Task 2

Starting with the following function expressed in maxterms:

$$f(d,c,b,a) = \prod (M_0, M_1, M_5, M_7, M_8, M_{10}, M_{14}, M_{15})$$

Taking d, c, b, a as input variables. For simplify, the same function is expressed in minterms to operate later:

$$f(d,c,b,a) = \sum (m_2, m_3, m_4, m_6, m_9, m_{11}, m_{12}, m_{13})$$

Starting with it, we build the function without simplifying:

$$f(d,c,b,a) = (\overline{d}\cdot\overline{c}\cdot b\cdot\overline{a})+(\overline{d}\cdot\overline{c}\cdot b\cdot a)+(\overline{d}\cdot c\cdot\overline{b}\cdot\overline{a})+(\overline{d}\cdot c\cdot b\cdot\overline{a})+(d\cdot\overline{c}\cdot\overline{b}\cdot a)+(d\cdot\overline{c}\cdot b\cdot a)+(d\cdot c\cdot\overline{b}\cdot\overline{a})+(d\cdot c\cdot\overline{b}\cdot a)$$

We grouped by common factor in a convenient way:

$$\begin{aligned} f(d,c,b,a) \quad &= \quad \underbrace{(\overline{d}\cdot\overline{c}\cdot b\cdot\overline{a})+(\overline{d}\cdot\overline{c}\cdot b\cdot a)}+\underbrace{(\overline{d}\cdot c\cdot\overline{b}\cdot\overline{a})+(\overline{d}\cdot c\cdot b\cdot\overline{a})}+\underbrace{(d\cdot\overline{c}\cdot\overline{b}\cdot a)+(d\cdot\overline{c}\cdot b\cdot a)}+ \\ &\longrightarrow \quad \underbrace{(d\cdot c\cdot\overline{b}\cdot\overline{a})+(d\cdot c\cdot\overline{b}\cdot a)} \end{aligned}$$

$$f(d,c,b,a) = [\overline{d}\cdot\overline{c}\cdot b\cdot\underbrace{(\overline{a}+a)}_{1}] + [\overline{d}\cdot c\cdot\overline{a}\cdot\underbrace{\overline{b}+b)}_{1}] + [d\cdot\overline{c}\cdot a\cdot\underbrace{(\overline{b}+b)}_{1}] + [d\cdot c\cdot\overline{b}\cdot\underbrace{(\overline{a}+a)}_{1}]$$

$$\boxed{f(d,c,b,a) = (\overline{d}\cdot\overline{c}\cdot b) + (\overline{d}\cdot c\cdot\overline{a}) + (d\cdot c\cdot\overline{b}) + (d\cdot\overline{c}\cdot a)}$$

Analogously, starting with the expresssion in minterms we reduce the function through a map of Karnaugh:

dc \ ba	00	01	11	10
00	0	0	1	1
01	1	0	0	1
11	1	1	0	0
10	0	1	1	0

From the first group (first row) we have that d, c and b are constantes, thus the first factor stays the way $\overline{d}\overline{c}b$.

From the second group (second row) we have d, c and a as constants, thus this factor stays as $\overline{d}c\overline{a}$.

From the third group (third row) remain constant d, c and b, thus this factor stays as $dc\overline{b}$.

Finally, from the last row, in the group d, c and a stay constant, thus this factor stays the way $d\overline{c}a$.

Adding the partial termns we get the simplified function:

$$\boxed{f(d,c,b,a) = (\overline{d}\cdot\overline{c}\cdot b) + (\overline{d}\cdot c\cdot\overline{a}) + (d\cdot c\cdot\overline{b}) + (d\cdot\overline{c}\cdot a)}$$

Wich is the same obtained by simplification by boolean algebra.

Taking this function, it was implemented in a logical circuit by AND, OR and NOT gates, as shown below.

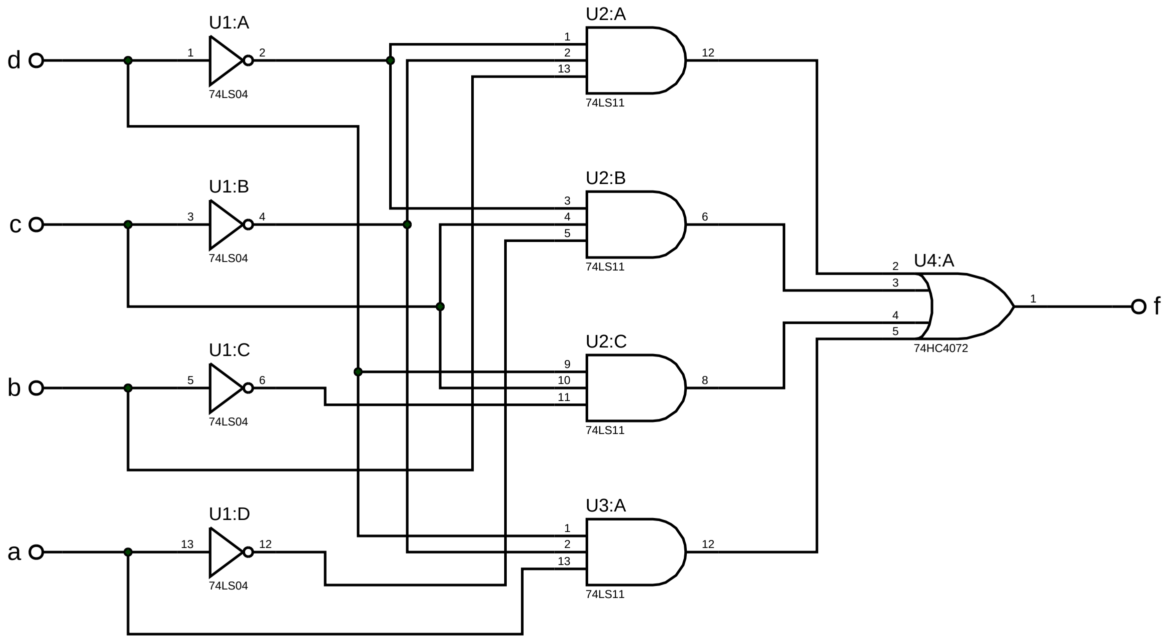

Figure 1: Logical circuit that implements $f(d,c,b,a)$ - Made in Proteus 7.8

For implementation using only NOR gates, first we have to work with the obtained function applyin boolean algebra properties. Taking the function:

$$f(d,c,b,a) = (\overline{d}\cdot\overline{c}\cdot b) + (\overline{d}\cdot c\cdot\overline{a}) + (d\cdot c\cdot\overline{b}) + (d\cdot\overline{c}\cdot a)$$

We twice denied the terms separated by sums, for keeping the equal:

$$f(d,c,b,a) = \overline{\overline{(\overline{d}\cdot\overline{c}\cdot b)}} + \overline{\overline{(\overline{d}\cdot c\cdot\overline{a})}} + \overline{\overline{(d\cdot c\cdot\overline{b})}} + \overline{\overline{(d\cdot\overline{c}\cdot a)}}$$

Next we deny the factors only once, for turning the products into sums (property of De Moivre):

$$\boxed{f(d,c,b,a) = \overline{(d+c+\overline{b})} + \overline{(d+\overline{c}+a)} + \overline{(\overline{d}+\overline{c}+b)} + \overline{(\overline{d}+c+\overline{a})}}$$

Having the expression in terms of sums, it is possible to implement the circuit using only NOR gates, as shown below.

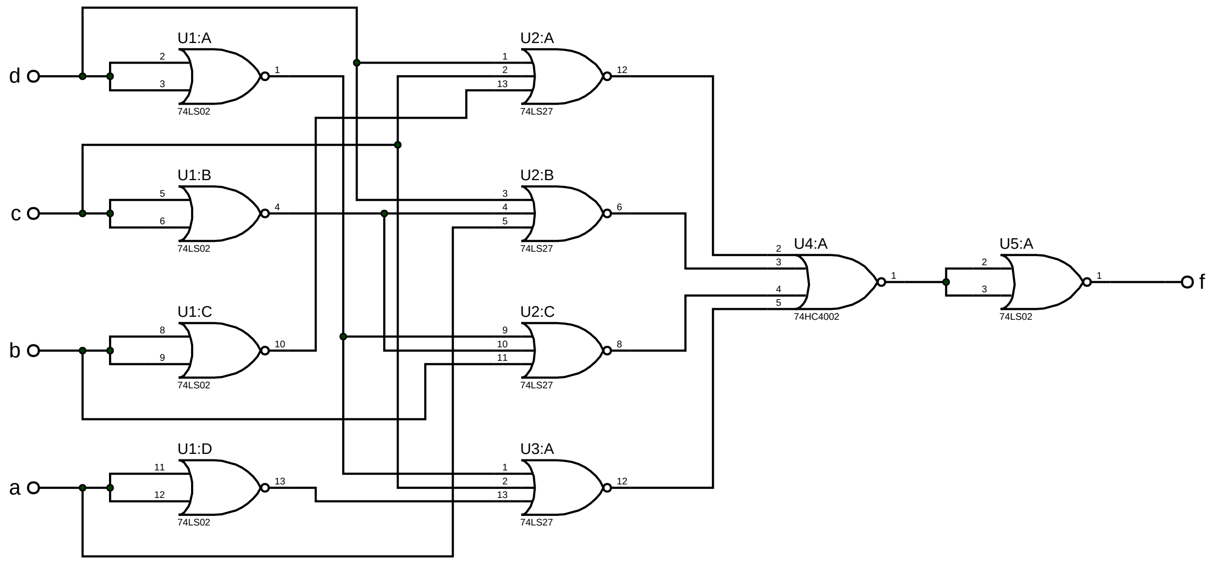

Figure 2: Logical circuit that implements $f(d,c,b,a)$ with NOR gates - Made in Proteus 7.8