TM Homework 5

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Feb 2024

Link back to GitHub

1 Task 1

1.1 Task description

The legend shall speak that this situation was in WW2. There are two actors in this story: a sniper and an officer. Both knew about each other's existence. There was a river between them. The officer was always sitting in a trench, but the sniper knew his location and already calculated the distance to the target (L meters). After a while cargo ship appeared, which blocked the direct vision of the trench. The officer decided to stand up to stretch his legs. The sniper assumed that it might happened and make a shot, hitting the officer. Let's check this story.

Formal description: Considering the bullet as a material point and taking into account its weight and the force of wind resistance, we need to solve the following problems:

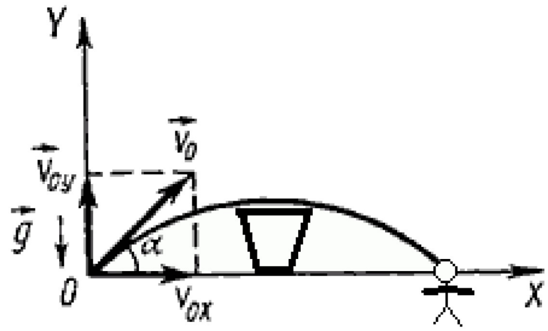
1. Find the α (initial angle of overhang) that is required to hit the target. Output the value in degrees.

2. At this angle, what is the maximum height the bullet will reach? Output the value.

3. Plot $y(x), F_c(t)$.

The problem should be solved in 2 ways. When air resistance is not taken into account, and when $F_c(V^2) = -kV\vec{V}$. The second problem can only be solved by numerical integration. All specifications about Mosin rifle such as bullet weight m, bullet velocity at departure V_0, effective firing distance are taken from the official documentation.

$m = 13.6g, L = 1500m, k = 1.3 \cdot 10^{-5}, V_0 = 870m/s.$



1.2 Solution

Part 1, WITHOUT air drag force

RO: particle (bullet), planar motion.

Conditions:

$$\begin{aligned} t_0 &= 0, & t_f &=? \\ x_0 &= 0, & x_f &= L \\ y_0 &= 0, & y_f &= 0, \\ \dot{x}_0 &= V_0\cos(\alpha), & \dot{x}_f &= V_0\cos(\alpha) \\ \dot{y}_0 &= V_0\sin(\alpha), & \dot{y}_f &=? \\ \ddot{x}_0 &= 0, & \ddot{x}_f &= 0 \\ \ddot{y}_0 &= -g, & \ddot{y}_f &= -g \end{aligned}$$

Force analysis: only gravity force $\vec{g}$, since we do not take into account air drag.

Now, let's solve it.

We can create the system of equations:

$$\begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = -mg \end{cases}$$

Let's integrate:

$$\begin{cases} \dot{x} = C_1 \\ \dot{y} = -gt + C_2 \end{cases}$$

Let's integrate once more time:

$$\begin{cases} x = C_1 + C_3 \\ y = -\frac{1}{2}gt^2 + C_2t + C_4 \end{cases}$$

If we substitute the initial conditions we get this:

$$\begin{cases} C_1 = V_0 \cos(\alpha) \\ C_2 = V_0 \sin(\alpha) \\ C_3 = 0 \\ C_4 = 0 \end{cases}$$

Substituting these values into the system gives:

$$\begin{cases} 0 = L - V_0 \cos(\alpha) t \\ 0 = -\frac{1}{2} g t^2 + V_0 \sin(\alpha) t \end{cases}$$

Wow, we got the equation for ballistic trajectory!

Since we did a lot of coding before, we can also code it and solve using python. Well, we solved it and got that angle $\alpha_1 = 0.556982768154159 \approx 0.557^o$. And there is another solution, that is also acceptable. $\alpha_2 = 89.44301723184584 \approx 89.443^o$.

Now we need to find the max altitude the bullet could reach.

For this we just need to find the time, when the bullet reaches the highest point, and find the y(max time). We can get the formula from the internet or derive it via calculating the extremum.

$$t_{max} = \frac{V_0 \sin(\alpha)}{g}$$

As we did before, this time we'll use python again.

And we got that time when height is max $t_{max_1} = 0.862109700484324$, $t_{max_2} = 88.6808248943596$ seconds, and max height is $y_{max_1} = 3.64555853045729$ and $y_{max_2} = 38574.3360928457$ meters respectively. So, it means that the cargo ship could be about 3.6 meters height or 38.5 km. It seems that it is not a big ship OR it is very big ship. It happens because we do not consider air drag force, and the only force that slows down the bullet is gravity.

Part 2, WITH air drag force

RO: particle (bullet), planar motion.

Conditions:

$$\begin{aligned} t_0 &= 0, & t_f &=? \\ x_0 &= 0, & x_f &= L \\ y_0 &= 0, & y_f &= 0, \\ \dot{x}_0 &= V_0 \cos(\alpha), & \dot{x}_f &=? \\ \dot{y}_0 &= V_0 \sin(\alpha), & \dot{y}_f &=? \\ \ddot{x}_0 &= 0, & \ddot{x}_f &= 0 \\ \ddot{y}_0 &= -g, & \ddot{y}_f &=? \end{aligned}$$

Force analysis: gravity force $\vec{g}$ and air drag $\vec{F_c}$
Now, let's solve it.
We can create the system of equations for axes:

$$\begin{cases} m\ddot{x} = -k\sqrt{\dot{x}^2 + \dot{y}^2}\dot{x} \\ m\ddot{y} = -mg - k\sqrt{\dot{x}^2 + \dot{y}^2}\dot{y} \end{cases}$$

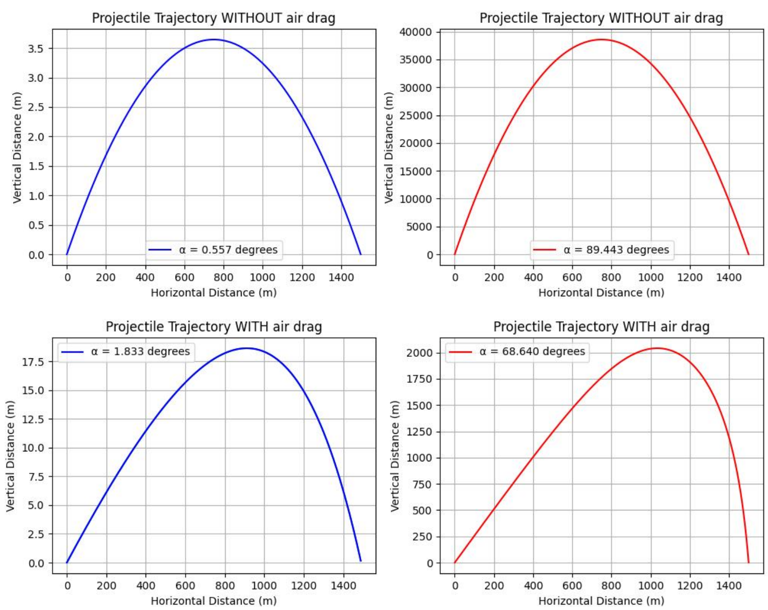
The system is the same as

$$\begin{cases} ma_x = -k\sqrt{\dot{x}^2 + \dot{y}^2}V_x \\ ma_y = -mg - k\sqrt{\dot{x}^2 + \dot{y}^2}V_y \end{cases}$$

Since it was written that we need to solve it numerically, then let's believe and do it in python. See the .ipynb file in the folder.

All the necessary things are found using python. See the .ipynb file.

1.3 Plots



1.4 Answers

HIGHLIGHTED ANSWERS ARE HERE

Part 1, WITHOUT air drag force

$$\alpha_1 = 0.557^o$$
$$y_{max_1} = 3.646 \text{ meters}$$
$$\alpha_2 = 89.443^o$$
$$y_{max_2} = 38574.336 \text{ meters}$$

Part 2, WITH air drag force

$$\alpha_1 = 1.833^o$$
$$y_{max_1} = 18.634 \text{ meters}$$
$$\alpha_2 = 68.64^o$$
$$y_{max_2} = 2041.379 \text{ meters}$$

2 Task 2

2.1 Task description

A particle M (mass m) is moving inside of the cylindrical channel of the moving object D. The object D has a radius r. No friction between M and D. Determine the equation of the relative motion of this particle $x = f(t)$. Also you need to find the pressure force the particle acting on the channel wall. At the end, you should provide:

1. simulate this mechanism (obtain all positions);
2. show all acceleration components, inertial forces, gravity force and N;
3. plot of the particle $x(t)$, till the time, while point won't leave the channel;
4. plot $N(t)$, till the time, while point won't leave the channel.

Needed variables: $m = 0.02, \omega = \pi, a = 60, \alpha = 45^o$;
Initial conditions: $t_0 = 0, x_0 = 0, x_0 = 0.4$.

2.2 Solution

2.3 Answers

HIGHLIGHTED ANSWERS ARE HERE

3 MEME