TM Homework 1

Timur Islamov

Jan 2024

Link to the folder with files

1 Task 1 (Coding)

1.1 Task description

You should find:

1. simulate the move of $\vec{O}$ for $t = [0...10]$;

$$\vec{O} = \begin{cases} x = 3cos(2t)cos(t) + 0.82 \\ y = 3cos(2t)sin(t) + 0.82 \end{cases}$$

2. find and draw plots $v, a, a_n, a_\tau, \kappa$ (Osculating circle) respect to t;
3. find $y(x)$, $\vec{v}$, $\vec{a}$, $\vec{a_n}$, $\vec{a_\tau}$ and show it on the simulation.

1.2 Solution

Here is the link for simulation (.ipynb file is in the folder) SIMULATION

Before we find what we want, let's find derivatives.

First derivative is the velocity

$$\dot{x} = -3sin(t)cos(2t) - 6cos(t)sin(2t)$$
$$\dot{y} = 3cos(t)cos(2t) - 6sin(t)sin(2t)$$

Second derivative is the acceleration

$$\ddot{x} = 12sin(t)sin(2t) - 15cos(t)cos(2t)$$
$$\ddot{y} = -12cos(t)sin(2t) - 15sin(t)cos(2t)$$

We know the formula for velocity (V), that is

$$|\vec{V}| = \sqrt{\dot{x}^2 + \dot{y}^2}$$

Substituting first derivatives into the formula gives the magnitude

$$|\vec{V}| = 3\sqrt{3sin(2t)^2 + 1}$$

Also we know the formula for acceleration (a), that is

$$|\vec{a}| = \sqrt{\ddot{x}^2 + \ddot{y}^2}$$

Substituting second derivatives into the formula gives the magnitude

$$|\vec{a}| = 3\sqrt{16 + 9cos(2t)^2}$$

We know the tangential acceleration formula

$$a_\tau = \frac{\vec{V} \cdot \vec{a}}{|\vec{V}|}$$

If we substitute the values we get something like this

$$a_\tau = \frac{\begin{bmatrix} -3sin(t)cos(2t) - 6cos(t)sin(2t) \\ 3cos(t)cos(2t) - 6sin(t)sin(2t) \end{bmatrix} \cdot \begin{bmatrix} 12sin(t)sin(2t) - 15cos(t)cos(2t) \\ -12cos(t)sin(2t) - 15sin(t)cos(2t) \end{bmatrix}}{3\sqrt{3sin(2t)^2 + 1}}$$

Solving this, we get

$$a_\tau = \frac{9sin(4t)}{\sqrt{1 + 3sin(2t)^2}}$$

If we need tangential acceleration as a vector $\vec{a_\tau}$, then we can found it using the formula

$$\vec{a_\tau} = a_\tau \cdot \frac{\vec{V}}{|\vec{V}|} = \frac{3sin(4t)}{1 + 3sin(2t)^2} \cdot \begin{bmatrix} -3sin(t)cos(2t) - 6cos(t)sin(2t) \\ 3cos(t)cos(2t) - 6sin(t)sin(2t) \end{bmatrix}$$

Now let's move to the normal acceleration. The known formula is

$$\vec{a_n} = \frac{\vec{a} \times \vec{V}}{|\vec{V}|}$$

$$\vec{a_n}(t) = \frac{\begin{bmatrix} 12sin(t)sin(2t) - 15cos(t)cos(2t) \\ -12cos(t)sin(2t) - 15sin(t)cos(2t) \end{bmatrix} \times \begin{bmatrix} -3sin(t)cos(2t) - 6cos(t)sin(2t) \\ 3cos(t)cos(2t) - 6sin(t)sin(2t) \end{bmatrix}}{3\sqrt{3sin(2t)^2 + 1}}$$

Finally, we get this such a big formula

$$\vec{a_n}(t) = \frac{1}{3\sqrt{1 + 3sin(2t)^2}} \cdot \begin{bmatrix} 0 \\ 0 \\ -27sin(2t)^2 - 45 \end{bmatrix}$$

And the magnitude is

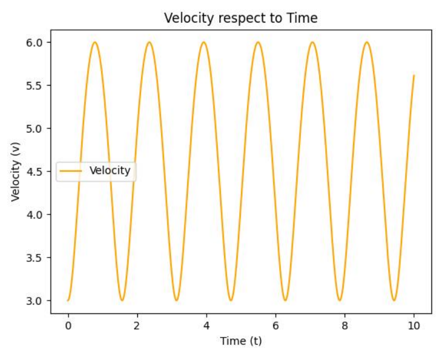
$$a_n(t) = \frac{9sin(2t)^2 + 15}{\sqrt{1 + 3sin(2t)^2}}$$

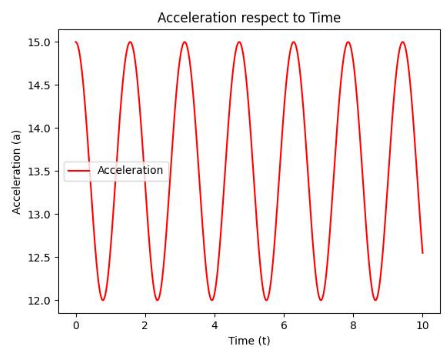
Curvature we find using a simple formula

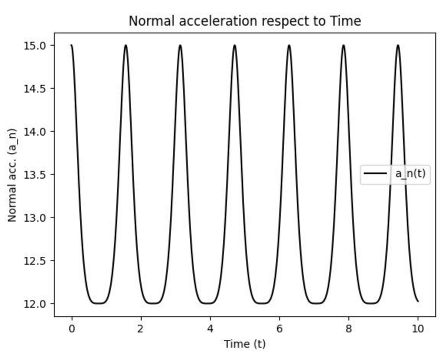
$$\kappa = \frac{a_n}{V^2} = \frac{3sin(2t)^2 + 5}{3(1 + 3sin(2t)^2)\sqrt{1 + 3sin(2t)^2}}$$

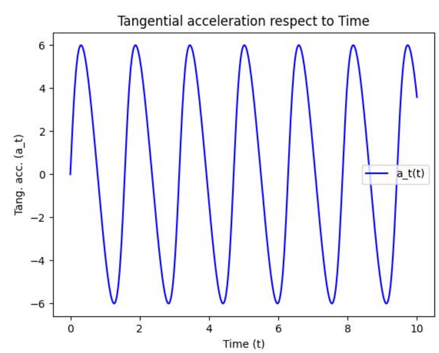
It was not impossible to find $y(x)$. However, here is Cartesian form

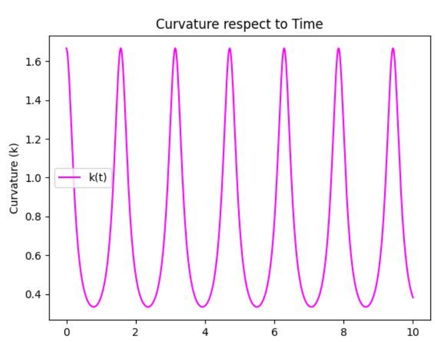
$$((x - 0.82)^2 + (y - 0.82)^2)^3 = 9((x - 0.82)^2 - (y - 0.82)^2)^2$$











1.3 Answers

HIGHLIGHTED ANSWERS ARE HERE

$$V = 3\sqrt{3sin(2t)^2 + 1}$$

$$\vec{V} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -3sin(t)cos(2t) - 6cos(t)sin(2t) \\ 3cos(t)cos(2t) - 6sin(t)sin(2t) \end{bmatrix}$$

$$a = 3\sqrt{16 + 9cos(2t)^2}$$

$$\vec{a} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 12sin(t)sin(2t) - 15cos(t)cos(2t) \\ -12cos(t)sin(2t) - 15sin(t)cos(2t) \end{bmatrix}$$

$$a_n = \frac{9sin(2t)^2 + 15}{\sqrt{1 + 3sin(2t)^2}}$$

$$\vec{a_n}(t) = \frac{1}{3\sqrt{1 + 3sin(2t)^2}} \cdot \begin{bmatrix} 0 \\ 0 \\ -27sin(2t)^2 - 45 \end{bmatrix}$$

$$a_\tau = \frac{9sin(4t)}{\sqrt{1 + 3sin(2t)^2}}$$

$$\vec{a_\tau} = \frac{3sin(4t)}{1 + 3sin(2t)^2} \cdot \begin{bmatrix} -3sin(t)cos(2t) - 6cos(t)sin(2t) \\ 3cos(t)cos(2t) - 6sin(t)sin(2t) \end{bmatrix}$$

$$\kappa = \frac{3sin(2t)^2 + 5}{3\sqrt{1 + 3sin(2t)^2}^3}$$

Cartesian form of the original parametric equation

$$((x - 0.82)^2 + (y - 0.82)^2)^3 = 9((x - 0.82)^2 - (y - 0.82)^2)^2$$

2 Task 2 (Coding)

2.1 Task description

You should solve the task, till the M point travels s:

1. simulate this mechanism (obtain all positions of bodies 1, 2, 3);
2. velocity for M(draw plots for magnitudes and show vectors on simulation);
3. accelerations (tangent, normal, overall) for M(draw plots for magnitudes and show vectors on simulation);
4. draw plots of angular velocities for 2, 3 bodies.

If $R_2 = 40, r_2 = 30, R_3 = 15, x = x(t) = 3 + 80t^2, s_M = [0, 5]$

2.2 Solution

Here is the link for simulation (the same .ipynb file is in the folder)

SIMULATION

The most easiest part is to find velocity $V_x(t)$ of object 1 (or object x) since that is just a derivative

$$V_x = \dot{x} = 160t$$

Since we are smart guys, we know the formula

$$V_x = \omega_2 R_2,$$

where the left - velocity of body 1, and to the right - velocity of body 2.

Using this we can find angular velocity of body 2 (For circles R_2 and r_2)

$$\omega_2 = \frac{V_x}{R_2} = \frac{160t}{40} = 4t$$

However, they have different linear velocities. Circle R_2 has the same linear velocity as body 1 (V_x). Linear velocity of circle r_2 is:

$$V_{r_2} = \omega_2 r_2 = 120t$$

Body 3 and circle r_2 have the same linear velocities (since they touch each other), but different angular velocities (due to different radii). So,

$$V_M = V_{r_2} = 120t$$

As always, we know such formulas for normal acceleration (a_n^M), tangential acceleration (a_τ^M), and overall acceleration (a_M) for point M

$$a_n^M = \frac{V_M^2}{R_3} = \frac{(120t)^2}{15} = 960t^2$$

$$a_\tau^M = R_3 \epsilon_M$$

But first we need to find ϵ_M that is

$$\epsilon_M = \dot{\omega}_M = 8$$

Wow, we've found it!

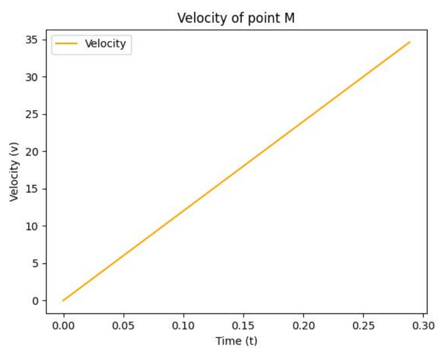
$$a_\tau^M = R_3 8 = 120 \tag{1}$$

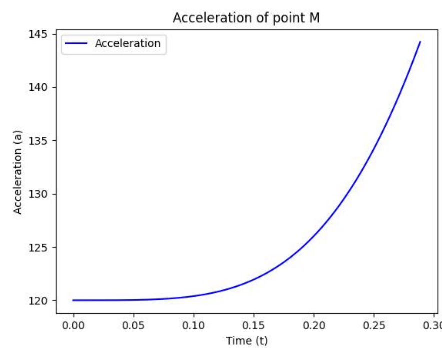
Next we need angular velocity of point M

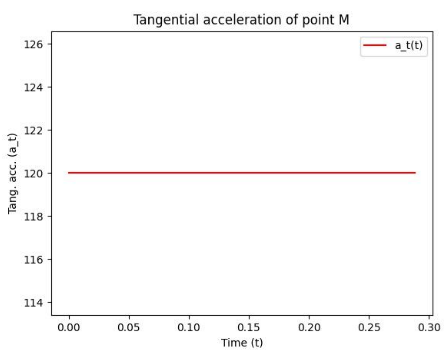
$$\omega_M = \frac{V_M}{R_3} = \frac{120t}{15} = 8t$$

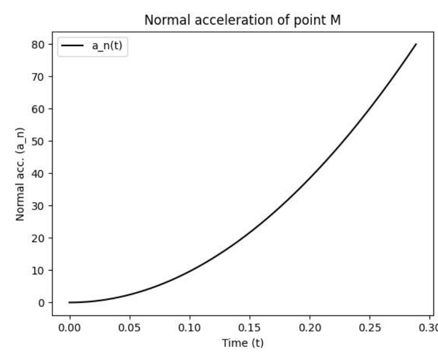
Total acceleration of point M can be evaluated through the formula

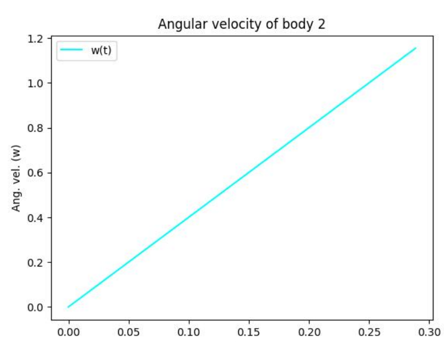
$$a_M = \sqrt{(a_\tau^M)^2 + (a_n^M)^2} = \sqrt{14400 + 960^2 t^4} = 120\sqrt{1 + 64t^4} \tag{2}$$

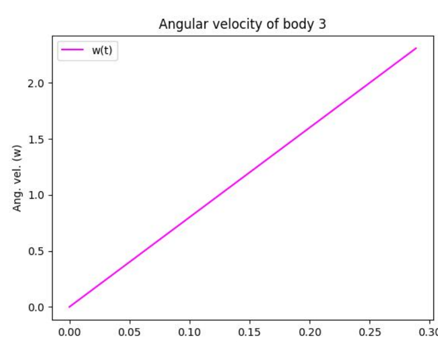












2.3 Answers

HIGHLIGHTED ANSWERS ARE HERE

$$
\begin{aligned}
V_M &= 120t \\
a_M &= 120\sqrt{1+64t^4} \\
a_\tau^M &= 120 \\
a_n^M &= 960t^2 \\
\omega_2 &= 4t \\
\omega_M = \omega_3 &= 8t
\end{aligned}
$$

3 Task 3 (Coding)

3.1 Task description

You should find:

1. simulate this mechanism (obtain all positions.) $(x_i(t), y_i(t)$, where i is A, B, C point)
2. velocities for B, C (draw plots for magnitudes and show vectors on simulation);
3. accelerations for B and C (draw plots for magnitudes and show vectors on simulation);
4. draw a plot of angular velocity of body BA.

If $y_A(t) = 22.5 + 10sin(\frac{\pi}{5} * t); t = [0..10]$ sec.;
$AB = 45, BC = 30$.

3.2 Solution

Here is the link for simulation (the same .ipynb file is in the folder)
SIMULATION

The final stage of the simulation is in progress. I've found very interesting video how does this work. And also I've learned that it is called "Elliptical Trammel mechanism". Here is the link to YouTube animation.

As we know from lectures, there is a cool thing, called ICV. ICV is perpendicular to axes of points A and B. Let's name the ICV as point I.

From the picture we can see that $AI = OB = \sqrt{AB^2 - y_A^2} = \sqrt{45^2 - y_A^2}$ and $BI = OA = y_A$.

$$OB = \sqrt{45^2 - y_A^2} = \frac{5\sqrt{243 - 72sin(\frac{\pi x}{5}) - 16sin(\frac{\pi x}{5})^2}}{2}$$

Then we can find V_A, V_B

$$V_A = \dot{y}_A = 2\pi cos(\frac{\pi t}{5})$$

$$V_B = \dot{O}B = -\frac{18\pi cos(\frac{\pi x}{5}) + 4\pi sin(\frac{2\pi x}{5})}{\sqrt{243 - 72sin(\frac{\pi x}{5}) - 16sin(\frac{\pi x}{5})^2}}$$

To find acceleration of $a_A(t)$ we just take the derivative

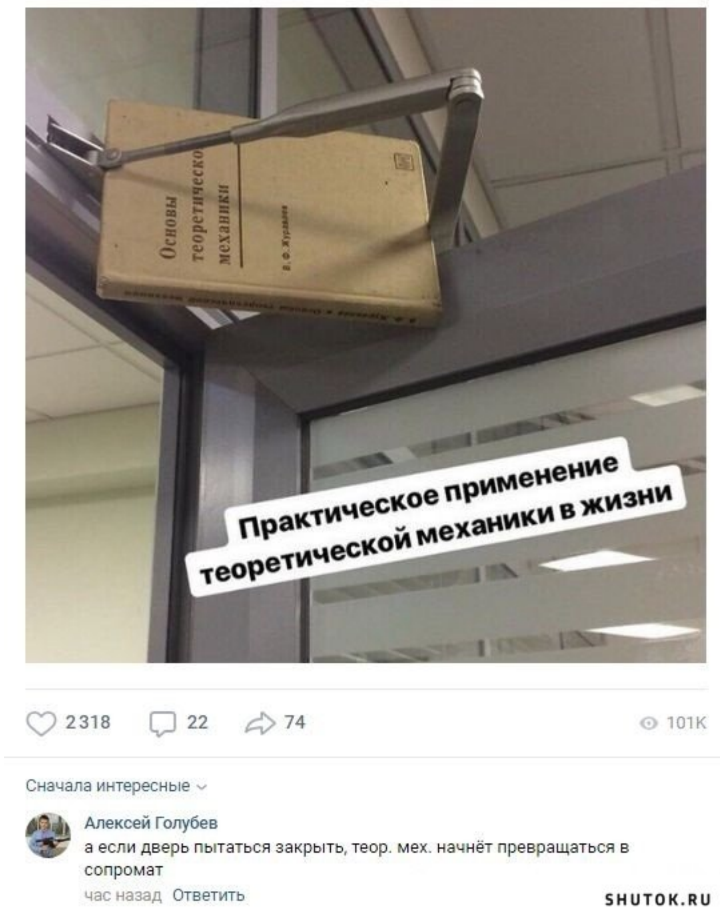
$$a_A = \dot{V}_A = -\frac{2\pi^2 sin(\frac{\pi x}{5})}{5} \tag{3}$$

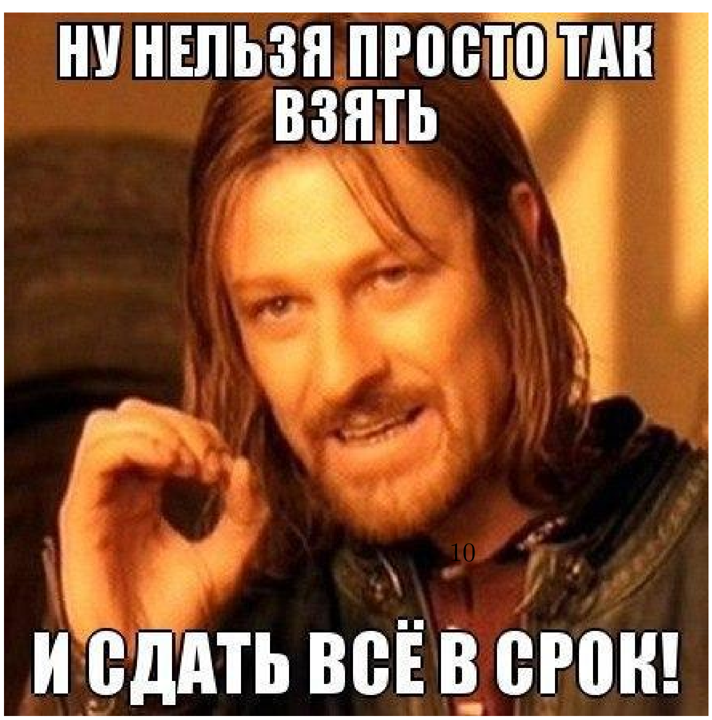
However, for $a_B(t)$ the derivative is very long, so that it seems that it shouldn't be like that.

3.3 Answers

$$V_B = -\frac{18\pi cos(\frac{\pi x}{5}) + 4\pi sin(\frac{2\pi x}{5})}{\sqrt{243 - 72sin(\frac{\pi x}{5}) - 16sin(\frac{\pi x}{5})^2}}$$

4 MEME


Основы
теоретическо
механики
Практическое применение
теоретической механики в жизни
2318
22
74
101K
Сначала интересные
Алексей Голубев
а если дверь пытаться закрыть, теор. мех. начнёт превращаться в сопромат
час назад Ответить
SHUTOK.RU


НУ НЕЛЬЗЯ ПРОСТО ТАК ВЗЯТЬ
И СДАТЬ ВСЁ В СРОК!