

## Problem Set 3-b

**Problem 1** For each node  $v$  in a tree  $T$ , let  $\text{pre}(v)$  be the rank of  $v$  in the preorder traversal of  $T$ ,  $\text{post}(v)$  the rank of  $v$  in the postorder traversal of  $T$ ,  $\text{depth}(v)$  the depth of  $v$ , and  $\text{desc}(v)$  the number of descendants of  $v$ , not counting  $v$  itself. Derive a formula defining  $\text{post}(v)$  in terms of  $\text{desc}(v)$ ,  $\text{depth}(v)$ , and  $\text{pre}(v)$ , for each node  $v$  in  $T$ .

**Problem 2** Give an  $O(n)$ -time algorithm for computing the depths of all the nodes of a tree  $T$ , where  $n$  is the number of nodes of  $T$ .

**Problem 3** Describe, in pseudo code, a nonrecursive algorithm for performing the preorder traversal on a binary tree in linear time.

**Problem 4** Describe, in pseudo code, a nonrecursive algorithm for performing the inorder traversal on a binary tree in linear time.

**Problem 5** Describe, in pseudo code, a nonrecursive algorithm for performing the postorder traversal on a binary tree in linear time.

**Problem 6** In the inorder traversal on a binary tree, a node  $v_i$  is the immediate inorder predecessor of a node  $v_j$ , if  $v_j$  is visited immediately after  $v_i$  in the inorder traversal. Describe a linear time algorithm for finding the immediate inorder predecessor of a node.

**Problem 7** Let  $T$  be a binary tree. Define a **Roman node** to be a node  $v$  in  $T$ , such that the number of descendants in  $v$ 's left subtree differs from the number of nodes in  $v$ 's right subtree by at most 5. Describe a linear-time algorithm for finding each node  $v$  of  $T$ , such that  $v$  is not a Roman node, but all of  $v$ 's descendants are Roman nodes.

**Problem 8** Let  $T$  be a binary tree with  $n$  nodes. Define the lowest common ancestor (LCA) of two nodes  $v$  and  $w$  as the lowest node in  $T$  that has both  $v$  and  $w$  as descendants (where we allow a node to be a descendant of itself). Given two nodes  $v$  and  $w$ , describe an efficient algorithm for finding the LCA of  $v$  and  $w$ . What is the running time of your algorithm?