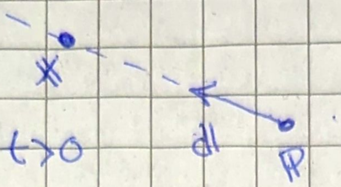
19- Ray tracing

Ray

$\mathbf{x} = \mathbf{p} + t\mathbf{d}$



In ray tracing, we're only interested i $t>0$.

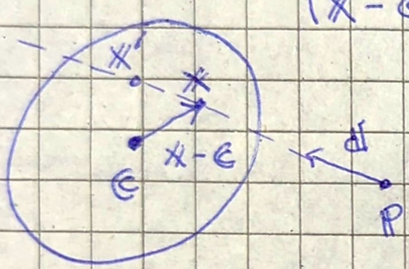
Ray intersection

Implicit surface: $f(\mathbf{x}) = f(\mathbf{p}+t\mathbf{d}) = 0,\ \exists t \Rightarrow$ Hit!
$\neq 0\ \forall t \Rightarrow$ Miss!

Ray-sphere intersection

$$f(\mathbf{x}) = x^2+y^2+z^2-r^2 = (\mathbf{x}-\mathbf{c})\cdot(\mathbf{x}-\mathbf{c})-r^2 = 0$$

$$|\mathbf{x}-\mathbf{c}| = r \Rightarrow \sqrt{(\mathbf{x}-\mathbf{c})\cdot(\mathbf{x}-\mathbf{c})} = r$$
$$(\mathbf{x}-\mathbf{c})\cdot(\mathbf{x}-\mathbf{c}) = r^2$$



$$\begin{aligned} f(\mathbf{x}) &= (\mathbf{x}-\mathbf{c})\cdot(\mathbf{x}-\mathbf{c})-r^2 \\ &= (\mathbf{p}-\mathbf{c}+t\mathbf{d})\cdot(\mathbf{p}-\mathbf{c}+t\mathbf{d})-r^2 \\ &= \underbrace{(\mathbf{d}\cdot\mathbf{d})}_{a}t^2 + \underbrace{2\mathbf{d}\cdot(\mathbf{p}-\mathbf{c})}_{b}t + \underbrace{(\mathbf{p}-\mathbf{c})\cdot(\mathbf{p}-\mathbf{c})-r^2}_{c} \\ &= 0 \end{aligned}$$

Two roots $\Rightarrow \mathbf{x}, \mathbf{x}'$
One root $\Rightarrow \mathbf{x}$ tangent

$at^2+bt+c=0,\quad t_{1,2} = \frac{-b\pm\sqrt{\Delta}}{2a}$

$\Delta < 0 \Rightarrow$ Miss!
$\Delta \geq 0 \Rightarrow$ Hit!

$\boxed{\Delta = b^2-4ac}$

$\mathbf{x}$ is $t = \frac{-b-\sqrt{\Delta}}{2a}$ ← only one we're interested in, because it's smaller

$\mathbf{x}'$ is $t = \frac{-b+\sqrt{\Delta}}{2a}$

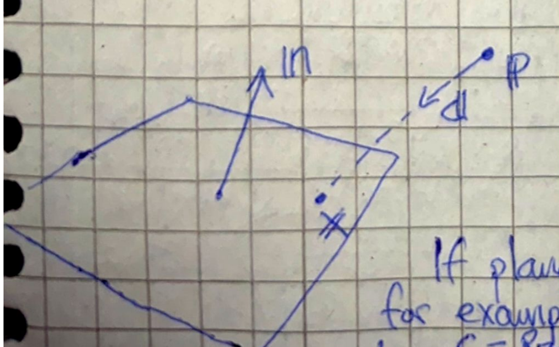
$\mathbf{x}-\mathbf{c}$ is surface normal at $\mathbf{x}$.

Ray-plane intersection

$\mathbf{x} = \mathbf{p}+t\mathbf{d}$

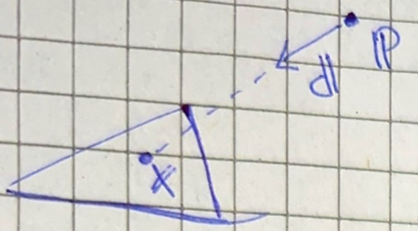
$$f(\mathbf{x}) = \mathbf{x}\cdot\mathbf{n}-c = (\mathbf{p}+t\mathbf{d})\cdot\mathbf{n}-c = 0$$

$$\Rightarrow t = \frac{c-\mathbf{p}\cdot\mathbf{n}}{\mathbf{d}\cdot\mathbf{n}}$$ ← if $\mathbf{d}\cdot\mathbf{n}=0$, $\mathbf{d}$ is perpendicular to plane. Need to check for that.



If plane is axis-aligned, for example $\mathbf{n} = \begin{bmatrix}0\\0\\1\end{bmatrix}$, then $t = \frac{c-p_z}{d_z}$ for the example.

Ray-triangle intersection



We can see the triangle as a plane, and then use the same method for finding the intersection. We however also need to check that the intersection is inside the triangle using barycentric coordinates.

Ray tracing algorithm

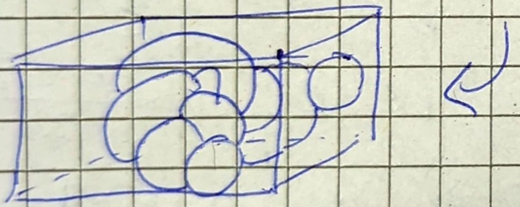
```
for each ray
    for each primitive
        if ray intersects primitive
            if closest hit
                store
```

Massively slow

(will be used in project)

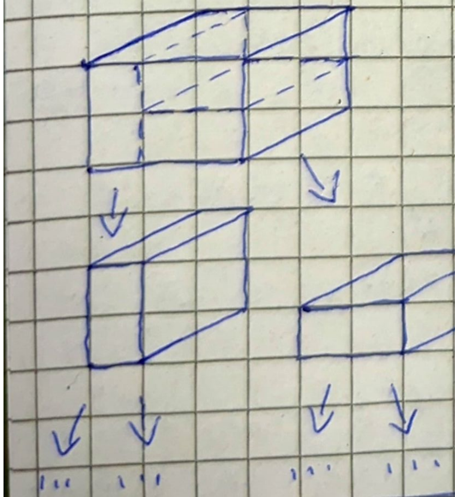
Ray tracing acceleration (acceleration structures/space partitioning

Axis-Aligned Bounding Box

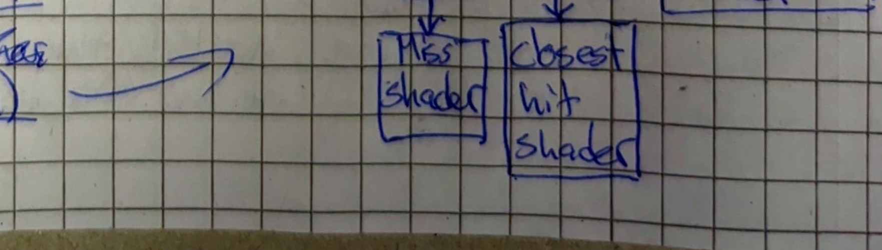


only needs to check if ray intersects with the box or not.

Bounding volume hierarchy (BVH)



First checks outer box, then inner etc. creates a tree structure, ie. acceleration structure.



GPU ray tracing (hardware acceleration)

Software ray tracing on the GPU

Uses the rasterization pipeline to just draw a quad filling the screen. Then we can implement ray tracing in the fragment shader.

Can also interpolate the primary rays in the rasterizer.