N_1 = f^{M_1}$$

$$X_1 = 0 \qquad N_2 = 0$$

$$X_1 = 0 \qquad N_3 = f^{M_2}$$

$$X_1 = 0 \qquad N_4 = 0$$



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Eigenvecton $x_i = \begin{bmatrix} a \\ a \end{bmatrix}$

? Find the eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

 $|\mathbf{A} - \mathbf{\Lambda} \mathbf{I}| = 0 \implies \begin{vmatrix} 1 - \lambda & 2 \\ 2 & 1 - \lambda \end{vmatrix} = 0$

Kr- xprace as Rit valco

 $\left(1-\lambda\right)^2-4=0$

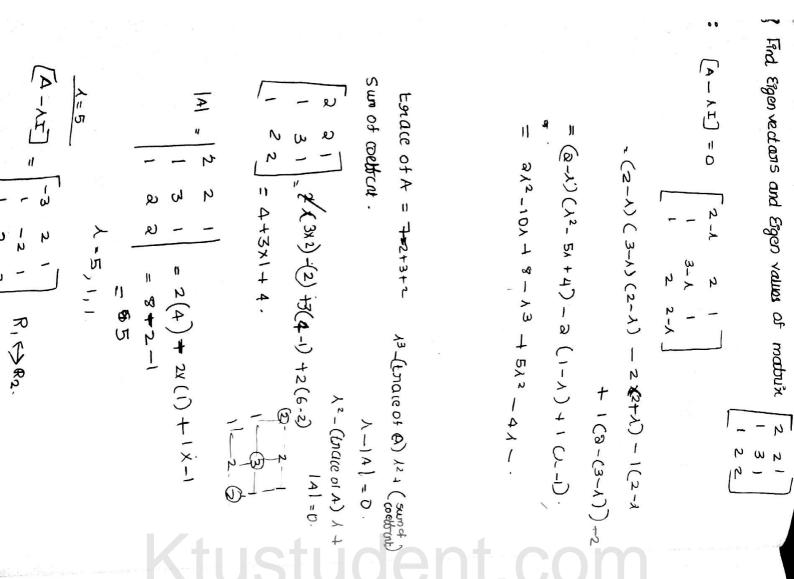
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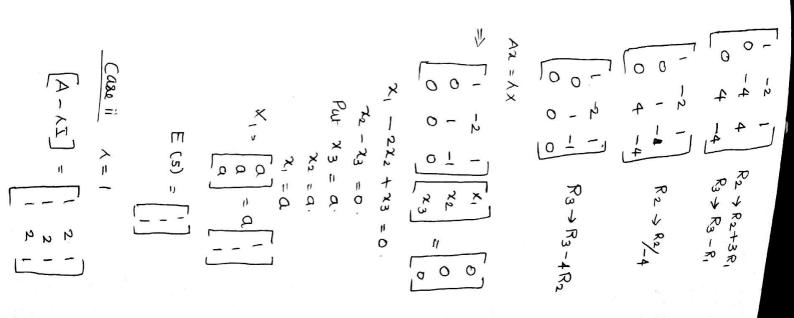
 $|-21+1^2-4=0$

1 = 3, -1

(4-3)(4+1) =0

12-04-3=0





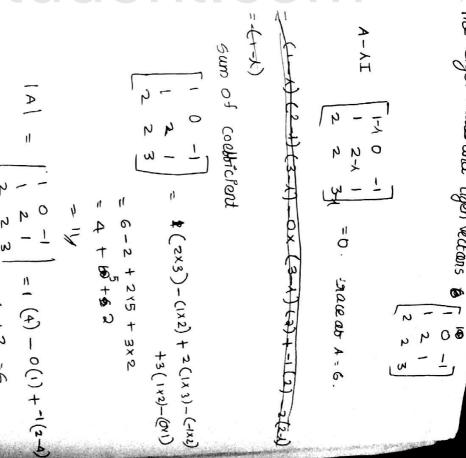
$$|A| = \begin{cases} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{cases} = 4 + 2 = 6,$$

$$|A| = \begin{cases} 1 & 2 & 1 & 1 \\ 2 & 2 & 3 \\ 3 & 4 & 4 & 2 \\ 4 & 4 & 4 & 4 \end{cases}$$

$$|A| = \begin{cases} 1 & 2 & 1 & 4 \\ 2 & 2 & 3 \\ 4 & 4 & 2 & 6 \end{cases}$$

$$|A| = \begin{cases} 1 & 2 & 1 & 4 \\ 2 & 2 & 3 \\ 4 & 4 & 2 & 6 \end{cases}$$

$$|A| = \begin{cases} 1 & 2 & 1 & 4 \\ 2 & 2 & 3 \\ 4 & 4 & 2 & 6 \end{cases}$$



2 Find the Eigen Yalus and ligen vectous & 1

$$(\lambda - 2)(2\lambda - 3)\lambda = 2, 3, 1$$

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$$= \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

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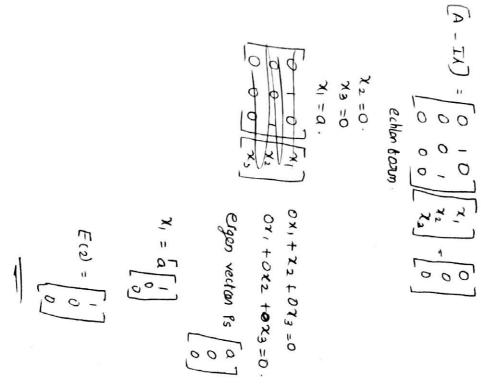
$$= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

here the given mother is a upper tollangular malaira . Its ergen values are its diagonal etts 1=222



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Find the eigen value and vectors at matrix [2 10]

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The chanadeustic een is (1-2)3 =0

Diagonalise the modul
$$z$$
 $a = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

A^T = A

i. A is supmoetable
$$\begin{vmatrix} (2-i)((2-i)(2-i) - 0) & 0 & 1 \\ 1 & 0 & 2-i & 0 \\ 1 & 0 & 2-i \end{vmatrix} = 0$$

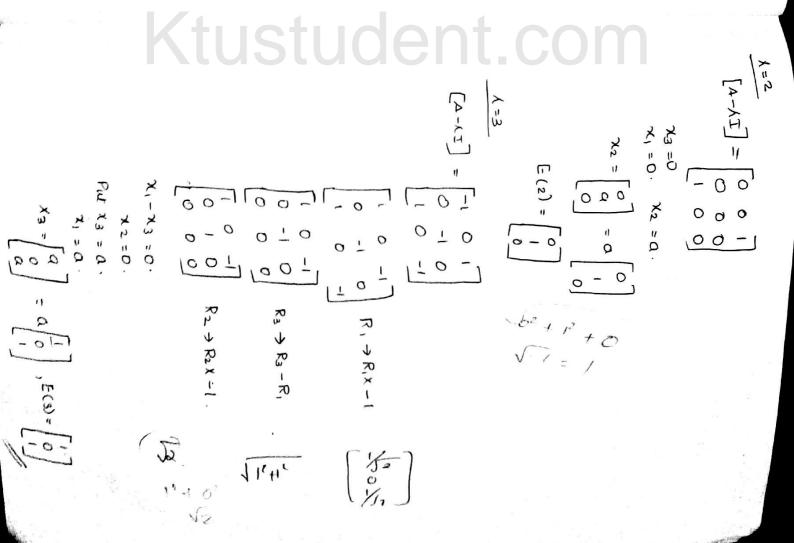
$$(2-i)((2-i)(2-i) - 0) - 0 + 1 (0 - (2-i)) = 0$$

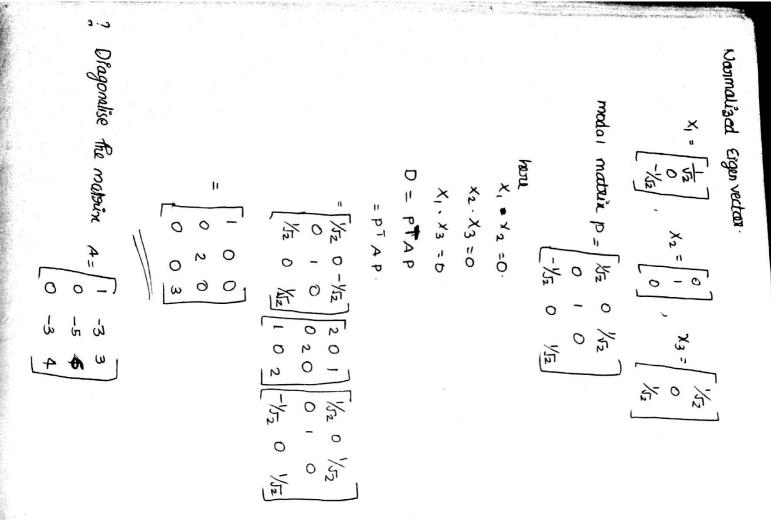
$$(2-i)[a - 4i + i^2 - 1] = 0$$

$$(2-i)[a - 4i + i^2 - 1] = 0$$

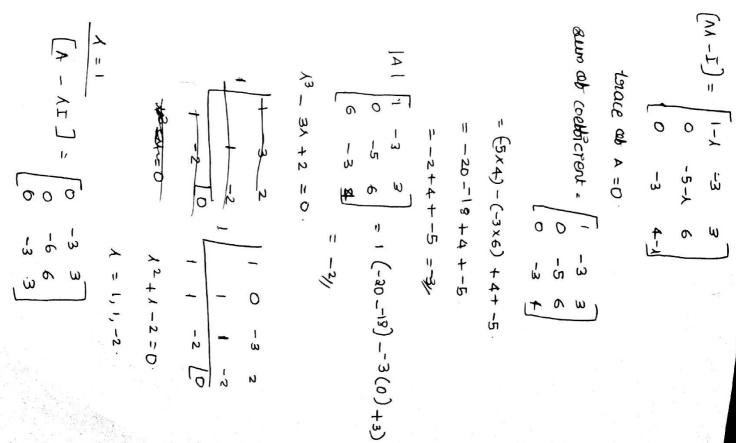
$$(2-i)[a - 3i(-1)] = 0$$

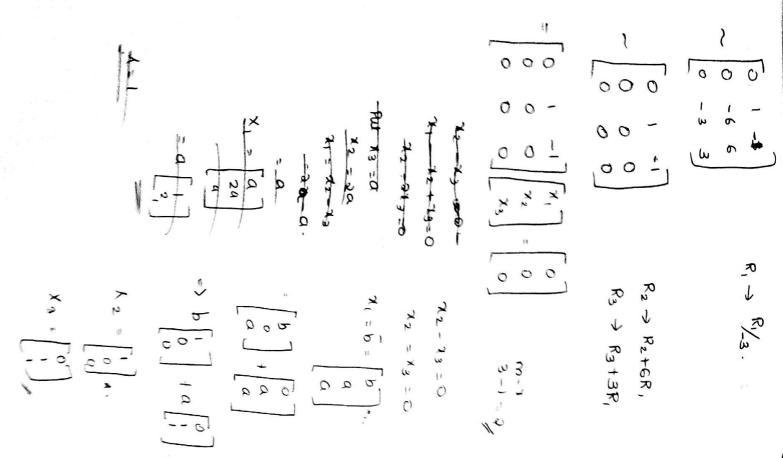
$$(3-i)[a - 3$$

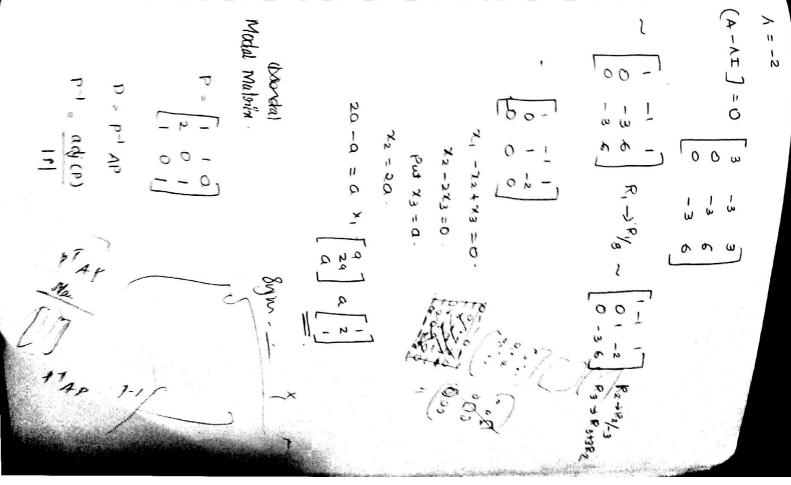




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Examine whether the material
$$\begin{bmatrix} 2 & 2 \\ 0 & 2 \\ -1 & 2 \end{bmatrix}^{2}$$
 of deagonalisable.

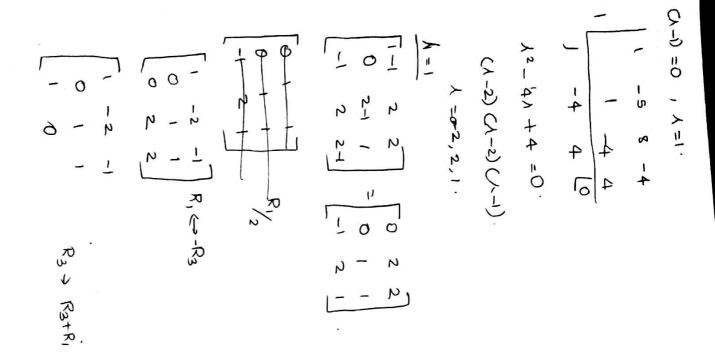
AT = $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & 2 \\ 2 & 1 & 2 \end{bmatrix}$, AT \neq A an asymmetric $\begin{bmatrix} AA - T \end{bmatrix} = \begin{bmatrix} 1 - A & 2 & 2 \\ -1 & 2 & 2 \\ -1 & 2 & 2 - A \end{bmatrix}$

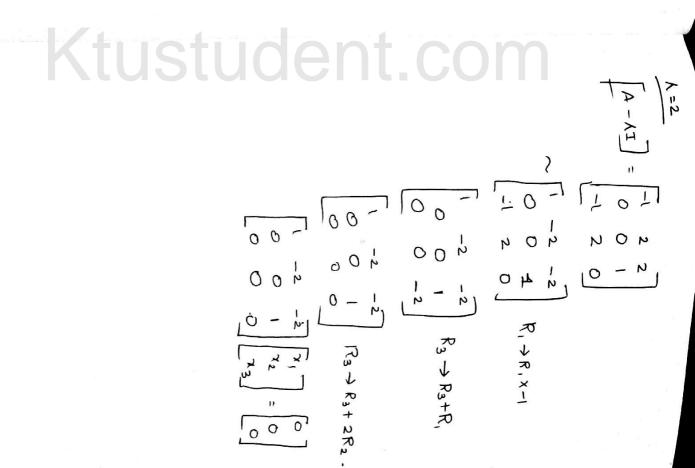
Lea are at $A - 5$.

Sum at coestident = $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 2 & 2 \\ -1 & 2 & 2 \end{bmatrix}$

$$= \underbrace{(2 + 2)}_{=} + \underbrace{(2 + 2)}_{=} + \underbrace{(2 - 2$$

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? If a is an Eigen value of
$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \end{bmatrix}$$
 without using its characteristic eqn. Bind the other Eigen values also bind the eigen values

Let $\lambda_1 = 2$

Let $\lambda_1 = 2$
 $\lambda_1 + \lambda_2 + \lambda_3 = 1$
 $\lambda_2 + \lambda_3 + \lambda_3 = 1$
 $\lambda_3 + \lambda_3 + \lambda_3 = 1$
 $\lambda_4 + \lambda_2 + \lambda_3 = 1$
 $\lambda_5 + \lambda_5 + \lambda_5 = 1$

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Vector spare:

space ower a bread F, if it satisfies the bollowing condition and scalar multiplication denoted by '+ and '.' I respectively is a vector of known empty set V with two openations called addition

) If u, ve v then u+v ev (closure property under addition

2) Let u, v, w E v been any vectors . u+(v+a) = (u+v)+w cassociative ph)

3) Them is a vector OEV, but any UEV O+ U= U+0 = U (existence at additive Educative)

4) For every uer, those is -uers.i- u+-u=-u+u=0 (existence ab additive goverse)

5) Foot any u, v, ev U+v=v+u (Commutative 1912ty).

6) Fam any nevana cer cinev colosura ppy under scalar multi plication)

a) from any u, vev and cer c, cutv) = c, u + c, v.

8) from any nev and a, b & F (ab). u = a. (bu)

10) Fan any UEV YEAD CHYSLAM FIGE. 5, I'L = U a) from any user and assert a, bef (a+b). u= a.u+bu

check notherhose the set of setal numbers with equal addition and multiplication 15 a vector securite ou not

i) Foot any u, v & IR U+v & R (closume pphy southshed)

11) For how ER (U+V)+W = U+(V+W) Cassociative ppty scales from uer 2 oer 5 uto =otu = u

v) ben u, ver , as atv=v+u. ton uer, -uer & u+-u=-u+u=0.

bon uer, lef cuer

i igs bet of polynomials together with 300 vii) bon any user and CEF C. CHY)= C.U+C.Y

(ab) u = a (ba) a (u+v) = a·u+a·V ucr.bu=u

perymorated is a vector space. multiplication is a rectorcipice. set at all myn matrixes north matrix addition and

? A two dimensional Euclidian plane is a vectous pue culd Eucledian plane

1) UVE R 2 Ut V C IR2

11 U, V, W E IR2 CUTYTW) = UT (V+W).

III) LIGHT O = CON) ER? U+O =O+U=U.

UEIR2 - UEIR2 . U+-11 = -4+11

Vi) UERZ COF CIUERZ.

ツ リ、ソの民2 ロチャーンサム・

vii) a, b cr u elR2 Ca+b.u = a.u + b.u

a, b e f u e 182 (ab) u = a (b u).

IEF , UER2 I·U=U ace une eles acutu) = a·uta·v.

? check whethen this utremen sect v at all the seal no with spau on not the aperations utv=uv and eu = uc is a vector

1) u, v, w & Rt (u+v)+w= (uv)+w=uvw=ucvw) 3) UNCIRAT WYSUVERT = u+(vw) = u+(v+w).

iii) ucle+ 16R+ u+1 = 1+u=u·1=u

CEF CIU- WCERT

A) HP aber (a+b). u=ua+b=ua. Ub=ua+lb=au+b.u

with a feet a, bef (ab). u = uab = (ub) = a (ub) = a · (bu).

V.D. D=V46H = 604H) = (V+W) = (V+B) = 0.014P = 0.014P. x) u E 1 P2 | EF | 1 L = U = U .

Plineous Independence and dependence

theas combination

element in vithen any vector x in v is a linear combination ab x, 12, 13 X, . It there oxist scalars a, a2 an . 5. Let V be a vector space ower a steld F and lat E.F., Fn

2 = 97+922+····+2020

to be linearly dependend. let V be a vectorspace own a field F the vectors is said

the voction is said to be linearly independent $48 \, a_1 \, x_1 + a_2 \, x_2 + \cdots + a_n \, x_n = 0 \Rightarrow a_1 = a_2 = \cdots = a_n = 0$ spanning Set:

set at the rectanspace v. 16 each rectan in v can be at scalans f the subset 1+(v) is said to be a span in expressed us a linear combination cet ets at the Let # be a subset at a releanspace owen the steld

blasts and dimension at v.s

v.5, 18 it is both line couly independent and spenning set at the vectors spuce A Bubset Haba V5 'V' 15 cauld the books at the

The no: at eles in the busis is called dimension at vis

? Exposes the vector v = (1, -2, 5) as a linear combination ab the vectoons v, = (1,1,1) , v2 = (62,3), v3 = (2,-1,1). V= V1 91 + 1292 + V303.

(1,-2,5) = a, (1,1,1) + a2(1,2,3) +a3(2,-1,1) = (9,,9,,9) + (92,292, 302) +(203,43,9)

00 5 70 (1) -(1) -2 73 73 上る 7

0012 3 -3 Rg -> R3/5

which is in echlon-boom. by back substitu

0a, + a= -3a3 = -3 $0a_1 + 002 + a_3 = 2$ a, + a2 + 2a3 = 1 q2 = -3 +343 = -3+6=3 a, = 1-92-893 91+202-03=-2 91+92+203=1 03=2 91+302+93=5 *1-3-2X2

V= (2,-5,3) cannot be expressed as a lineau combination out the vectors (1,-3,2), (2,-4,-1), (1,-6,7) · (1,-2,5)=-6(1,1,1)+3(1,2,3)+2(2,-1,1) V= 9, V1 + 02 V2 + 03 V3

⟨ (2) - 5, 3) = a, (1, -3, a) +a2(2,4,-1) +a3(1,-5,7) =(a1,-3a1, a41)+(a02, 402,-02) + (a3, -5ag, 7a3)

917202+03=2. = (9,+202+03), =301+402+-593, 29, + - 92 + 703

-3a1 + 4 a2-5a3 =-5

201 - 02 + 703 = 3

unearly independent 5-2+2.

unearly endependent. = 5 to them Yalus one

Show that the rectains (1,1,1), (1,2,3) & (2,1,1) are G G 0 00 9 which is in Echlon bown a, +x2+ a3 = 2. a2-a3 = 1/2 Q2 = 1/2+Q3. R2 -> R2+3R1 R3 > R3 - 2R, R2 -> Re/2 1 (2+3)-1 (1+1)+2(3-2) R3 V R3 # 5 R

dependent and bend a edation connecting them. powe that the vectors (1,2,1), (6,1,4) and (4,5,6) are unearly

the given set of vertigs is threatly dependent

13-44,+3 (x21/-3)=0.

X3 - 4 X1 - (xz1) =0

13 - 2x1 - x2 -0

x3 -4x1 -x2+2x1 =0

X3-4x1-(x2-2x1)=0

X31+3X2" = 0.

uneany dependent prove that the vectors (2,3,0), (1,2p). (1,3,0) on

? Brown that the vectors (1,-1,1), (0,1,2) and (3,0,-1) beam a basis bon R

here mank = 3 - an dimension = 3. 01-10 $R_3 \rightarrow R_3 - 3R_2$ $R_3 \rightarrow R_3 - 3R_1$

Find the busis of the vectors (1,1,1), (1,2,3), (2,-1,1) and (3,0,-1) bom a basis boon R3 busis boon Rn, hence the thruce independent vection (1,-1,1), (0,1,2) we know that any linearly independent redoors at Rh Born a

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-4 - 31 -4-6

A =
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix}$$
 Row seedweed echlon boarm
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$
 $R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 5 \end{bmatrix} \xrightarrow{R_3 \to R_3 + 3R_2} R_3 + 3R_2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3/5} R_3/5.$$

Rank = 3.

basis: {(1,0,1), (0,1,2), (0,0,1)}