



QUESTION PAPER WITH SOLUTION

Reg. No. _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER BTECH DEGREE EXAMINATION JULY 2017

Course Code: CS 201

Course Name: **DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max Marks: 100

Duration: 3 hrs

PART A*Answer all questions, 3marks each.*

1. Draw the Hasse diagram for the divisibility relation on the set $A = \{2, 3, 6, 12, 24, 36\}$
2. Define equivalence relation? Give an example of a relation that is not an equivalence relation?
3. In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?
4. Define homomorphism and isomorphism?

PART B*Answer any 2 Questions, 9 marks each.*

5. (a) Consider f, g and h are functions on integers $f(n) = n^2$, $g(n) = n + 1$, $h(n) = n - 1$. Determine
 i) $fogoh$ ii) $gofoh$ iii) $hofog$ (6)
- (b) State Pigeonhole Principle. Prove that at least two of the children were born on the same day of the week. (3)
6. (a) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$ (6)
 (b) Show that set of all integers is countable (3)
7. (a) Find the number of integers between 1 and 250 both inclusive that are not divisible by any of integers 2, 3, 5 and 7. (5)
 (b) If $*$ is the binary operation on the set R of real numbers defined by $a*b=a+b+2ab$. Find if $\{R, *\}$ is a semigroup. Is it commutative? (4)

PART C*Answer all questions, 3marks each.*

8. Show that the set $\{1, 2, 3, 4, 5\}$ is not a group under addition modulo 6.
9. Prove that every distributive lattice is modular.
10. Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation $*$ defined by $a*b = \frac{1}{2}ab$; $a, b \in Q^+$
11. Simplify the Boolean expression $a'b'c + ab'c + a'b'c'$

PART D*Answer any 2 Questions, 9 marks each.*

12. For the set $I_4 = \{0, 1, 2, 3\}$, show that modulo 4 system is a ring. (9)
13. a) Prove that the order of each sub group of a finite group G is a divisor of the order of group G. (6)
 b) If $A = (1 \ 2 \ 3 \ 4 \ 5)$ and $B = (2 \ 3)(4 \ 5)$. Find product of permutation AB. (3)
14. If (L, \leq) is a lattice, then for any $a, b, c \in L$, the following properties hold. If $b \leq c$,
 then i) $a \vee b \leq a \vee c$ ii) $a \wedge b \leq a \wedge c$ (9)

PART E*Answer any 4 Questions, 10 marks each.*

15. (a) Show that $(P \rightarrow Q) \wedge (Q \rightarrow P)$ is logically equivalent to $P \leftrightarrow Q$. (5)
 (b) Suppose x is a real number. Consider the statement "If $x^2 = 4$, then $x = 2$."
 Construct the converse, inverse, and contrapositive (5)
16. (a) Prove that $(P \wedge Q) \rightarrow (P \leftrightarrow Q)$ is a tautology. (5)
 (b) Show that $(a \vee b)$ follows logically from the premises
 $\text{pvq}, (\text{pvq}) \rightarrow \neg r, \neg r \rightarrow (\text{sa} \neg t) \text{ and } (\text{sa} \neg t) \rightarrow (a \vee b)$ (5)
17. (a) Represent the following sentence in predicate logic using quantifiers i) All men are mortal. ii) Every apple is red iii) Any integer is either positive or negative. (6)
 (b) Use the truth table to determine whether $p \rightarrow (q \wedge \neg q)$ and $\neg p$ are logically equivalent. (4)
18. Show that the premises "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high paying job" imply the conclusion "Someone in this class can get a high paying job" (10)
19. (a) Prove the following statement by contraposition:
 If a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10. (6)
 (b) Express the negation of the following statement in English using quantifiers
 i) If the teacher is absent, then some students do not keep quiet ii) All students keep quiet and teacher is present. (4)
20. (a) Prove that $\sqrt{2}$ is irrational using proof by contradiction. (6)
 (b) Find the truth table $(\sim Q \Rightarrow \sim P) \Rightarrow (P \Rightarrow Q)$ (4)

SOLUTION

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION JULY 2017

CS201: DISCRETE COMPUTATIONAL STRUCTURES(CS, IT)

Max. Marks:100

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ANSWER KEY

Part A(Answer all, 3 marks each)

1.



2. Equivalence relation - reflexive($a R a$, for all $a \in X$), symmetric($a R b$ implies $b R a$, for all $a, b \in X$) and transitive($a R b$, $b R c$ implies $a R c$, for all $a, b, c \in X$) (2)

$R = \{(1,1), (2,1), (1,3), (3,3)\}$ is not an equivalence relation on set $X = \{1,2,3\}$ (1)

3. MATHEMATICS - total 11 letters; considering all the vowels together as one letter we have 8 letters out of which two letters(M, T) are repeating twice. Thus, no. of different ways is

$$\frac{8!}{2!*2!} = 10080. \quad (3)$$

4. A function $f : (X, *) \rightarrow (Y, +)$ is a homomorphism if $f(a * b) = f(a) + f(b)$. (2)

A bijective homomorphism is an isomorphism. (1)

Part B(Answer any 2, 9 marks each)

5. (a) i) $fogoh = f(g(h(n))) = f(g(n-1)) = f((n-1)+1) = f(n) = n^2$ (2)

ii) $gofoh = g(f(h(n))) = g(f(n-1)) = g((n-1)^2) = (n-1)^2 + 1 = n^2 - 2n + 2$ (2)

iii) $hofog = h(f(g(n))) = h(f(n+1)) = h((n+1)^2) = (n+1)^2 - 1 = n^2 + 2n$ (2)

(b) Pigeonhole principle: Suppose there are k pigeonholes and n pigeons to be placed in them. If $n > k$, then at least one pigeonhole contains at least two pigeons. In mathematics, if n items are put into m containers, with $n > m > 0$, then at least one container must contain more than one item. (2)

$365 = 52 * 7 + 1$ implies remainder = 1, thus at least two children born on same day (1)

6. (a) Ch. eqn.: $r^2 - 4r + 4 = 0$; roots - 2, 2; thus, $a_h = (c_1 + c_2 n)2^n$; $a_p = An^2 2^n$ thus, total solution $a_n = a_h + a_p = (c_1 + c_2 n)2^n + An^2 2^n$ (6)

(b) Define $f: \mathbb{N} \rightarrow \mathbb{Z}$ such that $f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ -\frac{n-1}{2}, & \text{if } n \text{ is odd} \end{cases}$, then f is bijective. Hence, $\mathbb{Z} \approx \mathbb{N}$ and so \mathbb{Z} is denumerable which implies \mathbb{Z} is countable. (3)

7. (a) $|A| = |\text{divisible by 2}| = 125$; $|B| = |\text{divisible by 3}| = 83$; $|C| = |\text{divisible by 5}| = 50$; $|D| = |\text{divisible by 7}| = 35$; $|A \cap B| = 41$; $|A \cap C| = 25$; $|A \cap D| = 17$; $|B \cap C| = 16$; $|B \cap D| = 11$; $|C \cap D| = 7$; $|A \cap B \cap C| = 8$; $|A \cap B \cap D| = 5$; $|B \cap C \cap D| = 2$; $|A \cap C \cap D| = 3$; $|A \cap B \cap C \cap D| = 1$ (2)

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D| = 229 \quad (2)$$

thus not divisible by 2, 3, 5, 7 = $250 - 229 = 21$ (1)

(b) $a * b = a + b + 2ab \in \mathbf{R} \rightarrow$ closed (1)

$$(a * b) * c = a + b + c + 2ab + 2bc + 2ac + 4abc = a * (b * c) \rightarrow \text{associative} \quad (1)$$

thus, $(\mathbf{R}, *)$ is a semigroup (1)

$$a * b = a + b + 2ab = b + a + 2ba = b * a \rightarrow \text{commutative} \quad (1)$$

Part C(Answer all, 3 marks each)

8. $5 + 1 \pmod{6} = 0 \in \{1, 2, 3, 4, 5\} \rightarrow$ not closed \rightarrow not a group (3)

9. $a \wedge (b \vee (a \wedge d)) = (a \wedge b) \vee (a \wedge (a \wedge d)) = (a \wedge b) \vee (a \wedge d) \rightarrow$ modular lattice (3)

10. $a * b = (1/2)ab \in \mathbf{Q}^+ \rightarrow$ closed; $(a * b) * c = (1/4)abc = a * (b * c) \rightarrow$ associative;
 $a * b = (1/2)ab = (1/2)ba = b * a \rightarrow$ commutative. (1)

$$\text{identity} = 2 \in \mathbf{Q}^+ ; \text{inverse} = 4/a \in \mathbf{Q}^+ \quad (2)$$

$$11. a'b'c + ab'c + a'b'c' = (a' + a)b'c + a'b'c' = 1.b'c + a'b'c' = b'(c + a'c') = b'(c + a') \quad (3)$$

Part D(Answer any 2, 9 marks each)

12. $+(mod 4)$	0	1	2	3	$*(mod 4)$	0	1	2	3
0	0	1	2	3	0	0	0	0	0
1	1	2	3	0	1	0	1	2	3
2	2	3	0	1	2	0	2	0	2
3	3	0	1	2	3	0	3	2	1

closed, associative, commutative, identity and inverse exist, distributive; thus a ring (9)

13. (a) Lagrange's theorem: H - subgroup of G; $o(H) = m$, $o(G) = n$; every coset of H has same no. of elements as H. Let r be no. of blocks in the partition of G into left cosets of H. Then $n = rm$. (6)

$$(b) \quad A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 5 & 4 \end{pmatrix} \quad AB = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 1 & 5 \end{pmatrix} = (1 \ 2 \ 4) \quad (3)$$

14. (9)

Part E(Answer any 4, 10 marks each)

15. (a) (5)

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$	$P \leftrightarrow Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

(b) p: $x^2 = 4$; q: $x = 2$; $p \rightarrow q$: If $x^2 = 4$, then $x = 2$. Converse - $q \rightarrow p$: If $x = 2$, then $x^2 = 4$. Inverse - $\neg p \rightarrow \neg q$: If $x^2 \neq 4$, then $x \neq 2$. Contrapositive - $\neg q \rightarrow \neg p$: If $x \neq 2$, then $x^2 \neq 4$. (5)

16. (a) (5)

P	Q	$P \wedge Q$	$P \leftrightarrow Q$	$(P \wedge Q) \rightarrow (P \leftrightarrow Q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	T	T

(b) (5)

Steps	Reasons
1 $p \vee q$	Premise
2 $(p \vee q) \rightarrow \neg r$	Premise
3 $\neg r$	Modus Ponens(1, 2)
4 $\neg r \rightarrow (s \wedge \neg t)$	Premise
5 $s \wedge \neg t$	Modus Ponens(3, 4)
6 $(s \wedge \neg t) \rightarrow (a \vee b)$	Premise
7 $a \vee b$	Modus Ponens(4, 5)

17. (a) i) $p(x)$: x is mortal; $\forall x p(x), x \in \{\text{men}\}$ (2)

ii) $p(x)$: x is red; $\forall x p(x), x \in \{\text{apple}\}$ (2)

iii) $p(x)$: x is positive, $q(x)$: x is negative; $\forall x [p(x) \vee q(x)], x \in \mathbf{Z}$ (2)

(b)

(4)

P	Q	$\neg Q$	$Q \wedge \neg Q$	$P \rightarrow (Q \wedge \neg Q)$	$\neg P$
T	T	F	F	F	F
T	F	T	F	F	F
F	T	F	F	T	T
F	F	T	F	T	T

18. $p(x)$: x know to write programs in JAVA. $q(x)$: x get a high paying job.

(10)

Steps	Reasons
1 $\exists x p(x)$	Premise
2 $p(c)$ for some c	Existential Specification
3 $\forall x p(x) \rightarrow q(x)$	Premise
4 $p(c) \rightarrow q(c)$	Universal Specification
5 $q(c)$	Modus Ponens(2, 4)
6 $\exists x q(x)$	Existential Generalisation

19. (a) Assume the two nos. are less than or equal to 10. Then product can only be maximum 100.

Thus by contraposition, at least one of them should be greater than 10.

(6)

(b) p : teacher is present. $q(x)$: x keep quiet.

i) $\neg[\neg p \rightarrow \exists x \neg q(x)] = \neg[p \vee \exists x \neg q(x)] = \neg p \wedge \forall x q(x)$: The teacher is absent and all students keep quiet. (2)

ii) $\neg[\forall x q(x) \wedge p] = [\exists x \neg q(x) \vee \neg p]$: Some students do not keep quiet or teacher is absent. (2)

20. (a) Assume $\sqrt{2}$ is rational, then $\sqrt{2} = \frac{p}{q}$ where p, q are relatively prime with $q \neq 0$. Squaring, $p^2 = 2q^2 \rightarrow p^2$ is even $\rightarrow p$ is even $\rightarrow p = 2k$, for some integer k . Then, $(2k)^2 = 2q^2 \rightarrow q^2 = 2k^2 \rightarrow q^2$ is even $\rightarrow q$ is even. This contradicts the assumption and hence $\sqrt{2}$ is irrational. (6)

(b)

(4)

P	Q	$P \rightarrow Q$	$\neg P$	$\neg Q$	$\neg Q \rightarrow \neg P$	$(\neg Q \rightarrow \neg P) \rightarrow (P \rightarrow Q)$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

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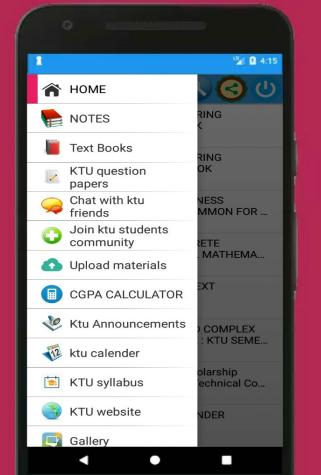
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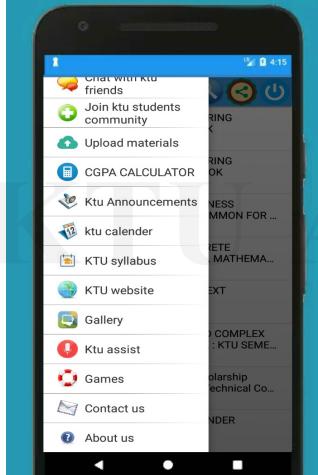
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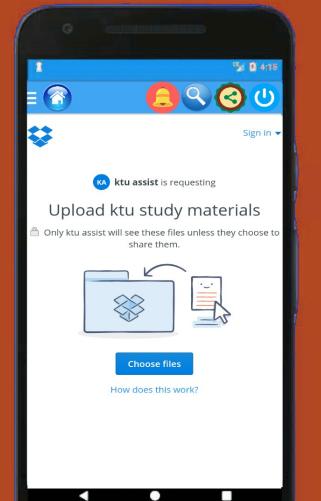
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