Reg. No._____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DEC 2016

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration:3. Hours

PART A

(Answer any *two* questions)

1.a Show that $u = y^3 - 3x^2y$is harmonic and hence find its harmonic conjugate. (8)

b Find the image of $\left|z - \frac{1}{2}\right| \leq \frac{1}{2}$under the transformation $= \frac{1}{z}$. Also find the fixed points of the transformation $w = \frac{1}{z}$ (7)

2.a Define an analytic function and prove that an analytic function of constant modulus is constant. (8)

b Find the linear fractional transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$onto $w_1 = -1,\ w_2 = -i, w_3 = 1$ respectively. (7)

3.a Show that $f(z) = e^{-x}cosy - ie^{-x}siny$ is differentiable everywhere. Find its derivative. (8)

b Find the image of the lines $x = c$ and $y = k$, where c&$k$are constants, under the transformation $w = sinz$. (7)

PART B

(Answer any *two* questions)

4.a Evaluate $\int_C Re\ (z)\, dz$ where C is a straight line from 0 to $1 + 2i$. (7)

b Show that $\int_0^\infty \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$ (8)

5.a Integrate $\frac{z^2}{z^2-1}$ counterclockwise around the circle $|z - 1 - i| = \frac{\pi}{2}$ by Cauchy's Integral Formula. (7)

b Evaluate $\int_C \frac{z-23}{z^2-4z-5}dz$ where C is $|z - 2 - i| = 3.5$ by Cauchy's Residue Theorem (8)

6.a If $f(z) = \frac{1}{z^2}$ find the Taylor series that converges in $|z - i| < R$and the Laurent's series that converges in $|z - i| > R$. (8)

b Define three types of isolated singularities with an example for each. (7)

PART C

(Answer any *two* questions)

7.a Solve by Gauss Elimination:

$$x_1 - x_2 + x_3 = 0,$$
$$-x_1 + x_2 - x_3 = 0,$$
$$10\ x_2 + 25\ x_3 = 90,$$
$$20\ x_1 + 10\ x_2 = 80.$$

(5)

b Find the rank. Also find a basis for the row space and column space for

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$$

(5)

c Find out what type of conic section the quadratic form $Q = 17\ x^2 - 30\ xy + 17\ y^2 = 128$ represents and transform it to the principal axes. (10)

8.a Find whether the vectors $[1 \quad 2{-}1 \quad 3]$, $[2 \quad -13 \quad 2]$ *and* $[-1 \quad 8{-}9 \quad 5]$ are linearly dependent. (5)

b Show that the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$ is symmetric. Find the spectrum. (5)

c Diagonalise $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (10)

9. a. Determine whether the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & {}^{1}/_{\sqrt{2}} & {}^{-1}/_{\sqrt{2}} \\ 0 & {}^{1}/_{\sqrt{2}} & {}^{1}/_{\sqrt{2}} \end{bmatrix}$ is orthogonal? (5)

b. Find the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ (5)

c. Define a Vector Space with an example. (10)