

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**THIRD SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018**

**Course Code: CS201**

**Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks*

Marks

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|---|--|-----|
| 1 | Let $X = \{1,2,3,4\}$ and $R = \{ \langle x,y \rangle \mid x > y \}$ . Draw the graph of R and also give its matrix.         | (3) |
| 2 | Define countable and uncountable set. Prove that set of real numbers are uncountable.  | (3) |
| 3 | State Pigeonhole principle. A school has 550 students. Show that at least two of them were born on the same day of the year. | (3) |
| 4 | How many 4-digit numbers can be formed from six digits 1, 2, 3, 5, 7, 8. Also find how many numbers are less than 4500.      | (3) |

**PART B**

*Answer any two full questions, each carries 9 marks*

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| 5 | a) Let Z be the set of integers and R be the relation called congruence modulo 3 defined by $R = \{ \langle x,y \rangle \mid x \text{ and } y \text{ are elements in } Z \text{ and } (x-y) \text{ is divisible by } 3 \}$ . Determine the equivalence classes generated by the elements of Z.  | (5) |
|   | b) Let A be the set of factors of a particular positive integer m and let $\leq$ be the relation divides, ie relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagrams for $m=30$ and $m=45$ .  | (4) |
| 6 | a) Let $f(x) = x+2$ , $g(x) = x-2$ and $h(x) = 3x$ for x is in R, where R is the set of real numbers. Find $\text{gof}$ , $\text{fog}$ , $(\text{foh})\text{og}$ , $\text{hog}$ .   | (4) |
|   | b) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology and 30 do not study any of the three subjects.<br>i) Find the number of students studying all three subjects.<br>ii) Find the number of students studying exactly one of three subjects. | (5) |
| 7 | a) Solve the recurrence equation $a_r + 5a_{r-1} + 6a_{r-2} = 42 \cdot 4^r$ where $a_2 = 278$ and $a_3 = 962$ .   | (4) |
|   | b) Define Monoid. Show that the algebraic systems $\langle Z_m, +_m \rangle$ and $\langle Z_m, *_m \rangle$ are monoids where $m = 6$ .   | (5) |

**PART C**

*Answer all questions, each carries 3 marks*

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| 8  | Define Abelian group. Prove that the algebraic structure $\langle Q^+, * \rangle$ is an abelian group. $*$ defined on $Q^+$ by $a * b = (ab)/2$ . | (3) |
| 9  | Define Cosets and Lagrange's theorem.   | (3) |
| 10 | Draw the diagram of lattices $\langle S_n, D \rangle$ for $n = 15$ and $n = 45$ . Where $S_n$ be the set  | (3) |

of all divisors of  $n$  and  $D$  denote the relation 'divides'.

- 11 Define sub Boolean algebra. Give one example. (3)

### PART D

*Answer any two full questions, each carries 9 marks*

- 12 a) Show that the set  $\{0, 1, 2, 3, 4, 5\}$  under addition and multiplication modulo 6 is group or not. (5)  
 b) Find all the subgroups of  $\langle \mathbb{Z}_{12}, +_{12} \rangle$  (4)
- 13 a) Define ring and field. Give one example to each. (5)  
 b)  $A = \{2, 3, 4, 6, 12, 18, 24, 36\}$  with partial order of divisibility. Determine whether the POSET is a lattice or not. (4)
- 14 a) Show that the lattice  $\langle S_n, D \rangle$  for  $n = 216$  is isomorphic to the direct product of lattices  $n = 8$  and  $n = 27$ . (5)  
 b) Define complemented lattice and distributive lattice. Give one example to each. (4)

### PART E

*Answer any four full questions, each carries 10 marks*

- 15 a) Prove that  $p \vee (q \wedge r)$  and  $(p \vee q) \wedge (p \vee r)$  are logically equivalent (5)  
 b) Show that  $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$  (5)
- 16 a) Show that  $s \vee r$  is tautologically implied by  $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow s)$  (5)  
 b) Show that  $r \wedge (p \vee q)$  is a valid conclusion from the premises  $p \vee q$ ,  $q \rightarrow r$ ,  $p \rightarrow m$ , and  $\sim m$  (5)
- 17 a) "If there are meeting, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time. There was no meeting". Show that the statements constitute a valid argument. (6)  
 b) Construct truth table for  $\sim (p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$ . Determine whether it is tautology or not. (4)
- 18 a) Show that  $(x) (P(x) \rightarrow Q(x)) \wedge (x) (Q(x) \rightarrow R(x)) \Rightarrow (x) (P(x) \rightarrow R(x))$  (5)  
 b) Prove that  $(\exists x) (P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$  (5)
- 19 a) Symbolize the statements: (4)  
 i) All the world loves a lover ii) All men are giants.  
 b) Show that  $(\exists x) M(x)$  follows logically from the premises  $(x) (H(x) \rightarrow M(x))$  and  $(\exists x) H(x)$  (6)
- 20 a) Prove by contradiction that if  $n^2$  is an even integer then  $n$  is even. (5)  
 b) Prove that  $23^n - 1$  is divisible by 11 for all positive integers  $n$ . (5)

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