

$$\therefore M_t = \left\lceil \frac{-y_{n(t)} W_t^T x_{n(t)}}{|x_{n(t)}|^2} \right\rceil = \frac{-y_{n(t)} W_t^T x_{n(t)}}{|x_{n(t)}|^2} + \epsilon, \text{ where } 0 \leq \epsilon < 1$$

$$\therefore W_{t+1} = W_t + y_{n(t)} x_{n(t)} M_t = W_t + y_{n(t)} x_{n(t)} \left(\frac{-y_{n(t)} W_t^T x_{n(t)}}{|x_{n(t)}|^2} + \epsilon \right)$$

$$\therefore |W_{t+1}|^2 = |W_t|^2 + |x_{n(t)}|^2 \left(\frac{-y_{n(t)} W_t^T x_{n(t)}}{|x_{n(t)}|^2} + \epsilon \right)^2 + 2y_{n(t)} W_t^T x_{n(t)} \left(\frac{-y_{n(t)} W_t^T x_{n(t)}}{|x_{n(t)}|^2} + \epsilon \right)$$

$$= |W_t|^2 + |x_{n(t)}|^2 \left(\frac{|W_t^T x_{n(t)}|^2}{|x_{n(t)}|^4} + \epsilon^2 + \frac{-2\epsilon y_{n(t)} W_t^T x_{n(t)}}{|x_{n(t)}|^2} \right) + \left(\frac{-2|W_t^T x_{n(t)}|^2}{|x_{n(t)}|^2} + 2\epsilon y_{n(t)} W_t^T x_{n(t)} \right)$$

$$= |W_t|^2 + \frac{|W_t^T x_{n(t)}|^2}{|x_{n(t)}|^2} + \epsilon^2 |x_{n(t)}|^2 - 2\epsilon y_{n(t)} W_t^T x_{n(t)} + \frac{-2|W_t^T x_{n(t)}|^2}{|x_{n(t)}|^2} + 2\epsilon y_{n(t)} W_t^T x_{n(t)}$$

$$= |W_t|^2 + \frac{|W_t^T x_{n(t)}|^2}{|x_{n(t)}|^2} + \epsilon^2 |x_{n(t)}|^2 + \frac{-2|W_t^T x_{n(t)}|^2}{|x_{n(t)}|^2}$$

$$= |W_t|^2 + \epsilon^2 |x_{n(t)}|^2 + \frac{-|W_t^T x_{n(t)}|^2}{|x_{n(t)}|^2}$$

$$\leq |W_t|^2 + \epsilon^2 |x_{n(t)}|^2$$

$$\leq |W_t|^2 + \max |x_n|^2$$

$$\therefore |W_{t+1}|^2 \leq |W_t|^2 + \max |x_n|^2, W_0 = 0$$

\therefore Applying the inequality t times, we get:

$$|W_t|^2 \leq t * \max |x_n|^2$$

$$|W_t| \leq \sqrt{t} * \max |x_n| \quad \text{-----(1)}$$

$$\therefore y_{n(t)} W_f^T x_{n(t)} \geq \min(y_n W_f^T x_n) > 0$$

$$\therefore W_f^T W_{t+1} = W_f^T (W_t + y_{n(t)} x_{n(t)} M_t) \geq W_f^T W_t + \min(y_n W_f^T x_n) M_t$$

$$\therefore \min(M_t) = 1$$

$$\therefore W_f^T W_{t+1} \geq W_f^T W_t + \min(y_n W_f^T x_n)$$

\therefore Applying the inequality t times, we get:

$$W_f^T W_t \geq t * \min(y_n W_f^T x_n)$$

$$\frac{W_f^T W_t}{|W_f| |W_t|} \geq \frac{t * \min(y_n W_f^T x_n)}{|W_f| |W_t|} \quad \text{-----(2)}$$

\therefore Then combine inequation(1) and (2), we get:

$$1 \geq \frac{W_f^T W_t}{|W_f| |W_t|} \geq \frac{\sqrt{t} * \min \left(y_n \frac{W_f^T}{|W_f|} x_n \right)}{\max |x_n|}$$

$$\therefore \sqrt{t} \leq \frac{\max |x_n|}{\min \left(y_n \frac{W_f^T}{|W_f|} x_n \right)}$$

$$\therefore t \leq \left(\frac{\max |x_n|}{\min \left(y_n \frac{W_f^T}{|W_f|} x_n \right)} \right)^2$$

\therefore The data set is linear separable

\therefore t is finite