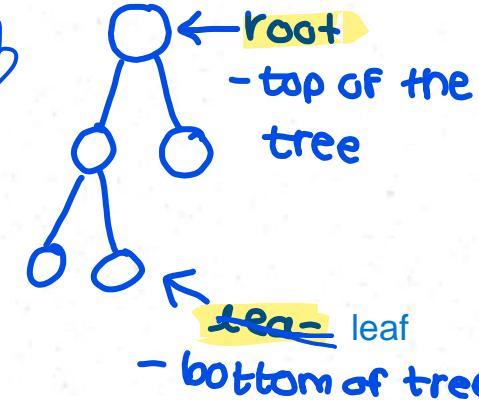
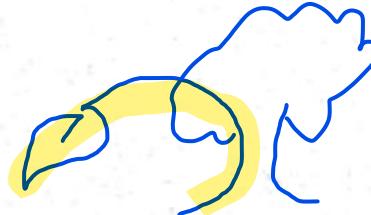


Tree Data Structure

What is a tree?



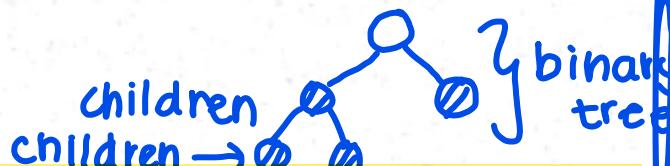
Like a tree there is only one way to get to a certain leaf

A tree consist of nodes connected by edges.

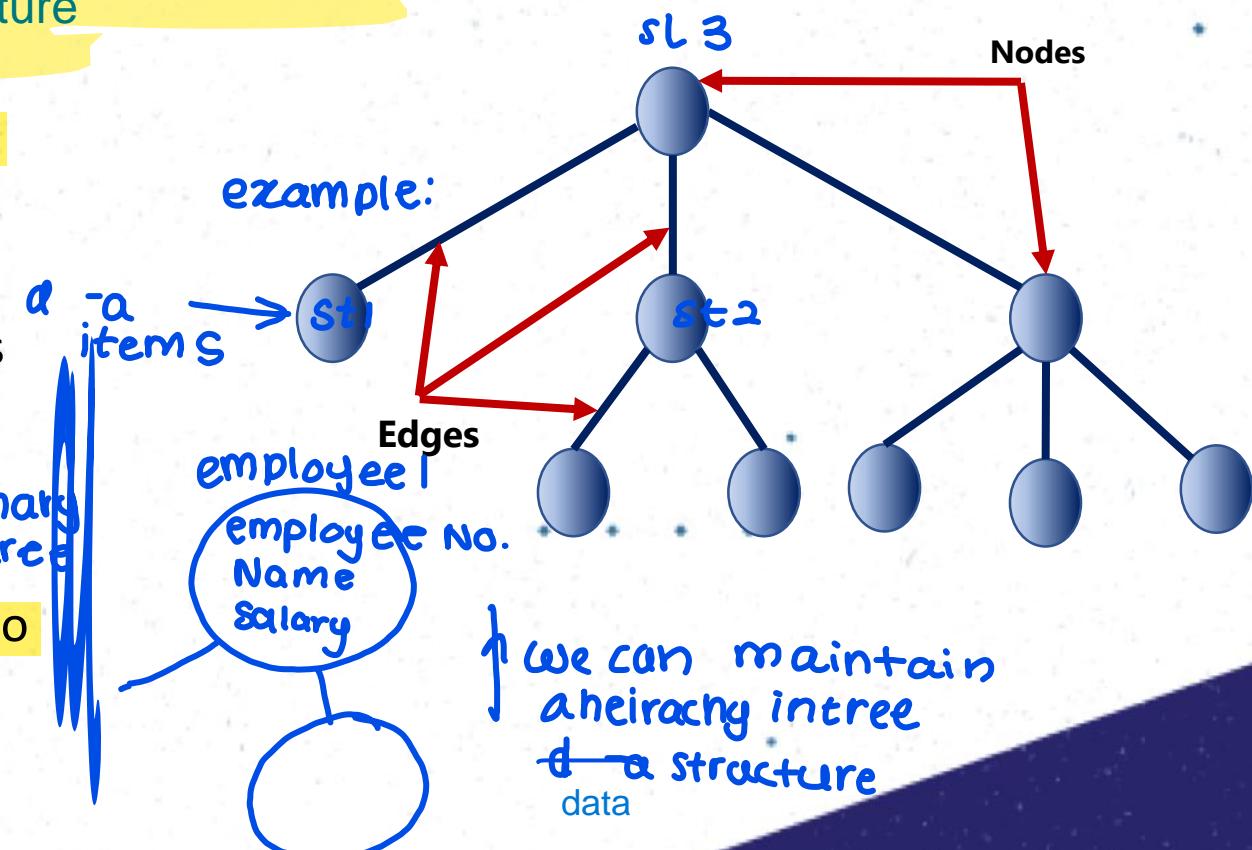
Google search engine refers this sort of structure

In a picture of a tree the nodes are
represented as circles and the edges as lines
connecting the circles.

In a tree, the nodes represent the data items
and the edges represent the way the nodes
are related.



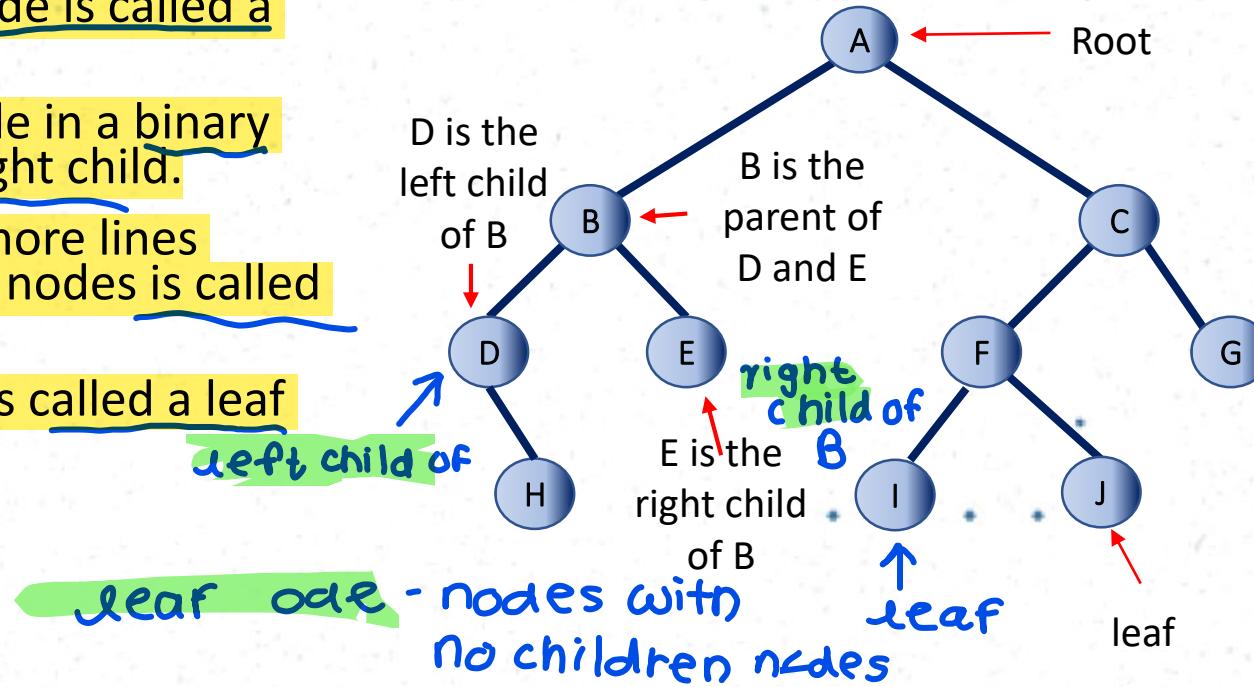
A tree with nodes which has maximum of two
children is called a **binary tree**.



What is root, parent and children in

- The node at the top of the tree is called root.
- Any node which has exactly one edge running upwards to other node is called a child.
- The two children of each node in a binary tree is called left child and right child.
- Any node which has one or more lines running downwards to other nodes is called a parent.
- A node that has no children is called a leaf node or leaf.

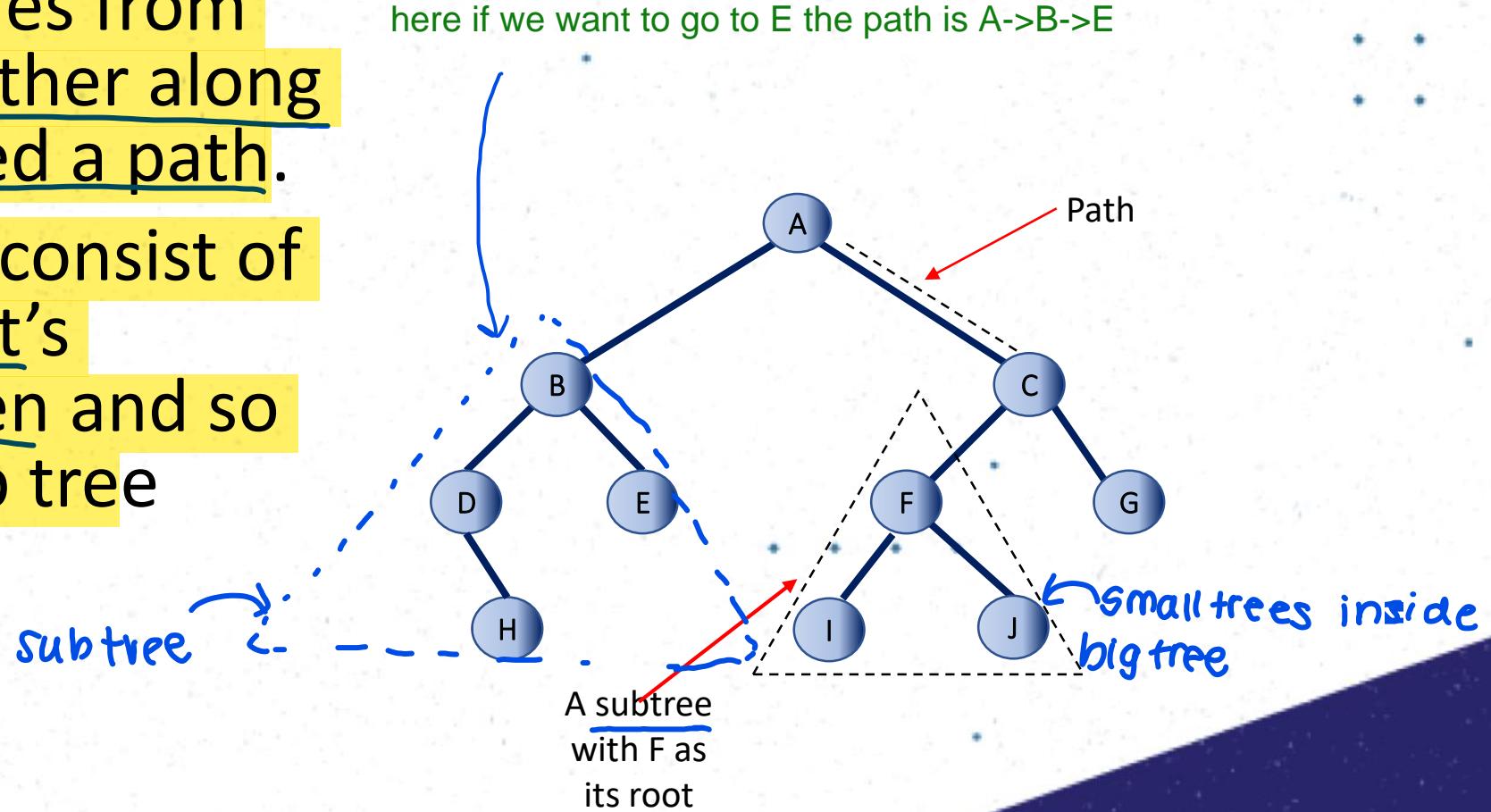
Root node = A
Leaf node = G/A/J/E/H(has no children)



What is path and subtree in a tree

We always start the journey from one node

- Sequence of nodes from one node to another along the edges is called a path.
- Any node which consist of its children and it's children's children and so on is called a sub tree



based on number of children's a tree can be
a) Binary Tree- any node can have max of two children
if we apply value of left child < value of node <= value of right child
it becomes a binary search tree (the binary tree may have random nodes at random places)

Key value of a node

- Each node in a tree stores objects containing information.
- Therefore one data item is usually designated as a key value.
it can be any value
- The key value is used to search for a item or to perform other operations on it.
- eg: person object – social security number
car parts object– part number

Operations of Binary Search Tree

-There are four main operations perform in binary search tree ,

Find - Find a node with a given key

Insert - insert new node

Delete - delete a node

Traverse - visit all nodes



Traverse can be known as displaytree() / here there are many ways

Binary Search Tree

- Binary Search Tree

- ► is a tree that has at most two children.

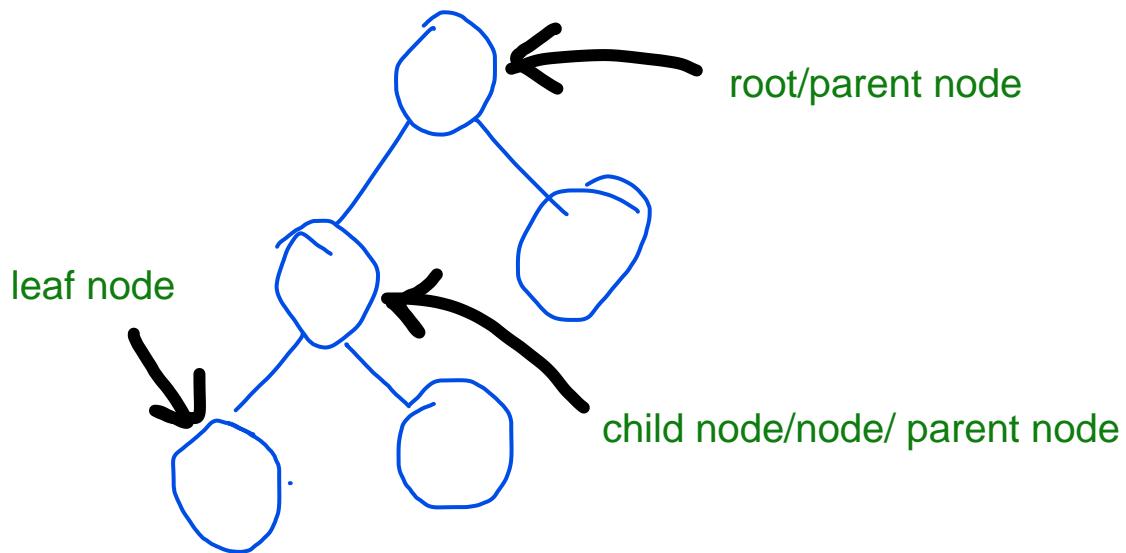
This tree can be a random number given to a node but the rules below must apply

► a node's left child must have a key less than its parent and node's right child must have a key greater than or equal to its parent.

condition for Key values in binary search tree

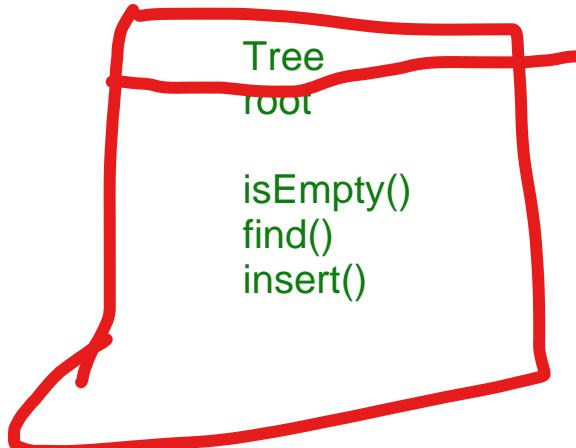
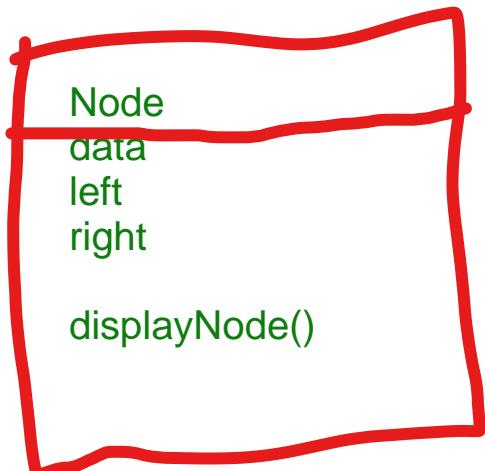
Condition for binary search tree =

Left Node < Node(root) <= Right Node



here the searching of nodes may be more efficient than the linked list structure

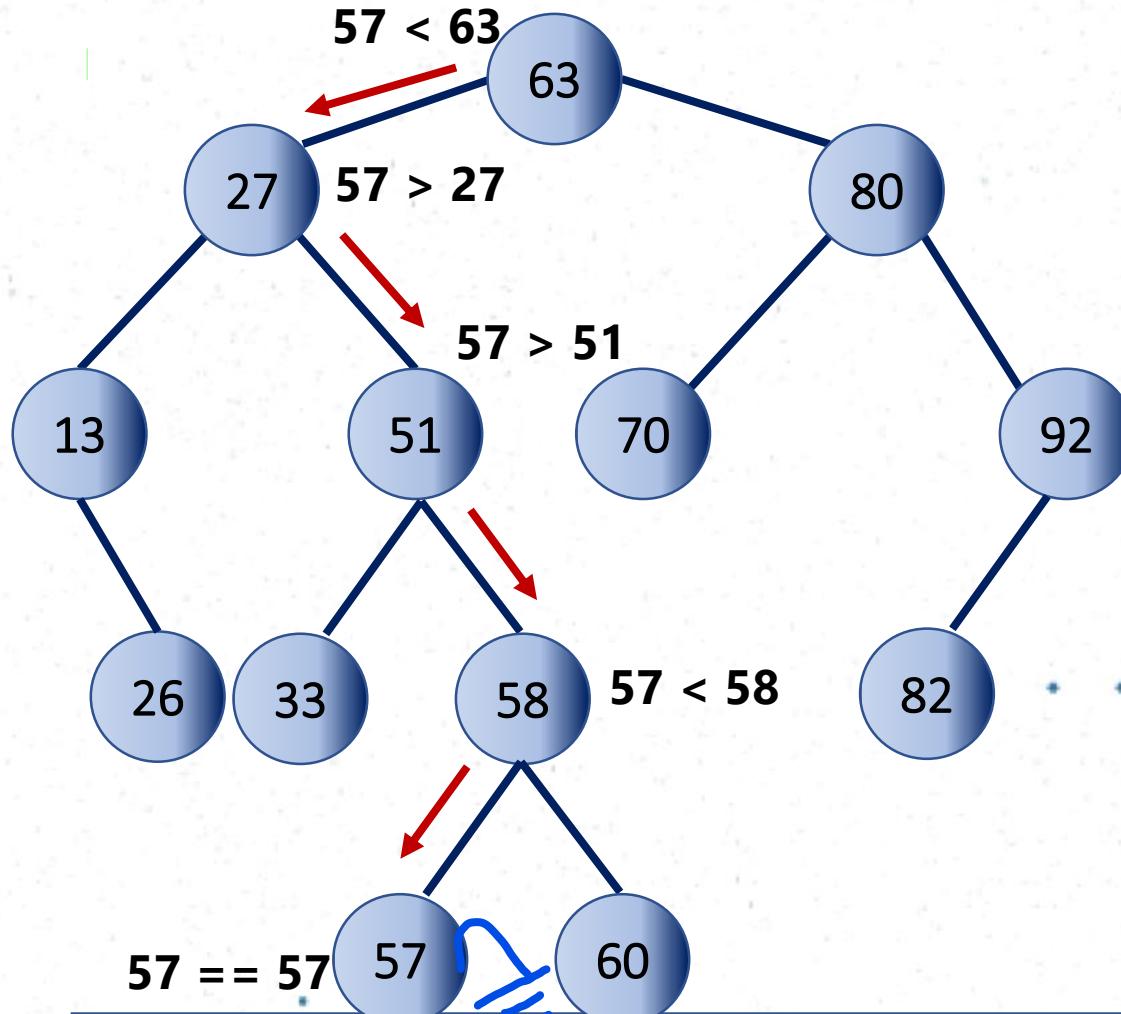
you can use characters for the key of the node as well



Operations - Find

- Find always start at the root.
- Compare the key value with the value at root.
- If the key value is less, then compare with the value at the left child of root.
- If the key value is higher, then compare with the value at the right child of root
- Repeat this, until the key value is found or reach to a leaf node.

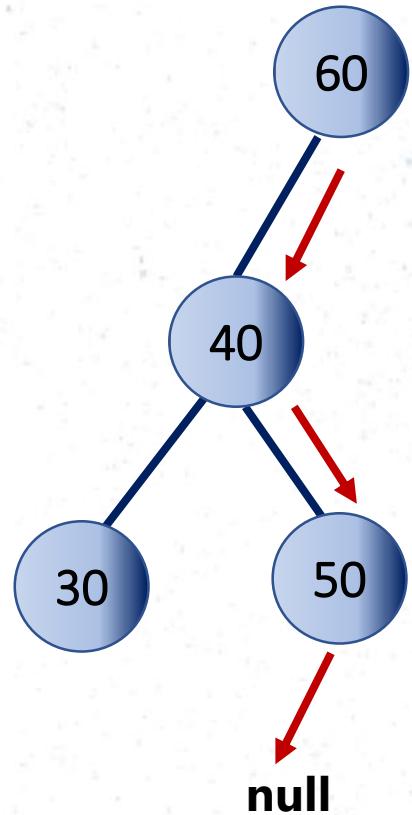
Find value 57



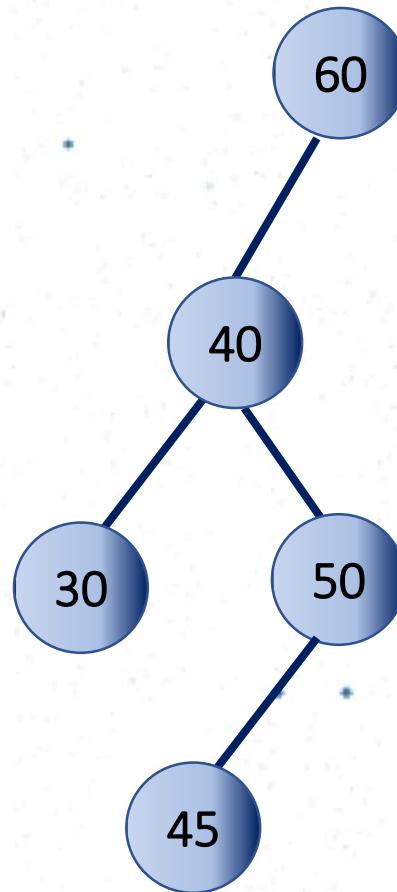
Operations - Insert

- Create a new node.
- Find the place (parent) to insert a new node.
- When the parent is found, the new node is connected as its left or right child, depending on whether the new node's key is less than or greater than that of the parent.

Insert value 45



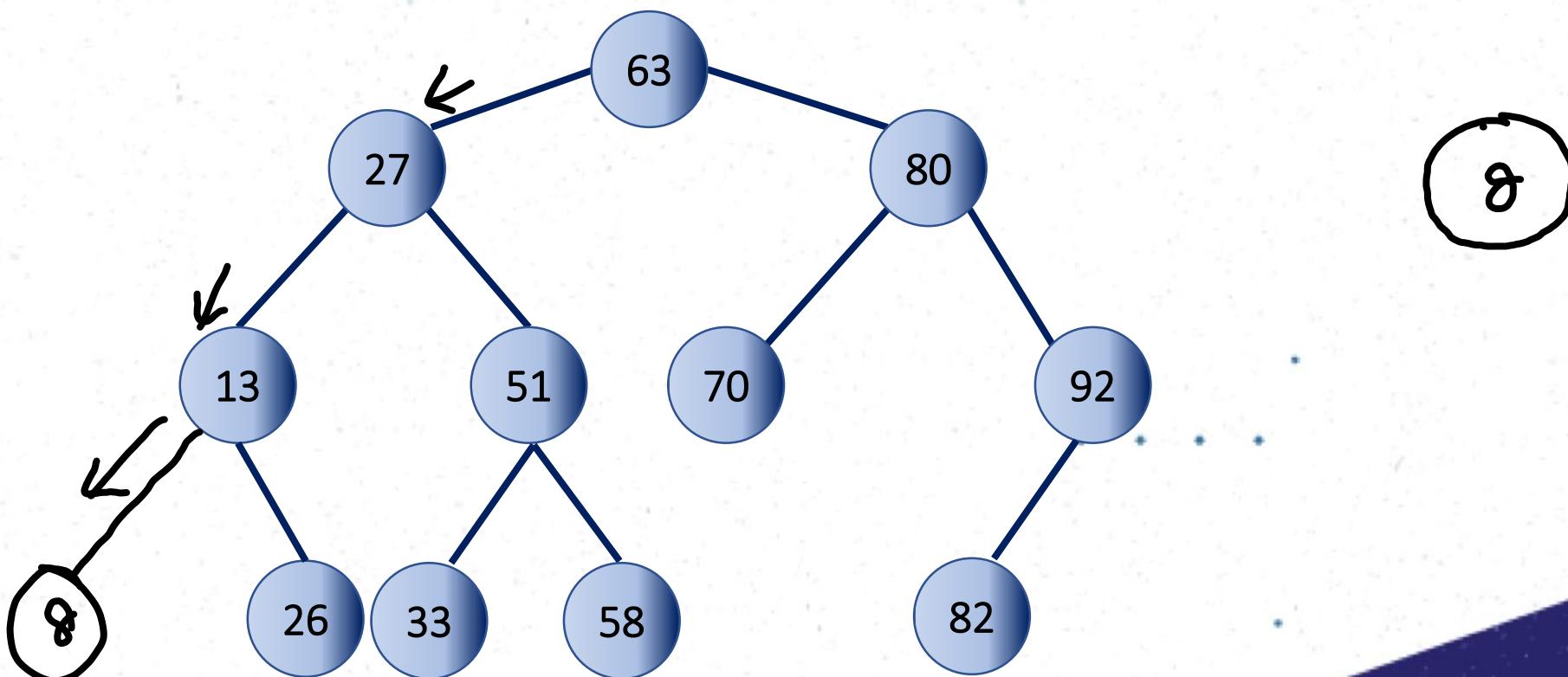
a) Before insertion



b) After insertion

Question 1

- Draw a tree after inserting number 8



BFS vs DFS for Binary Tree

A Tree is typically traversed in two ways:

Traversal is visiting all nodes

- Breadth First Traversal (Or Level Order Traversal)
- Depth First Traversals
 - 1. Inorder Traversal (Left-Root-Right)
 - 2. Preorder Traversal (Root-Left-Right)
 - 3. Postorder Traversal (Left-Right-Root)

Traversal = visiting all nodes
the ways of which this can
be done is called traversal

level by level
from left to right

BFS and DFSs of above Tree

Breadth First Traversal : 1 2 3 4 5

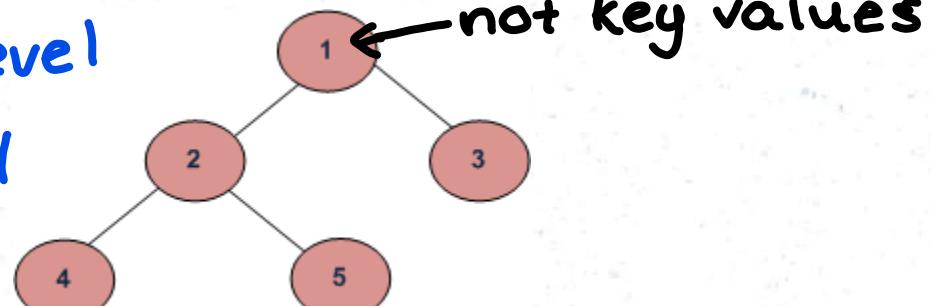
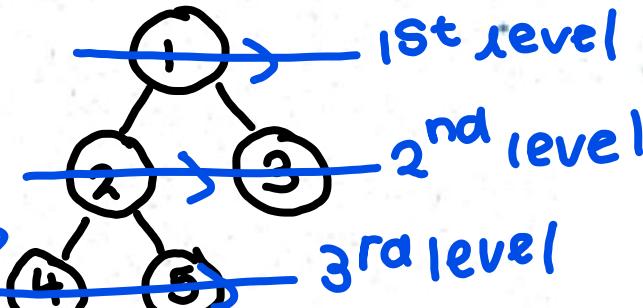
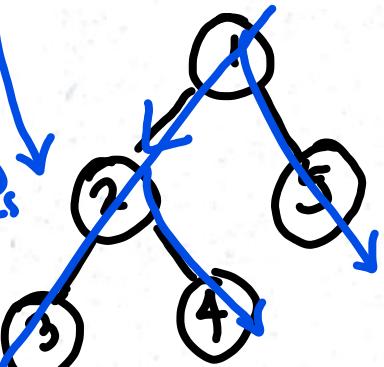
Depth First Traversals:

Preorder Traversal : 1 2 4 5 3

Inorder Traversal : 4 2 5 1 3

Postorder Traversal : 4 5 2 3 1

go along one
root till you
meet an end
after that
complete all
subroots along
that root as
well



Traversing

- Traverse a tree means to visit all the nodes in some specified order.
- There are three common ways to traverse a tree
 - Pre order
 - In order
 - Post order

InOrder(root) visits nodes in the following order:

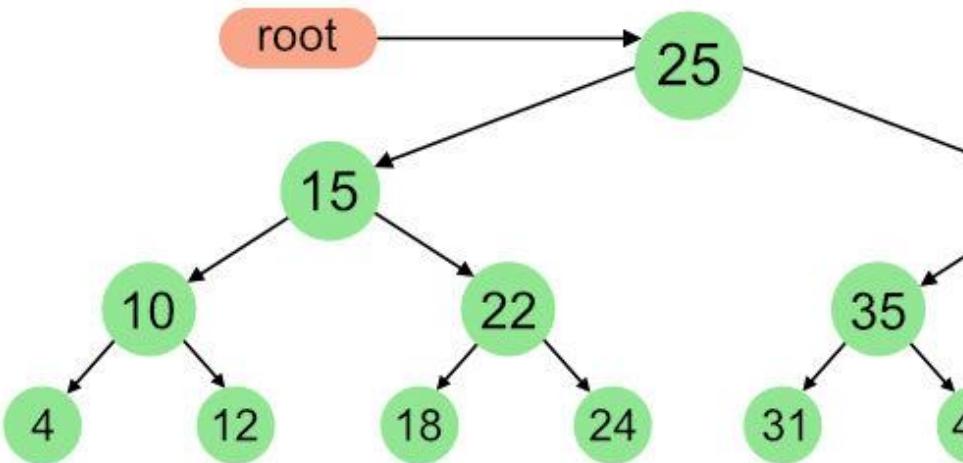
4, 10, 12, 15, 18, 22, 24, 25, 31, 35, 44, 50, 66, 70, 90

A Pre-order traversal visits nodes in the following order:

25, 15, 10, 4, 12, 22, 18, 24, 50, 35, 31, 44, 70, 66, 90

A Post-order traversal visits nodes in the following order:

4, 12, 10, 18, 24, 22, 15, 31, 44, 35, 66, 90, 70, 50, 25



Pre order

25 15 10 4 12 22 18
24 50 35 31 44 70 66
90

in order

4 10 12 15 18 22
24 25 31 35 44 50

66 70 90

Postorder

4 12 10 18 24 22 15
31 44 35 66 90 70
50 25

Inorder traversing

By the theory here is first display the left subtree of a node then the node, after right subtree

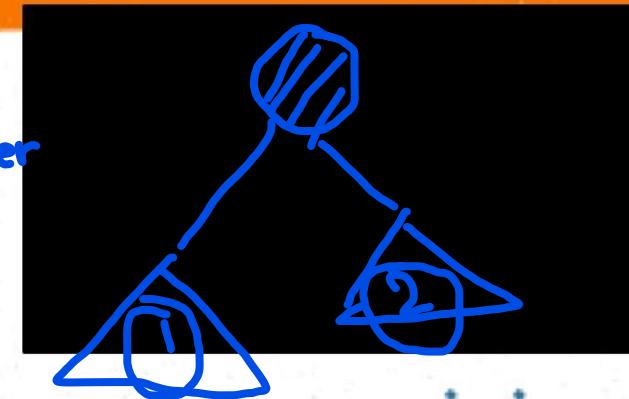
- Call itself to traverse the node's left subtree
- Visit the node
- Call itself to traverse the node's right subtree

depth first transversal

pre-order

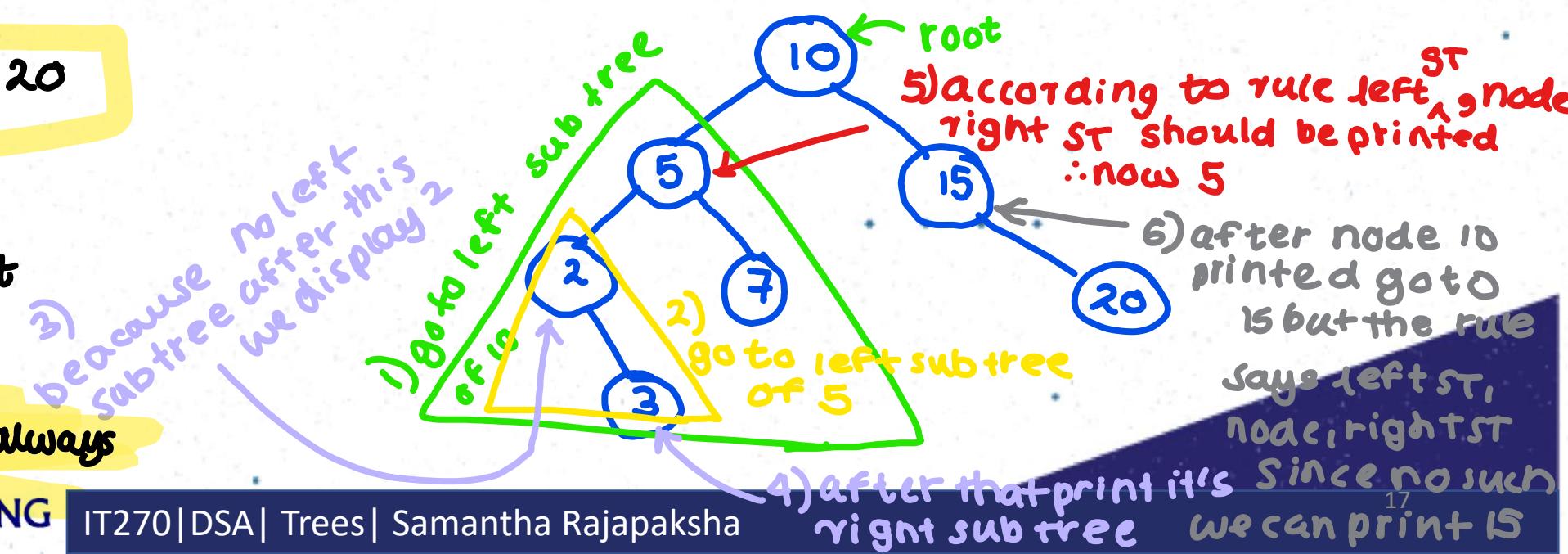
in order

post order

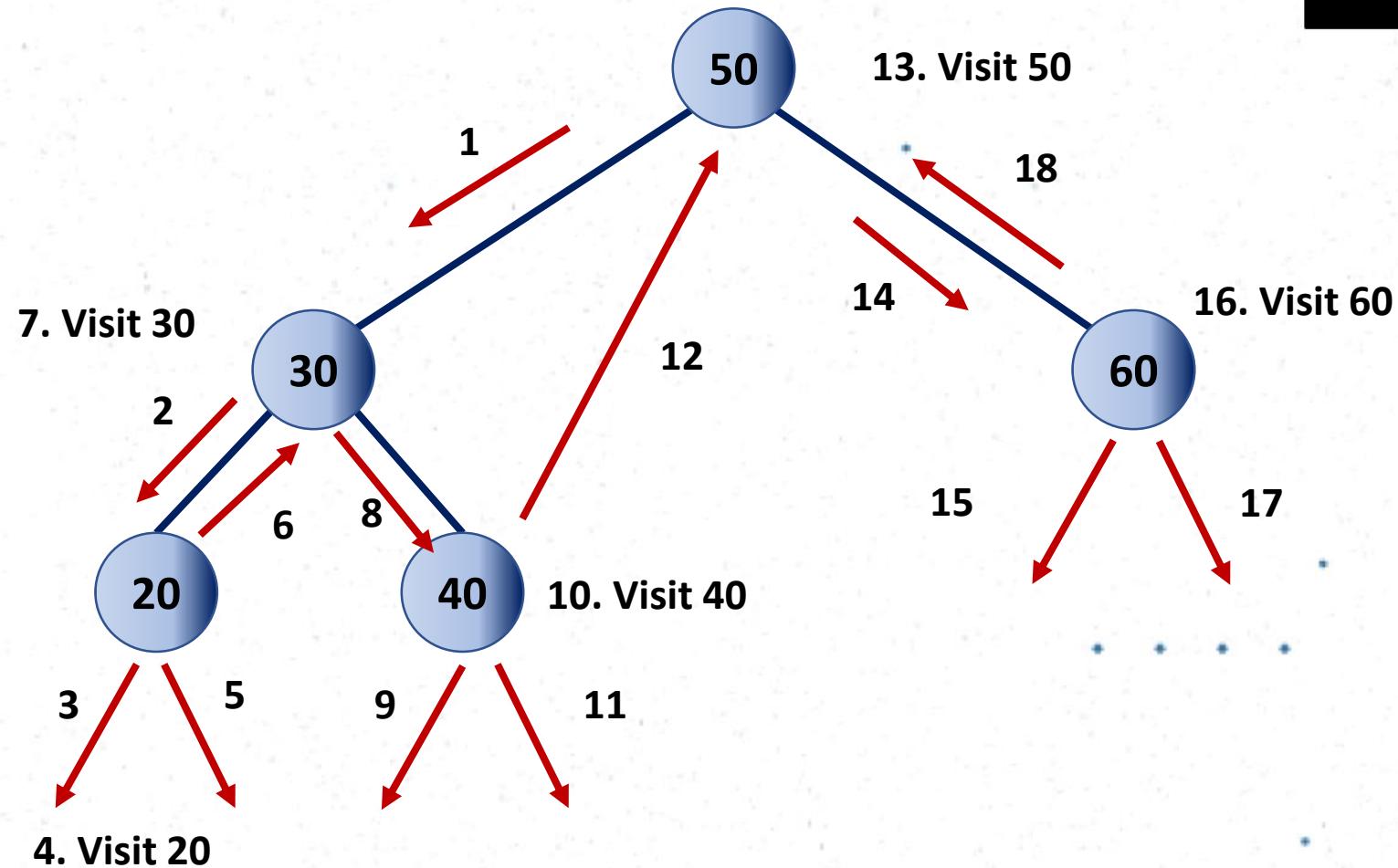


2 3 5 7 10 15 20

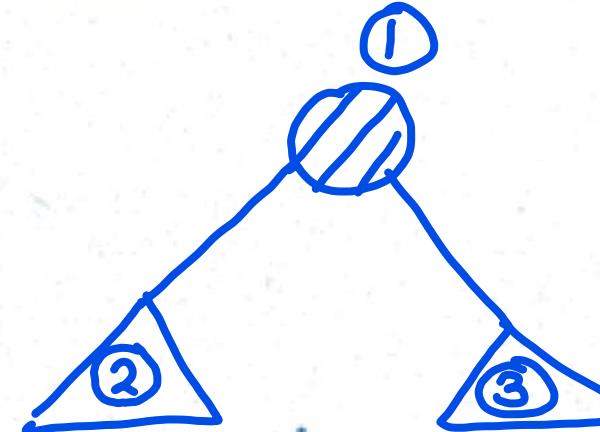
it is easy to print in order traversal because output is in ascending order always



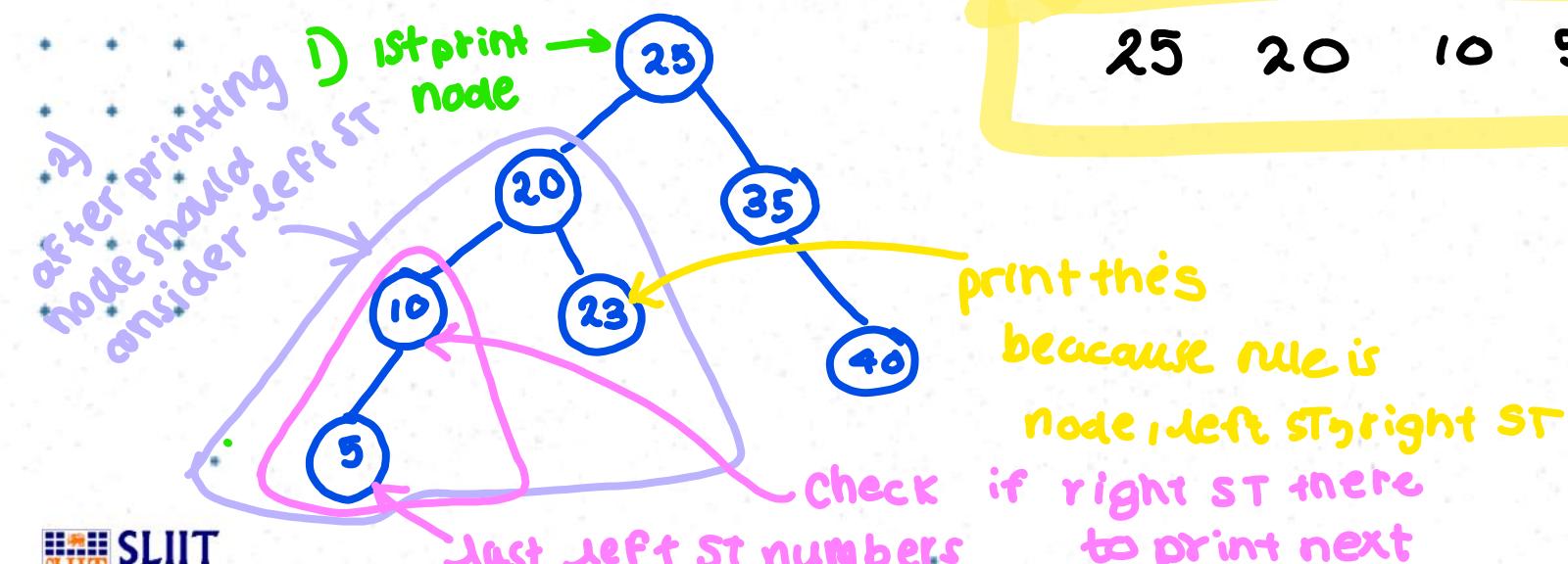
Inorder traversing cont...



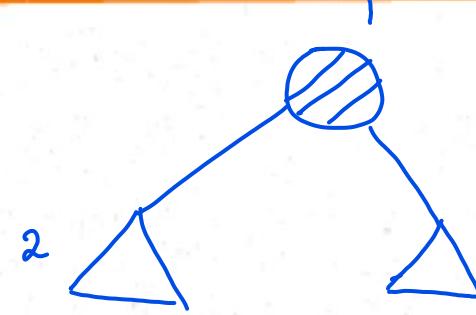
Preorder traversing



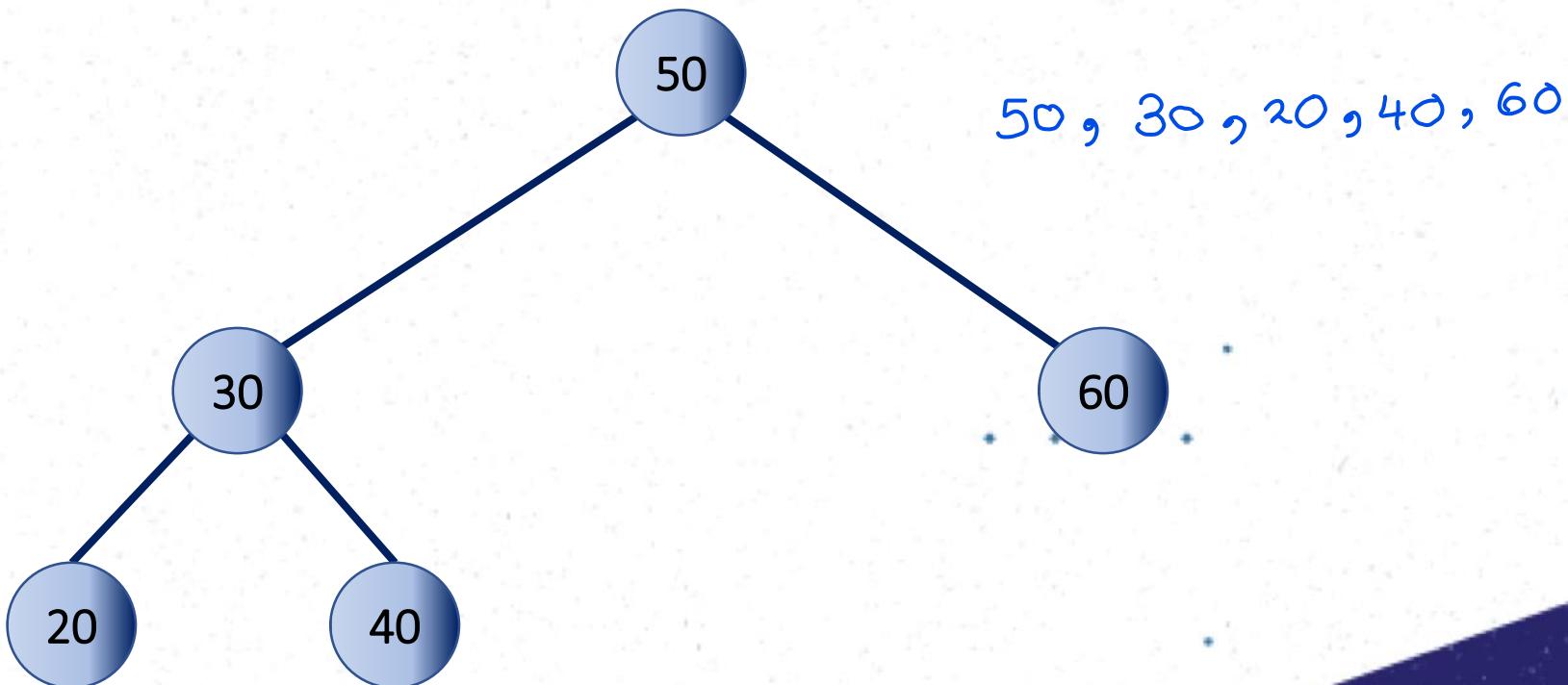
- Visit the node
- Call itself to traverse the node's left subtree
- Call itself to traverse the node's right subtree



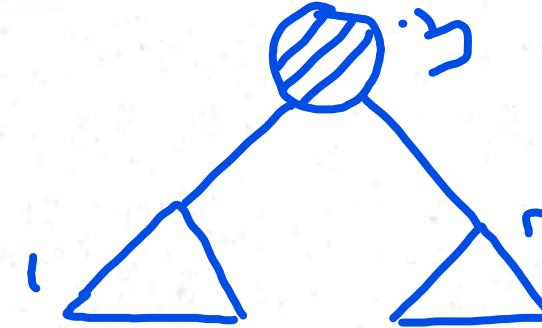
Question 2



- Write the output, if the following tree is traverse in preorder.

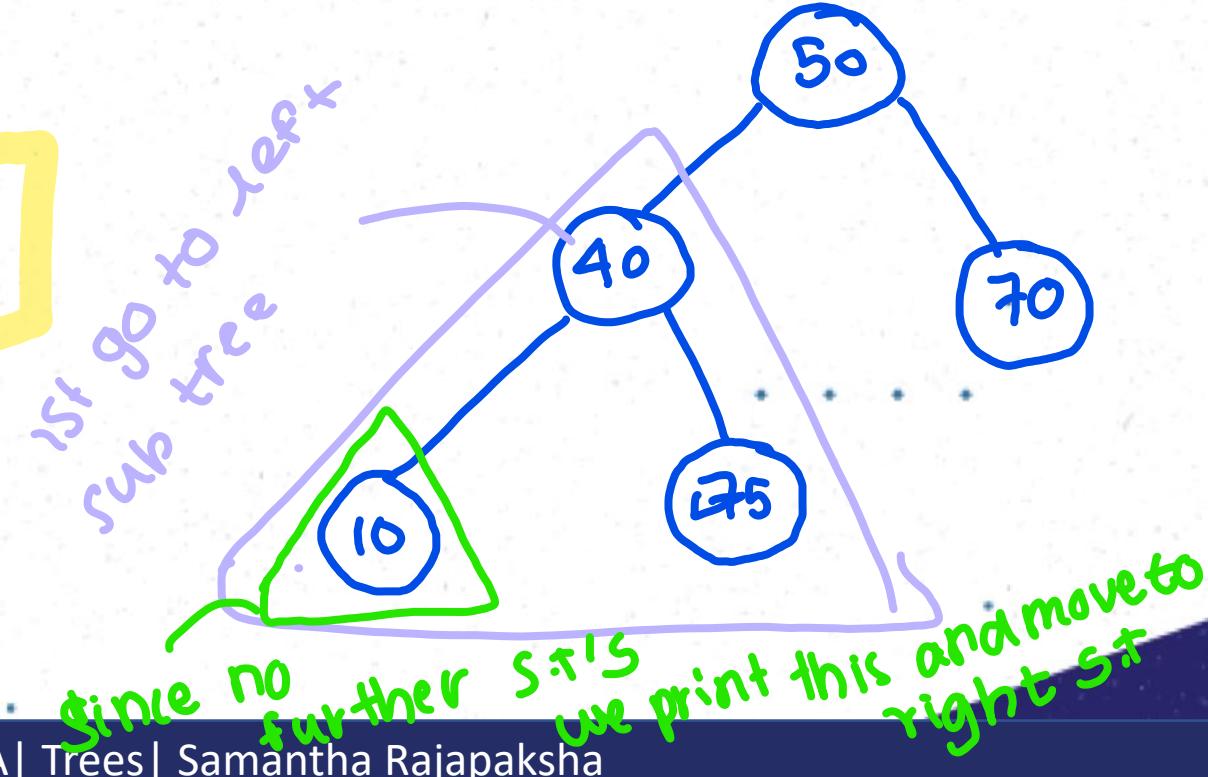


Postorder traversing

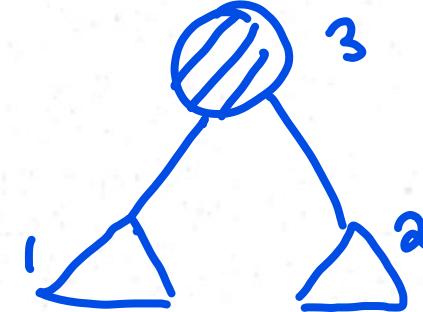


- Call itself to traverse the node's left subtree
- Call itself to traverse the node's right subtree
- Visit the node

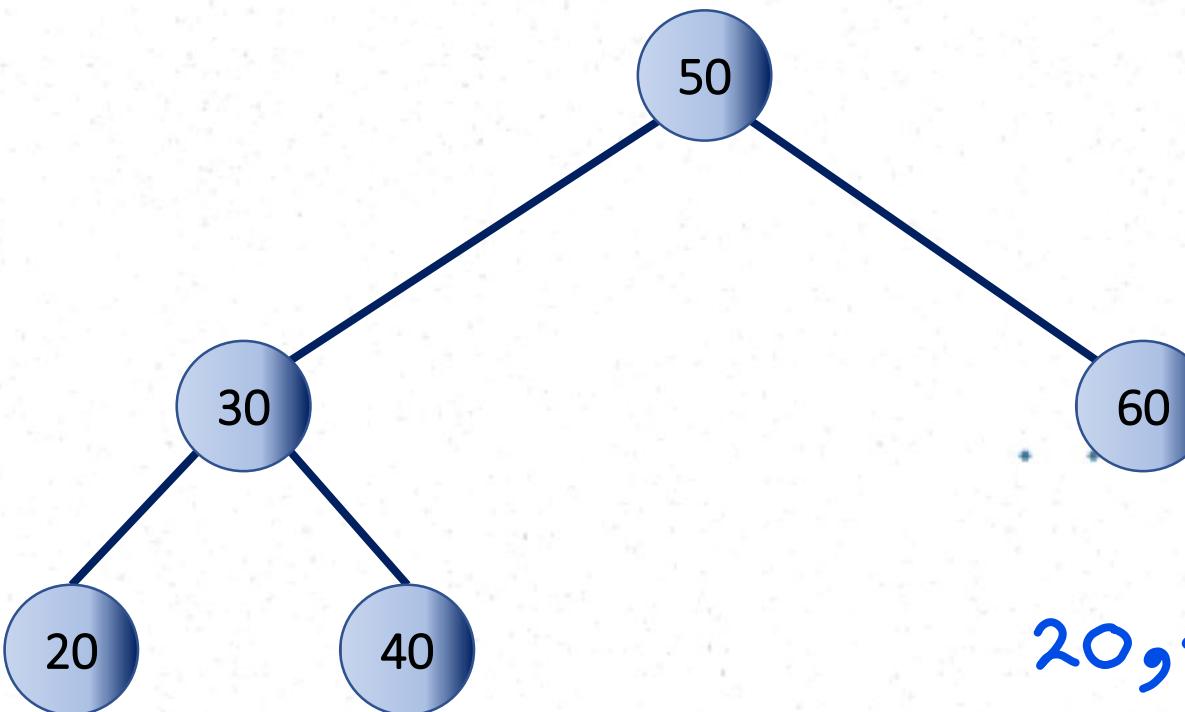
10 45 40 70 50



Question 3



- Write the output, if the following tree is traverse in postorder.



20, 40, 30, 60, 50

Binary search tree - Implementation

in this implementation like Link List we
need two classes as

Node Class

- Node contains the info of an object.
- Each node should have a key, data and reference to left and right child.

```
class Node
{
    public int iData; // data item (used as key value)
    public double dData; //other data
    public Node leftChild; // this node's left child
    public Node rightChild; //this node's right child

    public void displayNode(){
        System.out.print("{");
        System.out.print(iData);
        System.out.print(", ");
        System.out.print(dData);
        System.out.print( " }");
    }
}
```

key //in example

displayData

constructor () {
when initializing
left and right child
inside constructor
we must == null
y

Binary search tree - Implementation

Tree Class

```
class Tree
{
    private Node root; // first node of tree

    public Tree(){
        ↗ no parameter
        root = null;   ↗ at the initial moment no
                        nodes :: no root
    }

    public void insert (int id, double dd){
    }

    public boolean delete (int id){
    }

    public Node find (int key){
    }
}
```

Binary search tree - Implementation

Tree Class – Find Method

```
class Tree
{
    private Node root;
    public Tree(){
        root = null;
    }
    public void insert (int id, double dd){
    }
    public void delete (int id){
    }
    public Node find(int key){
        Node current = root;
        while (current.iData != key)
        {
            if(key < current.iData)
                current = current.leftChild;
            else
                current = current.rightChild;
            if (current == null)
                return null;
        }
        return current;
    }
}
```

Binary search tree - Implementation

```
class Tree{  
    private Node root;  
    .....  
    .....  
  
    public void insert ( int id , double dd){  
        Node newNode = new Node();  
        newNode.iData = id;  
        newNode.dData = dd;  
        if (root == null) // no node in root  
            root = newNode;  
        else // root occupied  
        {  
            Node current = root; //start at root  
            Node parent;  
            while (true)  
            {  
                parent = current;  
                .....  
            }  
        }  
    }  
}
```

Method

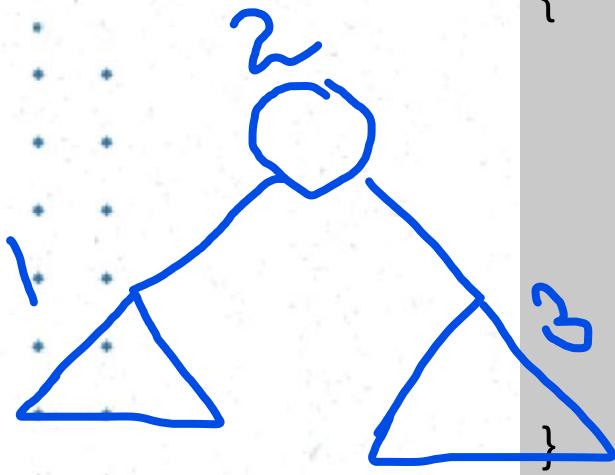
```
        if (id < current.iData) // go left  
        {  
            current = current.leftChild;  
            if (current == null) {  
                parent.leftChild = newNode;  
                return;  
            }  
        }  
        else // go right  
        {  
            current = current.rightChild;  
            if (current == null){  
                parent.rightChild = newNode;  
                return;  
            }  
        }  
    }  
}
```



Binary search tree - Implementation

Tree Class – Inorder Traversing Method

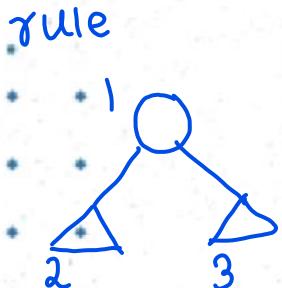
```
private void inOrder(Node localRoot)
{
    if (localRoot != null)
    {
        inOrder(localRoot.leftChild);
        localRoot.displayNode();
        inOrder(localRoot.rightChild());
    }
}
```



N L R

Binary search tree - Implementation

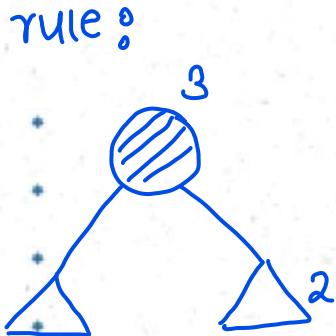
Tree Class – Preorder Traversing Method



```
private void preOrder(Node localRoot)
{
    if (localRoot != null)
    {
        localRoot.displayNode(); N
        preOrder(localRoot.leftChild()); L
        preOrder(localRoot.rightChild()); R
    }
}
```

Binary search tree - Implementation

Tree Class – Postorder Traversing Method



```
private void postOrder(Node localRoot)
{
    if (localRoot != null)
    {
        postOrder(localRoot.leftChild); L
        postOrder(localRoot.rightChild); R
        localRoot.displayNode(); N
    }
}
```

Operations - Delete

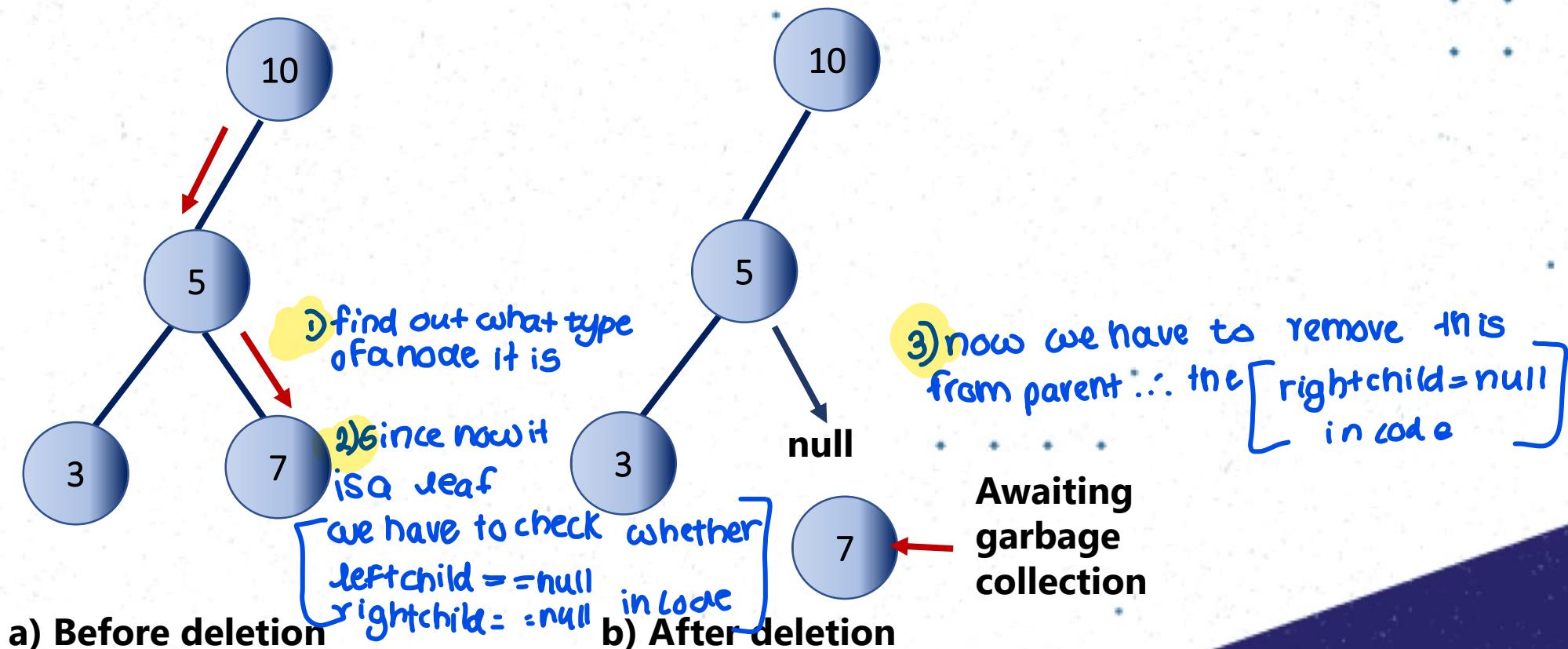
here we don't consider the code just the concept

- First find the node to be deleted.
- If the node to be deleted is found there are three cases to be considered. Whether
 - The node to be deleted is a leaf
 - The node to be deleted has one child
 - The node to be deleted has two children.

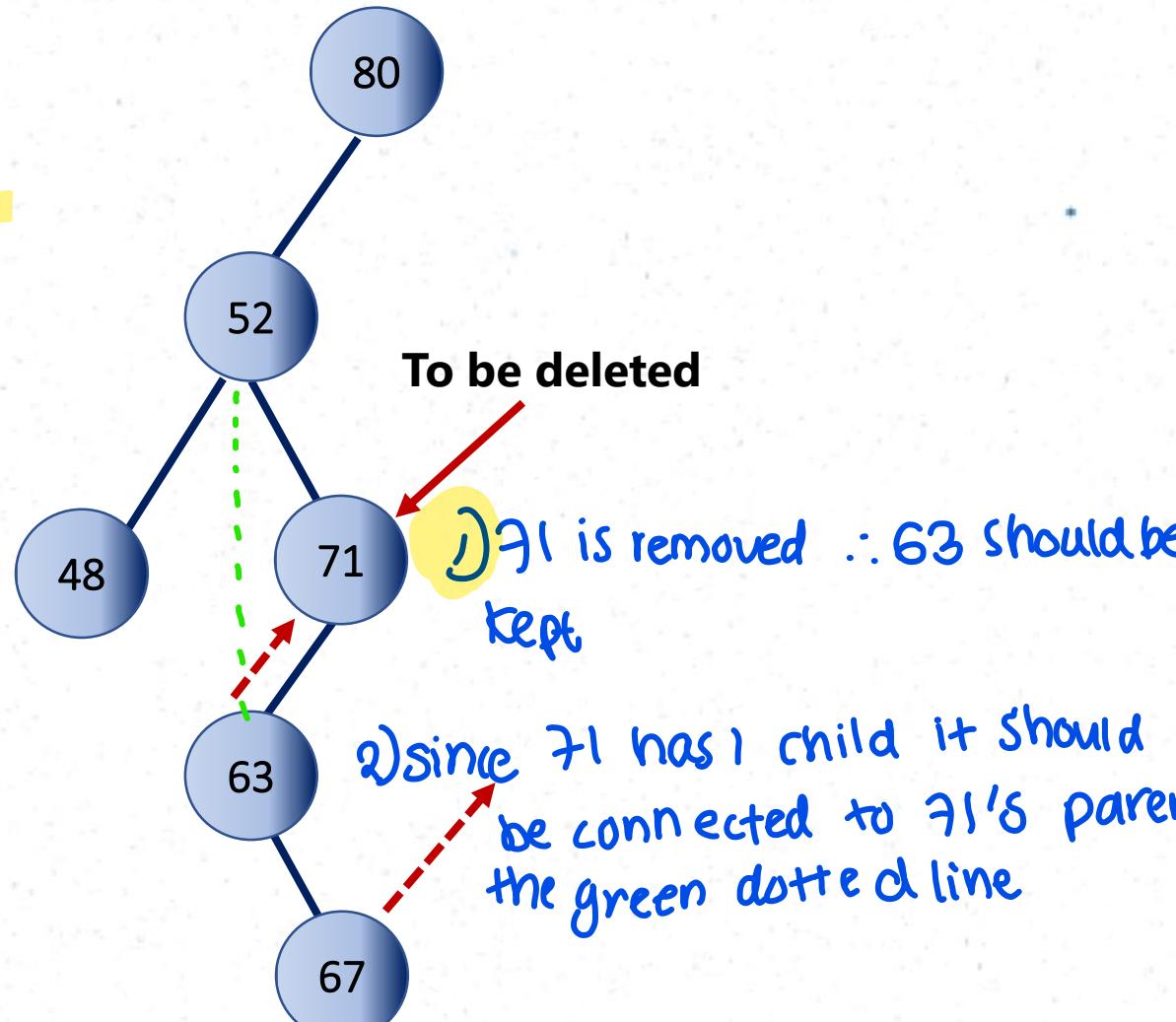
Case 1 : The node to be deleted has no children

no

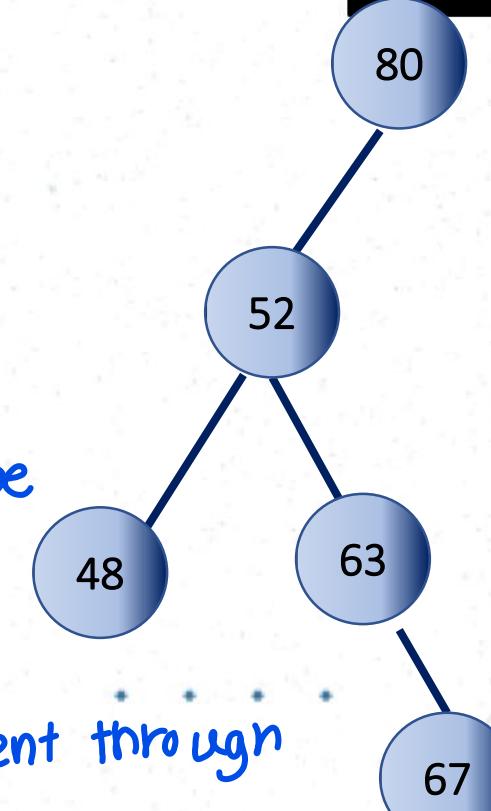
Delete 7



Case 2 : The node to be deleted has one child

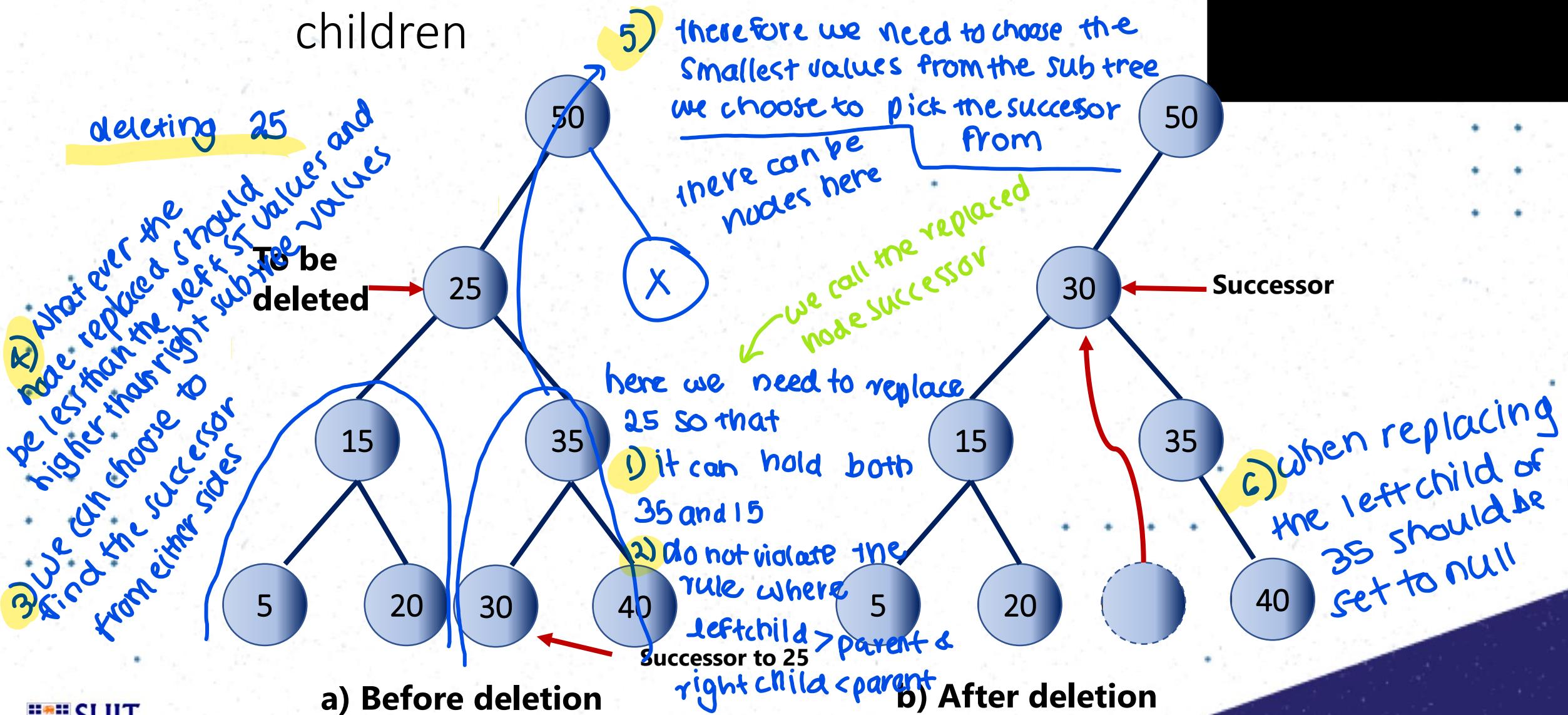
deleting 71:

a) Before deletion



b) After deletion

Case 3 : The node to be deleted has two children



if we are replacing a node with another, we call it a successor

How to find a successor of a node?

here there are two options are to choose from either side in the tree

- a) Choose the minimum value in the right sub tree and replace it with the node to be deleted
- b) Choose the maximum value in the left sub tree and replace it with the node to be deleted

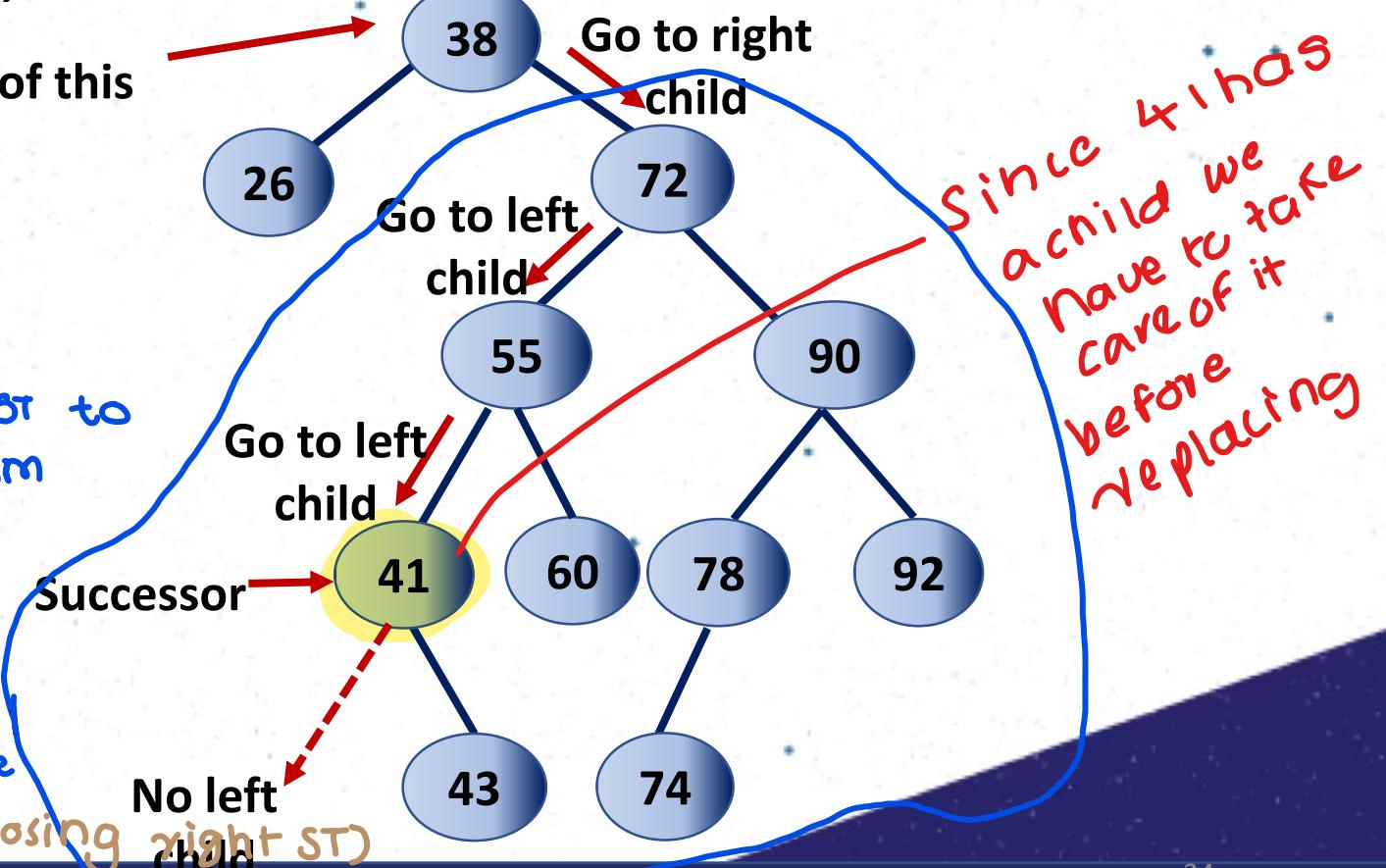
- In a Binary Search Tree, successor of a node is a node with next-highest key

To find the successor of this node

deleting 38°

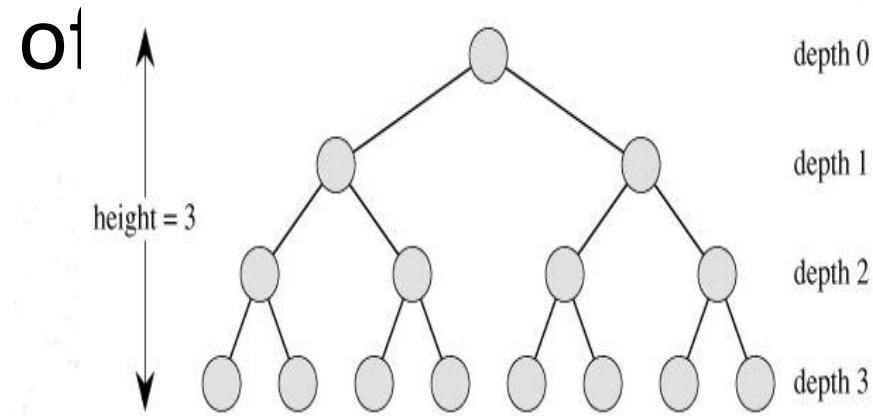
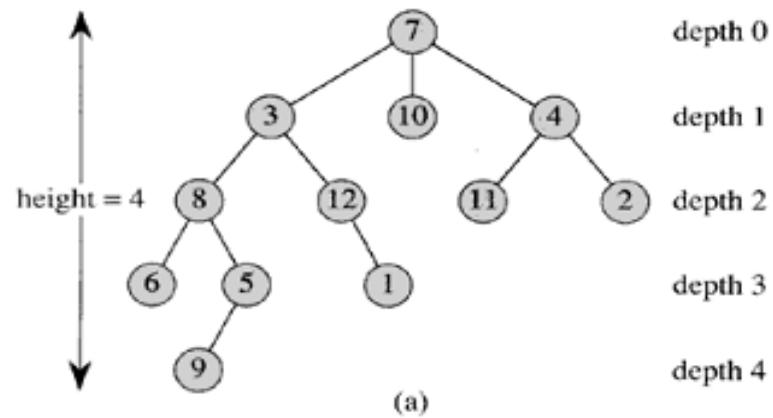
if choosing left ST choose the maximum value from there

- 1) here we chose right ST to choose the successor from
- 2) we need to choose the smallest value from the tree chosen
- 3) usually it is the left most value that contains the smallest value (if choosing right ST)



Tree Terminology

- **Degree of a node:** The number of children it has
- **Depth:** The depth of x in a tree is the length of the path from the root to a node x . *← levels* root depth is always 0
- **Height:**



The maximum depth is taken as the height of the tree

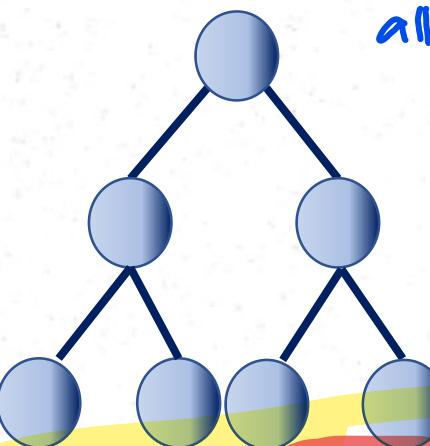
Height = the largest depth of any node in a tree

Full Binary Tree

a full binary tree is when all the levels are full

- A Full binary tree of height h that contains exactly $2^{h+1}-1$ nodes

all levels are filled here



Height, $h = 2$,

$$\text{nodes} = 2^{2+1}-1 = 7$$

$$2^{h+1} - 1$$

if you want it to be a binary tree it should fill from left to right

Complete Binary Tree

- It is a Binary tree where each node is either a leaf or has degree ≤ 2 .

- Completely filled, except possibly for the bottom level

- 2) • Each level is filled from **left to right**

- All nodes at the lowest level are as far to the left as possible

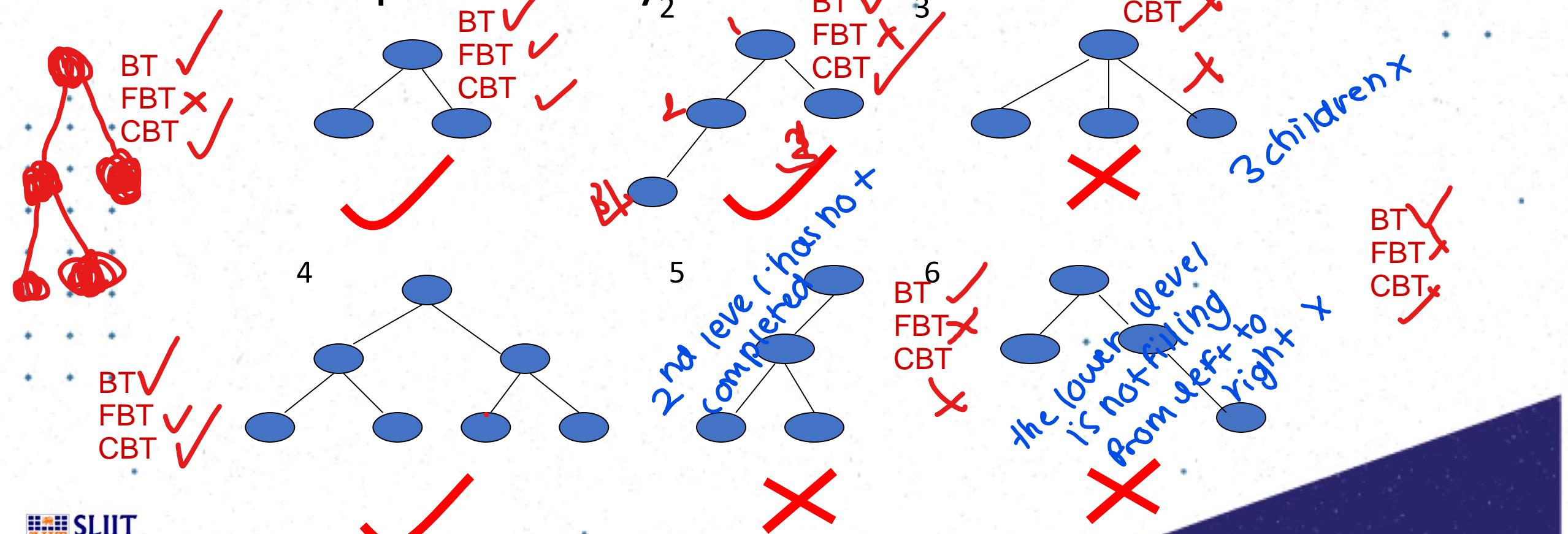
- Full binary tree is also a complete binary tree

All complete binary trees are not full binary trees

Question

- in order to check for binary trees
- should have two children= BINARY TREE
 - all levels should completely fill with node= FULL BINARY TREE
 - should fill from left to right=COMPLETE BINARY TREE

- Find Complete Binary Trees

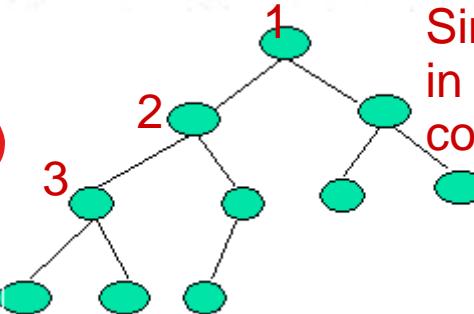


Height of a complete binary tree

• Height of a complete binary tree that contains n elements is $\lfloor \log_2(n) \rfloor$

- Example

$$h = \lfloor \log_2 n \rfloor = \lfloor \log_2 10 \rfloor = 3 \dots$$



Since maximum depth is the height in a full binary tree this answer is correct (3. something)

This is 3. something because 2^3 is 8 (closest)

Above is a Complete Binary Tree with height = 3

No of nodes: $n = 10$

$$\text{Height} = \lfloor \log_2(n) \rfloor = \lfloor \log_2(10) \rfloor = 3$$

answers
correct

level 3's are
complete