Analytical Formulation and Python Implementation for an Extended Carrión/Arroyo SCUC/SCED Optimization Formulation

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1 Introduction

Centrally-managed wholesale power markets in the United States rely on *Security-Constrained Unit Commitment (SCUC)* and *Security Constrained Economic Dispatch (SCED)* optimizations to determine commitments and scheduled dispatch levels for generating units for the provision of power in future real-time operating conditions. In an earlier report [2], a complete analytically-formulated combinec SCUC/SCED optimization was presented that extends the well-known SCUC/SCED optimization model developed by Carrión and Arroyo [3] in five key ways:

- Inclusion of non-dispatchable generation
- Inclusion of energy storage units
- Inclusion of nodal power balance constraints with possible transmission congestion
- Inclusion of zonal as well as system-wide reserve requirements
- Inclusion of penalty terms in the objective function for slack in power balance and reserve constraints

In addition, the earlier report [2] discusses a software implementation of this extended SCUC/SCED optimization formulation by means of the *Python Optimization Modeling Objects (Pyomo)* package [4, 5, 6]. The Pyomo package is an open-source tool for optimization applications. This Pyomo software implementation has been incorporated into the AMES Wholesale Power Market Test Bed [7], starting with AMES V4.0 [8]. It has also been used to implement the agent-based 8-Zone ISO-NE Test System developed by Krishnamurthy, Li, and Tesfatsion [9, 10], and an extension of this test system by Li and Tesfatsion [11] that incorporates wind power in the form of physically-modeled wind turbine agents.

The current report provides a substantially revised version of the earlier report [2] in order to improve the readability and understanding of the extended SCUC/SCED optimization formulation. First, the ordering of presented materials has been changed to facilitate the logical progression of ideas. Second, the presentation of technical model terms (nomenclature) has been augmented with explanatory notes to facilitate understanding of these terms. Third, the presentation of technical model components (objective function, choice variables, constraints) has been augmented with detailed notes to explain the meaning and/or derivation of these components. Fourth, sections discussing possible stochastic (scenario-based) extensions of the extended SCUC/SCED optimization formulation have been omitted.

Hereafter, the extended SCUC/SCED optimization formulation presented in this report will be referred to as the *Extended Carrión-Arroyo (ECA)* Model. Also, the Pyomo software implementation of the ECA Model will be referred to as the *Pyomo Model*.

Section 3 provides complete nomenclature for the ECA Model, grouped by similar elements with accompanying explanatory notes. Section 4 provides analytical formulations for the ECA Model objective function and system constraints, again with accompanying explanatory notes. The manner in which the Pyomo Model implements piece-wise linear approximations for the total production cost functions of dispatchable generators is explained in Section 5. Pyomo Model determination of locational marginal prices is discussed in Section 6. Pyomo Model flags permitting the user to implement the SCUC/SCED optimization either with or without energy storage units, and either with or without non-dispatchable generation, are discussed in Section 7. Section 8 provides a mapping among corresponding input names for the ECA Model, AMES V5.0, and the Pyomo Model, together with Pyomo Model input default values.

Finally, an important terminological clarification needs to be stressed. Throughout these notes, *power output* refers to the amount of power (MW) that a generator is injecting into a transmission grid at a particular point in time. In contrast, *total power generation* refers to the total amount of power (MW) that a generator is producing at a particular point in time. This total power generation can include local (behind-the-meter) power that the generator needs to produce and use locally in order to bring itself into a "synchronized state". A *synchronized state* is an operating state in which a generator is ready to inject power into the grid but is not yet injecting any power into the grid.

2 The ECA Model: Overview

The ECA Model provides a complete analytical formulation for a SCUC/SCED optimization undertaken by an *Independent System Operator (ISO)* tasked with management of wholesale power system operations. The participants in the SCUC/SCED optimization include dispatchable and non-dispatchable generator units, energy storage units, and load-serving entities functioning as intermediaries for retail power customers.

Given initial system conditions, together with forecasts for loads and non-dispatchable generation, the SCUC/SCED optimization determines cost-minimizing solution values for dispatchable generator unit commitments, energy storage unit commitments, dispatchable generator power outputs, energy storage power outputs (discharge levels), energy storage power absorptions (charge levels), and locational marginal prices (LMPs) over a finite planning horizon subject to system constraints. These system constraints include:

- transmission line power constraints;
- power balance constraints;
- generator capacity constraints;
- dispatchable generator ramp constraints for start-up, normal, and shut-down conditions;
- dispatchable generator minimum up-time/down-time constraints;
- dispatchable generator hot-start constraints;
- dispatchable generator start-up/shut-down cost constraints;
- storage unit limit constraints;

- storage unit charge/discharge constraints;
- storage unit ramping constraints;
- storage unit energy conservation constraints;
- storage unit end-point constraints;
- system-wide reserve requirement constraints;
- zonal reserve requirement constraints.

The ECA Model formulation for the power balance constraints relies on a standard DC Optimal Power Flow (DC-OPF) approximation. Consequently, it relies on the following three assumptions. First, the resistance for each transmission line is negligible compared to the reactance, hence the resistance for each transmission line is set to 0. Second, the voltage magnitude at each bus is equal to a common base voltage magnitude. Third, the voltage angle difference $\Delta\theta(\ell)$ across any line ℓ is sufficiently small that the following approximations can be used: $\cos(\Delta\theta(\ell)) \approx 1$ in size and $\sin(\Delta\theta(\ell)) \approx \Delta\theta(\ell)$ in size.

3 Nomenclature for the ECA Model

3.1 Sets and Subsets

$\mathcal{B}$	Set of grid buses b
$\mathcal{B}(z) \subseteq \mathcal{B}$	Subset of buses constituting reserve zone z
$\mathcal{G}$	Set of dispatchable generators j
$\mathcal{G}(b) \subseteq \mathcal{G}$	Subset of dispatchable generators located at bus b
$\mathcal{G}(z) \subseteq \mathcal{G}$	Subset of dispatchable generators located in reserve zone z
K	Set of indices $k = 1, \ldots, \lvert K \rvert$ for successive operating periods
$\mathcal{L} \subseteq \mathcal{B} \times \mathcal{B}$	Set of transmission lines ℓ
$\mathcal{L}_O(b) \subseteq \mathcal{L}$	Subset of lines outgoing at bus b
$\mathcal{L}_I(b) \subseteq \mathcal{L}$	Subset of lines incoming at bus b
$\mathcal{NG}$	Set of non-dispatchable generators i
$\mathcal{NG}(b) \subseteq \mathcal{NG}$	Subset of non-dispatchable generators located at bus b
$\mathcal{NG}(z) \subseteq \mathcal{NG}$	Subset of non-dispatchable generators located in zone z
$\mathcal{S}$	Set of energy storage units s
$\mathcal{S}(b) \subseteq \mathcal{S}$	Subset of energy storage units located at bus b
$\mathcal{Z}$	Set of indices $z = 1, \ldots, \lvert Z \rvert$ for reserve zones $\mathcal{B}(z)$, which form a partition of $\mathcal{B}$, i.e., $\cup_{z \in \mathcal{Z}} \mathcal{B}(z) = \mathcal{B}$, and $\mathcal{B}(z_i) \cap \mathcal{B}(z_j) = \emptyset$ for any z_i and z_j in $\mathbb{Z}$ with $i \neq j$.

3.2 External Forcing Terms: Net Load Forecasts

$NL(b,k)$	Net load forecast at bus b for period k
$NL(z,k)$	Net load forecast at zone z for period k, given by $\sum_{b \in \mathcal{B}(z)} NL(b,k)$
$NL(k)$	System-wide net load forecast for period k, given by $\sum_{b \in \mathcal{B}} NL(b,k)$

Remarks on Net Load Forecasts: All *load* in the ECA model is assumed to be must-have (non-price-sensitive) power usage. As will be explained more carefully in Section 3.7, below, *net load* at a particular location is defined to be load at that location minus *non-dispatchable generation (NDG)* at that location. Thus, NDG is treated as negative load.

3.3 User-Specified Parameters

User-Specified Parameters for Dispatchable Generation Physical Attributes:

DT_j	Minimum down-time (h) for generator $j \in \mathcal{G}$
UT_j	Minimum up-time (h) for generator $j \in \mathcal{G}$
NRD_j	Nominal ramp-down rate (MW/τ) for generator $j \in \mathcal{G}$
NRU_j	Nominal ramp-up rate (MW/τ) for generator $j \in \mathcal{G}$
NSD_j	Shut-down ramp rate (MW/τ) for generator $j \in \mathcal{G}$
NSU_j	Start-up ramp rate (MW/τ) for generator $j \in \mathcal{G}$
$\overline{P}_j$	Maximum sustainable power output (MW) for generator $j \in \mathcal{G}$
$\underline{P}_j$	Minimum sustainable power output (MW) for generator $j \in \mathcal{G}$

User-Specified Parameters for Dispatchable Generator Start-Up/Shut-Down Costs:

CSC_j	Cold-start cost (\$) for generator $j \in \mathcal{G}$
CSH_j	Cold-start hours (h) for generator $j \in \mathcal{G}$
HSC_j	Hot-start parameter (\$) for generator $j \in \mathcal{G}$ (required to satisfy $HSC_j \leq CSC_j$)
SDC_j	Shut-down cost (\$) for generator $j \in \mathcal{G}$

Remarks on the Cold-Start Hours Parameter: The cold-start hours parameter CSH_j has the following meaning. If a dispatchable generator j at the beginning of an operating period k has been off-line for at least CSH_j consecutive hours *immediately prior* to k, then j in operating period k is in a *cold-start state* and any start-up of j in k incurs the cold-start cost CSC_j. Otherwise, j in operating period k is in a *hot-start state* and j incurs no start-up cost in k.[1]

User-Specified Parameters for Dispatchable Generator Total Production Cost Functions:

a_j	Production cost function coefficient (\$) for generator $j \in \mathcal{G}$
b_j	Production cost function coefficient (\$/MW) for generator $j \in \mathcal{G}$
c_j	Production cost function coefficient (\$/(MW)^2) for generator $j \in \mathcal{G}$
NS	Number of time segments for piecewise-linear approximation of generator production cost functions
T_{ij}	Maximum possible sustainable power output (MW) for generator $j \in \mathcal{G}$ in segment $i = 1, \ldots, NS$
G_{ij}	Total production cost (\$) for generator $j \in \mathcal{G}$ at sustained power output T_{ij}, $i = 1, \ldots, NS$

Remarks on the Formulation of the Total Production Cost Function:

As explained more fully in Section 5, the ECA Model defines the total production cost for a dispatchable generator $j \in \mathcal{G}$ in any period k to be the total cost (\$) incurred by j in order to maintain a power output $p_j(k)$ during period k in accordance with dispatch instructions. Total production cost is assumed to be a convex function of $p_j(k)$ for each generator j during each period k. The

[1] Carrión and Arroyo [3, Sec. II] propose a "stairwise startup function" to model the manner in which start-up costs increase for a dispatchable generator j as a function of the number of consecutive hours immediately prior to k during which j was offline.

Pyomo Model implementation of the ECA Model permits users to approximate these convex total production cost functions as piecewise linear functions using *either* an automated approximation option *or* a user-directed approximation option. The first option only requires admissible user-specified settings for the production function coefficients $\{a_j, b_j, c_j \mid j \in \mathcal{G}\}$ and the total number NS of time segments. The second option additionally requires admissible user-specified settings for the parameters T_{ij} and G_{ij} for each $i = 1, \ldots, NS$ and $j \in \mathcal{G}$.

User-Specified Parameters for Energy Storage Units $s \in \mathcal{S}$:

$EPSOC_s$	Target charge state (decimal percent) for storage unit s at end of planning horizon
$\overline{ES}_s$	Maximum energy storage capacity (MWh) of storage unit s during each operating period
$NRDIS_s$	Nominal charge ramp-down rate (MW/τ) for storage unit s
$NRUIS_s$	Nominal charge ramp-up rate (MW/τ) for storage unit s
$NRDOS_s$	Nominal discharge ramp-down rate (MW/τ) for storage unit s
$NRUOS_s$	Nominal discharge ramp-up rate (MW/τ) for storage unit s
$\overline{PIS}_s$	Maximum charge power (MW) for storage unit s
$\underline{PIS}_s$	Minimum charge power (MW) for storage unit s
$\overline{POS}_s$	Maximum discharge power (MW) for storage unit s
$\underline{POS}_s$	Minimum discharge power (MW) for storage unit s
$\underline{SOC}_s$	Minimum state of charge (decimal percent) for storage unit s
η_s	Round-trip efficiency (decimal percent) for storage unit s

User-Specified Parameters for Reserve Requirements:

$RD(k)$	System-wide reserve requirement (decimal percent) for down-power in period k
$RU(k)$	System-wide reserve requirement (decimal percent) for up-power in period k
$ZRD(z,k)$	Zonal reserve requirement (decimal percent) for down-power at reserve zone z in period k
$ZRU(z,k)$	Zonal reserve requirement (decimal percent) for up-power at reserve zone z in period k

Remarks on Reserve Requirements:

The system-wide down/up reserve requirements $RD(k)$ and $RU(k)$ appear on the right-hand side of the system-wide down/up reserve requirement constraints (40) and (41) for period k as percentages of system-wide forecasted net load $NL(k)$ for period k. The zonal down/up reserve requirements $ZRD(z,k)$ and $ZRU(z,k)$ appear on the right-hand side of the zonal down/up reserve requirement constraints (42) and (43) at zone z for period k as percentages of the forecasted net load $NL(z,k)$ at zone z for period k.

Other User-Specified Parameters:

$BF(\ell)$	Start-bus for transmission line ℓ
$BT(\ell)$	Terminal bus for transmission line ℓ
$RE(\ell)$	Reactance (ohms) on transmission line ℓ, restricted to be non-zero
$TL(\ell)$	Capacity limit (MW) for transmission line ℓ
V_o	Base voltage magnitude (kV)
Λ	Penalty weight ($/MW) for non-zero slack variables in the total cost objective function
τ	Length (h) of each operating period k

3.4 Derived Parameters (Calculated from User-Specified Parameters)

A_j	Min total production cost (\$) for generator $j \in \mathcal{G}$ in any period k for which it is committed
$B(\ell)$	Inverse of reactance (pu) on transmission line ℓ
$re(\ell)$	Reactance (pu)
SDT_j	Scaled minimum down-time (number of operating periods) for generator $j \in \mathcal{G}$
SUT_j	Scaled minimum up-time (number of operating periods) for generator $j \in \mathcal{G}$
$SNRDIS_s$	Scaled nominal charge ramp-down limit (MW) for storage unit s (ramp-down per operating period)
$SNRUIS_s$	Scaled nominal charge ramp-up limit (MW) for storage unit s (ramp-up per operating period)
$SNRDOS_s$	Scaled nominal discharge ramp-down limit (MW) for storage unit s (ramp-down per operating period)
$SNRUOS_s$	Scaled nominal discharge ramp-up limit (MW) for storage unit s (ramp-up per operating period)
SRD_j	Scaled nominal ramp-down limit (MW) for generator $j \in \mathcal{G}$
SRU_j	Scaled nominal ramp-up limit (MW) for generator $j \in \mathcal{G}$
SSD_j	Scaled shut-down ramp limit (MW) for generator $j \in \mathcal{G}$
SSU_j	Scaled start-up ramp limit (MW) for generator $j \in \mathcal{G}$
Z_o	Base impedance (ohms)

Calculations for Derived Parameters:

- $A_j = a_j + b_j\underline{P}_j + c_j\underline{P}_j^2$
- $B(\ell) = 1/re(\ell)$
- $re(\ell) = RE(\ell)/Z_o$
- $SDT_j = \texttt{round}(DT_j/\tau)$
- $SUT_j = \texttt{round}(UT_j/\tau)$
- $SNRDIS_s = \tau \times NRDIS_s$
- $SNRUIS_s = \tau \times NRUIS_s$
- $SNRDOS_s = \tau \times NRDOS_s$
- $SNRUOS_s = \tau \times NRUOS_s$
- $SRD_j = \min\{\overline{P}_j, \tau \times NRD_j\}$
- $SRU_j = \min\{\overline{P}_j, \tau \times NRU_j\}$
- $SSD_j = \min\{\overline{P}_j, \tau \times NSD_j\}$
- $SSU_j = \min\{\overline{P}_j, \tau \times NSU_j\}$
- $Z_o = (V_o)^2/S_o$

Remarks on the "Round" Function: In the above calculations for SUT_j and SDT_j, "`round`" denotes Python's function round(), used to round a number to a certain decimal point. Round() takes in two numbers as inputs. The first number is interpreted as the number to be rounded, and the second number is interpreted as the number of decimal places to be included in this rounding. The number 5 is the cut-off for rounding up. Thus, for example, round(17.750,1) = 17.8 whereas round(17.749,1) = 17.7. If nothing is received for the second number input, round() rounds off the first number input to the nearest integer. For example, round(15.59159) = 16 whereas round(15.49321) = 15.

3.5 User-Specified Initial State Conditions

$p_j(0)$	Initial power output (MW) for dispatchable generator $j \in \mathcal{G}$
$\hat{v}_j(0)$	Initial up-time/down-time status (number of hours) for dispatchable generator $j \in \mathcal{G}$
$SOC_s(0)$	Initial state of charge (decimal percent) for storage unit s
$\bar{x}_s(0)$	Initial power output (MW) for storage unit s
$\underline{x}_s(0)$	Initial power absorption (MW) for storage unit s

Remarks on the Meaning of $\hat{v}_j(0)$: If the value of $\hat{v}_j(0)$ is positive (negative) for some dispatchable generator $j \in \mathcal{G}$, it indicates the number of consecutive hours prior to *and including* operating period 0 that j has been turned on (off). Note that $\hat{v}_j(0)$ cannot be zero, by definition.

3.6 Derived Initial State Conditions

ITF_j	Number of operating periods dispatchable generator $j \in \mathcal{G}$ must be offline *initially*
ITO_j	Number of operating periods dispatchable generator $j \in \mathcal{G}$ must be online *initially*
$v_j(0)$	Initial ON/OFF (1/0) status for dispatchable generator $j \in \mathcal{G}$

Calculations for Derived Initial State Conditions:

- If $\hat{v}_j(0) < 0$, $ITF_j = \min(|K|, \max(0, \texttt{round}((DT_j + \hat{v}_j(0))/\tau)))$; otherwise, $ITF_j = 0$.
- If $\hat{v}_j(0) > 0$, $ITO_j = \min(|K|, \max(0, \texttt{round}((UT_j - \hat{v}_j(0))/\tau)))$; otherwise, $ITO_j = 0$.
- If $\hat{v}_j(0) > 0$, $v_j(0) = 1$; otherwise, $v_j(0) = 0$.

3.7 Net Load Forecast Specification with NDG Treated as Negative Load

Step 1: *Specify Load Forecasts*

$D(b,k) \geq 0$	Forecasted load (MW) at bus b for period k, $\forall b \in \mathcal{B},\ k \in K$
$D(z,k) = \sum_{b \in z} D(b,k)$	Forecasted load (MW) at zone z for period k, $\forall z \in \mathcal{Z},\ k \in K$
$D(k) = \sum_{b \in \mathcal{B}} D(b,k)$	System-wide forecasted load (MW) for period k, $\forall\ k \in K$

Step 2: *Specify Forecasts for Non-Dispatchable Generation (NDG)*

$np_i(k) \geq 0$	Forecasted power output (MW) of NDG i for period k, $\forall i \in \mathcal{NG},\ k \in K$
$NDG(b,k) = \sum_{i \in \mathcal{NG}(b)} np_i(k)$	Forecasted NDG power output (MW) at bus b for period k, $\forall b \in \mathcal{B},\ k \in K$
$NDG(z,k) = \sum_{i \in \mathcal{NG}(z)} np_i(k)$	Forecasted NDG power output (MW) at zone z for period k, $\forall z \in \mathcal{Z},\ k \in K$
$NDG(k) = \sum_{i \in \mathcal{NG}} np_i(k)$	System-wide forecasted NDG power output (MW) for period k, $\forall\ k \in K$

Step 3: *Specify Net Load Forecasts*

$NL(b,k) = [D(b,k) - NDG(b,k)]$	Forecasted net load (MW) at bus b for period k, $\forall b \in \mathcal{B},\ k \in K$
$NL(z,k) = [D(z,k) - NDG(z,k)]$	Forecasted net load (MW) at zone z for period k, $\forall z \in \mathcal{Z},\ k \in K$
$NL(k) = [D(k) - NDG(k)]$	System-wide forecasted net load (MW) for period k, $\forall\ k \in K$

Remarks on Net Load Forecasts:

- A SCUC/SCED optimization is a forward-market planning tool for ensuring suitable resource availability for subsequent real-time operations. If a SCUC/SCED optimization is conducted several hours in advance of real-time operations, it would generally not be credible to assume real-time loads and non-dispatchable generation are known with certainty at the time of this optimization.
- In US ISO/RTO-managed day-ahead markets, the ISO/RTO is required to use load-serving entity demand bids as forecasted next-day loads. Forecasts for non-dispatchable generation are typically formulated by the ISO/RTO itself. Reserve requirements are then included in the SCUC/SCED optimization constraints to protect against the possibility of forecast errors.
- The SCUC/SCED optimization formulated by Carrión and Arroyo [3] does not include non-dispatchable generation and does not consider transmission congestion. Consequently, the only external forcing term in each operating period k is "demand" $D(k)$, where $D(k)$ denotes system-wide forecasted must-have (non-price-sensitive) power usage for period k.

3.8 ISO Choice Variables and Derived Solution Variables

Binary-Valued ISO Choice Variables:

$v_j(k)$	1 if dispatchable generator $j \in \mathcal{G}$ is committed for operating period k; 0 otherwise
$hs_j(k)$	1 if dispatchable generator $j \in \mathcal{G}$ is in a hot-start state in period k; 0 otherwise
$\bar{u}_s(k)$	1 if storage unit s is committed for power output (discharge) in operating period k; 0 otherwise;
$\underline{u}_s(k)$	1 if storage unit s is committed for power absorption (charging) in operating period k; 0 otherwise.

Continuously-Valued ISO Choice Variables:

$p_j(k)$	Power output (MW) for dispatchable generator $j \in \mathcal{G}$ in operating period k
$\theta_b(k)$	Voltage angle (radians) for bus b during operating period k
$\bar{x}_s(k)$	Power output (MW) for storage unit s in operating period k
$\underline{x}_s(k)$	Power absorption (MW) for storage unit s in operating period k
$\alpha_b(k)$	Slack variable (MW) for power balance at bus b in operating period k

Solution Variables Derived from Choice Variables and System Constraints:

$c_j^P(k)$	Total production cost (\$) for dispatchable generator $j \in \mathcal{G}$ in operating period k
$c_j^u(k)$	Start-up cost (\$) for dispatchable generator $j \in \mathcal{G}$ in operating period k
$c_j^d(k)$	Shut-down cost (\$) for dispatchable generator $j \in \mathcal{G}$ in operating period k
$\overline{p}_j(k)$	Max possible power output (MW) of dispatchable generator $j \in \mathcal{G}$ in operating period k
$\underline{p}_j(k)$	Min possible power output (MW) of dispatchable generator $j \in \mathcal{G}$ in operating period k
$w_\ell(k)$	Power flow (MW) on transmission line ℓ in operating period k
$z_s(k)$	State of charge (decimal percent) for storage unit s in operating period $k = 0, \ldots, \vert K\vert$
$\alpha_b^+(k), \alpha_b^-(k)$	Power balance slack variable terms (MW) for bus b in operating period k

Remarks on the Slack Variable Terms: For any real variable x, there exist unique non-negative values x^+ and x^- satisfying $x^+ - x^- = x$ and $x^+ + x^- = |x|$. As will be seen below in Section 4, the total cost objective function (1) for the ECA Model decomposes the slack variable $\alpha_b(k)$ into $(\alpha_b^+(k), \alpha_b^-(k))$ in order to impose equal penalties on positive and negative deviations from power balance at bus b in period k.

4 ECA Model SCUC/SCED Optimization Formulation

4.1 Overview

The ECA Model is an analytical formulation for a SCUC/SCED optimization in which an ISO selects admissible choice variables to minimize total cost ($) over operating periods $k = 1, \ldots, |K|$ subject to system constraints. This section provides the analytical formulations for the total cost objective function and the system constraints, making use of the nomenclature presented in Section 3. The start-up and shut-down cost functions $c_j^u(k)$ and $c_j^d(k)$ included in the total cost objective function are carefully formulated in the constraint relationships (28) and (29); the production cost function $c_j^P(k)$ included in the total cost objective function is carefully explained in Section 5.

4.2 SCUC/SCED Objective Function

$$\sum_{k \in K} \sum_{j \in \mathcal{G}} c_j^P(k) + c_j^u(k) + c_j^d(k) + \Lambda \Big(\sum_{b \in \mathcal{B}} \sum_{k \in K} |\alpha_b(k)| \Big) \tag{1}$$

The SCUC/SCED objective function (1) is constructed as the summation of production cost, start-up cost, shut-down cost, and penalty terms imposing costs for any non-zero slack variables appearing in the power balance constraints.[2]

4.3 ISO Admissible Choice Variables

$$v_j(k) \in \{0, 1\} \qquad \forall j \in \mathcal{G}, k \in K \tag{2}$$
$$hs_j(k) \in \{0, 1\} \qquad \forall j \in \mathcal{G}, k \in K \tag{3}$$
$$\bar{u}_s(k) \in \{0, 1\} \qquad \forall s \in \mathcal{S}, k \in K \tag{4}$$
$$\underline{u}_s(k) \in \{0, 1\} \qquad \forall s \in \mathcal{S}, k \in K \tag{5}$$
$$p_j(k) \geq 0 \qquad \forall j \in \mathcal{G}, k \in K \tag{6}$$
$$\theta_b(k) \in [-\pi, \pi] \qquad \forall b \in \mathcal{B}, k \in K \tag{7}$$
$$\bar{x}_s(k) \geq 0 \qquad \forall s \in \mathcal{S}, k \in K \tag{8}$$
$$\underline{x}_s(k) \geq 0 \qquad \forall s \in \mathcal{S}, k \in K \tag{9}$$
$$\alpha_b(k) \in \mathbb{R} \qquad \forall b \in \mathcal{B}, k \in K \tag{10}$$

4.4 System Constraints

Line power constraints: For all $\ell \in \mathcal{L}$ and $k \in K$,

$$w_\ell(k) = S_0 B(\ell) \left[\theta_{BF(\ell)}(k) - \theta_{BT(\ell)}(k) \right] ; \tag{11}$$
$$-TL(\ell) \leq w_\ell(k) \leq TL(\ell) \, . \tag{12}$$

[2] The SCUC/SCED optimization formulated by Carrión and Arroyo [3] does not include slack variables and penalty terms.

Power balance constraints (with slack variables): For all $b \in \mathcal{B}$ and $k \in K$,

$$\sum_{j \in \mathcal{G}(b)} p_j(k) + \Big[\sum_{\ell \in \mathcal{L}_I(b)} w_\ell(k) \; - \sum_{\ell \in \mathcal{L}_O(b)} w_\ell(k) \Big] + \Big[\sum_{s \in \mathcal{S}(b)} \big(\overline{x}_s(k) - \underline{x}_s(k) \big) \Big] \; + \; \alpha_b(k) \quad = \quad NL(b,k) \, .$$

Dispatchable generator capacity constraints: For all $j \in \mathcal{G}$ and $k \in K$,

$$\underline{p}_j(k) \;\le\; p_j(k) \;\le\; \overline{p}_j(k) \, ; \tag{13}$$

$$\overline{p}_j(k) \;\le\; \overline{P}_j v_j(k) \, ; \tag{14}$$

$$\underline{p}_j(k) \;\ge\; \underline{P}_j v_j(k) \, . \tag{15}$$

Dispatchable generator ramping constraints for start-up, normal, and shut-down conditions:

$$\bar{p}_j(k) \le \; p_j(k-1) \; + \; SRU_j v_j(k-1) \; + \; SSU_j[v_j(k) - v_j(k-1)] \; + \; \bar{P}_j[1 - v_j(k)] \, , \quad \forall j \in \mathcal{G}, \; \forall k \in K \, ; \tag{16}$$

$$\bar{p}_j(k) \le \; \bar{P}_j v_j(k+1) \; + \; SSD_j[v_j(k) - v_j(k+1)] \, , \quad \forall j \in \mathcal{G}, \; \forall k = 1 \cdots |K| - 1 \, ; \tag{17}$$

$$p_j(k-1) - \underline{p}_j(k) \le \; SRD_j v_j(k) \; + \; SSD_j[v_j(k-1) - v_j(k)] \; + \; \bar{P}_j[1 - v_j(k-1)] \, , \quad \forall j \in \mathcal{G}, \; \forall k \in K \, . \tag{18}$$

Dispatchable generator minimum up-time constraints:

$$\sum_{k=1}^{ITO_j} [1 - v_j(k)] = 0 \;\text{ for all } j \in \mathcal{G} \text{ with } ITO_j \ge 1 \, ; \tag{19}$$

$$\sum_{n=k}^{k+SUT_j-1} v_j(n) \ge SUT_j[v_j(k) - v_j(k-1)], \quad \forall j \in \mathcal{G}, \; \forall k = ITO_j + 1, \cdots, |K| - SUT_j + 1 \, ; \tag{20}$$

$$\sum_{n=k}^{|K|} \{ v_j(n) - [v_j(k) - v_j(k-1)] \} \ge 0, \quad \forall j \in \mathcal{G}, \; \forall k = |K| - SUT_j + 2, \cdots, |K| \, . \tag{21}$$

Remarks on the derivation of the minimum up-time constraints: To derive these constraints, consider the following. If $ITO_j \ge 1$ for generator j, then by definition of ITO_j it must hold that $v_j(k) = 1$ for all operating periods k satisfying $1 \le k \le ITO_j$. For $ITO_j + 1 \le k$, suppose a start-up event occurs for generator j in period k; i.e., suppose $v_j(k-1) = 0$ and $v_j(k) = 1$, implying generator j is turned off in period $k-1$ and on in period k. Then, by definition of SUT_j, generator j must remain on for $SUT_j - 1$ additional periods, or until the end of the final modeled period $|K|$ if $|K| \le k + SUT_j - 1$. The above minimum up-time constraints express these requirements in concise form.

Dispatchable generator minimum down-time constraints:

$$\sum_{k=1}^{ITF_j} v_j(k) = 0 \;\; \forall j \in \mathcal{G} \text{ with } ITF_j \geq 1\,; \tag{22}$$

$$\sum_{n=k}^{k+SDT_j-1} [1 - v_j(n)] \geq SDT_j[v_j(k-1) - v_j(k)], \quad \forall j \in \mathcal{G}, \; \forall k = ITF_j + 1, \cdots, |K| - SDT_j + 1\,; \tag{23}$$

$$\sum_{n=k}^{|K|} \left[1 - v_j(n) - [v_j(k-1) - v_j(k)]\right] \geq 0, \quad \forall j \in \mathcal{G}, \; \forall k = |K| - SDT_j + 2, \cdots, |K|\,. \tag{24}$$

Remarks on the derivation of the minimum down-time constraints: The derivation of the above minimum down-time constraints is similar to the derivation of the minimum up-time constraints, except that one considers shut-down events with $v_j(k-1) = 1$ and $v_j(k) = 0$ rather than start-up events.

Dispatchable generator hot-start constraints:

$$hs_j(k) = 1, \qquad \forall j \in \mathcal{G}, 1 \leq k \leq CSH_j : k - CSH_j \leq \hat{v}_j(0)\,; \tag{25}$$

$$hs_j(k) \leq \sum_{t=1}^{k-1} v_j(t), \qquad \forall j \in \mathcal{G}, 1 \leq k \leq CSH_j : k - CSH_j > \hat{v}_j(0)\,; \tag{26}$$

$$hs_j(k) \leq \sum_{t=k-CSH_j}^{k-1} v_j(t), \qquad \forall j \in \mathcal{G}, k = CSH_j + 1, \ldots, |K|\,. \tag{27}$$

Remarks on Generator Hot-Start Constraints: Constraint (25) ensures that, if k does not exceed CSH_j, then generator j is in a hot-start state ($hs_j(k) = 1$) as long as j was "on" either during operating period 0 or during an operating period prior to operating period 0 that is sufficiently close to operating period 0. Constraint (26) ensures that, if k does not exceed CSH_j, and j was *not* "on" during operating period 0 or during an operating period prior to operating period 0 that is sufficiently close to operating period 0, then j is in a cold-start state ($hs_j(k) = 0$) unless j was on during some operating period between 1 and $k-1$. Finally, constraint (27) ensures that, for k larger than CSH_j, j will be in a cold-start state ($hs_j(k) = 0$) if j was not committed during any of the CSH_j operating periods immediately preceding operating period k. For reasons explained in the remarks following the next set of constraints (i.e., the generator start-up cost constraints), if generator j has a positive cold-start cost CSC_j, the cost-minimizing ISO will set $hs_j(k) = 1$ unless the generator hot-start constraints (25) through (27) force the ISO to set $hs_j(k) = 0$.

Dispatchable generator start-up cost constraints: For all $j \in \mathcal{G}$ and $k \in K$,

$$\begin{aligned} c_j^u(k) &= \max\{0, U(k)\}\,; \\ U(k) &= CSC_j - [CSC_j - HSC_j]hs_j(k) - CSC_j\left[1 - [v_j(k) - v_j(k-1)]\right]. \end{aligned} \tag{28}$$

Remarks on Dispatchable Generator Start-Up Cost: Definitions of a cold-start state versus a hot-start state for any dispatchable generator j are provided in Section 3.3. Also recall from this previous section that the user-specified parameters CSC_j and HSC_j are required to satisfy $\text{CSC}_j \geq \text{HSC}_j$. Consequently, (28) implies for any operating period k that $c_j^u(k) = \text{CSC}_j$ if j is in a cold-start state in k and $c^u_j(k) = 0$ if j is in a hot-start state in k. Thus, assuming $\text{CSC}_j > 0$, in attempting to minimize total costs the ISO will strive to avoid starting up generator j in a cold-start state, all else equal. In particular, unless ruled out by the hot-start constraints, the cost-minimizing ISO will set $hs_j(k) = 1$ if it commits generator j for operating period k.

Dispatchable generator shut-down cost constraints: For all $j \in \mathcal{G}$ and $k \in K$,

$$\begin{aligned} c^d_j(k) &= \max\{0, D(k)\}\,; \\ D(k) &= SDC_j[v_j(k-1) - v_j(k)]\,. \end{aligned} \tag{29}$$

Storage unit limit constraints: For all $s \in \mathcal{S}$ and $k \in K$,

$$\underline{u}_s(k)\underline{PIS}_s \leq \underline{x}_s(k) \leq \underline{u}_s(k)\overline{PIS}_s\,; \tag{30}$$

$$\bar{u}_s(k)\underline{POS}_s \leq \bar{x}_s(k) \leq \bar{u}_s(k)\overline{POS}_s\,. \tag{31}$$

Storage unit charge/discharge constraint (cannot charge and discharge at same time):

$$\underline{u}_s(k) + \bar{u}_s(k) \leq 1,\ \forall s \in \mathcal{S}, k \in K\,. \tag{32}$$

Storage unit ramping constraints: For all $s \in \mathcal{S}$ and $k \in K$,

$$\bar{x}_s(k) \leq \bar{x}_s(k-1) + SNRUOS_s\,; \tag{33}$$

$$\bar{x}_s(k) \geq \bar{x}_s(k-1) - SNRDOS_s\,; \tag{34}$$

$$\underline{x}_s(k) \leq \underline{x}_s(k-1) + SNRUIS_s\,; \tag{35}$$

$$\underline{x}_s(k) \geq \underline{x}_s(k-1) - SNRDIS_s. \tag{36}$$

Storage unit energy conservation constraints: For all $s \in \mathcal{S}$ and $k \in K$,

$$z_s(k) = z_s(k-1) + \frac{[-\bar{x}_s(k) + \eta_s \underline{x}_s(k)] \cdot \tau}{\overline{ES}_s}\,; \tag{37}$$

$$z_s(0) = SOC_s(0)\,. \tag{38}$$

Storage unit end-point constraints: For all $s \in \mathcal{S}$,

$$z_s(|K|) = EPSOC_s\,. \tag{39}$$

System-wide down/up reserve requirement constraints: For all $k \in K$,

$$\sum_{j \in \mathcal{G}} \underline{p}_j(k) \ \leq\ [1 - RD(k)] \cdot NL(k)\,; \tag{40}$$

$$\sum_{j \in \mathcal{G}} \overline{p}_j(k) \;\geq\; [1 + RU(k)] \cdot NL(k)\,. \tag{41}$$

Zonal down/up reserve requirement constraints: For all $z \in \mathcal{Z}$ and $k \in K$,

$$\sum_{j \in \mathcal{G}(z)} \underline{p}_j(k) \;\leq\; [1 - ZRD(z,k)] \cdot NL(z,k)\,; \tag{42}$$

$$\sum_{j \in \mathcal{G}(z)} \overline{p}_j(k) \;\geq\; [1 + ZRU(z,k)] \cdot NL(z,k)\,. \tag{43}$$

Voltage angle specification at angle reference bus 1: For all $k \in K$,

$$\theta_1(k) = 0\,. \tag{44}$$

5 Pyomo Formulation for ECA Model Total Production Costs

The ECA Model formulation for the total production cost incurred by a dispatchable generator $j \in \mathcal{G}$ during any operating period k relies on five critical assumptions. First, the operating periods k are all of equal length. Second, the range $\mathcal{P}_j = [\underline{P}_j, \overline{P}_j]$ of sustainable power outputs for each dispatchable generator j is the same for each operating period k. Third, the ISO dispatches power in energy-block form; i.e., the dispatch instructions conveyed to each committed generator j for any operating period k consist of a single power output $p_j(k) \in \mathcal{P}_j$ to be maintained during k. Fourth, the total production cost $TPC_j(p)$ incurred by a dispatchable generator j as a result of maintaining a power output $p \in \mathcal{P}_j$ during an operating period k is the same for all k. Fifth, $TPC_j(p)$ is a convex function of p.

Specifically, in accordance with these five assumptions, the ECA Model assumes that the total production cost function for each dispatchable generator j for any operating period k takes the following convex form:

$$TPC_j(p) = a_j + b_j p + c_j p^2 \;, \quad \forall p \in \mathcal{P}_j = [\underline{P}_j, \overline{P}_j] \;, \tag{45}$$

where a_j, b_j, and c_j are non-negative user-set parameters. The cost $TPC_j(p)$ (\$) denotes the total production cost incurred by generator j as a result of maintaining the constant power output p (MW) during any operating period k. As clarified below, $TPC_j(p)$ can incorporate no-load cost and lost opportunity cost as well as dispatch cost.

On the other hand, as seen in Section 4.2, the total production cost for each dispatchable generator j during each operating period k appears in the objective function (1) for the ECA Model SCUC/SCED optimization as $c_j^P(k)$. As will next be explained, the Pyomo Model implements these cost terms $c_j^P(k)$ as approximations for true total production costs, assumed to be given by (45).

More precisely, as depicted in Fig. 1 and carefully explained below, the cost terms $c_j^P(k)$ are based on piecewise-linear approximations for the true total production cost functions (45). These piecewise-linear functions are constructed by connecting successive points in the MW-\$ plane corresponding to successive time segments labeled $i = 1, ..., NS$. The points take the form of power-cost pairs (T_{ij}, G_{ij}) whose components are determined by one of two options: (i) settings automatically generated by internal Pyomo Model calculations; or (ii) settings directly supplied by the user.

Specifically, for each dispatchable generator j and operating period k, the cost term $c_j^P(k)$ in the SCUC/SCED

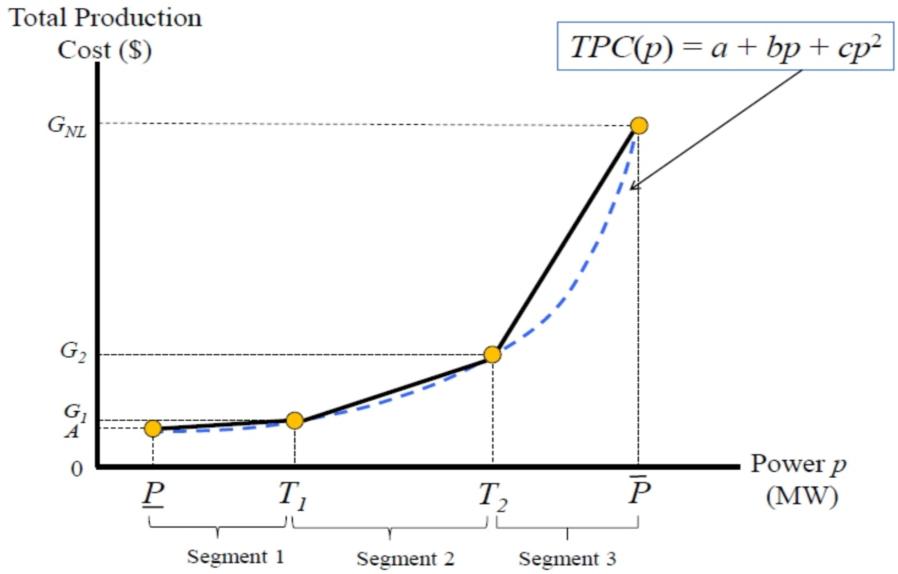

Figure 1: Illustration of the Pyomo Model's piecewise-linear approximation for the total production cost function (45) for a dispatchable generator during any operating period k. In this depiction the user-specified number of time segments is set to NS=3 and the time segments are of unequal user-specified lengths.

objective function (1) is represented as follows:[3]

Total Production Cost Approximation Method:

$$c_j^P(k) = A_j v_j(k) + \sum_{i=1}^{NS} F_{i,j}\delta_{i,j}(k) \tag{46}$$

$$p_j(k) = \sum_{i=1}^{NS} \delta_{i,j}(k) + \underline{P}_j v_j(k) \tag{47}$$

$$\delta_{1,j}(k) \leq T_{1j} - \underline{P}_j \tag{48}$$

$$\delta_{i,j}(k) \leq T_{ij} - T_{i-1,j}, \;\; \forall i = 2 \cdots NS - 1 \tag{49}$$

$$\delta_{NS,j}(k) \leq \bar{P}_j - T_{NS-1,j} \tag{50}$$

$$\delta_{i,j}(k) \geq 0, \;\; \forall i = 1 \cdots NS \tag{51}$$

where

$$A_j \;\; = \;\; a_j + b_j\underline{P}_j + c_j\underline{P}_j^2\,; \tag{52}$$

$$F_{1j} \;\; = \;\; \frac{G_{1j} - A_j}{T_{1j} - \underline{P}_j}\,; \tag{53}$$

$$F_{ij} \;\; = \;\; \frac{G_{ij} - G_{i-1,j}}{T_{ij} - T_{i-1,j}}, \;\; i = 2, \ldots, NS - 1\,; \tag{54}$$

$$F_{NS,j} \;\; = \;\; \frac{G_{NS,j} - G_{NS-1,j}}{\overline{P}_j - T_{NS-1,j}}\,, \tag{55}$$

[3]See [3, Sec. II.A] for additional details regarding the manner in which the Pyomo Model implements a piecewise-linear approximation for each dispatchable generator's convex total production cost function (45).

The 1/0 binary variable $v_j(k)$ in (46) indicates whether generator j is committed (1) or not (0) for operating period k. The variable $p_j(k)$ in (47) denotes the power output to be sustained by generator j during period k if j is committed for k.[4] The marginal cost for generator j, producing at a sustained power output in an interior time segment $i = 2, ..., NS-1$, is approximated by F_{ij} in (54). For the initial time segment (1) this marginal cost is approximated by (53), and for the final time segment NS this marginal cost is approximated by (55). The variables $\delta_{i,j}(k)$ appearing in (46)-(51) are introduced into the SCUC/SCED optimization as additional ISO choice variables.

Suppose, for example, that generator j is committed for operating period k; and suppose there exists a time segment $n \in (1, NS)$ such that $\delta_{i,j}(k)$ takes on its maximum possible value for $i = 1, \ldots, n$ and $\delta_{i,j}(k) = 0$ for $i = n+1, \ldots, NS$. Then $c_j^P(k) = G_{n,j}$. On the other hand, suppose generator j is committed for period k but $\delta_{i,j}(k) = 0$ for all $i = 1, \ldots, NS$. Then $c_j^P(k) = A_j \equiv a_j + b_j\underline{P}_j + c_j\underline{P}_j^2$.

As will next be explained, the Pyomo Model offers users two different options for specifying the power-cost combinations $(T_{i,j}, G_{i,j})$ appearing in the Total Production Cost Approximation Method.

Automated Approximation Option:

The user selects the Automated Approximation Option by specifying admissible (non-negative) settings for the production-function coefficients (a_j, b_j, c_j) for each dispatchable generator $j \in \mathcal{G}$ together with an admissible (positive integer) setting for the segment number NS. The Pyomo Model then uses these specifications to compute power-cost combinations $(T_{i,j}, G_{i,j})$ for the Total Production Cost Approximation Method, as follows: (a) The width of each time segment $i = 1, \ldots, NS$ is set equal to $w_j = (\overline{P}_j - \underline{P}_j)/NS$; (b) the maximum sustainable power output for each time segment $i = 1, \ldots NS-1$ is set equal to $T_{ij} = \underline{P}_j + iw_j$; (c) the maximum sustainable power output for the final time segment NS is set equal to $T_{NS,j} = \overline{P}_j$; and (d) the total production cost corresponding to T_{ij} is set equal to $G_{ij} = a_j + b_jT_{ij} + c_jT_{ij}^2$ for each i and j. This piecewise-linear approximation is accomplished via a Pyomo `Piecewise` construct.

User-Directed Approximation Option:

Alternatively, a user selects the User-Directed Approximation Option by specifying production-function coefficients $\{a_j, b_j, c_j \mid j \in \mathcal{G}\}$, a segment number NS, and power-cost combinations $\{T_{ij}, G_{ij} \mid i = 1, \ldots NS, j \in \mathcal{G}\}$ subject to the following admissibility conditions: (i) The production-function coefficients must be non-negative; (ii) the total number NS of time segments i must be a positive integer; (iii) the maximum sustainable power output T_{ij} for time segment i must lie within generator j's feasible sustainable power output range $[\underline{P}_j, \overline{P}_j]$, for each i and j; (iv) the maximum sustainable power output T_{ij} for time segment i must be non-decreasing in i, for each j; (v) the maximum sustainable power output for the final time segment NS must be set equal to $T_{NS,j} = \overline{P}_j$, for each j; and (vi) the total production cost corresponding to T_{ij} must be set equal to $G_{ij} = a_j + b_jT_{ij} + c_jT_{ij}^2$ for each i and j.

Figure 2 illustrates the manner in which the Pyomo Model approximates the energy requirements for a dispatchable generator that has been instructed to deliver a constant power output level $p(k) = T_2$ during an operating period $k = [k^{\mathsf{s}}, k^{\mathsf{e}})$. In this figure, g^{sync} denotes the non-negative power generation level at which the generator attains a synchronized state, ready to inject power into the grid but not yet injecting any power into the grid. The generator's total power generation level sustained during period k is thus given by $g(t) = T_2 + g^{\mathsf{sync}}$ for all $t \in k$.

[4] The SCUC/SCED constraints presented in Section 4.4 imply that $c_j^P(k)$ is zero in any period k for which j is not committed. That is, if $v_j(k) = 0$, then $p_j(k) = 0$, hence $c_j^P(k) = 0$.

The amount of energy that the generator expends locally (behind-the-meter) during period k in order to maintain itself in a synchronized state is denoted by NoLoad(g^{sync}) in Fig. 2. The amount of energy that the generator injects into the grid during operating period k in order to maintain its minimum sustainable power output $\underline{P}$ during period k is denoted by MinRun($\underline{P}$). The generator's remaining dispatched energy delivery during period k is denoted by NetDispatch.

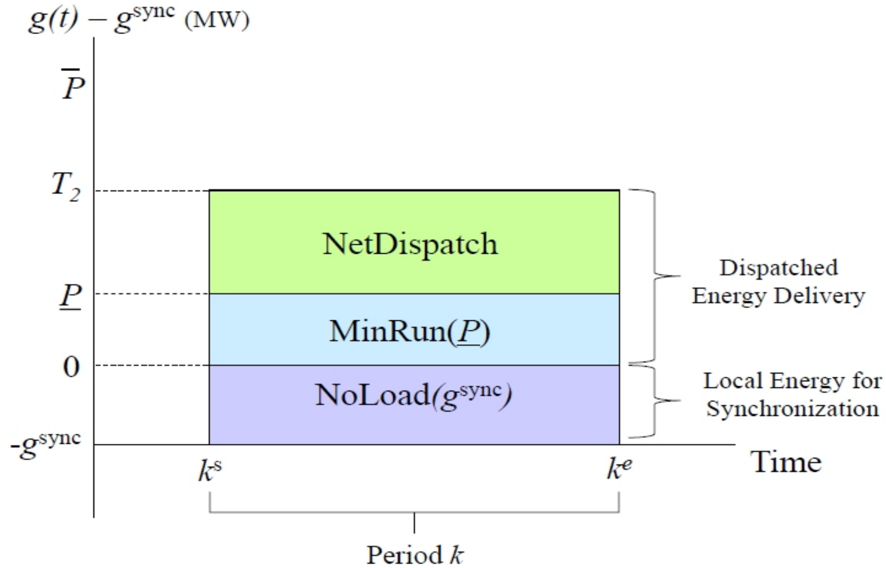

Figure 2: Illustrative depiction of the energy requirements for a dispatchable generator j during an operating period $k = [k^s, k^e)$ that maintains a constant power output $p_j(k) = T_2$ during k, as determined by the Pyomo Model's Total Production Cost Approximation method.

Suppose a dispatchable generator j has been committed for an operating period k for a maintained power output $p_j(k)$. Given the assumptions of the ECA Model, the costs incurred by the generator as a result of this commitment can be classified as follows. The generator's *no-load cost* (\$) is the cost it incurs to maintain itself in a synchronized state during k. The generator's *lost opportunity cost* (\$) is the net earnings that the generator could have obtained from the use of its committed generation capacity in a next-best alternative during k. Finally, the generator's *dispatch cost* (\$) is the cost it incurs for dispatched energy delivery during k.

For the case depicted in Fig. 2, generator j's no-load cost for period k includes the cost incurred for the energy requirement NoLoad(g^{sync}). The total production cost function (45) can account for this no-load cost, along with any lost opportunity costs, by appropriate specification of the production cost function coefficient a_j (\$). Generator j's dispatch cost for period k includes the cost of the energy requirements MinRun($\underline{P}$) and NetDispatch. These costs can be accounted for by appropriate specification of the remaining portion of the total production cost function (45): namely, $b_j p_j(k) + c_j[p_j(k)]^2$ (\$), where $p_j(k) = T_2 > \underline{P} > 0$.

6 Pyomo Model Calculation of Locational Marginal Prices

The Pyomo Model implementation of the ECA Model SCUC/SCED optimization problem set out in Section 4 determines unit commitments and scheduled dispatch levels for successive future operating periods $k = 1, \dots |K|$. Consistent with actual practice, settlements for these scheduled dispatch levels can be determined in accordance with locational marginal pricing, i.e., the pricing of power in accordance with the location and timing of its injection into, or withdrawal from, a physical grid.

Specifically, *Locational Marginal Prices (LMPs)* can be derived as follows from a Pyomo Model SCUC/SCED optimal solution. First, fix all unit commitment variables at their optimal binary (0/1) solution values. Second, re-run the optimization as a pure SCED optimization, conditional on these optimal unit commitment solution values. Third, calculate the LMP for each bus b in each operating period k as the dual variable for the power balance constraint (13) corresponding to this b and k.

The dual variable for a power balance constraint in a SCED optimization measures the change in the optimized SCED objective function with respect to a change in the constraint constant for this power balance constraint. This constraint constant is typically taken to be the possibly-zero forecasted amount of fixed (non-price-sensitive) net load appearing in the power balance constraint. The existence of a *unique* dual variable for a particular power balance constraint with constraint constant cc requires the optimized SCED objective function to be a differentiable function of cc at the optimal solution point.[5]

7 Pyomo Model Flag for Storage

The Pyomo Model permits the user to implement the SCUC/SCED optimization either with or without energy storage units. This is done by setting a flag named StorageFlag either to 1 (*with* storage) or to 0 (*without* storage). This flag setting is used by the Pyomo Model either to include or exclude the appearance of storage variables and storage constraints in the implemented SCUC/SCED optimization.

8 Input Name Mapping

Each of the user-specified inputs (parameters, initial state conditions, and external forcing terms) named in Section 3 for the analytical ECA Model has a corresponding name in the Pyomo Model implementation of this ECA Model. Table 1, reproduced from ref. [2, Sec. 3], provides a partial mapping between these names, and also between these names and named variables in AMES V5.0. In addition, Table 1 lists any default values assigned by the Pyomo Model for these inputs. If no default value is assigned, the field is left blank.

In the Pyomo Model, `model` is the name of the variable used to define the model. All variable names are defined as instance variables of the model and may be programmatically accessed via `model.VariableName`. The "`model.`" syntax is elided for both brevity and clarity.

The order in Table 1 groups the parameters for dispatchable generator units before the parameters for energy storage units, which differs from their ordering in the Pyomo Model. This change in ordering, permitting related elements to be displayed together, is made to facilitate understanding.

ECA Model	**AMES V5.0**	**Pyomo Model**	**Pyomo Default**
$NL(b,k)$	NetLoadForecast[6]	NetLoadForecast[6]	0.0
$RE(\ell)$	Reactance	Reactance	
$RD(k)$	ReserveDownPercent[7]	ReserveDownPercent	0.05
$RU(k)$	ReserveUpPercent[7]	ReserveUpPercent	0.05

[5] For detailed discussions of LMP determination in U.S. ISO/RTO-managed wholesale power markets, see [1, 12].

[6] See Section 3.7.

[7] Currently this variable is written by AMES V5.0 into ReferenceModel.dat for the Pyomo Model. If its value needs to be changed, the user should change this value inside the AMES V5.0 software

$ZRD(z,k)$	ZonalReserveDownPercent	ZonalReserveDownPercent	0.05
$ZRU(z,k)$	ZonalReserveUpPercent	ZonalReserveUpPercent	0.05
$TL(\ell)$	MaxCap	ThermalLimit	
$\|K\|$	NumTimePeriods[7]	NumTimePeriods	
τ		TimePeriodLength	1
Λ		LoadMismatchPenalty	1.0×10^6
Dispatchable Generation Parameters			
DT_j	MinDownTime	MinimumDownTime	0
UT_j	MinUpTime	MinimumUpTime	0
NRD_j	NominalRampDown	NominalRampDownLimit	
NRU_j	NominalRampUp	NominalRampUpLimit	
NSD_j	ShutdownRampLim	ShutdownRampLimit	
NSU_j	StartupRampLim	StartupRampLimit	
$\overline{P}_j$	capU	MaximumPowerOutput	0.0
$\underline{P}_j$	capL	MinimumPowerOutput	0.0
$p_j(0)$	PowerT0	PowerGeneratedT0	
$\hat{v}_j(0)$		UnitOnT0State	
$v_j(0)$	UnitOnT0[8]	UnitOnT0	
CSC_j		ColdStartCost	0.0
CSH_j		ColdStartHours	0
HSC_j		HotStartCost	0.0
SDC_j		ShutdownCostCoefficient	0.0
a_j	FCost	ProductionCostA0	10.0
b_j	a	ProductionCostA1	0.0
c_j	b	ProductionCostA2	0.0
NS		NumGeneratorCostCurvePieces	2
T_{ij}		PowerGenerationPiecewisePoints[9]	
G_{ij}		PowerGenerationPiecewiseValues[9]	
Energy Storage Parameters			
$EPSOC_s$		EndPointSocStorage	0.5
$\overline{ES}_s$		MaximumEnergyStorage	0.0
$NRDIS_s$		NominalRampDownLimitStorageInput	

[8]This value is computed from $\hat{v}_j(0)$, hence it does not need to be specified.
[9]As explained in Section 5, these values do not need to be specified if (a_j, b_j, c_j) and NS are specified.

$NRUIS_s$		NominalRampUpLimitStorageInput	
$NRDOS_s$		NominalRampDownLimitStorageOutput	
$NRUOS_s$		NominalRampUpLimitStorageOutput	
$\overline{PIS}_s$		MaximumPowerInputStorage	0.0
$\underline{PIS}_s$		MinimumPowerInputStorage	0.0
$\overline{POS}_s$		MaximumPowerOutputStorage	0.0
$\underline{POS}_s$		MinimumPowerOutputStorage	0.0
$\underline{SOC}_s$		MinimumSocStorage	0.0
$SOC_s(0)$		StorageSocOnT0	0.5
η_s		EfficiencyEnergyStorage	1.0
$\overline{x}_s(0)$		StoragePowerOutputOnT0	0.0
$\underline{x}_s(0)$		StoragePowerInputOnT0	0.0
Flag for Storage			
		StorageFlag	0

Table 1: Input name-mapping for the ECA Model, AMES V5.0, and the Pyomo Model, plus default values for the Pyomo Model

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