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MODULE - 5

Proporsitional Logic

Proporsitions.

A proposition is a statement which is either true or false but not the both

ex: Jawaharlal Nehru is the 1st Prime Minister of India - It is a proposition

Why is your name - not proposition

If $x^2 = 13$. what is the value of x - not propositn

Remark :-

The two truth values are 'True' & 'False' and can be denoted by the symbols 'T' or '1' and 'F' or '0' respectively.

Remark:-

The propositions are usually denoted by the lowercase letters starting with 'p'

?. Classify the following statements as propositions or non-proposion

- The population of India goes upto 100 million in the year 2000. - propositn
- $x+y=30$ - not propositn.

- Come here - not propostn.

- **Truth Table**

A truth table displays the relationship b/w the truth values of compound propositions constructed from simpler propositions

Logical Connectives

1) **Conjunction**

The conjunction of the proposition 'p' & 'q' denoted by 'p∧q' and it is read. 'p AND q'. Conjunction will have truth value 'T' or '1' when both 'p' & 'q' have the truth value 'T' or '1' In all other cases it will be false.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

2) **Disjunction**

The expression for disjunction is given to be 'p∨q' where 'p' & 'q' are propositions and it is read 'p OR q'. The disjunction will have the truth value 'T' or '1' when either one or both 'p' & 'q' are true & is false when both 'p' & 'q' are false.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

3) Negation

Negation means the opposite of the original proporsition. Negation of p which is denoted by '~p' or '¬p' is a proposition which is true when 'p' is false & is false when 'p' is true.

p	¬p
T	F
F	T

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4) Implication or Conditional Connective.

We say that "p ~~implies~~" q & is denoted by "p→q" where p is called the hypothesis & q is called the conclusion. This implication have the truth value false only when p is true & q is false and in all other cases the truth value will be true. We can denote the implication by

i) if p, then q

ii) p is sufficient for q

iii) p is a sufficient condition for q
iv) q is necessary for p
v) q is a necessary condition for p.
vi) p only if q

Truthtable

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Different Types of Implications.

i) Contrapositive

The proposition $\neg q \to \neg p$ is called contrapositive of $p \to q$

ii) Converse

The proposition $q \to p$ is called the converse of $p \to q$

iii) Inverse

The proposition $\neg p \to \neg q$ is called the iverse of $p \to q$.

5) Biconditional

The biconditional of two statement p & q is denoted by $p \leftrightarrow q$ which is read "p if and only if q" or "p is necessary and sufficient for q"

The proposition $p \leftrightarrow q$ have the truth value false, if p and q donot have the same truth values and is true when both p and q have same true values.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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? Consider the propositions p: He is a rich man and q : He is not greedy. Write the contrapositive, converse & inverse of the implication $q \rightarrow p$.

A. Implication

Here $q \rightarrow p$ is the preposition if ~~p~~
If he is not greedy he is a rich man.

Contrapositive.

$\neg p \rightarrow \neg q$.

If he is not a rich man then he is greedy which is the proposition as above.

Converse

$p \rightarrow q$.

If. he is a rich man then. he is not greed.

Inverse $\neg q \rightarrow \neg p$ which is the proposition
If he is ~~a~~ greedy they he is not a rich man.

? Draw the truth table for the propositions.

$p \vee (q \wedge r)$ & $(p \vee q) \wedge r$.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	F
F	F	T	F	F
T	T	F	F	T
F	T	T	T	T
T	F	T	F	T
F	F	F	F	F

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	T	F	F
T	T	F	T	F
F	T	T	T	T
T	F	T	T	T
F	F	F	F	F.

? Draw the truth table of $(p \to q) \leftrightarrow (\neg q \to \neg p)$

$\neg p$	$\neg q$	p	q	$p \to q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
F	F	T	T	T	T	T
F	T	T	F	F	F	T
T	F	F	T	T	T	T
T	T	F	F	T	T	T

? A preposition p is called to be tautology if it is true under all circumstances. That means it contains only the truth value 'T' in the final column of the truth table. The above question is an example for tautology.

If it is all false then it is called contradiction.

A compound statement ie, neither a tautology nor a condridiction is called a contingency.

? Check whether the preposition $(p \wedge \neg p)$ belongs to tautology, contridiction, contigency.

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

It is a contridiction.

Logical Equalence

Two propositions are said to be logically equalent if they have exactly the same truth values under all circumstances. It is denoted by '$\cong$' or '$\equiv$'

? Check whether $(\neg p \vee \neg q) \cong \neg(p \wedge q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \wedge q$	$\neg(p \wedge q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

$\therefore (\neg p \vee \neg q) \cong \neg(p \wedge q)$

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? ~~Consider~~ ~~proposition p such that p. He is a~~

~~ft~~

Remark : We use the denotion T_0 tautology & F_0 for contradiction.

Remark : Simple proposition are also called as primitive statements.

Precedance of Logical Operators.

1) The bracketed expressions are always evaluated first & normally we do our evaluation from left to right.

2) The negation operator before all other operators.

3) The conjunction operator is to be applied before disjunction.

4) The implication operation is done before bicondition

Laws of Logic

For any primitive statements p, q & r and any tautology T_0 & for contradiction F_0 we have following laws.

1. Law of Double Negation

$\neg(\neg p) \Leftrightarrow p.$

2. De-Morgan's Law

$\sim(p \wedge q) \Leftrightarrow \neg p \vee \neg q.$

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

3. Commutative Law.

$$p \vee q \Leftrightarrow q \vee p.$$

$$p \wedge q \Leftrightarrow q \wedge p.$$

4. Associative Law

$$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r.$$

$$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$$

5. Distributive Law

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

6. Idempotent Law.

$$p \vee p \Leftrightarrow p.$$

$$p \wedge p \Leftrightarrow p$$

7. Identity Law.

$$p \vee F_0 \Leftrightarrow p.$$

$$p \wedge T_0 \Leftrightarrow p$$

8. Inverse Law.

$$p \vee \neg p \Leftrightarrow T_0$$

$$p \wedge \neg p \Leftrightarrow F_0$$

9. Domination Law.

$$p \vee T_0 \Leftrightarrow T_0.$$

$$p \wedge F_0 \Leftrightarrow F_0.$$

10. Absorption Law.

$$p \vee (p \wedge q) \cong p$$

$$p \wedge (p \vee q) \cong p.$$

? Prove that $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$ without using truth table.

A. LHS = $(p \vee q) \wedge \neg(\neg p \wedge q)$ — given

$\Leftrightarrow (p \vee q) \wedge [\neg(\neg p) \vee \neg q]$ — Demorgan's law

$\Leftrightarrow (p \vee q) \wedge (p \vee \neg q)$ — Double negation.

$\Leftrightarrow p \vee (q \wedge \neg q)$ — Distributive.

$\Leftrightarrow p \vee F_0$ — Inverse

$\Leftrightarrow p$ = RHS. — Identity.

Dual of the Proposition.

Let 's' be a statements. If s contains no logical connectives other than conjuction & disjunction. Then the dual of s is denoted by s^d and is obtained by replacing the symbol disjunction by conjunction, conjuction by disjunction T_0 by F_0 and F_0 by T_0.

? Given the primitive statements p, q, r and the compound statements $s: (p \wedge \neg q) \wedge (r \wedge T_0)$ Write the dual of s.

A. The dual $s^d : (p \vee q) \vee (r \vee F_0)$

11/10/17 **Principle of Duality**

Let 's' & 't' be the statements that contains no logical connectives other than conjunction & disjunction.

If $s \Leftrightarrow t$, then the ~~dual~~ $s^d \Leftrightarrow t^d$

? Prove that $\neg[\neg[(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow q \wedge r$ without truth table.

		Reason	
LHS =	$\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$	given	Do negation all first.
	$\Leftrightarrow \neg.\neg[(p \vee q) \wedge r] \wedge \neg\neg q$	Demorgan's law.	
	$\Leftrightarrow [(p \vee q) \wedge r] \wedge q$	Double negation.	
	$\cancel{(p \wedge r) \vee (q \wedge r)}$		
	$\Leftrightarrow (p \vee q) \wedge (r \wedge q)$	Associative law.	
	$\Leftrightarrow (p \vee q) \wedge (q \wedge r)$	Commutative law	
	$\Leftrightarrow [(p \vee q) \wedge q] \wedge r$	Associative law.	
	$\Leftrightarrow \cancel{q \vee p}\, [q \wedge (p \vee q)] \wedge r$	Commutative law.	
	$\Leftrightarrow [q \wedge (q \vee p)] \wedge r$	Commutative law	
	$\Leftrightarrow \underline{q \wedge r}$	Absorption. law	

Logical Implication - Validity of Argument

Let us consider the implication $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to q$ where n is a positive integer. The statements $p_1, p_2 \ldots p_n$ are called premises of the argument and the statement q is the conclusion of the argument.

The preceding argument is called valid if whenever each of the premises $p_1, p_2 \ldots p_n$ is true then the conclusion is likewise true.

Note that if anyone of the premises $p_1, p_2 \ldots p_n$ is false then the hypothesis $p_1 \wedge p_2 \wedge \cdots p_n$ is false and then the implication $p_1 \wedge p_2 \wedge \cdots \wedge p_n \to q$ is automatically true.

Consequently one way to establish the validity of the given argument is to show that the statement or argument $p_1 \wedge p_2 \wedge \cdots \wedge p_n \leftrightarrow q$ is a tautology.

? Let p, q, r be the premitive statements such that p: Rojer studies well q: Rojer plays racket ball r: Rojer passes in all subjects. Now let p_1, p_2 & p_3 denote the premises, p_1: If Rojer studies well then he will pass in all subjects.

p_2: If Rojer doesn't play racket ball then he will study well. p_3: Rojer failed in all subjects. Show that $p_1 \wedge p_2 \wedge p_3 \to q$ is a valid argument.

A. $P_1 : p \to r$

$P_2 : \neg q \to p$

$P_3 : \neg r$

Here the premises can be rewritten as above

$$(P_1 \wedge P_2 \wedge P_3) \to q$$

We want to check $(P_1 \wedge P_2 \wedge P_3) \to q$ is a valid argument ie, $[(p \to r) \wedge (\neg q \to p) \wedge \neg r] \to q$

This substitution is a valid argument. For the validity we will be checking $[(p \to r) \wedge (\neg q \to p) \wedge \neg r] \to q$ is a tautology with the help of truth table

p	q	r	$\neg q$	$\neg r$	$p \to r$	$\neg q \to p$	$(p \to r) \wedge (\neg q \to p)$	$(p \to r) \wedge (\neg q \to p) \wedge \neg r$	$\to q$
T	T	T	F	F	T	T	T	F	T
T	F	F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	F	F	T
F	T	F	F	T	T	F	T	T	T
T	T	F	F	T	T	T	F	F	T
F	T	T	F	F	F	T	T	F	T
T	F	T	T	F	T	T	T	F	T
F	F	F	T	T	T	F	F	F	T

The given argument is a valid.

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If p and q are arbitrary statements such that $p \to q$ is a tautology then we say that p logically implies q or $p \to q$ is a logical implication and is denoted by ~~$p \Leftrightarrow q$~~ $p \Rightarrow q$.

If an expression is said to be tautologically imply another expression then the logical implication of the two expression will be a tautology

Rule of Inference

1. p
   $p \to q$
   $\therefore q$ (modus ponens)

Rule of Inference	Related Logical Implication	Name of the Rule
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	modus ponens or Rule of detachment
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Law of syllogism
$p \to q$ $\sim q$ $\therefore \sim p$	$[(p \to q) \wedge \sim q] \to \sim p$	Modus tollens
p q $\therefore p \wedge q$	$(p \wedge q) \to (p \wedge q)$	Rule of conjunction

$p \vee q$ $\sim p$ $\therefore q$	$[(p \vee q) \wedge \sim p] \to q$	Rule of disjunctive syllogism
$\sim p \to F_0$ $\therefore p$	$[\sim p \to F_0] \to p$	Rule of contradiction
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Rule of conjunctive simplification.
p $\therefore p \vee q$	$p \to (p \vee q)$	Rule of disjunctive simplification
$p \wedge q$ $p \to (q \to r)$ $\therefore r$	$[(p \wedge q) \wedge (p \to (q \to r))] \to r$	Rule of conditional proof.
$p \to r$ $q \to r$ $\therefore (p \vee q) \to r$	$[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$	Rule ofor proof by cases.
$p \to q$ $r \to s$ $p \vee r$ $\therefore q \vee s$	$[(p \to q) \wedge (r \to s) \wedge (p \vee r)] \to (q \vee s)$	Rule of the constructive. Dilemma
$p \to q$ $r \to s$ $\sim q \vee \sim s$ $\therefore \sim p \vee \sim r$	$[(p \to q) \wedge (r \to s) \wedge (\sim q \vee \sim s)] \to \sim p \vee \sim r$	Rule of destructive Dilemma

? Check the validity of the argument

"Reeta is baking a cake. If Reeta is baking a cake then she is not practicing her flute. If Reeta is not practicing her flute then her father will not buy her a car. Therefore Reeta's father will not buy her a car"

A. In this problem first we have to write the given argument with the help of premitive statements & logical connectives. ie.

p : Reeta is baking a cake
q : Reeta is practicing her flute
r : Reeta's father will buy her a car.

Now the premises will be.

$$\begin{array}{l} p \\ p \to \neg q \\ \neg q \to \neg r \\ \hline \therefore \neg r \end{array}$$

we have to check the validity of this argument This can be done either by the truth table method or by the rule of inference.

1) <u>By truth table method</u>

For doing this method we have to write the related logical expression & check whether it is a tautology.

The related logical expression is.

$$[p \wedge (p \to \neg q) \wedge (\neg q \to \neg r)] \to \neg r$$

p	q	r	$\neg q$	$\neg r$	$p \to \neg q$	$p \wedge (p \to \neg q)$	$\neg q \to \neg r$	$r \wedge (\neg q \to \neg r)$	$q \to \neg r$
T	T	T	F	F	F	F	T	F	T
T	T	F	F	T	F	F	T	F	T
T	F	T	T	F	T	T	F	F	T
F	T	T	F	F	T	F	T	F	T
T	F	F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	T	F	T
F	F	T	T	F	T	F	F	F	T
F	F	F	T	T	T	T	T	T	T

only once

→ By using rules of inference.

Steps.	Reasons.
1) p	premise
2) $p \to \neg q$	premise
3) $\neg q$	step 1 & 2 with modus pones.
4) $\neg q \to \neg r$.	premise
5) $\neg r$	step. 3 & 4 with modus pones.

? Check the validity of the following

$p \to \neg q$.
p
$\therefore \neg q$

A.

Steps	Reasons
1) $p \to \neg q$	premises.
2) p	Premises
3) $\neg q$	steps 1 & 2 with modus ponens

? $(p \vee q) \rightarrow r$

$\neg r$

$\therefore \neg p$

A.

Steps.	Reason.
1) $(p \vee q) \rightarrow r$	premise
2) $\neg r \rightarrow \neg(p \vee q)$	contrapositive
3) $\neg r \rightarrow \neg p \vee \neg q$	demorgan's law.
4) $\neg r$	premise
5) $\neg p \wedge \neg q$	step 4 & 3 with modus ponens
6) $\neg q$	step 5 with rule of conjunctive

'or'

Steps	Reason
1) $(p \vee q) \rightarrow r$	premise
2) $\neg r$	premise
3) $\neg(p \vee q)$	1 & 2 with modus tollens
4) $\neg p \wedge \neg q$.	demorgan
5) $\neg p$	

? $p \rightarrow q$

$q \rightarrow r$

$r \rightarrow s$

p

$\therefore s$

Steps	Reasons
1) $p \rightarrow q$	premise
2) $q \rightarrow r$	premise

3) $p \to r$	1 & 2 law of syllogism
4) $r \to s$	premise
5) $p \to s$	3 & 4 law of syllogism
6) p	premise
7) s	5 & 6 law of modus ponens

'OR'

Steps	Reasons
1) $p \to q$	premise
2) p	premise
3) q	1 & 2 modus ponens
4) $q \to r$	premise
5) r	3 & 4 modus ponens
6) $r \to s$	premise
7) s	5 & 6 modus ponens

?

$p \to \neg q$
$r \to q$
r
$\overline{\quad\quad\quad}$
$\neg p$

2) $p \to q$
$q \to r$
$r \to s$
$\neg s$
$\overline{\quad\quad\quad}$
$\neg t$

3) $p \to q$
$p \wedge r$
$\overline{\quad\quad\quad}$
q

4) $(p \to q) \wedge (r \to s)$
$(p \vee r) \wedge (q \wedge r)$

Steps	Reason $q \vee s$
$p \to \neg q$	premise.
$r \to q$	premise.
$\neg q \to \neg r$	contrapositive
$p \to \neg r$	step 1 & 3 law of syllogism.
$\neg\neg r \to \neg p$	
$r \to \neg p$	
r	
$\neg p$	

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? Determine the validity of the argument by truth table & also by rule of inference "If I study then I will pass examination. If I donot go to picnic then I will study. But I failed examination. Therefore I went to picnic.

A. p : I study.

q : I will pass examination.

r : I go to picnic.

~~s : I failed examination~~

$p \to q$.

$\neg r \to p$.

$\neg q$.

$\therefore r$.

Steps.	Reasons.
1) $p \to q$.	premise
2) $\neg q$ ~~$\neg r \to p$~~	premise
~~$p \to \neg r$~~	~~converse~~
3) $\neg p$	step 1 & 2 modus tollens.
~~$p \to q$~~.	
4) $\neg r \to p$.	premise.
5) $\neg\neg r$	Step 3 & 4 modus tollens
6) r.	double negation

Truth Table

$$[(p \to q) \wedge (\neg r \to p) \wedge \{\neg q\}] \to r$$

p	q	r	$p \to q$	$\neg r$	$\neg q$	$\neg r \to p$	$\overset{x}{p \to q \wedge \neg r \to p}$	$\underbrace{x \wedge \neg q}_{y}$	$y \to$
F	F	F	T	T	T	F	F	F	T
F	F	T	T	F	T	T	T	T	T
F	T	F	T	T	F	F	F	F	T
F	T	T	T	F	F	T	T	F	T
T	F	F	F	T	T	T	F	F	T
T	F	T	F	F	T	T	F	F	T
T	T	F	T	T	F	T	T	F	T
T	T	T	T	F	F	T	T	F	T