TM Homework 3

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Link back to GitHub

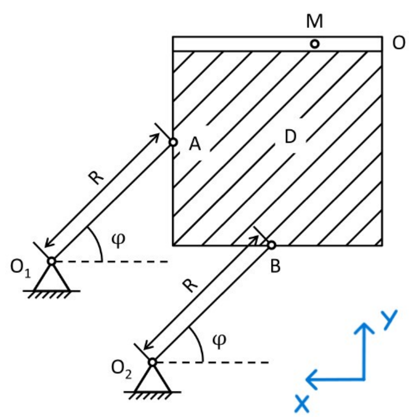
1 Task 1

1.1 Task description

You should find an absolute velocity (V_{full}) and Coriolis acceleration (a_{cor}), and absolute acceleration (a_{full}) of particle M at the time $t = t_1$.

Needed variables: $OM = s_r(t) = f_3(t) = 2t^3 + 3t$;

$\phi(t) = f_2(t) = \frac{\pi t^2}{24}; t_1 = 2, R = 15.$



1.2 Solution

First of all, it is necessary to say that there is not Coriolis acceleration due to the fact that the body is not rotating (doesn't have the axis of rotation). The motion of the body is called translatory motion. That is because of the same length of rods O_1A and O_2B. So, $a_{cor} = 0$

From the formula of the angle ϕ we can find angular velocity and angular acceleration:

$$\omega(t) = \dot{f}_2(t) = \frac{\pi t}{12}$$

$$\epsilon = \dot{\omega}(t) = \frac{\pi}{12}$$

We are given with the formula of the distance of point M from point O (OM). From this we can find relative velocity and relative acceleration of point M:

$$V_{rel}^M(t) = \dot{f}_3(t) = 6t^2 + 3$$
$$a_{rel}^M = \dot{V}_{rel}(t) = 12t$$

Now let's find some stuff for point A. We already know some formulas: First, for the next calculations we say that $\omega_{tr}^A(t) = \omega(t)$

$$V_{tr}^A(t) = \omega_{tr}^A R$$

$$a_n^A = \frac{V_A^2}{R} = \frac{(\omega_{tr}^A R)^2}{R} = (\omega_{tr}^A)^2 R$$
$$a_\tau^A = \epsilon R$$

And we know formulas for full velocity and acceleration:

$$V_{full} = \sqrt{V_x^2 + V_y^2} \text{ and } a_{full} = \sqrt{a_x^2 + a_y^2}$$

For our figure we define the axes like X-axis to the left and Y-axis up. So, we find projections of velocities and accelerations on the axes:

$$\begin{cases} V_x : V_{rel}^M + V_{tr}^A \cos(\pi/2 - \phi) \\ V_y : V_{tr}^A \sin(\pi/2 - \phi) \end{cases}$$

Moving further we substitute it and find V_{full} (see the answer below).

$$\begin{cases} a_x : a_{rel}^M + a_\tau^A \cos(\pi/2 - \phi) + a_n^A \sin(\pi/2 - \phi) \\ a_y : 0 + a_\tau^A \cos(\pi/2 - \phi) - a_n^A \sin(\pi/2 - \phi) \end{cases}$$

And to find a_{full} we substitute the formulas above to the formula for finding full acceleration.

And for $t = 2$ we get:

$$\phi = \phi(2) = \frac{\pi 2^2}{24} = \frac{\pi}{6}$$

$$\omega_{tr}^A(2) = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$a_{rel}^M(2) = 12 * 2 = 24$$

$$a_\tau^A = \frac{\pi}{12} * 15 = \frac{5\pi}{4}$$

$$a_n^A(2) = (\frac{2\pi}{12})^2 * 15 = \frac{5\pi^2}{12}$$

$$V_{tr}^{A}(2) = \frac{\pi * 2 * 15}{12} = \frac{5\pi}{2}$$
$$V_{rel}^{M} = 6 * 2^3 + 3 = 27$$
$$V_x = 27 + \frac{5\pi}{2}\cos(\pi/3) \approx 30.93$$
$$V_y = \frac{5\pi}{2}\sin(\pi/3) \approx 6.8$$
$$V_{full} = \sqrt{30.93^2 + 6.8^2} \approx 31.67$$
$$a_x = 24 + \frac{5\pi}{4}\cos(\pi/3) + \frac{5\pi^2}{12}\sin(\pi/3) \approx 29.5248$$
$$a_y = 0 + \frac{5\pi}{4}\sin(\pi/3) - \frac{5\pi^2}{12}\cos(\pi/3) \approx 1.34$$
$$a_{full} = \sqrt{29.52^2 + 1.34^2} \approx 29.55$$

1.3 Answers

HIGHLIGHTED ANSWERS ARE HERE

No Coriolis acceleration $(a_{cor} = 0)$
$$a_{full} \approx 29.55$$
$$V_{full} \approx 31.67$$

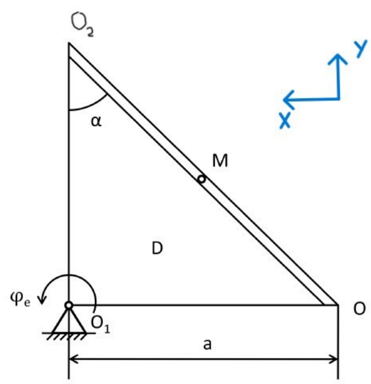
2 Task 2 (Coding)

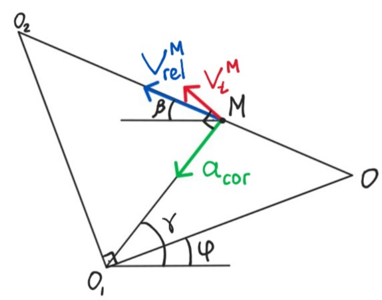
2.1 Task description

You should find:

1. simulate this mechanism (obtain all positions);
2. Find absolute, transport and relative velocities and accelerations for M;
3. Find t, when M reaches O point;
4. draw plots $v_{rel}, v_{tr}, a_{tr}, a_{rel}, a$ respect to time.

Needed variables:
$\phi_e = f_1(t) = 0.2t^3 + t$;
$OM = s_r = f_2(t) = 5\sqrt{2}(t^2 + t)$
$a = 60, \alpha = 45^o$





2.2 Solution

Here is the link for simulation (.ipynb file is in the folder) SIMULATION

Let's find what we can immediately:

The easiest part is to find the time when the point M reaches the end of the tube (point O_2). The time can be found if we equate formula for OM to the length OO_2. Let's do it:

$5\sqrt{2}(t^2+t) = 60\sqrt{2},$ which gives the roots $t_1 = -4, t_2 = 3,$ and we choose the positive time $t = 3$

From formula for angle ϕ we obtain the following:

$$\omega_{tr}(t) = \dot{f_1}(t) = 0.6t^2 + 1, \epsilon_{tr}(t) = \dot{\omega_{tr}}(t) = 1.2t$$

From the $f_2(t)$ we find formula for V^M_{rel}:

$$V^M_{rel} = \dot{OM} = 10\sqrt{2}t + 5\sqrt{2}$$

And from that formula we get the relative acceleration:

$$a^M_{rel} = \dot{V}^M_{rel} = 10\sqrt{2}$$

For transport velocity:

$$V_{tr}^{M} = O_1 M \omega_{tr}$$

For transport acceleration:

$$a_{tr}^{M} = \dot{V}_{tr}^{M} \text{ and } |a_{tr}^{M}| = \sqrt{(a_{tr}^{\tau M})^2 + (a_{tr}^{nM})^2}$$

Where

$$a_{tr}^{\tau M} = \epsilon O_1 M = 1.2t O_1 M, \text{ and } a_{tr}^{nM} = \frac{V_{tr}^{M^2}}{O_1 M} = O_1 M \omega_{tr}^2$$

To find full velocity and full acceleration:

$$V_{full} = \sqrt{V_x^2 + V_y^2} \text{ and } a_{full} = \sqrt{a_x^2 + a_y^2}$$

Let's project on the axes:

$$\begin{cases} V_x : V_{tr}^{M} \cos(\pi/2 - \gamma) + V_{rel}^{M} \cos(\beta) \\ V_y : V_{tr}^{M} \sin(\pi/2 - \gamma) + V_{rel}^{M} \sin(\beta) \end{cases}$$

It is necessary to say that $\beta = \pi/4 - \phi$. If we consider $\Delta O_1 O_2 M$ we get that $\pi/4 + (\gamma + \beta) + (\pi/2 + \phi - \gamma) = \pi$ (Sum of the angles equals to π). From this we find β.

Now we move to the full acceleration:

$$\begin{cases} a_x : a_{tr}^{\tau M} \sin(\gamma) + a_{tr}^{nM} \cos(\gamma) + a_{rel}^{M} \cos(\beta) + a_{cor} \cos(\pi/2 - \beta) \\ a_y : a_{tr}^{\tau M} \cos(\gamma) - a_{tr}^{nM} \sin(\gamma) + a_{rel}^{M} \sin(\beta) - a_{cor} \sin(\pi/2 - \beta) \end{cases}$$

$$\vec{a}_{corr} = 2\vec{\omega}_{tr} \times \vec{V}_{rel}^{M}$$

$$a_{corr} = 2\omega_{tr} V_{rel}^{M} \sin(\pi/2) = 2(0.6t^2+1)(10\sqrt{2}t+5\sqrt{2}) = 10\sqrt{2}(1.2t^3+0.6t^2+2t+1)$$

One of the task is to find all positions. Well, let's do it!

Let point O_1 be the origin (0,0).

Point O and O_2 (the end of the tube where point M moves) are moving along the circumference with radius $R = O_1 O = O_1 O_2$. So, the formula for these point is:

$$O = \begin{bmatrix} O_1 O \cos(\phi) \\ O_1 O \sin(\phi) \end{bmatrix} = \begin{bmatrix} 60 \cos(\phi) \\ 60 \sin(\phi) \end{bmatrix}$$

$$O_2 = \begin{bmatrix} O_1 O_2 \cos(\phi + \pi/2) \\ O_1 O_2 \sin(\phi + \pi/2) \end{bmatrix} = \begin{bmatrix} 60 \cos(\phi + \pi/2) \\ 60 \sin(\phi + \pi/2) \end{bmatrix}$$

Let's consider $\Delta O O_1 M$.

The length of OM we can find from the given formula, $O_1 O$ is given (60), but $O_1 M$ we have to find by ourselves. Using cosine theorem we get:

$$O_1M = \sqrt{O_1O^2 + OM^2 - 2 * O_1O * OM\cos(\alpha)} = \sqrt{60^2 + OM^2 - 60\sqrt{2} * OM}$$

And the most complicated is point M. We know all the sides of the triangle, and we can apply sine theorem:

$$\frac{OM}{\sin(\angle OO_1M)} = \frac{O_1M}{\sin(\angle O_1OM)}$$

And from this we can find $\angle OO_1M$:

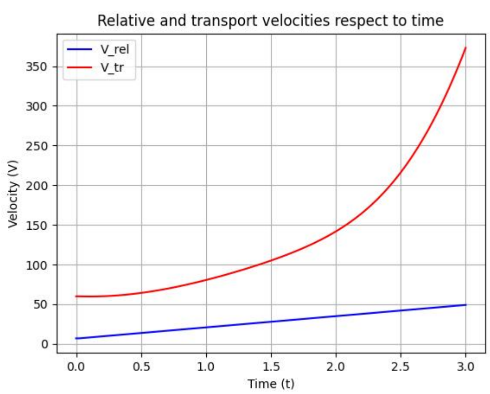
$$\angle OO_1M = \arcsin\left(\frac{\sin(\angle O_1OM)OM}{O_1M}\right)$$

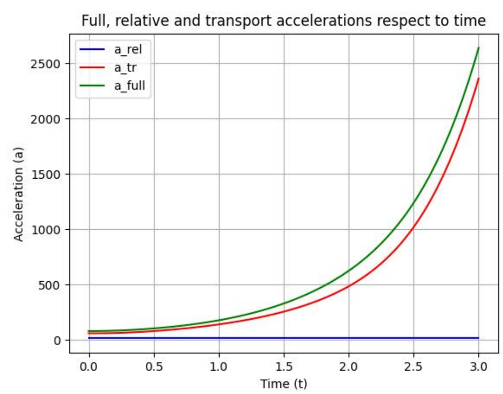
So, using this angle we can easily find position of point M at any time. We can say that this angle is $\gamma - \phi$. Then

$$M = \begin{bmatrix} O_1M\cos(\gamma - \phi) \\ O_1M\sin(\gamma - \phi) \end{bmatrix}$$

There was another approach how to find coordinates of point M. You may see it in the file **anotherApproach.ipynb**. It was quite interesting.

2.3 Plots





2.4 Answers

HIGHLIGHTED ANSWERS ARE HERE

$$t = 3$$

$$O = \begin{bmatrix} 60\cos(\phi) \\ 60\sin(\phi) \end{bmatrix}$$

$$O_2 = \begin{bmatrix} 60\cos(\phi + \pi/2) \\ 60\sin(\phi + \pi/2) \end{bmatrix}$$

$$M = \begin{bmatrix} O_1M\cos(\gamma-\phi) \\ O_1M\sin(\gamma-\phi) \end{bmatrix}$$

$$V^M_{rel} = 10\sqrt{2}t + 5\sqrt{2}$$

$$a^M_{rel} = 10\sqrt{2}$$

$$V^M_{tr} = O_1M\omega_{tr}$$

$$a^M_{tr} = \sqrt{(1.2tO_1M)^2 + O_1M\omega^2_{tr}}$$

$$V_{full} = \sqrt{(V_x : V^M_{tr}\cos(\pi/2-\gamma) + V^M_{rel}\cos(\beta))^2 + (V_y : V^M_{tr}\sin(\pi/2-\gamma) + V^M_{rel}\sin(\beta))^2}$$

$$a_{full} = \sqrt{a_x^2 + a_y^2},\ \text{where} \begin{cases} a_x : a^{\tau M}_{tr}\sin(\gamma) + a^{nM}_{tr}\cos(\gamma) + a^M_{rel}\cos(\beta) + a_{cor}\cos(\gamma) \\ a_y : a^{\tau M}_{tr}\cos(\gamma) - a^{nM}_{tr}\sin(\gamma) + a^M_{rel}\sin(\beta) - a_{cor}\sin(\gamma) \end{cases}$$

For the formulas above to substitute:

$$O_1M = \sqrt{60^2 + OM^2 - 60\sqrt{2} * OM}$$

3 MEME