

## Problem Set 5

**Problem 1** Show how to implement a stack ADT using only a priority queue and one additional integer variable.

**Problem 2** Write an algorithm for updating the key of an item in a priority queue, and analyse its time complexity.

**Problem 3** Given a heap  $T$  and a key  $k$ , give an algorithm to compute all the items in  $T$  with keys less than or equal to  $k$ . Your algorithm should run in time proportional to the number of items returned.

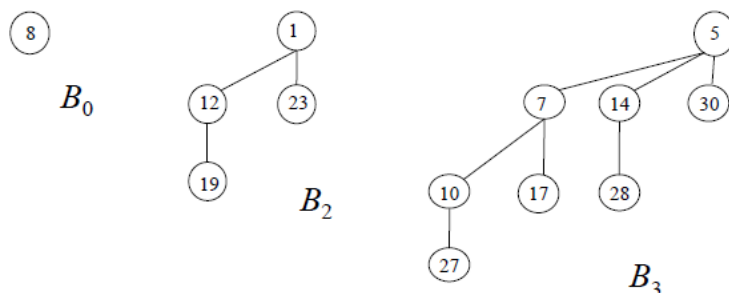
**Problem 4** Qantas Airlines wants to give a first-class upgrade coupon to their top  $\log n$  frequent flyers, based on the number of miles accumulated, where  $n$  is the total number of the airlines' frequent flyers. The algorithm they currently use, which runs in  $O(n \log n)$  time, sorts the flyers by the number of miles flown and then scans the sorted list to pick the top  $\log n$  flyers. Describe an algorithm that identifies the top  $\log n$  flyers in  $O(n)$  time.

**Problem 5** Suppose two binary trees,  $T_1$  and  $T_2$ , hold entries satisfying the heap-order property, where no entry in each tree exists in the other tree. Describe a method for combining  $T_1$  and  $T_2$  into a tree  $T$  such that  $T$ 's internal nodes hold the union of the entries in  $T_1$  and  $T_2$  and  $T$  also satisfies the heap-order property. Your algorithm should run in time  $O(h_1 + h_2)$  where  $h_1$  and  $h_2$  are the respective heights of  $T_1$  and  $T_2$ .

**Problem 6** Give an alternative analysis of the bottom-up heap construction algorithm.

**Problem 7** Prove that a binomial tree with  $2^n$  nodes has  $\binom{n}{i}$  nodes at depth  $i$  ( $0 \leq i \leq n$ ).

**Problem 8** Consider the following binomial heap. Draw the resulting binomial heaps after inserting the keys 7, 12, 20, 24, 25 and 25, respectively.



**Problem 9** In a computer game, all the players are divided into a number of groups. Each player can join one group only and is not allowed to join a different group later. Describe an algorithm for checking if two players are in the same group. What is the running time of your algorithm?