# ITU Artificial Intelligence/Machine Learning in 5G Challenge Site-Specific Channel Estimation with Hybrid MIMO Architectures

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## 1 Introduction

It is well known from real world channel measurement and modelling that the mmWave channel have only a few scattering components that lead to very less dominant components when compared to the number of transmit and receive antennas [1, 2]. In other words, mmWave channels are sparse in the virtual angle domain (beamspace) representation[1]. Thus the channel estimation problem in mmWave systems can be seen as a sparse recovery problem [3]. There are various well studied sparse recovery algorithms that can recover the sparse signal in a computationally efficient way. One of the algorithm is the Orthogonal Matching Pursuit (OMP) algorithm which is a greedy algorithm. In this project we propose a hybrid solution that utilizes the well defined problem structure to incorporate the sparse mmWave channel properties and also combines machine learning approach to learn some hyperparameters that are used by the traditional OMP type algorithm. Basically, the site-specific training dataset with channel realizations is used to learn the number of dominant paths in the channels ($L$) which is an important hyperparameter to the OMP type algorithm.

## 2 Proposed Solution

### 2.1 System Model and Algorithm Development

First we note that for a mmWave MIMO system with $N_r$ receive and $N_t$ transmit antennas, the frequency domain mmWave channels can be represented in the beamspace domain as [3, Eq. 6]

$$\boldsymbol{H}[k] = \boldsymbol{A}_r \boldsymbol{H}_v[k] \boldsymbol{A}_t^H \;\forall\; k = 1, 2, \ldots, N_c \tag{1}$$

where $\boldsymbol{H}[k] \in \mathbb{C}^{N_r \times N_t}$ is the frequency domain channel for the $k^{th}$ subcarrier and $\boldsymbol{H}_v[k] \in \mathbb{C}^{G_r \times G_t}$ is the virtual beamspace domain representation of the the channel. Since there are only a few dominant angle of arrival (AoA) and angle of departure (AoD), $\boldsymbol{H}_v[k]$ is a sparse matrix with only a few non-zero components. Further, $\boldsymbol{A}_r \in \mathbb{C}^{N_r \times G_r}$ and $\boldsymbol{A}_t \in \mathbb{C}^{N_t \times G_t}$ are the dictionary matrices that contain the transmitter and receiver array response vectors evaluated on a grid of size $G_r$ for the AoA and a grid of size $G_t$ for the AoD. After collecting all the received signals during pilot transmission phase, the equivalent linear measurement model is given by [3, Eq. 7-13]

$$\boldsymbol{y}[k] = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{h}_v[k] + \boldsymbol{n}_c[k] \;\forall\, k = 1, 2, \ldots, N_c \tag{2}$$

where $\boldsymbol{\Phi} \in \mathbb{C}^{ML_r \times N_r N_t}$ is the equivalent pilot matrix that depends on the transmit pilot signals, and transmit and receive precoders and combiner matrices. Here $M$ is the pilot length and $L_r$ is the number of receive RF chains and $\boldsymbol{\Psi} = \boldsymbol{A}_t^* \otimes \boldsymbol{A}_r \in \mathbb{C}^{N_r N_t \times G_r G_t}$. For each test data point we are provided with the received signal vector $\boldsymbol{y}[k] \in \mathbb{C}^{ML_r \times 1}$ (After vectorization operation) along with the matrices $\boldsymbol{\Phi}, \boldsymbol{\Psi}$, and the objective is to first recover the virtual angle domain channel $\boldsymbol{h}_v[k] \in \mathbb{C}^{G_r G_t \times 1}$ and then output the estimated mmWave channel in the original antenna domain, i.e, $\hat{\boldsymbol{H}}[k]$ for each of the sub-carriers $k = 1, 2, \ldots, N_c = 256$. We use a simultaneous weighted OMP (SWOMP) algorithm similar to the algorithm proposed in [3], where we use the fact that the virtual angle domain channels are highly correlated and they have a common support. Thus, all the received signals for the $N_c = 256$ subcarriers are used to identify the dominant components in a greedy manner. Note that the additive noise in (2) is correlated and hence we need to first whiten the received signal before employing the SWOMP algorithm [3]. The whitening matrix $\boldsymbol{D}_w \in \mathbb{C}^{ML_r \times ML_r}$ depends on the transmit precoder and receive combiner matrices as defined in [3, Sec. III-B]. The SWOMP greedy algorithm should either know the noise variance or the sparsity level of the unknown vector for the stopping criteria. In this competition the noise variance is unknown but we are provided with a site-specific channel realizations obtained from ray tracing. Thus, we can use the training data to learn the sparsity level which is an input to our proposed SWOMP algorithm. By using cross validation and minimizing the normalized mean squared error (NMSE) performance on the training data, we learn the optimum sparsity level for the 3 different test SNR ranges i.e, low SNR ($-20\text{dB} \leq \gamma < -11\text{dB}$) , moderate SNR ($-11\text{dB} \leq \gamma < -6\text{dB}$) and high SNR ($-6\text{dB} \leq \gamma < 0\text{dB}$), using the training datasets generated at $-15$dB, $-10$dB and $-5$dB respectively, where $\gamma$ denotes the true SNR of the signals in the test dataset which are unknown. The overall M-SWOMP algorithm is summarized in Algorithm 1. Note that the $\text{vec}^{-1}(\boldsymbol{a})$ operator reshapes a vector $\boldsymbol{a} \in \mathbb{C}^{N_r N_t \times 1}$ into a matrix $\boldsymbol{A} \in \mathbb{C}^{N_r \times N_t}$.

**Algorithm 1:** Modified Simultaneous Weighted OMP (M-SWOMP)

**Input:** $\boldsymbol{y}[k]\ \forall\, k = 1, \ldots, N_c$ , $\boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{D}_w, L$

**Output:** $\hat{\boldsymbol{H}}[k]\ \forall\, k = 1, \ldots, N_c$

Set $\boldsymbol{\Upsilon}_w = \boldsymbol{D}_w^{-*} \boldsymbol{\Phi} \boldsymbol{\Psi}$

Initialize the input and residual signals: $\boldsymbol{y}_w[k] = \boldsymbol{D}_w^{-*} \boldsymbol{y}[k]$, and $\boldsymbol{r}[k] = \boldsymbol{y}_w[k]\ \forall\, k = 1, \ldots, N_c$

Initialize support estimate: $\mathcal{T} = \{\emptyset\}$ and set $i = 0$

**while** $i \leq L$ **do**

- $i = i + 1$
- Distributed Correlation: $\boldsymbol{c}[k] = \boldsymbol{\Upsilon}_w^* \boldsymbol{r}[k],\ \forall\, k = 1, \ldots, N_c$
- Find maximum projection: $p^* = \arg\max\limits_{p} \sum_{i=1}^{N_c} |\{\boldsymbol{c}[k]\}_p|$
- Update Support Set: $\mathcal{T} = \mathcal{T} \cup p^*$
- Project input signal using WLS: $\boldsymbol{x}_{\mathcal{T}}[k] = \left( [\boldsymbol{\Upsilon}_w]_{:,\mathcal{T}} \right)^{\dagger} \boldsymbol{y}_w[k],\ \forall\, k = 1, \ldots, N_c$
- Update residual: $\boldsymbol{r}[k] = \boldsymbol{y}_w[k] - [\boldsymbol{\Upsilon}_w]_{:,\mathcal{T}}\, \boldsymbol{x}_{\mathcal{T}}[k],\ \forall\, k = 1, \ldots, N_c$

$\hat{\boldsymbol{h}}[k] = [\boldsymbol{\Psi}]_{:,\mathcal{T}}\, \boldsymbol{x}_{\mathcal{T}}[k]$ and $\hat{\boldsymbol{H}}[k] = \text{vec}^{-1}\left( \hat{\boldsymbol{h}}[k] \right),\ \forall\, k = 1, \ldots, N_c$

## 2.2 Optimized Value of Hyperparameter using Training Data

We use the grid sizes as $G_r = 4N_r$ and $G_t = 4N_t$. Note that the Algorithm 1 requires the hyperparameter $L$ that determines the sparsity of the unknown beam-space domain channel. The hyper-parameter $L$ is obtained by solving

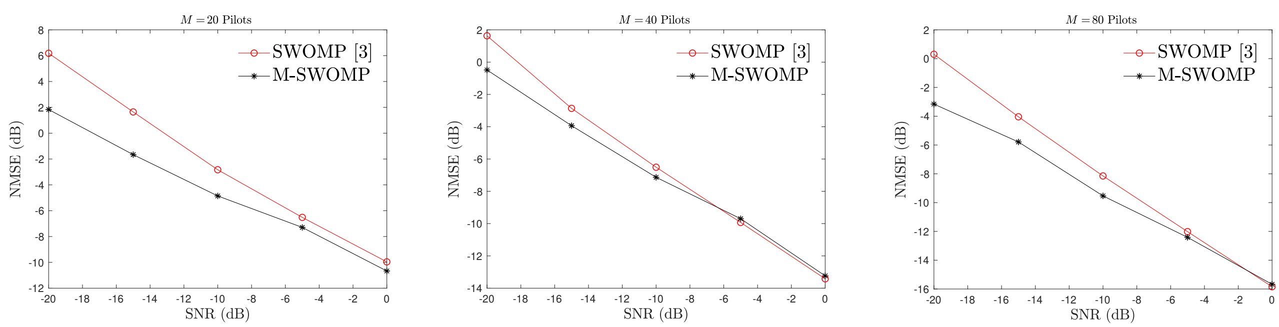
$$L^* = \arg\min_{L} \frac{1}{N_{\text{train}}} \left( \sum_{i=1}^{N_{\text{train}}} \frac{\sum_{k=1}^{N_c} \|\boldsymbol{H}^i[k] - \hat{\boldsymbol{H}}^i[k]\|_F^2}{\sum_{k=1}^{N_c} \|\boldsymbol{H}^i[k]\|_F^2} \right) \tag{3}$$

where $\boldsymbol{H}^i[k]$ is the true channel realization and $\hat{\boldsymbol{H}}^i[k]$ is the estimated channel by Algorithm 1 with hyper-parameter $L$ for the $k$-th sub-carrier and $i$-th sample in the training dataset of size $N_{\text{train}}$. After solving the problem in (3) for the three SNRs of the training data, the optimized value of $L$ is given by

$$L^* = \begin{cases} 0 & , \text{ for } \gamma_{\text{train}} = -15\text{dB } (-20\text{dB} \leq \gamma_{\text{test}} < -11\text{dB}) \\ 1 & , \text{ for } \gamma_{\text{train}} = -10\text{dB } (-11\text{dB} \leq \gamma_{\text{test}} < -6\text{dB}) \\ 3 & , \text{ for } \gamma_{\text{train}} = -5\text{dB } (-6\text{dB} \leq \gamma_{\text{test}} < 0\text{dB}) \end{cases} . \tag{4}$$

### 2.3 Performance Result on the Validation Dataset

In order to check the accuracy of our proposed solution, in this subsection we present the NMSE performance of our proposed Algorithm M-SWOMP and compare with the prior Algorithm SWOPM [3] on the validation dataset. The validation dataset (this is not used to solve (3)) is extracted from the provided dataset.




(a) $M = 20$ Pilots (b) $M = 40$ Pilots (c) $M = 80$ Pilots

Figure 1: The plots compare the NMSE performance of the proposed M-SWOMP algorithm and the previously proposed SWOMP algorithm in [3] on a validation dataset of size $N_{\text{val}} = 500$. Note that the SWOMP algorithm in [3] requires the knowledge of SNR whereas our M-SWOMP algorithm does not require the knowledge of SNR.

## References


[1] R. W. Heath, N. Gonzalez-Prelcic, S. Rangan, W. Roh, and A. M. Sayeed, "An overview of signal processing techniques for millimeter wave MIMO systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 3, pp. 436–453, 2016.

[2] J. P. González-Coma, J. Rodriguez-Fernandez, N. González-Prelcic, L. Castedo, and R. W. Heath, "Channel estimation and hybrid precoding for frequency selective multiuser mmWave MIMO systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 12, no. 2, pp. 353–367, 2018.

[3] J. Rodríguez-Fernández, N. González-Prelcic, K. Venugopal, and R. W. Heath, "Frequency-domain compressive channel estimation for frequency-selective hybrid millimeter wave MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 17, no. 5, pp. 2946–2960, 2018.