CHAPTER II: MECHANICS OF ORTHOGONAL CUTTING

To analyse various forces involved in an orthogonal metal cutting process and mainly to determine cutting force theoretically, a simplified scheme of chip formation is developed with the following assumptions:
a) The tool is perfectly sharp to avoid contact or rubbing of the tool flank with the machined surface
b) During formation of chips, no side flow of chips occurs, i.e., plane strain condition exists
c) Cutting velocity is uniform and constant
d) The chips produced are continuous without BUE
e) Free cutting condition exists, and
f) The chip is considered to be held in dynamic equilibrium by the action of the two equal and opposite resultant forces R and R' which are collinear. R' can be resolved into two mutually perpendicular forces F_s and N_s, and R may be resolved into another two mutually perpendicular forces F and N, where,
F_s = shear force acting along shear plane (i.e., the resistance to shear of the metal in forming the chip – reactive force)
N_s = normal force on shear plane
F = frictioinal force between chip and tool interface
N = normal reaction of the tool on the chip.

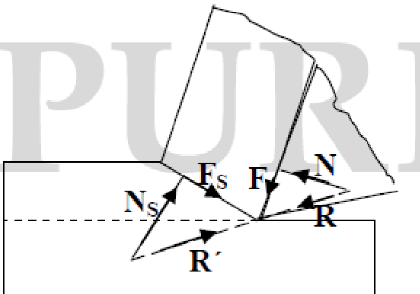

Fig. 32

Since, f, t, a_1 and a_2 are very small, R and R' may be considered to act at a point on the principal cutting edge. Again, these R and R' are nothing but the resultant of P_z and P_{xy}.

So, $\overline{R} = \overline{R'} = \overline{F} + \overline{N} = \overline{F_s} + \overline{N_s} = P_z + P_{xy}$

$P_{xy} = \sqrt{P_x^{\,2} + P_y^{\,2}}$ and in orthogonal 2-D system of second kind $P_{xy} = P_x$ or $P_{xy} = P_y$. However, P_{xy} and P_z are perpendicular to each other.

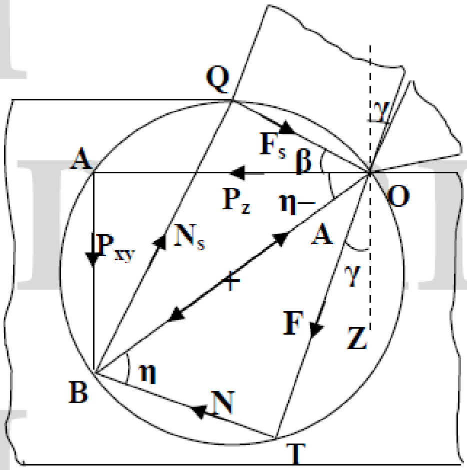

Fig. 33

In each of the three sets of forces, the common matter is one force is perpendicular to the other in a set. So, each set can be represented in a semi-circle. With this concept, M. E. Merchant suggested a circle diagram depicting all these forces in it and made possible to transform the forces of one set into that of other. This is known as Merchant's Circle Diagram.
In the above figure:
μ = frictional co-efficient between chips and tool face = $\tan\eta$ = F/N,
where η = friction angle = $\angle$OBT.
Now, $\angle$ AOZ = $\angle$ AOB + $\angle$ BOT + $\angle$ TOZ

$$\Rightarrow \frac{\pi}{2} = \angle AOB + (90^\circ - \eta) + \gamma\,; \qquad \Rightarrow \angle AOB = \eta - \gamma;$$

Also, $\angle BOQ = \angle QOA + \angle AOB = \beta + \eta - \gamma$; γ = rake angle and β = shear plane angle.
So, $F_s = R\cos(\beta+\eta-\gamma)$ and, $N_s = R\sin(\beta+\eta-\gamma)$; $\therefore N_s = F_s\tan(\beta+\eta-\gamma)$
$F = R\sin\eta$ and, $N = R\cos\eta$
$P_z = R\cos(\eta-\gamma)$ and, $P_{xy} = R\sin(\eta-\gamma)$. Therefore, $P_{xy} = P_z\tan(\eta-\gamma)$
$F_s = \tau_s.A_s$, and, $N_s = \sigma_n.A_s$; where, τ_s = the ultimate shear stress of the material being deformed, σ_n = normal stress on shear plane and A_s = area of the shear plane.

Case I: Finding F and N

Draw: a) OD′ ⊥ OT (or OD′ ∥ BT).
b) Join B and E. [$\therefore \angle BEO = \pi/2$.]
c) AD ⊥ OD′ ; d) AC ⊥ BE
Now, $\angle AOT + \angle ZOT = \pi/2$
$= \angle AOT + \angle AOD$
$\Rightarrow \angle ZOT = \gamma = \angle AOD$
But, $\angle AOD = \angle ABE = \gamma$
(Based on same arc AE)
So, **F** = OT = BE = BC + CE
= BC + AD
$= P_{xy}\cos\gamma + P_z\sin\gamma$ (i)

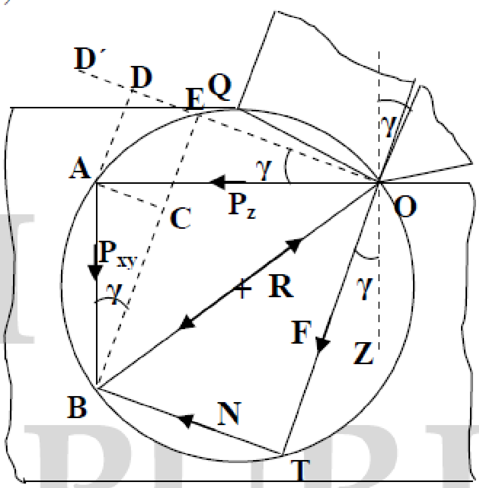

Fig. 34

And, N = OE = OD – DE

= OD – AC = $P_z \cos\gamma - P_{xy}\sin\gamma$. (ii)

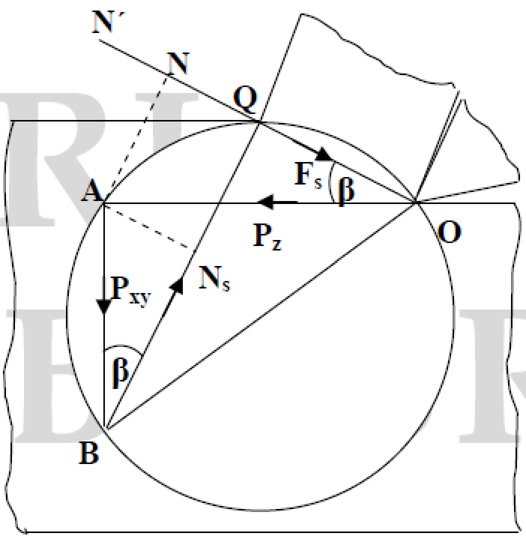

Fig. 35

Case II: Finding F_s and N_s

a) Increase OQ to ON′

b) Draw AN ⊥ ON′

c) Draw AS ⊥ BQ

Now, ∠AOQ = ∠ABQ

(based on same arc AQ)

But, ∠AOQ = β, ∴ ABQ = β

So, F_s = OQ = ON – NQ

= ON – AS

= $P_z\cos\beta - P_{xy}\sin\beta$ (iii)

N_s = BQ = BS + SQ

= BS + AN

= $P_{xy}\cos\beta + P_z\sin\beta$ (iv)

Thus, $\begin{bmatrix} N \\ F \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} P_z \\ P_{xy} \end{bmatrix}$ [from (i) & (ii)]

$\begin{bmatrix} F_s \\ N_s \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} P_z \\ P_{xy} \end{bmatrix}$ [From (iii) & (iv)]

Ernst and Merchant's Theory:

Ernst and Merchant established a relation among β, η and γ based on following assumptions:

(i) Cutting process is basically a plastic deformation process followed by shearing. Shear stress (τ_s) is maximum at the shear plane and it reaches the value of shear yield stress, i.e., $\tau_s = \tau_y = k$, during machining. This value remains constant and it is not effected by normal stress on shear plane (σ_n).

(ii) Shearing takes place in a direction such that the energy required for cutting is minimum i.e., shearing takes place in a direction in which cutting force is minimum ($W = P_z.V_c$). This is known as principle of minimum energy.

(iii) The friction angle (η) remains constant and it is independent of shear plane angle (β), i.e., η = constant. Hence, Friction coefficient (μ = tanη) between chip and tool rake face is constant.

(iv) The work material is perfectly rigid and ideally plastic.

Now, $\tau_s = \dfrac{F_s}{A_s} = \dfrac{F_s\sin\beta}{A_1}$; $F_s = R\cos(\beta+\eta-\gamma)$ and $P_z = R\cos(\eta-\gamma)$

$$\therefore \frac{F_s}{P_z} = \frac{\cos(\beta+\eta-\gamma)}{\cos(\eta-\gamma)}, \Rightarrow F_s = P_z.\frac{\cos(\beta+\eta-\gamma)}{\cos(\eta-\gamma)},$$

$$\text{Also, } \tau_s = \frac{F_s}{A_s} = \frac{F_s\sin\beta}{A_1} = \frac{F_s\sin\beta}{f.t} = \frac{P_z.\cos(\beta+\eta-\gamma).\sin\beta}{f.t.\cos(\eta-\gamma)} \qquad \text{(v)}$$

β is only variable on R.H.S. So, according to first assumption $\dfrac{d\tau_s}{d\beta} = 0$

[Also, $\dfrac{d^2\tau_s}{d\beta^2}$ = negative, $\Rightarrow -2\sin(\beta+\eta-\gamma)$ which is negative.]

$$\Rightarrow \frac{P_z}{f.t.\cos(\eta-\gamma)}.[\cos(\beta+\eta-\gamma).\cos\beta - .\sin(\beta+\eta-\gamma).\sin\beta] = 0$$

$\Rightarrow \cos(2\beta+\eta-\gamma) = 0$, i.e., $\mathbf{2\beta+\eta-\gamma} = \dfrac{\pi}{2}$.

This is called Ernst and Merchant's shear angle relationship.

Determining P_z, Cutting Pressure and P_{xy}:

$2\beta+\eta-\gamma=\pi/2$, Hence, $\eta-\gamma=(\pi/2-2\beta)$; and $\beta+\eta-\gamma=\pi/2-\beta$. Putting these in the following:

$$\frac{P_z}{F_s}=\frac{\cos(\eta-\gamma)}{\cos(\beta+\eta-\gamma)},\Rightarrow P_z=\frac{\tau_s.f.t.\cos(\frac{\pi}{2}-2\beta)}{\cos(\frac{\pi}{2}-\beta).\sin\beta}=\frac{\tau_s.ft.\sin 2\beta}{\sin^2\beta}=2\tau_s.ft.\cot\beta \qquad \text{(vii)}$$

Also, Cutting pressure $=\dfrac{P_z}{A_1}=2\tau_s\cot\beta$ (viii)

Again, $P_{xy}=P_z\tan(\eta-\gamma)=2\tau_s.ft.\cot\beta\tan(\pi/2-2\beta)$

$$=2\tau_s.ft.\cot\beta\cot 2\beta=2\tau_s.ft.\cot\beta\frac{\cot^2\beta-1}{2\cot\beta}=\tau_s.ft.(\cot^2\beta-1) \qquad \text{(ix)}$$

Merchant's second solution and 'machining constant'

In most of the cases Merchant found that the theoretical estimation of β using the relation '$2\beta+\eta-\gamma=\pi/2$' agrees poorly with practical values. The reasons for this discrepancy were supposed to be: a) non-compatibility with the actual deformation process, and b) non-cognizance of the dependency of β on η. Thus, the effect of actual plastic deformation and friction are reflected through a change in the normal force σ_n. In turn, the normal stress (σ_n) effects τ_s in the direction of shear. That time, P.W. Bridgeman [Piispanen in Shaw] established the relation between τ_s and σ_n as, $\boldsymbol{\tau_s=\tau_0+k.\sigma_n}$. This is known as Bridgeman's relation (Piispanen -as per Shaw) and Merchant utilized it.

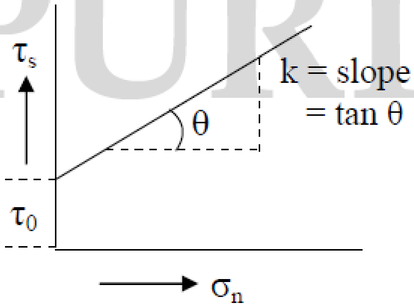

Fig. 36

We know, $N_s=F_s\tan(\beta+\eta-\gamma)$,

$\Rightarrow \sigma_n=\tau_s\tan(\beta+\eta-\gamma)$ [dividing both side by A_s]
Putting this in the above relation:
$\tau_s=\tau_0+k.\sigma_n=\tau_0+k.\tau_s.\tan(\beta+\eta-\gamma)$

$$\Rightarrow \tau_s-k\tau_s\tan(\beta+\eta-\gamma)=\tau_0,\Rightarrow \tau_s=\frac{\tau_0}{1-k\tan(\beta+\eta-\gamma)}$$

Now, $\dfrac{P_z}{F_s}=\dfrac{\cos(\eta-\gamma)}{\cos(\beta+\eta-\gamma)}$. Hence, $P_z=\dfrac{\tau_s.f.t.\cos(\eta-\gamma)}{\cos(\beta+\eta-\gamma).\sin\beta}$

$$=\frac{\tau_0.ft.\cos(\eta-\gamma)}{\{1-k\tan(\beta+\eta-\gamma)\}\cos(\beta+\eta-\gamma).\sin\beta}.$$

Now, considering principle of minimum energy,

$$\frac{dP_z}{d\beta}=0;\Rightarrow \frac{d}{d\beta}\{\sin\beta\cos(\beta+\eta-\gamma)-k\sin\beta.\sin(\beta+\eta-\gamma)\}=0$$

$\Rightarrow -\sin\beta.\sin(\beta+\eta-\gamma)+\cos(\beta+\eta-\gamma).\cos\beta-k.\sin\beta.\cos(\beta+\eta-\gamma)-$
$.\sin(\beta+\eta-\gamma).\cos\beta=0$
$\Rightarrow \cos(2\beta+\eta-\gamma)-k.\sin(2\beta+\eta-\gamma)=0,\ \Rightarrow \cot(2\beta+\eta-\gamma)=k$
$2\beta+\eta-\gamma=\cot^{-1}k=\text{constant}=C.$

'C' is called machining constant and this relation is known as Merchant's second solution. However, this modified relation agrees better with experimental results.

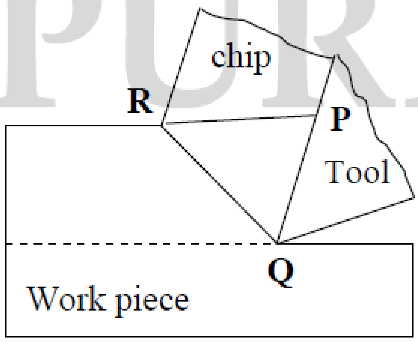

Fig. 38

Lee and Shaffer's Theory:

Lee and Shaffer's theory was the result of an attempt to apply the theory of plasticity to the problem of orthogonal machining. The assumptions made were as follows:

a) The shear plane coincides with the direction of maximum shear stress.

b) There exists a stress field (P-Q-R) within the chip, which transmits the cutting forces from the tool to the work piece. The chips are stress-free beyond this stress-field. The material within it subjected to a uniform state of shear yield stress k everywhere in parallel and perpendicular directions of shear plane.

c) The material in the stress field is ideally plastic, perfectly rigid and does not undergo strain-hardening during chip formation.

The relationship obtained by them, was: $\beta+\eta-\gamma=\pi/4$.

Now, if $\eta = 35^\circ$ and $\gamma = -10^\circ$ (quite possible values), then $\beta = 0$, which is quite impossible [$\Rightarrow \cot\beta = \infty$ and, $P_z = \dfrac{\tau_s.f.t.\cos(\eta-\gamma)}{\cos(\beta+\eta-\gamma).\sin\beta} = \infty$].

Hence, $\beta+\eta-\gamma=\pi/4$ was not an acceptable solution. They realized that the formation of BUE might have an effect on β. Considering this they arrived at a solution: $\beta+\eta-\gamma=\pi/4+\theta$, where θ is an angle depending on the size of BUE. 'θ' increases as the size of BUE increases.

COMMENT: β estimated by Ernst and Merchant's shear angle relationship ($2\beta+\eta-\gamma=\pi/2$) as well as from Lee and Shaffer's solution (i.e., $\beta = -(\eta-\gamma)+\pi/4$) do not agree with the experimental results. Fig. 37 shows a plot of them [$\beta = -(\eta-\gamma)/2+\pi/4$; $Y = -m.X + C$]. It is found that all the β's found experimentally lies below the merchant's solution and above the Lee and Shaffer's solution. Thus, Merchant's solution provides an over-estimate of 'β'. This solution is called an 'upper bound solution'. Also, 'β' estimated by Lee and Shaffer's solution is an under-estimate of 'β' and called 'lower bound solution'.

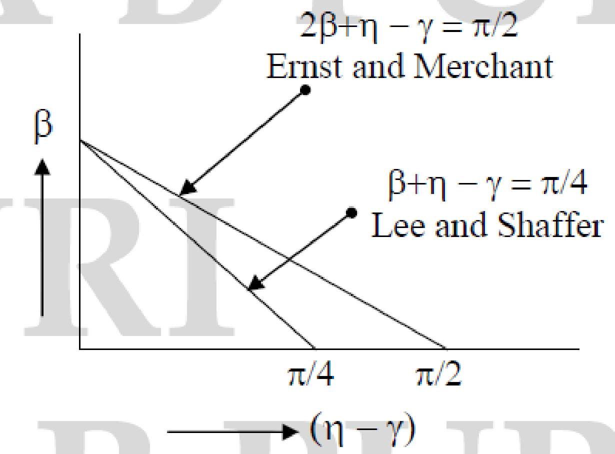

Fig. 37

Work done, Power required, MRR and Specific energy:

In metal cutting, the input energy is utilized in cutting and feeding (the tool).

So, $W = W_c + W_f$. Again, the same energy is utilized in shearing and overcoming friction. Hence, $W = W_s + W_F$

Now, $W_c = P_z \times V_c$; W_f = feed force $\times$ feed velocity,

But, feed force $\ll P_z$ and, feed velocity $\ll V_c$.

So, $W_f \ll W_c$ and $W \approx W_c = P_z \times V_c$

$W_s = F_s \times V_s$, F_s = shear force and V_s = shear or sliding velocity

$W_F = F \times V_f$, F = friction force between chip tool interface and V_f = chip flow velocity.

Therefore, $P_z \times V_c = F_s \times V_s + F \times V_f$; P_z is in kgf, and V_c is in m/min

$\therefore P_z \times V_c$ is in kgf-m/min

* 1 HP = 75 kgf-m/sec. Power required = $\dfrac{P_z \times V_c}{4500}$ HP.

* 1 kW = 1.36 HP [1 HP = 746 watt]

MRR: It is defined as volume (or mass) of material removal in unit time.

$A_1.V_c = A_2.V_f = a_1 b_1 V_c = f.t.V_c$ mm^3/ sec or mm^3/ min; f is in mm/rev.

Specific Energy of Cutting:

It is defined as the amount of energy consumed per unit amount of material removal. Thus, specific energy in cutting (U_C)may be defined as,

$$\frac{\text{Total work done in cutting in unit time}}{\text{Volume metal removal in unit time}} = \frac{P_z.V_c}{A_1.V_c} = \frac{P_z}{A_1} = \frac{P_z}{f.t}$$ [kgf/mm^2 or $HP/cm^3/min$].

It can be easily proved that, $U_C = U_S + U_F$. The specific energy in cutting is a very good yardstick for comparing the performance of various machining process and machine tool.