TM Homework 7

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Mar 2024

Link back to GitHub

1 Task 1

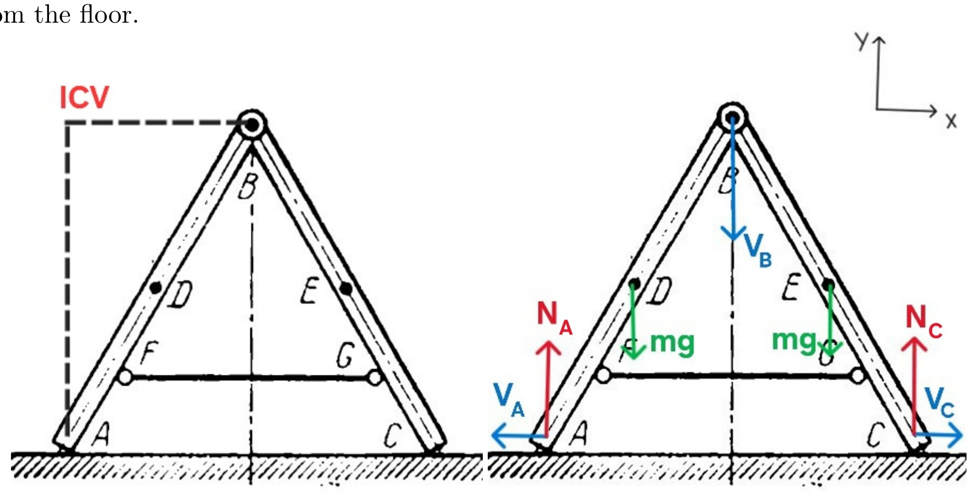
1.1 Task description

A step ladder ABC, hinged at B, rests on a smooth horizontal floor, as shown on the figure. AB = BC = 2l. The centres of gravity are at the midpoints D and E of the rods. The radius of gyration of each part of the ladder about the axis passing through the center of gravity is p.

The distance between B and the floor is h. At the certain moment the ladder collapses due to the rupture of a ling FG between the two halves of the ladder. Neglecting the effect of friction in the hinge, determine:

1. the velocity V_1 of the point B at the moment, when it hits the floor;

2. the velocity V_2 of point B at the moment, when it is at a distance $\frac{1}{2}$ h from the floor.



1.2 Solution

RO: system (2 rods: AB and BC)

Both undergo the planar motion.

Force analysis: As usual, gravity force: $m_{AB}g$, $m_{BC}g$, and normal forces: N_A, N_C

Conditions for subtask 1:

$$
\begin{aligned}
X_A^0 &= -\sqrt{4l^2 - h^2}, & X_A^f &= -2l \\
Y_A^0 &= 0, & Y_A^f &= 0 \\
X_B^0 &= 0, & X_B^f &= 0 \\
Y_B^0 &= h, & Y_B^f &= 0 \\
X_C^0 &= \sqrt{4l^2 - h^2}, & X_C^f &= 2l \\
Y_C^0 &= 0, & Y_C^f &= 0
\end{aligned}
$$

The conditions are pretty simple. Points A and C don't move in Y-axis. Point B doesn't move in X-axis, since the ladder is symmetrical. X-coordinates for A and C is just a Pythagorean theorem. On the right figure you may see the forces.

Let's say for now that $m_{AB} = m_{BC} = m$.

Well, let's solve it.

Firstly, we write the formula for kinetic energy:

$$
\begin{aligned}
T_{AB} &= \frac{I\omega_{AB}^2}{2} \\
T_{BC} &= \frac{I\omega_{BC}^2}{2}
\end{aligned}
$$

Then we obtain the inertia for the rods (the same for AB and BC):

$$I = ml^2 + mp^2$$

WOW, ML!

For simplicity let's say that ICV for the rod AB is at point A, and for the rod BC - at point C. Since we know the ICVs, we can find the angular velocities for rods AB and AC:

$$
\begin{aligned}
\omega_{AB} &= \frac{V_B}{2l} \\
\omega_{BC} &= \frac{V_B}{2l}
\end{aligned}
$$

The gravity did what it should do (work (rAbota)):

$$A_1 = \frac{mgh}{2} + \frac{mgh}{2} = mgh$$

here h/2 is the altitude of the CoM of rods AB and BC.
We know that

$$T_{AB} + T_{BC} = A_1$$

So, substituting what we got before gives us:

$$\frac{(ml^2 + mp^2)(\frac{V_B}{2l})^2}{2} + \frac{(ml^2 + mp^2)(\frac{V_B}{2l})^2}{2} = mgh$$

From this we get that

$$V_B = 2l\sqrt{\frac{gh}{l^2 + p^2}}$$

And now let's solve the second subtask.
Conditions for subtask 2:

$$\begin{aligned}
X_A^0 &= -\sqrt{4l^2 - h^2}, & X_A^f &= -\sqrt{4l^2 - \frac{h^2}{4}} - \frac{h}{2} \\
Y_A^0 &= 0, & Y_A^f &= 0 \\
X_B^0 &= 0, & X_B^f &= 0 \\
Y_B^0 &= h, & Y_B^f &= \frac{h}{2} \\
X_C^0 &= \sqrt{4l^2 - h^2}, & X_C^f &= \sqrt{4l^2 - \frac{h^2}{4}} + \frac{h}{2} \\
Y_C^0 &= 0, & Y_C^f &= 0
\end{aligned}$$

As in the previous subtask here the coordinates are pretty simple. Just need to say that on the left figure the ICV that is the intersection of perpendiculars to velocities.

Not much changed:

$$\omega_{AB} = \frac{V_B}{\sqrt{4l^2 - \frac{h^2}{4}}}$$

$$\omega_{BC} = \frac{V_B}{\sqrt{4l^2 - \frac{h^2}{4}}}$$

$$A_2 = \frac{mgh}{4} + \frac{mgh}{4} = \frac{mgh}{2}$$

Again, we substitute the obtained formulas into the:

$$T_{AB} + T_{BC} = A_2$$

$$\frac{(ml^2 + mp^2)(\frac{V_B}{\sqrt{4l^2 - \frac{h^2}{4}}})^2}{2} + \frac{(ml^2 + mp^2)(\frac{V_B}{\sqrt{4l^2 - \frac{h^2}{4}}})^2}{2} = \frac{mgh}{2}$$

From this we get that

$$V_B = \frac{1}{2}\sqrt{gh\frac{16l^2 - h^2}{2(l^2 + p^2)}}$$

1.3 Answers

HIGHLIGHTED ANSWERS ARE HERE

$$V_1 = 2l\sqrt{\frac{gh}{l^2 + p^2}}$$

$$V_2 = \frac{1}{2}\sqrt{gh\frac{16l^2 - h^2}{2(l^2 + p^2)}}$$

2 Task 2

2.1 Task description

You have a cart pole. Body 1 is a slider, mass m_1, it moves without friction. AB is a massless rod with length l. Body 2 with mass m_2 is connected to AB in point B.

It's a 2 DoF system. You should take x and ϕ as a representation of this system. The origin of each coordinate should be the same as on the picture.

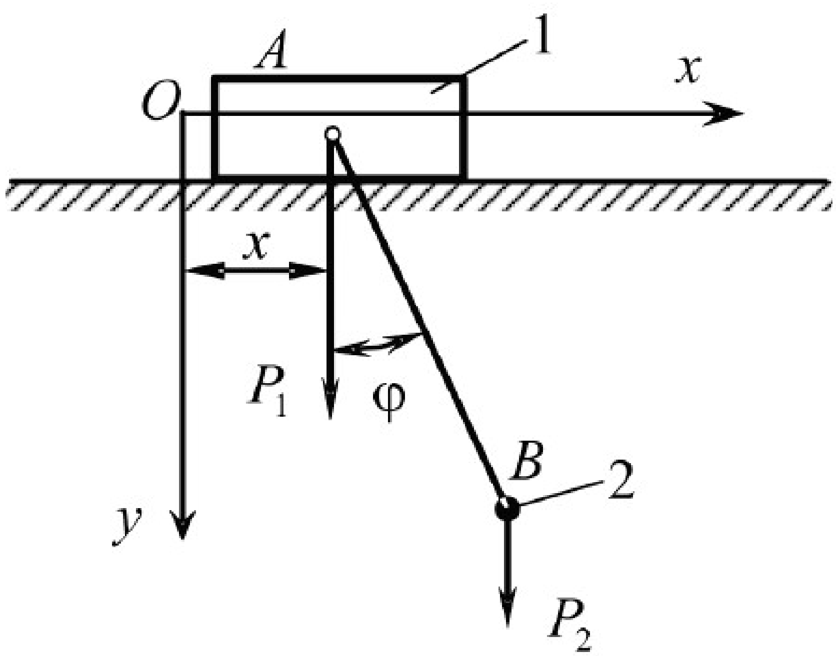
Initial conditions:

1.$x = 0, \phi = 10^o, \dot{x} = 0, \dot{\phi} = 0, t = 0;$

2.$x = 0.5, \phi = 45^o, \dot{x} = 0, \dot{\phi} = 0, t = 0;$

3.$x = 0.5, \phi = -135^o, \dot{x} = 0, \dot{\phi} = 0, t = 0;$

Parameters: $m_1 = 5kg, m_2 = 1kg, l = 1m$.



2.2 Solution

Link for the SIMULATION

You may see the .ipynb file in the folder.

RO: system (2 bodies: cart and let me call it ball)

Cart - translatory motion, ball - planar motion.

Force analysis:

$$
\begin{aligned}
m_1\ddot{x} &= T\sin(\phi)\\
m_2\left(-l\sin(\phi)\dot{\phi}^2 + l\cos(\phi)\ddot{\phi} + \ddot{x}\right) &= -T\sin(\phi)\\
-m_2 l\left(\sin(\phi)\ddot{\phi} + \cos(\phi)\dot{\phi}^2\right) &= -T\cos(\phi) + gm_2
\end{aligned}
$$

The first equation for the cart along X-axis, second and third - for X and Y of the ball respectively.

If we try to solve it using python we get long long formulas:

$$
\begin{aligned}
\ddot{\phi} = &-\frac{g\cdot m_1\sin(\phi(t))}{l\cdot m_1\sin(\phi(t))^2 + l\cdot m_1\cos(\phi(t))^2 + l\cdot m_2\sin(\phi(t))^2} -\\
&\frac{g\cdot m_2\sin(\phi(t))}{l\cdot m_1\sin(\phi(t))^2 + l\cdot m_1\cos(\phi(t))^2 + l\cdot m_2\sin(\phi(t))^2} -\\
&\frac{l\cdot m_2\sin(\phi(t))\cdot\cos(\phi(t))\cdot(\dot{\phi})^2}{l\cdot m_1\sin(\phi(t))^2 + l\cdot m_1\cos(\phi(t))^2 + l\cdot m_2\sin(\phi(t))^2}
\end{aligned}
$$

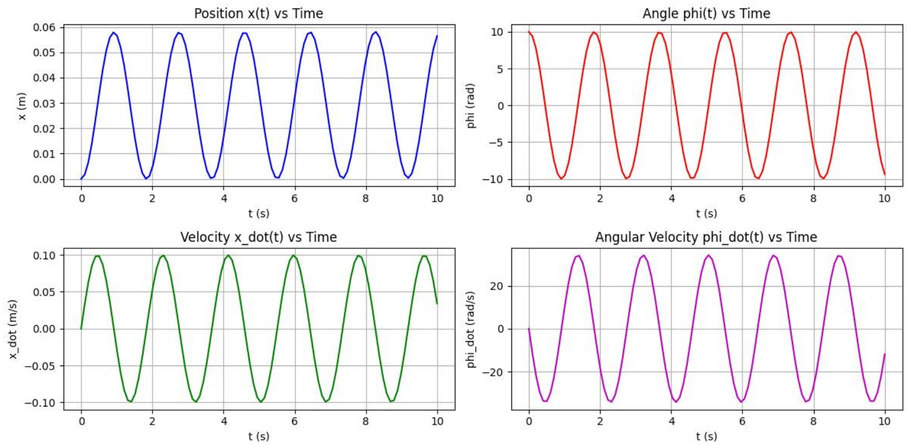
$$
\begin{aligned}
\ddot{x} = &\frac{g\cdot m_2\sin(\phi(t))\cdot\cos(\phi(t))}{m_1\sin(\phi(t))^2 + m_1\cos(\phi(t))^2 + m_2\sin(\phi(t))^2} +\\
&\frac{l\cdot m_2\sin(\phi(t))^3\cdot(\dot{\phi})^2}{m_1\sin(\phi(t))^2 + m_1\cos(\phi(t))^2 + m_2\sin(\phi(t))^2} +\\
&\frac{l\cdot m_2\sin(\phi(t))\cdot\cos(\phi(t))^2\cdot(\dot{\phi})^2}{m_1\sin(\phi(t))^2 + m_1\cos(\phi(t))^2 + m_2\sin(\phi(t))^2}
\end{aligned}
$$

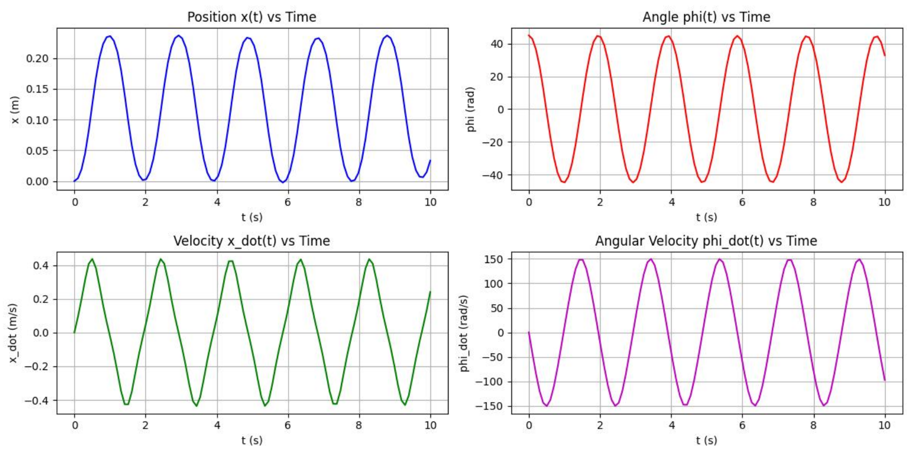
And since we (computer) cannot solve in analytically, we'll solve it numerically. You may see the calculations in the .ipynb file in the folder in GitHub.

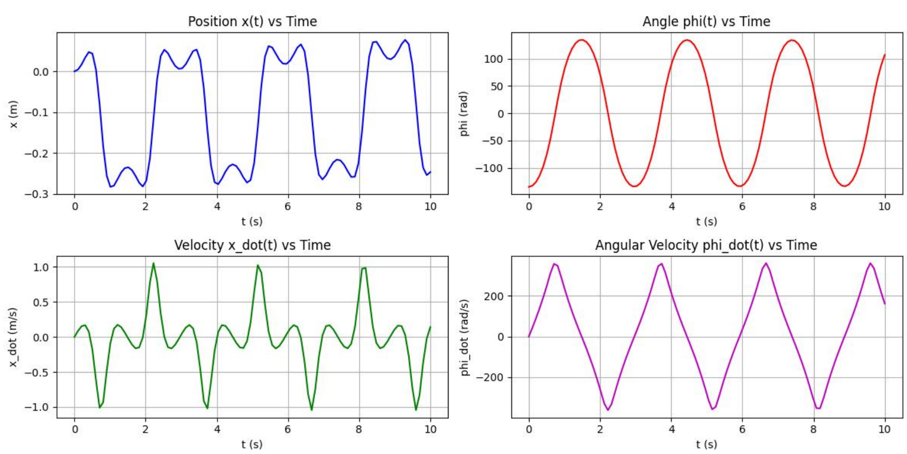
So, let's just see the plots. Also in the folder you may see the GIF's for each test. The plots are in the order: 10 degrees, 45 degrees, -135 degrees.

What about SimInTech? Well, I think it'd be better to use this time for preparation for the final. In the final I can get more points for this time.

2.3 Plots


Position x(t) vs Time
x (m)
t (s)
Angle phi(t) vs Time
phi (rad)
t (s)
Velocity x_dot(t) vs Time
x_dot (m/s)
t (s)
Angular Velocity phi_dot(t) vs Time
phi_dot (rad/s)
t (s)


Position x(t) vs Time
x (m)
t (s)
Angle phi(t) vs Time
phi (rad)
t (s)
Velocity x_dot(t) vs Time
x_dot (m/s)
t (s)
Angular Velocity phi_dot(t) vs Time
phi_dot (rad/s)
t (s)


Position x(t) vs Time
x (m)
t (s)
Angle phi(t) vs Time
phi (rad)
t (s)
Velocity x_dot(t) vs Time
x_dot (m/s)
t (s)
Angular Velocity phi_dot(t) vs Time
phi_dot (rad/s)
t (s)

2.4 Answers

HIGHLIGHTED ANSWERS ARE HERE

Answers are the plots. So, look a little bit higher.

3 MEME

В один день прорешали, а на следующий день нам кто-то поставил сердечко.