
Assignment $N^{\circ}1$

Electrónica 3 - 2018

Group 2

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1 EXERCISE 1: RESOLUTION AND RANGE OF A FIXED-POINT BINARY REPRESENTATION

1.1 WHAT IS THE FIXED-POINT BINARY REPRESENTATION

A fixed-point number has an integer part and a fractional part separated by a decimal point with a fixed position, as shown below:

$$(IntegerPart).(FractionalPart)$$

The integer part is formed by n bits and the fractional part is formed by m bits.

$$(bit\#1 \ bit\#2 \ \dots \ bit\#n).(bit\#1 \ bit\#2 \ \dots \ bit\#m)$$

1.2 WHAT IS RESOLUTION AND RANGE

1.2.1 RESOLUTION

The resolution of a number using the fixed point representation is the smallest unit that can be handled with it. Given a fixed-point number with m fractional bits, the resolution is 2^{-m} .

1.2.2 RANGE

The range is the difference between the biggest value that can be obtained with the fixed-point representation of a number with n bits in the integer part and with m bits in the fractional part, and the smallest number that can be represented.

1.3 MAKING USE OF THIS PROGRAM

1.3.1 INPUT

Three arguments must be entered through Command Line, separated by one space:

- 1 (indicating that the numeric representation of the binary number is signed) or 0 (indicating that the representation is unsigned).
- n : A positive integer (indicating the number of bits that correspond to the integer part of the number, which appears before the decimal point).
- m : A positive integer (indicating the number of bits that correspond to the fractional part of the number, which appears after the decimal point).

For example: "0 1 1".

1.3.2 OUTPUT

The result of this program is the resolution and range of the number that has n digits in the integer part and m digits in the fractional part.

```
Signed interpretation:
Resolution: 0.5
Range: 1.5
```

Figure 1.1: Output corresponding to the example input "0 1 1".

1.4 TESTING THE PROGRAM

2 EXERCISE 2: SIMPLIFICATION OF A MAXTERM EXPRESSION AND ITS CORRESPONDING LOGICAL CIRCUIT

Having the function in maxterms

$$f_1(A, B, C, D) = \prod (M_0, M_1, M_5, M_7, M_8, M_{10}, M_{14}, M_{15})$$

equivalent to

$$f_2(A, B, C, D) = \sum (m_2, m_3, m_4, m_6, m_9, m_{11}, m_{12}, m_{13})$$

using minterms, can be simplified by different ways and represented using logic gates.

2.1 SIMPLIFY: BOOLEAN ALGEBRA

Using the Boolean algebra properties

$$(A + B).(A + \overline{B}) = A \quad (1)$$

or

$$(AB) + (\overline{A}\overline{B}) = A \quad (2)$$

the function can be simplified using (1):

$$\begin{aligned} f_1(A, B, C, D) &= (A + B + C + D).(A + B + C + \overline{D}).(A + \overline{B} + C + \overline{D}).(A + \overline{B} + \overline{C} + \overline{D}). \\ &\quad (\overline{A} + B + C + D).(\overline{A} + B + \overline{C} + D).(\overline{A} + \overline{B} + \overline{C} + D).(\overline{A} + \overline{B} + \overline{C} + \overline{D}) \\ &= (A + B + C).(A + \overline{B} + \overline{D}).(\overline{A} + B + D).(\overline{A} + \overline{B} + \overline{C}) \end{aligned}$$

Which in minterms would be, using (2):

$$\begin{aligned} f_2(A, B, C, D) &= (\overline{A}\overline{B}\overline{C}\overline{D}) + (\overline{A}\overline{B}CD) + (\overline{A}B\overline{C}\overline{D}) + (\overline{A}BC\overline{D}) + \\ &\quad (\overline{A}\overline{B}CD) + (\overline{A}BCD) + (AB\overline{C}\overline{D}) + (ABC\overline{D}) \\ &= (\overline{A}\overline{B}D) + (\overline{A}B\overline{D}) + (\overline{A}\overline{B}C) + (ABC) \end{aligned}$$

2.2 SIMPLIFY: KARNAUGH MAP

A Karnaugh map is an easier way to simplify logic expressions when the function is too complex or too large to handle, because the Karnaugh map gives a more representative view for a faster analysis.

If the simplification is done with minterms, the groups should be of ones and powers of 2, adding each group in case there is more than one, and in each group the independent variables would be multiplied.

	AB			
CD \	00	01	11	10
00	0	1	1	0
01	0	0	1	1
11	1	0	0	1
10	1	1	0	0

Now grouping by colours we get that the function in minterms would be:

$$\begin{aligned}
 f_2(A, B, C, D) = & (\overline{A}\overline{B}D) \text{ (Red)} \\
 & + (\overline{A}B\overline{D}) \text{ (Blue)} \\
 & + (\overline{A}B\overline{C}) \text{ (Orange)} \\
 & + (A\overline{B}\overline{C}) \text{ (Green)}
 \end{aligned}$$

The same method could be done with maxterms; grouping zeroes in powers of 2, then multiplying groups in case there is more than one, and in each group the independent variables would be added.

2.3 LOGIC CIRCUIT: AND, OR AND NOT

Using the logic gates AND, OR and NOT the simplified version of the function could be represented by the figure below:

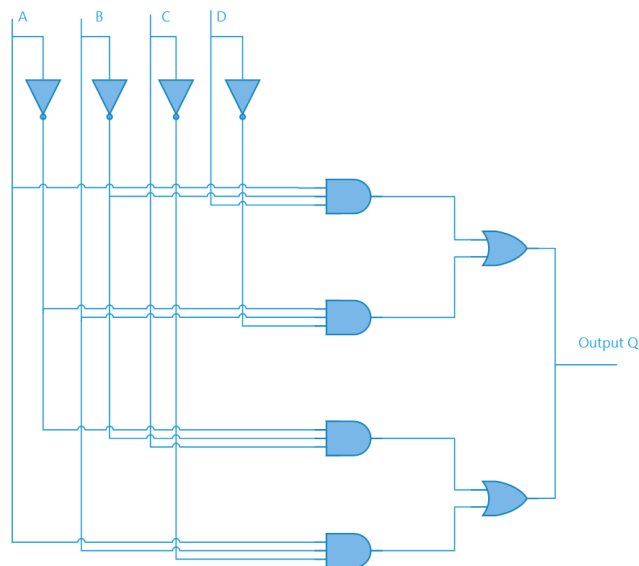


Figure 2.1: Logic circuit using AND, OR and NOT gates

2.4 LOGIC CIRCUIT: NAND

All the gates used above could be equivalent to a combination of NAND or NOR gates. Therefore, the simplified function can be drawn as the next figure:

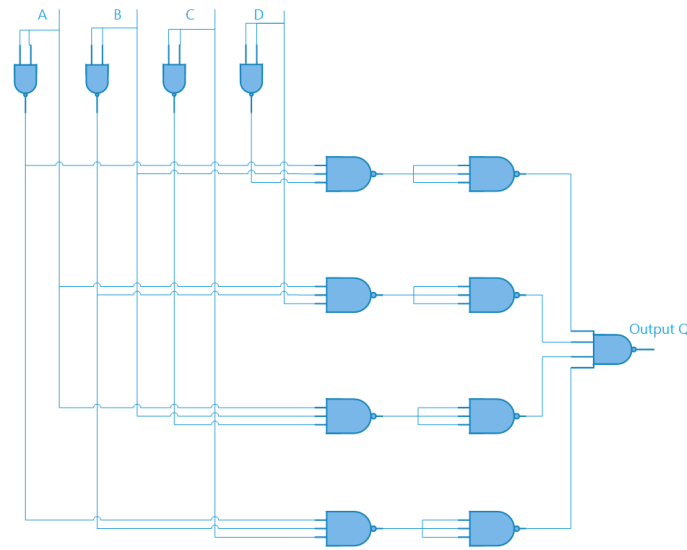


Figure 2.2: Logic circuit using NAND gates

3 EXERCISE 3: FOUR-ENTRY ENCODER AND FOUR-OUTPUT DEMUX USING VERILOG

Implement the following modules in Verilog:

- 4 inputs ENCODER
- 4 outputs DEMUX

3.1 4-INPUT ENCODER

3.1.1 DESCRIPTION

An encoder is an Application-Specific Integrated Circuit (ASIC) that converts information. In this case it receives a signal from a 4-bit input and returns the position of the Most Significant Bit that is currently on.

3.1.2 CODE IMPLEMENTATION

The Code Implementation of both the Module and its testbench can be found in the directory E3TP1/src/encoder.

3.1.3 MODULE TESTS

Testbench results can be found in Table 3.1.3

Input	Output	Value
0001	00	0
0010	01	1
0100	10	2
1000	11	3
0011	01	1
0101	10	2
1001	11	3
0110	10	2
1010	11	3
1100	11	3

Table 3.1.3 ENCODER Testbench Results

3.1.4 CONCLUSIONS

The module is working as expected, where it is taking only the Most Significant Bit as the value to be encoded.

3.2 4-OUTPUT DEMUX

3.2.1 DESCRIPTION

A DEMUX is an ASIC which receives an input signal and a selector signal. The selector signal determines through which output port the input signal is sent.

3.2.2 CODE IMPLEMENTATION

The Code implementation for the DEMUX and its testbench can be found in the directory E3TP1/src/demux.

3.2.3 MODULE TESTS

Input	Selector	Out_0	Out_1	Out_2	Out_3
1	0	1	0	0	0
1	1	0	1	0	0
1	2	0	0	1	0
1	3	0	0	0	1

Table 3.2.3 DEMUX Testbench Results

3.2.4 CONCLUSIONS

The module works as expected.

4 EXCERCISE 4

If we write every signle input bit according to the minterms, we get the following equations:

$$f_1(m_i) = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8$$

$$f_2(m_i) = m_1 + m_2 + m_3 + m_4 + m_9 + m_{10} + m_{11} + m_{12}$$

x_1	x_2	x_3	x_4	f_1	f_2	f_3	f_4
0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0
0	0	1	1	1	1	0	1
0	1	0	0	1	1	0	0
0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	0	0
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1

Figure 4.1: Two's Complement truth table for 4 bits

$$f_1(m_i) = m_1 + m_2 + m_5 + m_6 + m_9 + m_{10} + m_{13} + m_{14}$$

$$f_4(m_i) = m_1 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$$

Replacing the values of each minterm, we get the following:

$$f_1(x_1; x_2; x_3; x_4) = \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 + \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 + \bar{x}_1 \bar{x}_2 x_3 x_4 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 x_2 x_3 \bar{x}_4 + \bar{x}_1 x_2 x_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4$$

$$f_2(x_1; x_2; x_3; x_4) = \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 + \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 + \bar{x}_1 \bar{x}_2 x_3 x_4 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_1 \bar{x}_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 x_3 x_4 + x_1 x_2 \bar{x}_3 \bar{x}_4$$

$$f_3(x_1; x_2; x_3; x_4) = \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 + \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 x_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_1 \bar{x}_2 x_3 \bar{x}_4 + x_1 x_2 \bar{x}_3 x_4 + x_1 x_2 x_3 \bar{x}_4$$

$$f_4(x_1; x_2; x_3; x_4) = \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 + \bar{x}_1 \bar{x}_2 x_3 x_4 + \bar{x}_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 x_2 x_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_1 \bar{x}_2 x_3 x_4 + x_1 x_2 \bar{x}_3 x_4 + x_1 x_2 x_3 x_4$$

By simplification methods and properties, we can achieve this four formulas to describe each output bit according to the input bits:

$$f_1(x_1; x_2; x_3; x_4) = x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 (x_2 + x_3 + x_4)$$

$$f_2(x_1; x_2; x_3; x_4) = x_2 \bar{x}_3 \bar{x}_4 + \bar{x}_2 (x_3 + x_4)$$

$$f_3(x_1; x_2; x_3; x_4) = x_3 \bar{x}_4 + \bar{x}_3 x_4$$

$$f_4(x_1; x_2; x_3; x_4) = x_4$$

The previous formulas can be represented in logic gates' graphs as shown in figures 4.2, 4.3, 4.4 and 4.5.

Finally, this logic was implemented on verilog as shown in the figure 4.6 and by testing the code with test.v, we have got the output shown in figure 4.7, confirming that the code was executed correctly.

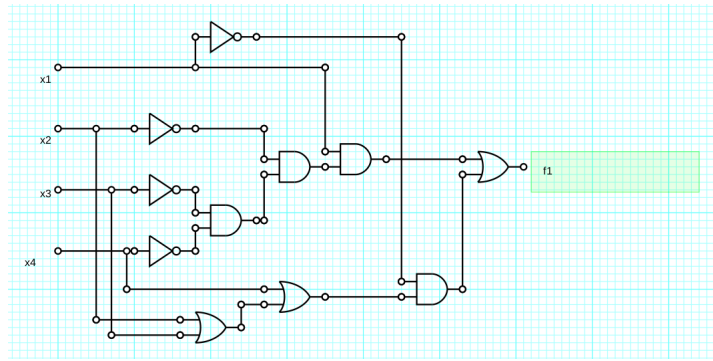


Figure 4.2: 1st Bit's logic gates graph

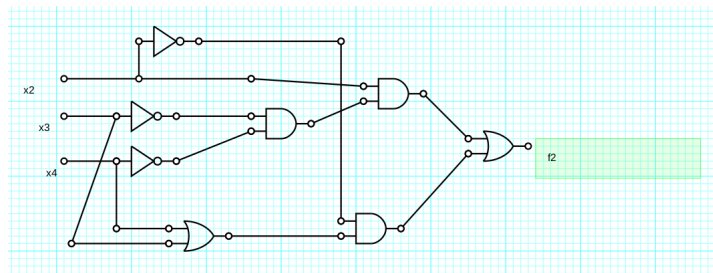


Figure 4.3: 2nd Bit's logic gates graph

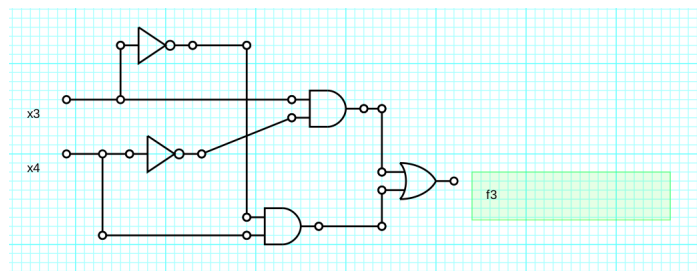


Figure 4.4: 3rd Bit's logic gates graph

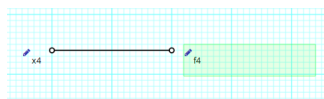


Figure 4.5: 4th Bit's logic gates graph

5 EXERCISE 5: BCD FORMAT ADDER

Implement a module that receives as inputs two one-digit numbers in Binary-Coded-Decimals (BCD) format and outputs a two-digit number in BCD format.

5.1 DESIGN CONSIDERATIONS

- BCD digits are comprised of 4 bits with a range of integer values between 0 and 9. Any value outside that range should be considered an error.
- It needs to make a simple addition. Given that the maximum value of the sum is 18, the result will be a 5-bit integer.


```

E4TP1.v
1  module twosComplement(x1,x2,x3,x4,f1,f2,f3,f4);
2      input x1, x2, x3, x4;
3      output f1, f2, f3, f4;
4      wire nx1,nx2,nx3,nx4;
5      not(nx1,x1);
6      not(nx2,x2);
7      not(nx3,x3);
8      not(nx4,x4);
9      //First Bit Logic
10     wire temp1, temp2, temp3;
11     and(temp1,x1,nx2,nx3,nx4);
12     or(temp2,x2,x3,x4);
13     and(temp3,temp2,nx1);
14     or(f1,temp1,temp3); //First Bit output
15
16     //Second Bit Logic
17     wire t1,t2,t3;
18     and(t1,x2,nx3,nx4);
19     or(t2,x3,x4);
20     and(t3,t2,nx2);
21     or(f2,t1,t3); //Second Bit Output
22
23     //Third Bit Logic
24     wire r1,r2;
25     and(r1,x3,nx4);
26     and(r2,nx3,x4);
27     or(f3,r1,r2); //Third Bit Output
28
29     //Four Bit Output
30     wire q;
31     and(f4,x4,x4);
32
33
34 endmodule
35

```

Figure 4.6: Verilog implementation

```

lan@Linux-Vaio:~/Desktop/Electro III/GIT TPS/tp1-team-2/E4TP1/code/src$ vvp a.out
Input values are: 0 0 0 0
Outs have changed! New values are: 0 0 0 0
Input values are: 0 0 0 1
Outs have changed! New values are: 1 1 1 1
Input values are: 0 0 1 0
Outs have changed! New values are: 1 1 1 0
Input values are: 0 0 1 1
Outs have changed! New values are: 1 1 0 1
Input values are: 0 1 0 0
Outs have changed! New values are: 1 1 0 0
Input values are: 0 1 0 1
Outs have changed! New values are: 1 0 1 1
Input values are: 0 1 1 0
Outs have changed! New values are: 1 0 1 0
Input values are: 0 1 1 1
Outs have changed! New values are: 1 0 0 1
Input values are: 1 0 0 0
Outs have changed! New values are: 1 0 0 0
Input values are: 1 0 0 1
Outs have changed! New values are: 0 1 1 1
Input values are: 1 0 1 0
Outs have changed! New values are: 0 1 1 0
Input values are: 1 0 1 1
Outs have changed! New values are: 0 1 0 1
Input values are: 1 1 0 0
Outs have changed! New values are: 0 1 0 0
Input values are: 1 1 0 1
Outs have changed! New values are: 0 0 1 1
Input values are: 1 1 1 0
Outs have changed! New values are: 0 0 1 0
Input values are: 1 1 1 1
Outs have changed! New values are: 0 0 0 1
lan@Linux-Vaio:~/Desktop/Electro III/GIT TPS/tp1-team-2/E4TP1/code/src$

```

Figure 4.7: Terminal's output

- The value of the sum must be returned in BCD format, so the 5-bit integer needs to be split back into 2 BCD digits.

Given these conditions, the module will need:

- 2 4-bit input ports
- 2 4-bit output ports
- 1 ERROR register

5.2 CODE IMPLEMENTATION

The BCD Adder was composed of the following modules:

- 2 BCD format "filters"
- 1 4-bit numbers adder
- 1 BCD format "decoder"

The code implementation for each of the modules can be found in their respective folders.

5.3 MODULE TESTBENCH

Testbench results for each submodule can be found in the the directory /E5TP1/tests.txt.

Testbench results for the main module can be found in the directory /E5TP1/final.txt.

5.4 CONCLUSIONS

Each sub-module is working as intended.

6 EXERCISE 6: ALU IMPLEMENTATION

For this exercise we were asked to implement a 4 bit Arithmetic Logic Unit (ALU). The operations we had to develop were SUM, SUBTRACTION, AND, OR, NOT, XOR, two's complement and shift left. For this, we decided to create a module responsible of adding 2 bits, and as output, it returned 2 bits, one bit for the answer, and another for the carry bit. By using this module, we decided now, to create a secondary module, responsible for adding 3 bits. This decision gave us a lot of simplification in the development of the module SUM for 4 bits. As you can see in the code "sum.v" found in the folder src, we commented the previous development without the module sum3Bits, and the new development with the module sum3Bits.

For the subtraction, we decided to re-use the module created on exercise 4 of two's complement, and utilizing it correctly with the module SUM, we had our SUBTRACTION module. For the operations AND, OR, NOT, XOR we chosen to use the predefined modules provided by verilog and utilize them bitwise.

6.1 DEFINITIONS

In this Arithmetic Logic Unit, we implemented with two accumulators (that we will call A and B), each one of four bits, three operational bits, four bits for the output accumulator (that we will call accumulator C) and one carry bit, ordered in the way they were mentioned.

To select the operation you want to make, you should turn the three operational bits in the following way:

- AND: Performs an AND operation bitwise between accumulators A and B and returns it on accumulator C, meanwhile, the carry bit is left to zero.
- NOT: Performs a logic NOT operation bitwise between accumulators A and B, and returns the answer in the accumulator C. The carry bit stays as zero
- OR: Performs a logic OR operation bitwise between accumulators A and B, and returns the answer in the accumulator C. The carry bit stays as zero.

	Bit 0	Bit 1	Bit 2
AND	0	0	0
NOT	0	0	1
OR	0	1	0
XOR	0	1	1
SHIFT LEFT	1	0	0
SUM	1	0	1
SUBTRACTION	1	1	0
TWO'S COMPLEMENT	1	1	1

Table 6.1: Representation of operational bits

- XOR: Performs a logic XOR operation bitwise between accumulators A and B, and returns the answer in the accumulator C. The carry bit stays as zero.
- SHIFT LEFT: It manages to move every bit in accumulator A, one space left, and inserts a logic zero to the less significant bit. The answer is given in the C accumulator and the carry bit will become 0 or 1 depending on the most significant bit of A.
- SUM: Performs a numeric sum of the binary values of accumulator A and B and the answer is given in accumulator C. Depending on the overflow, the carry bit will become 1 or 0.
- SUBTRACTION: Performs a numeric subtraction of the binary values of accumulator A and B and the answer is given in accumulator C. The carry bit will become 1.
- TWO'S COMPLEMENT: Performs a two's complement of the binary value of accumulator A and the answer is given in accumulator C. The carry bit will become 0.

7 APPENDIX

Input	Val	Output	Error
0000	0	0000	0
0001	1	0001	0
0010	2	0010	0
0011	3	0011	0
0100	4	0100	0
0101	5	0101	0
0110	6	0110	0
0111	7	0111	0
1000	8	1000	0
1001	9	1001	0
1010	10	1010	1
1011	11	1011	1
1100	12	1100	1
1101	13	1101	1
1110	14	1110	1

Figure 7.1: FilterBCD Module Testbench results