TM Homework 6

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Link back to GitHub

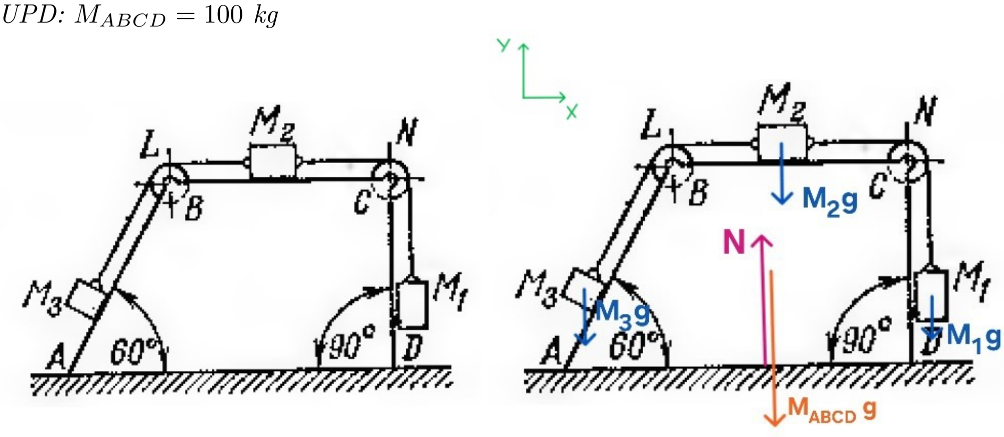
1 Task 1

1.1 Task description

There are 3 weights with masses $M_1 = 20$ kg, $M_2 = 15$ kg, $M_3 = 10$ kg. They are connected by ideal string. This string go through by two pulleys L, N. When the M_1 weight goes down on 1 meter, the body $ABCD$ shifts on some distance S.

The task is to find the distance of this movement according to the ground. Neglect the friction between the floor and ABCD.

UPD: $M_{ABCD} = 100$ *kg*



1.2 Solution

Well, after drawing the forces, let's solve the problem.

RO: system (4 bodies: ABCD, body 1, body 2, body 3). All bodies undergo translatory motion.

We will solve the problem via centre of masses.

Conditions:

Since the body ABCD does not move upward, we should consider only X direction. We know that the body will move on some distance s.

$$
\begin{aligned}
x^o_{ABCD} &= x_0, & x^f_{ABCD} &= x_0 + s \\
x^o_1 &= x_1, & x^f_1 &= x_1 + s \\
x^o_2 &= x_2, & x^f_2 &= x_2 + s + d_{M_2} \\
x^o_3 &= x_3, & x^f_3 &= x_3 + s + d_{M_2}\cos(\pi/3) \\
\dot{x}^o &= 0, & \dot{x}^f &= 0 \\
\ddot{x}^o &= 0, & \ddot{x}^f &= 0
\end{aligned}
$$

Force analysis: Gravity forces for all bodies: $\vec{G}_1$, $\vec{G}_2$, $\vec{G}_3$, $\vec{G}_{ABCD}$, $\vec{N}$

Let's write the equation for x-axis:

$$M\ddot{x}_c = 0$$

Double integration of this gives us that:

$$M\dot{x}_c = 0, => Mx_c = 0$$

It means that the center of mass didn't move from the beginning till the final position.

$$Mx^o_c = Mx^f_c$$

It means that:

$$\Sigma M_i x^o_I = \Sigma M_i x^f_i$$

If we substitute the values into M_i and x_i, we get long expression, but if we simplify it a bit we get:

$$M_1 s + M_2 d_{M_2} + M_2 s + M_3 d_{M_2}\cos(\pi/3) + M_3 s + M_{ABCD} s = 0$$

Since the load 1 goes down by 1 meter, therefore, the load 2 moves right by 1 meter. $d_{M_2} = 1$ meter.

From this we get:

$$s = -\frac{M_2 d_{M_2} + M_3 d_{M_2}\cos(\pi/3)}{M_1 + M_2 + M_3 + M_{ABCD}}$$

s $= -\frac{4}{29} \approx -0.138$ meters = 13.8 cm.

1.3 Answers

HIGHLIGHTED ANSWERS ARE HERE

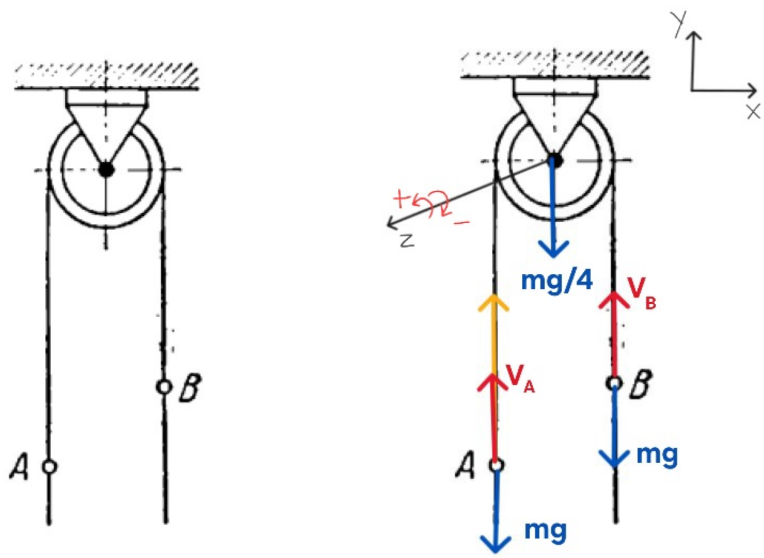
s = 13.8 cm to the left (or -13.8 cm along X-axis) $\approx$ 14 cm, as it was stated on the slide :)

2 Task 2

2.1 Task description

On the figure a pulley with a rope running over it is presented. A man (mass m) holds one side of the rope at A while a load, equal to the weight of the man, is attached to the other end of the rope at B. What would happen to the load if the man starts climbing up the rope with the velocity $\boldsymbol{a}$ relative to the rope. The weight of the pulley is equal 0.25. The mass of the pulley is distributed uniformly along its rim.

UPD: mass of the pulley is 0.25m



2.2 Solution

RO: system (pulley, man, load). A and B undergo rectilinear motion. And pulley undergo rotation.

Conditions:

$$
\begin{aligned}
V_A^o &= 0, & V_A^f &= a - V_B \\
V_B^o &= 0, & V_B^f &= -V_B \\
a_A^o &= -g, & a_A^f &= -g \\
a_B^o &= -g, & a_B^f &= -g \\
y_A^o &= y_A, & y_A^f &=? \\
y_B^o &= y_B, & y_B^f &=?
\end{aligned}
$$

Force analysis:

Gravity forces on the bodies: $\vec{G}_A, \vec{G}_B, \vec{G}_pulley$

Well, let's solve it via changing of anguLar momentum of the system.

The man climbed the rope and the momentum is $-mgr$, and the load moved upwards and the momentum should be the same, but with different sign. Well, here is the anguLar momentum:

$$L_A + L_B + L_{pulley} = 0$$

Since we know that the pulley is like a ring, so, the moment of inertia is MR^2. And the formula for $L_{pulley} = I\omega$. So, we get that:

$$L_{pulley} = -0.25mr^2 * \frac{V_b}{r}$$
$$L_A = m(a - V_B)r$$
$$L_B = -mV_Br$$

Then, we substitute into the equation and get that:

$$m(a - V_B)r - mV_Br - 0.25mr^2 * \frac{V_b}{r} = 0$$

We can divide it by mr:

$$-V_B + a - V_B - 0.25V_B = 0$$

From this we get that $V_B = \frac{4a}{9}$

2.3 Answers

HIGHLIGHTED ANSWERS ARE HERE

$V_B = \frac{4a}{9}$. The load will move upwards with this velocity.

3 MEME