Strive
SCHOOL

/ Fundamental Concepts

Sample Space {S}

It is the set of possible worlds of probability. For example, a set of students in our class or a set of cards in our deck.

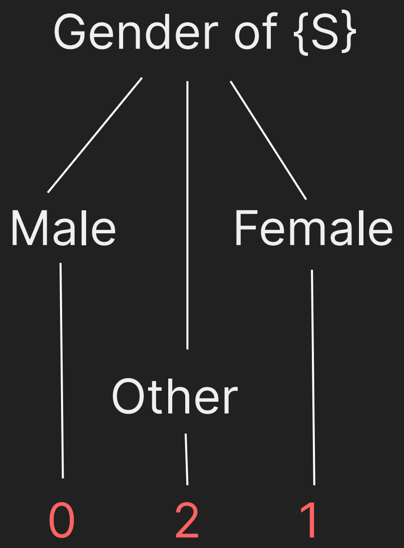
Once we are evaluating the set, we forget about the rest of probabilities that exist outside of it. {S=52 cards}

/ Random Variable {A}

It is a function defined over S. Ex: Gender of {S} → [M,F,O]

A random variable is a numerical description of the outcome of a statistical experiment.

A random variable that may assume only a finite number or an infinite sequence of values is said to be discrete; one that may assume any value in some interval on the real number line is said to be continuous.



Random Variable {A}

/ Discrete

> Countable

> Nothing in Between

> Digital (0-1)



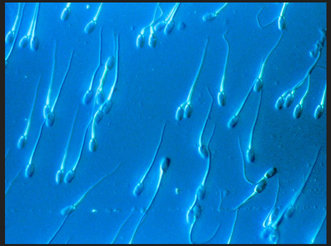

/ Continuous

> Infinite

> Always something in between

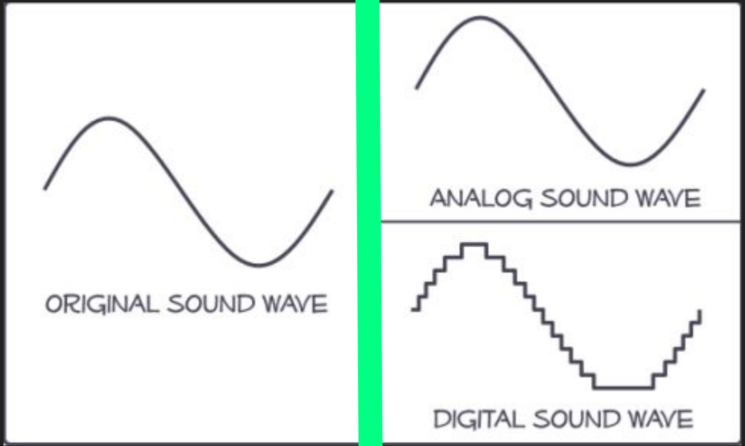
> Analog






Random Variable {A}

/ Digital



/ Analog

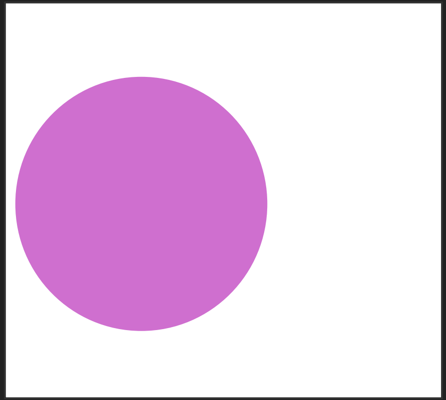
/ Event

An event is a set of outcomes of an experiment (a subset of the sample space) to which a probability is assigned.

A single outcome may be an element of many different events, and different events in an experiment are usually not equally likely, since they may include very different groups of outcomes.

An event defines a complementary event, namely the complementary set (the event not occurring).

Gender of {S}



/ Event

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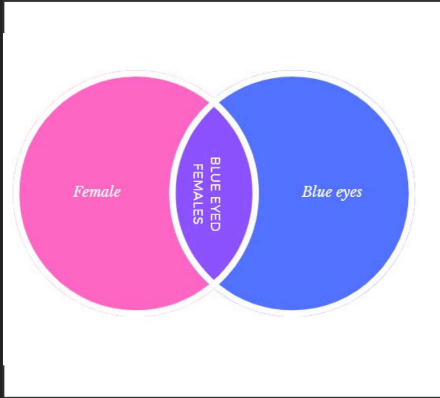
Traits of Europeans {S}



/ Probability P(A)

Our estimate or probability that a specific event takes place. In an informal way, it is the fraction of possible worlds in which A is true.

We are often interested in probabilities of specific events or from events that are conditioned on another event taking place.

Gender of {S:All humans}

Male

P(M)=0.501

Female

P(F)= 0.499

/ Probability P(A)

Our estimate or probability that a specific event takes place. In an informal way, it is the fraction of possible worlds in which A is true.

We are often interested in probabilities of specific events or from events that are conditioned on another event taking place.

Gender of {S:+75 humans}

Male

Female

P(M)=0 .36

P(F)= 0.64

/ Axioms of Probability

$$0 \leq P(A) \leq 1$$

$$P(True) = 1$$

$$P(False) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

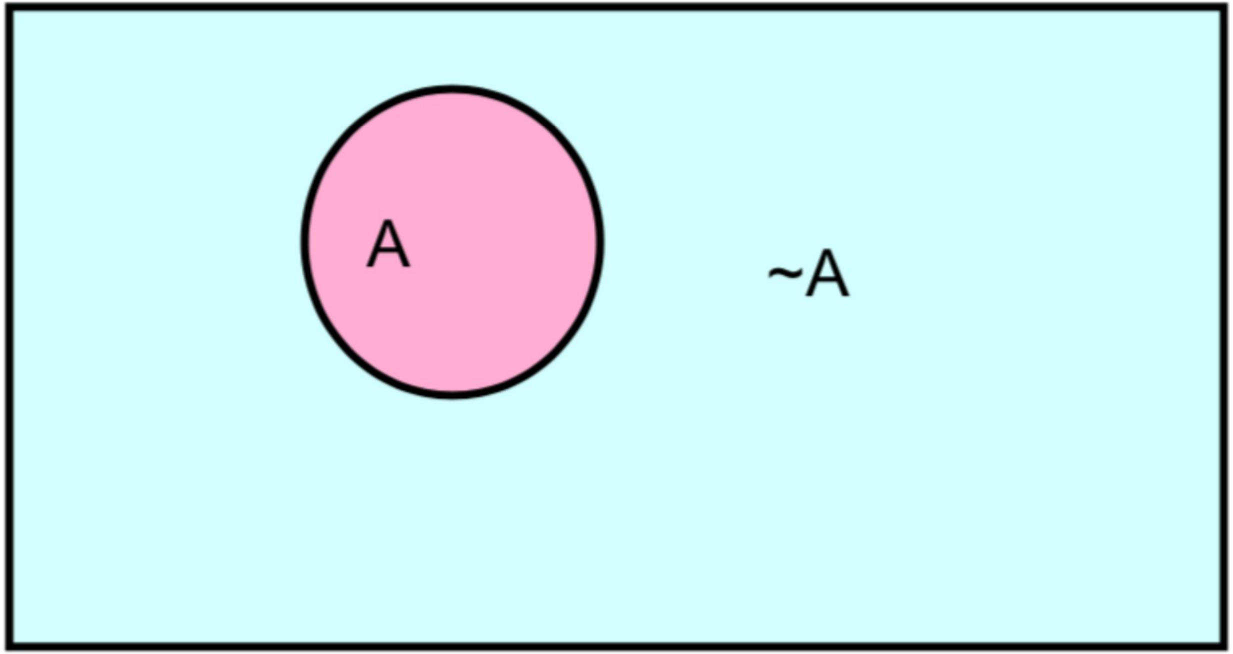
Gender of {S:+75 humans}

Male | Female

P(M)=0 .36 | P(F)= 0.64

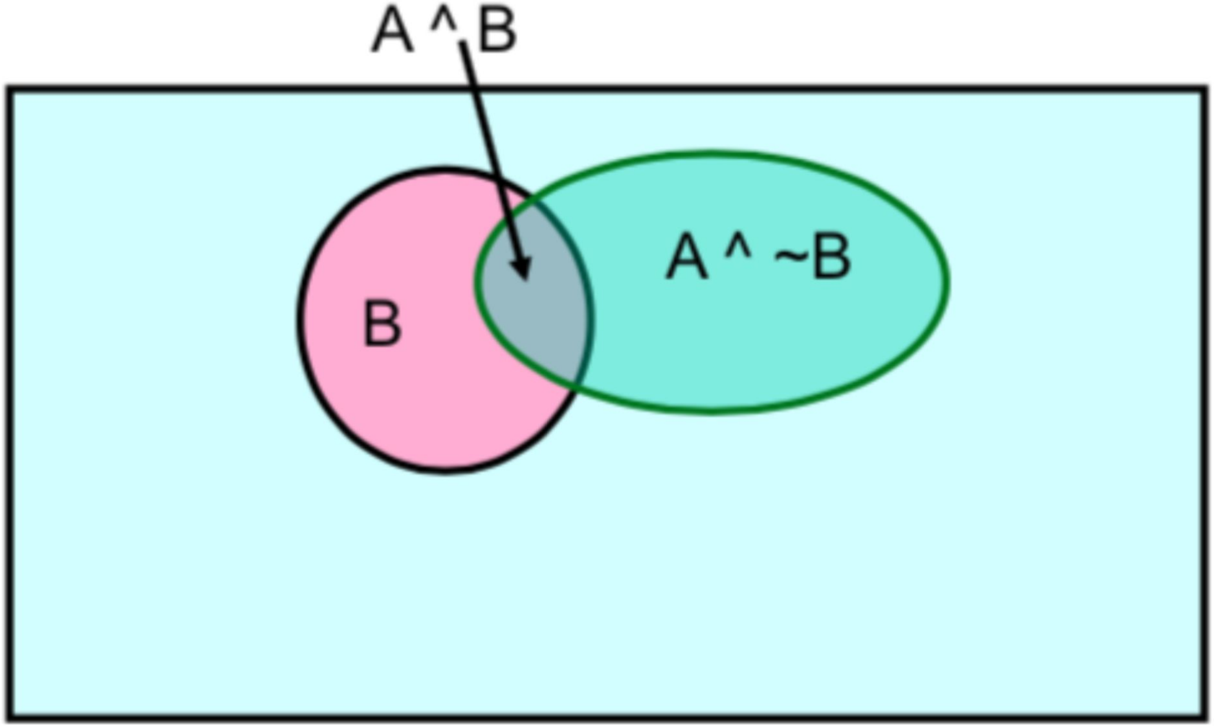
/ Visual Probability

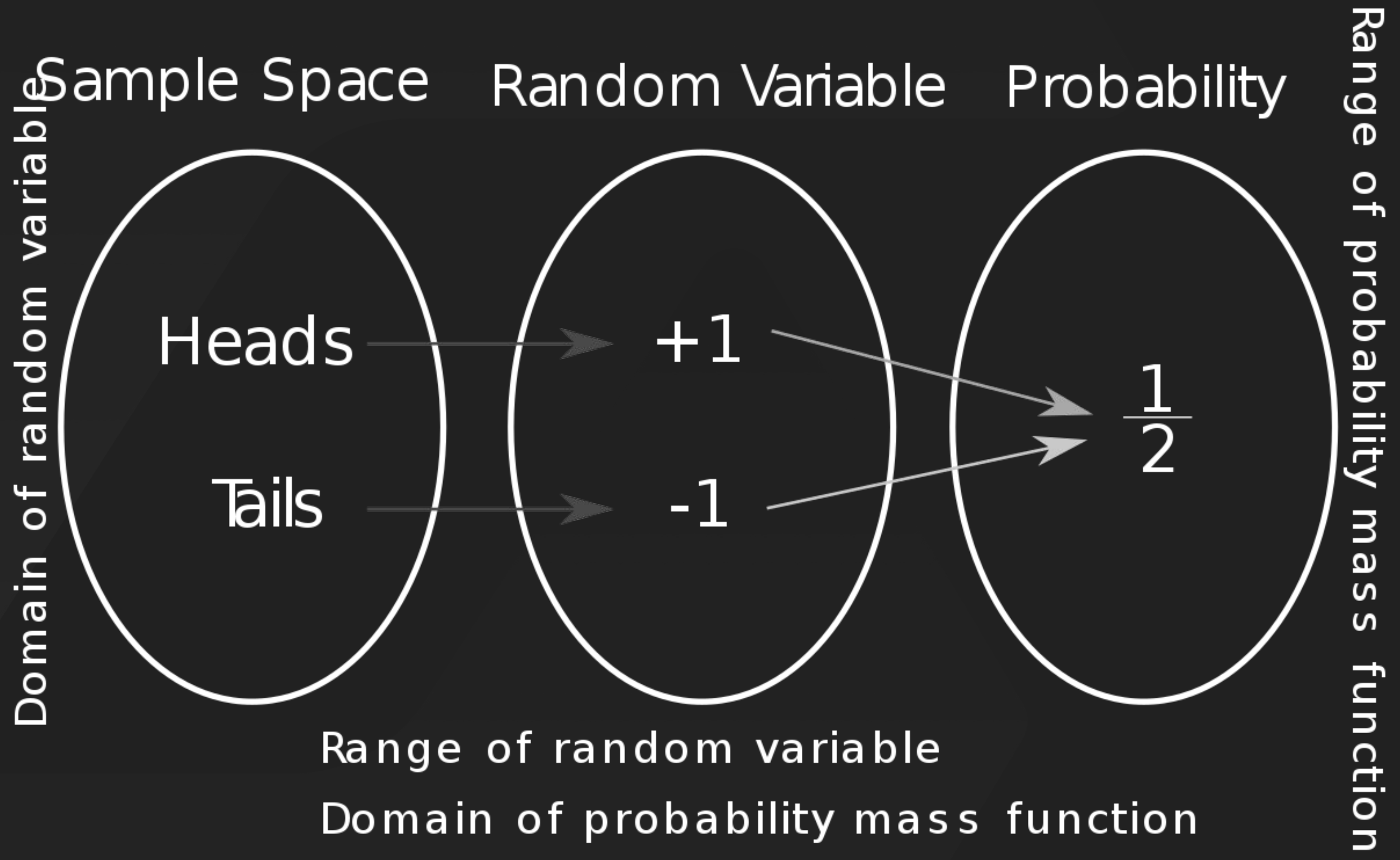
$$P(\neg A) + P(A) = 1$$



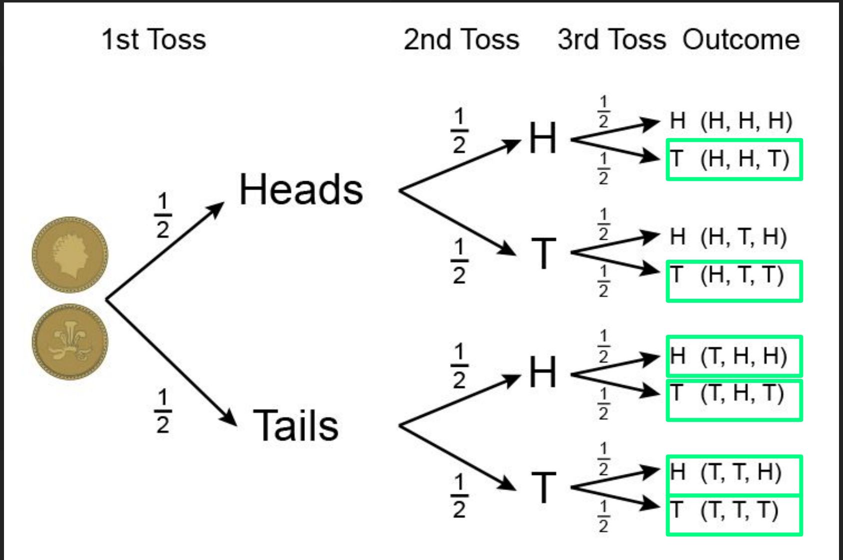
/ Visual Probability

$$P(A) = P(A \wedge B) + P(A \wedge \neg B)$$

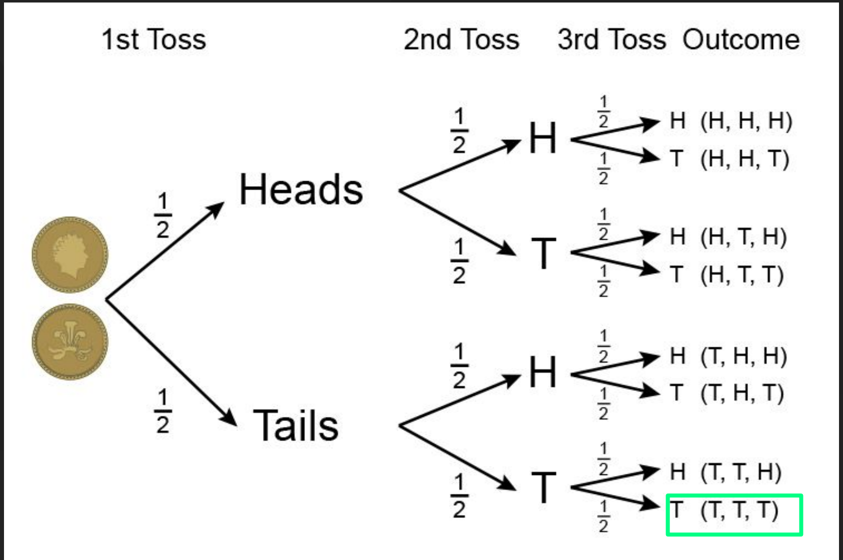



Sample Space
Random Variable
Probability
Domain of random variable
Range of probability mass function
Heads
Tails
+1
-1
1/2
Range of random variable
Domain of probability mass function

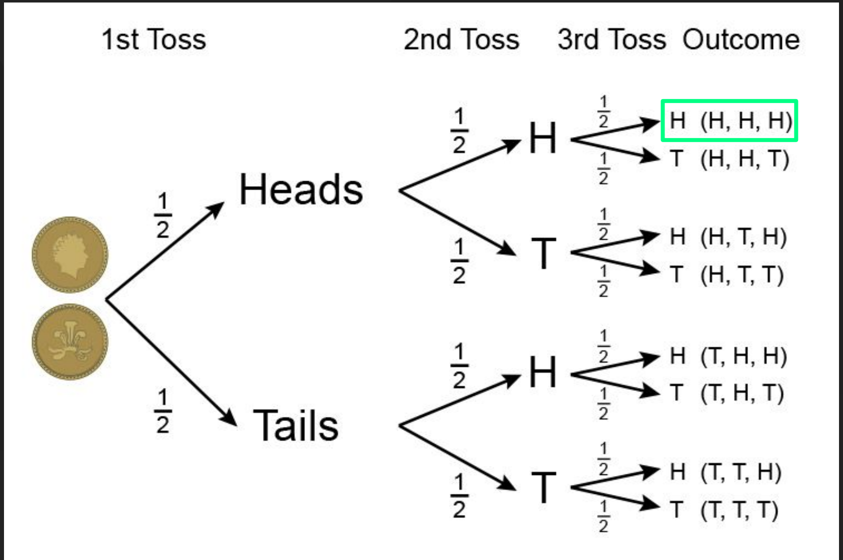
/ Probability of any Tails P(A)



/ Probability of all Tails P(A)



/ P(A) of a spec. combination



$$\underbrace{\left(\frac{1}{2}\right)^{10}}_{\substack{\textit{probability of} \\ \textit{ten heads in a row}}} \cdot \underbrace{\left(\frac{1}{2}\right)}_{\textit{followed by a tail}} = 0.0005$$

Can I offer you a bargain?

/ One coin virtual toss simulation.

Can I offer you a bargain?

/ We randomly pick someone in the class → What is the probability?
/ One coin virtual toss.

If you win, I donate $50 in Kiva.org to whoever you want.
If I win, you donate $25 to someone in Kiva.org that the class chooses.

/ If the person doesn't want to take the bet, we remove them and roll again. What are the updated probabilities?

/ Once we've done this once, same bet stands, but it is 5 tosses instead of 1.
If you win, I donate $100 in Kiva.org to whoever you want.
If I win, you donate $25 to someone in Kiva.org that the class chooses.

Can I offer you a bargain?

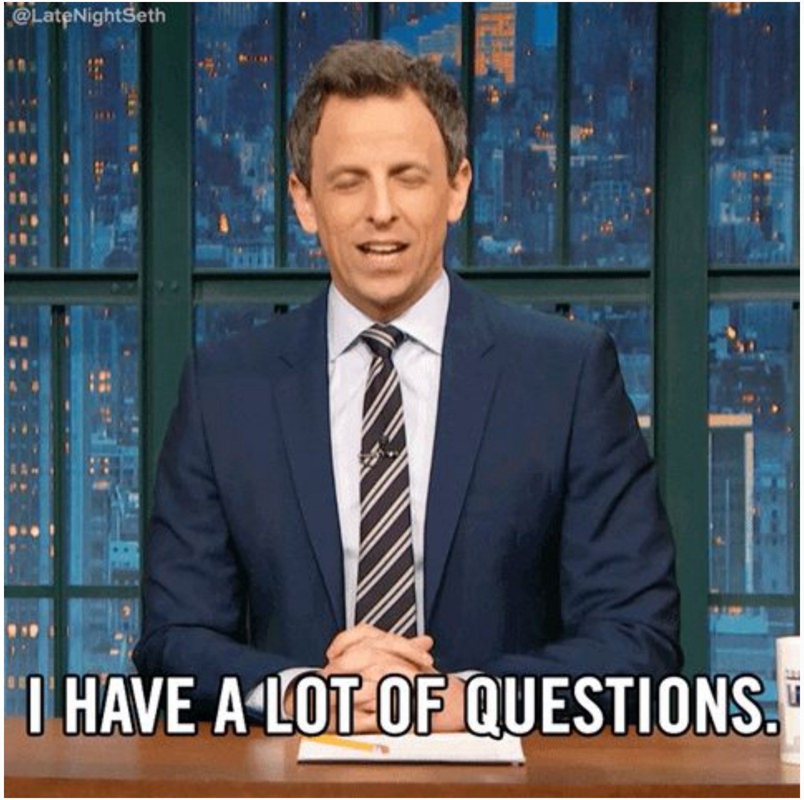
/ We randomly pick someone in the class → What is the probability?

/ Now, we get 4 rolls. I get to pick only one specific outcome.

There are 15 remaining outcomes.
If anyone of you wins, I donate $50 to someone in Kiva.org of your choice.
If I win, you must pledge to donate $25 (between all of you).
If we roll an outcome that no-one picked, we re-do the toss

/ What is my individual probability of winning?
/ What is your individual probability of winning?
/ What is your probability of winning as a class?

Any questions so far?



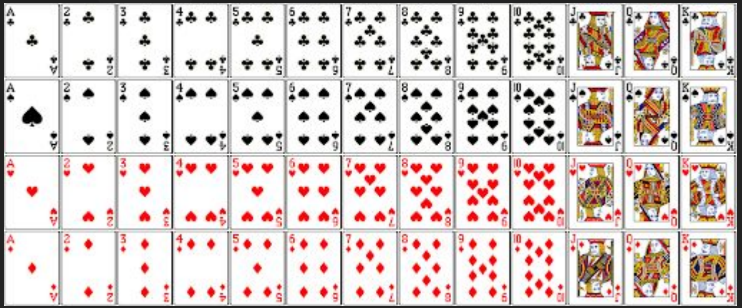
Marginal Probability

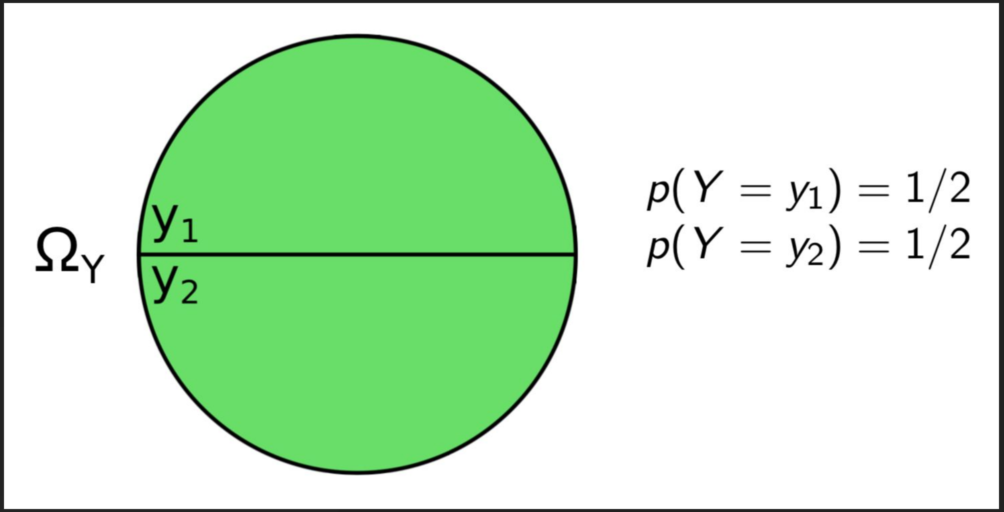
/ The marginal probability is the probability that a given event P(A) occurs. It is an unconditional probability, given that it is not tied to any other event.

- In a deck of 52 cards, the probability of pulling a red one is:

$$P(\,red) = P(\,hearts) + P(\,diamonds)$$

$$P(\,red) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2} = 0.5$$





Our Axioms must hold!!

/ Independently of the circumstances:

the sum of all the probabilities must be 1.

$$P(\,red) = P(\,hearts) + P(\,diamonds)$$

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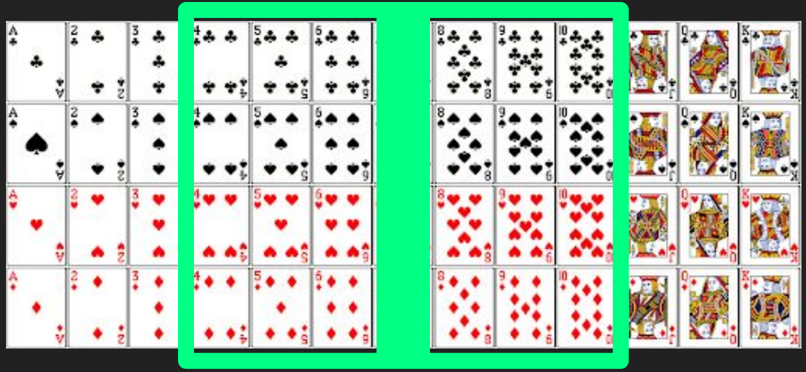

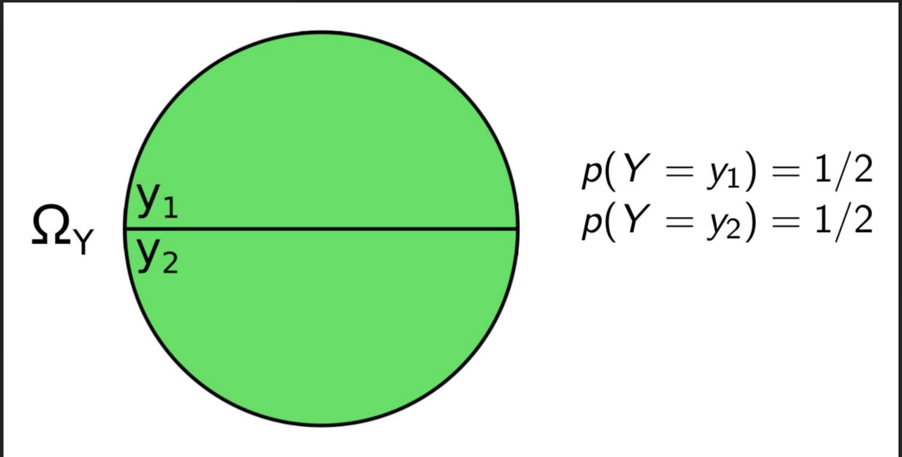

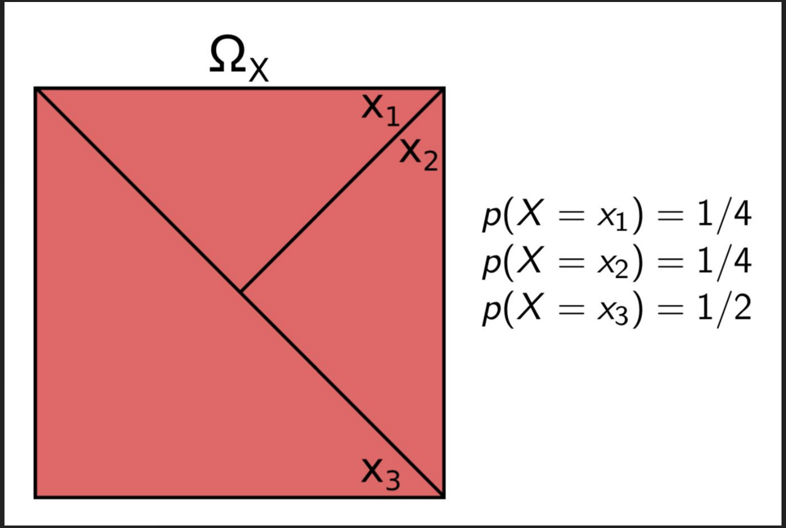

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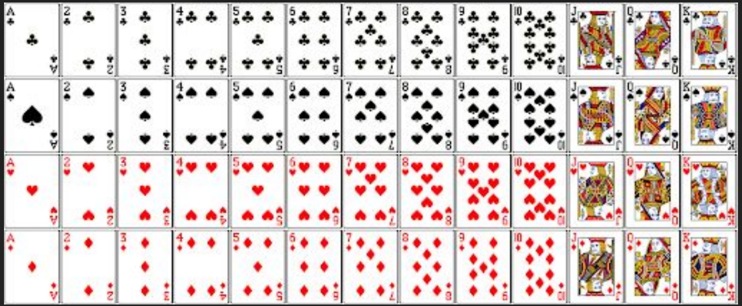
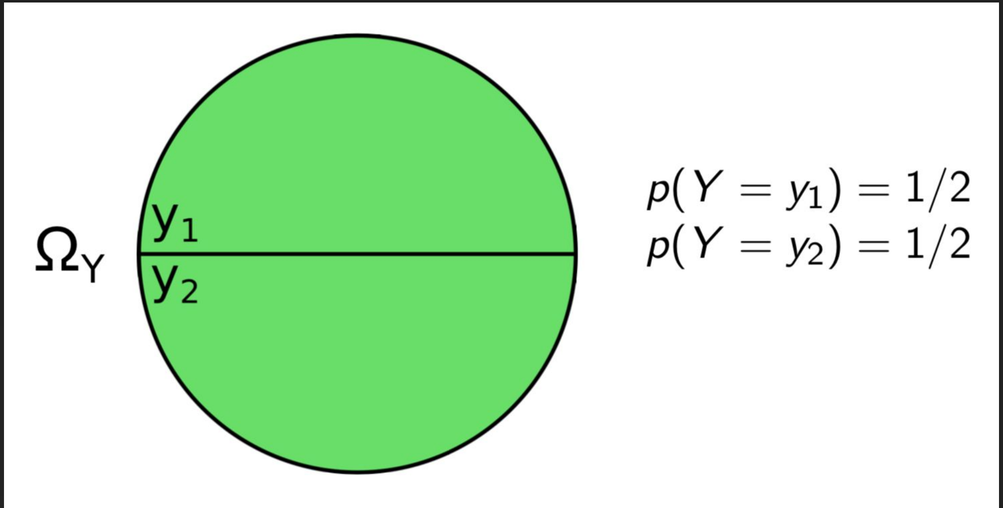


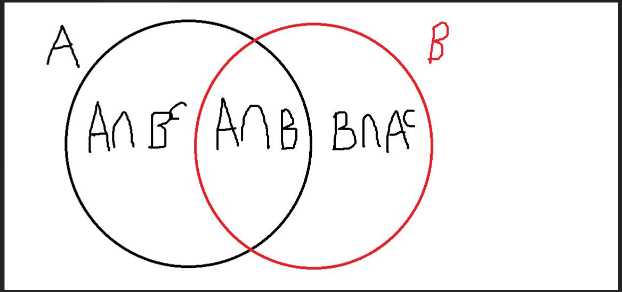


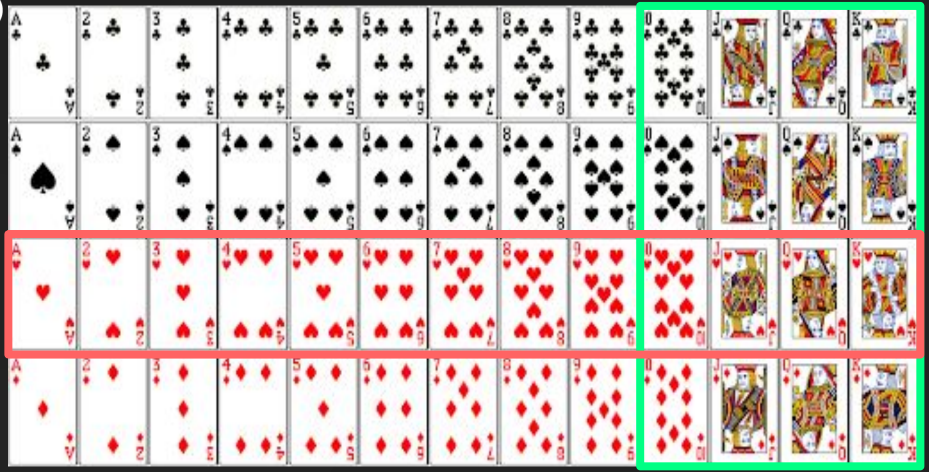
Joint Probability

/ The joint probability is the probability of one or several events occurring at the same time (eg: A **and** B). Thus, itt is the probability of the intersection of two or more events. **P(A and B) = P(A)*P(B)**

$$P(\,high\, \cap diamonds) \;=\; P(\,high) \cdot P(\,diamonds)$$

$$P(\,high\, \cap diamonds) \;=\; \frac{4}{13} \cdot \frac{13}{52} = \frac{4}{52}$$

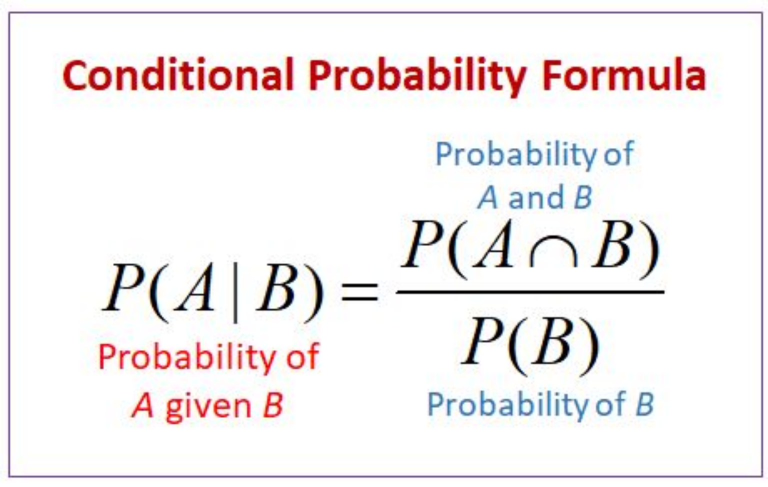


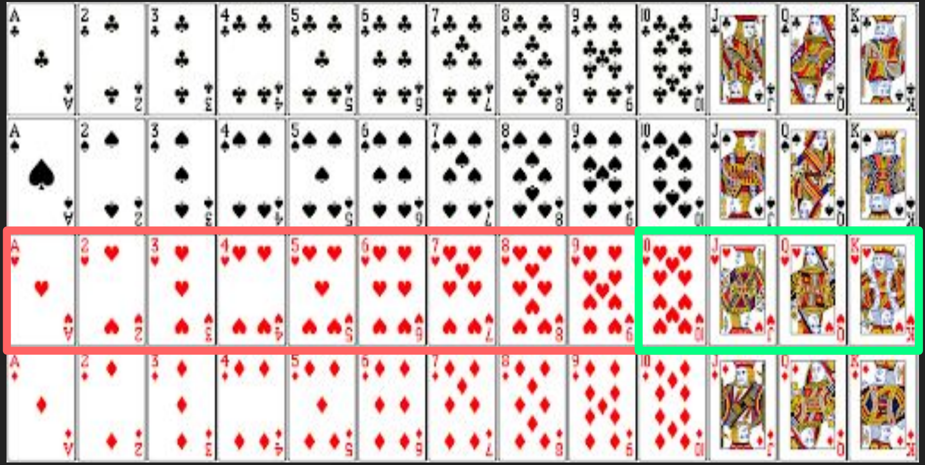


Conditional Probability

/ The conditional probability P(A|B) is the probability of the event A occurring, given than the event B has taken place.

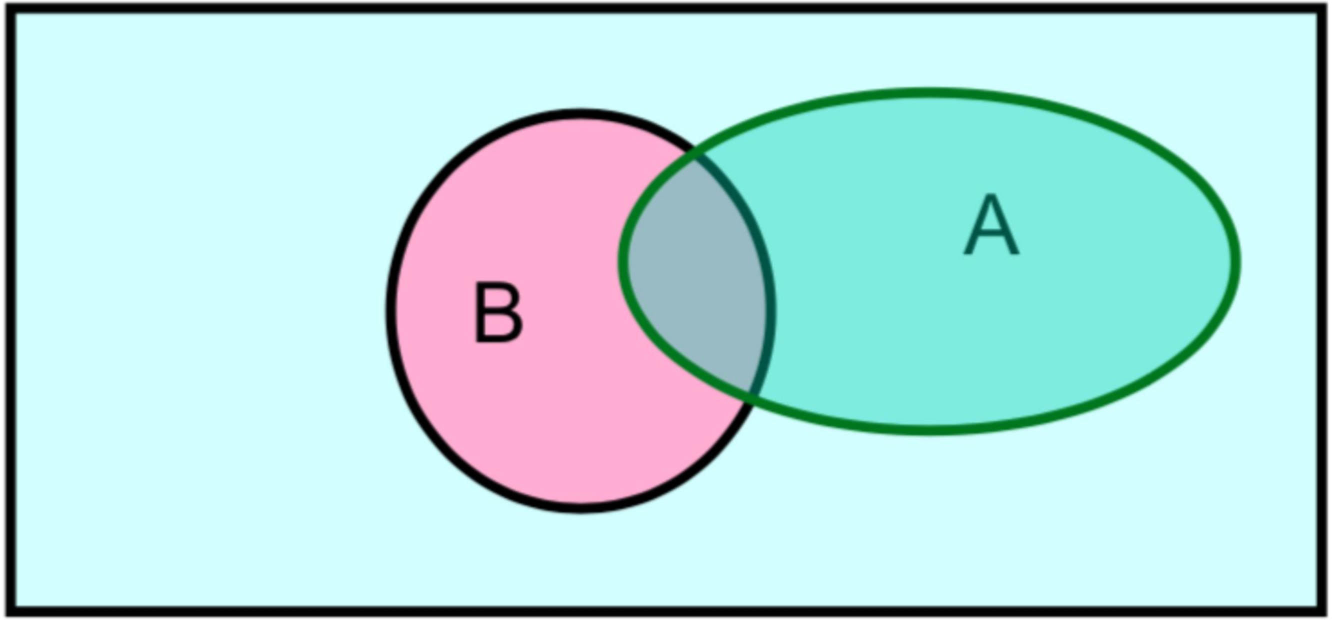
$$P(\,high \mid diamonds\,) = \frac{P(\,high \cap Diamonds\,)}{P(\,Diamonds\,)} = \frac{4/52}{13/52} = \frac{4}{52} \times \frac{52}{13} = \frac{4}{13}$$





Conditional Probability

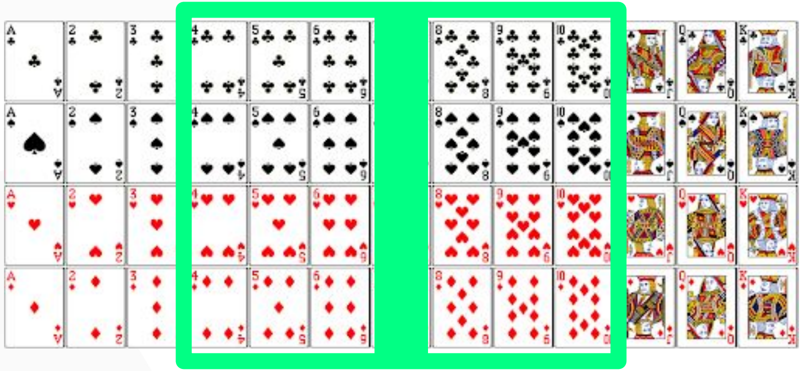
$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

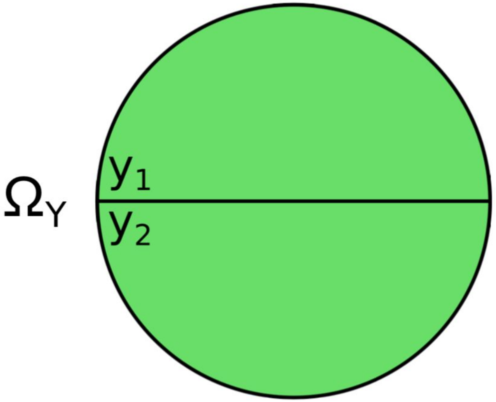


Conditional Probability

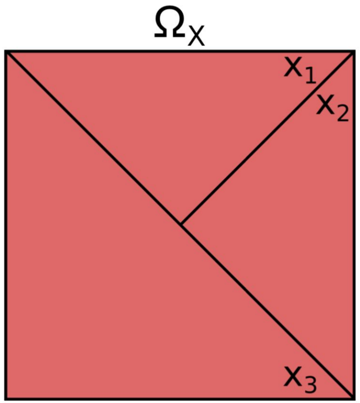
$$P(\,red) = P(\,hearts) + P(\,diamonds)$$

$$P(\,red) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2} = 0.5$$





$$p(Y = y_1) = 1/2$$
$$p(Y = y_2) = 1/2$$



$$p(X = x_1) = 1/4$$
$$p(X = x_2) = 1/4$$
$$p(X = x_3) = 1/2$$

Chain Rule (yes, also in probability!)

/ Given two random events A and B:

$$P(A \wedge B) = P(A \mid B) \cdot P(B)$$

Chain rule

For all $\boldsymbol{x}$ we have that

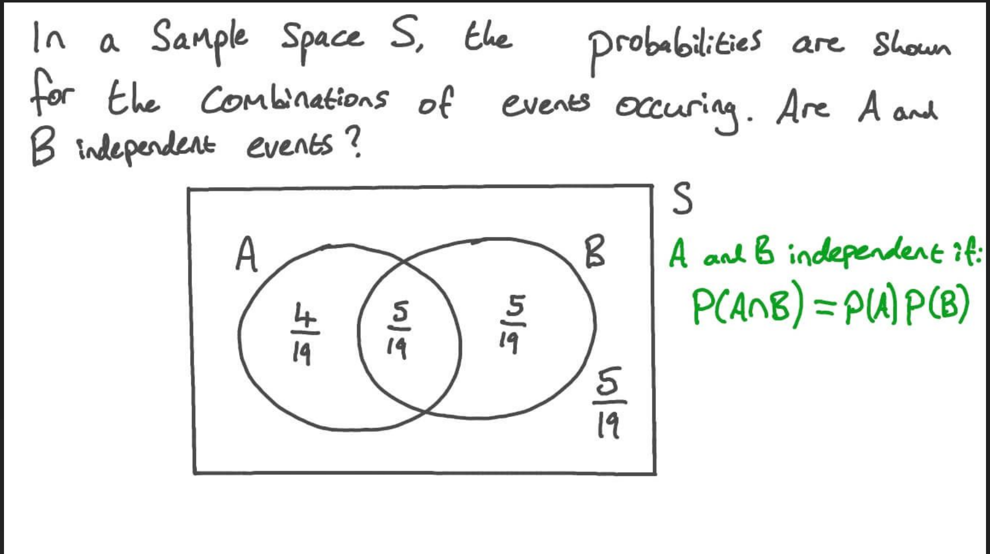
$$p(\boldsymbol{x}) = p(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} p(x_i | x_1, \ldots, x_{i-1})$$

** it holds for any ordering of $X_1, \ldots, X_n$ **

Chain Rule (yes, also in probability!)

/ Given two independent random events A and B, the outcome of A does not condition the outcome of B and vice versa. Thus: P(A and B) = P(A) * P(B)

Intuitively, one event does not hurt the odds of the other.



Examples of independent Events

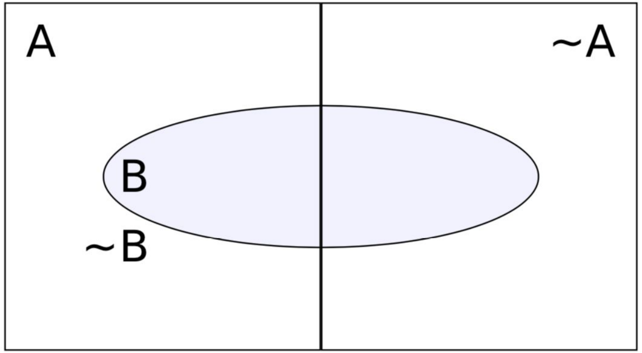
Definition

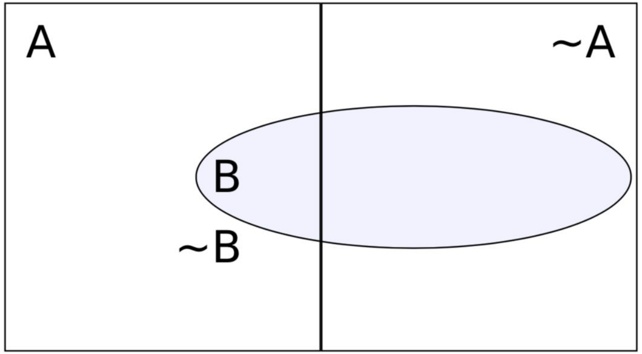
Two events A and B are independent if

$$P(A \wedge B) = P(A) * P(B)$$

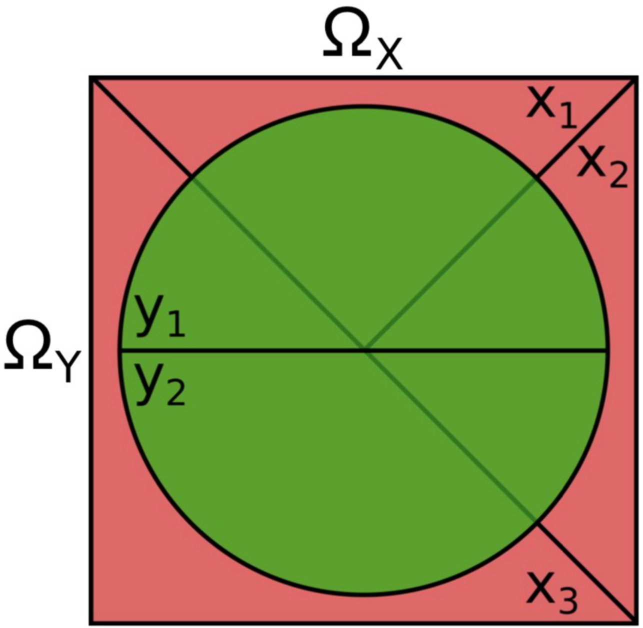
** Intuition **

Knowing A tells us nothing about the value of B (and vice versa)





Conditional Probability



$$p(Y = red \mid X = low) =$$

$$p(Y = black \mid X = low) =$$

$$p(Y = red \mid X = medium) =$$

$$p(Y = black \mid X = medium) =$$

$$p(Y = red \mid X = high) =$$

$$p(Y = black \mid X = high) =$$

Permutations

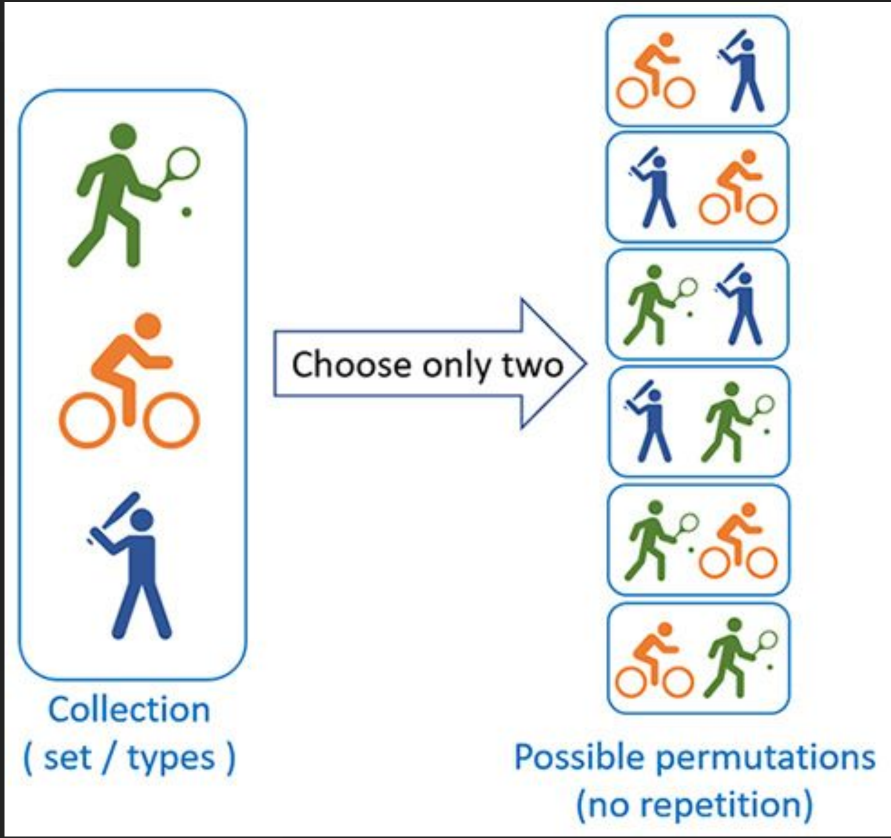
/ There are several ways in which we could select *r* sports our of *n* possibilities.

/ Repetitions are not allowed

/ Order matters

/ Clues: arrangement, schedule, order

$$P(n, r) = \frac{n!}{(n-r)!}$$



Combinations

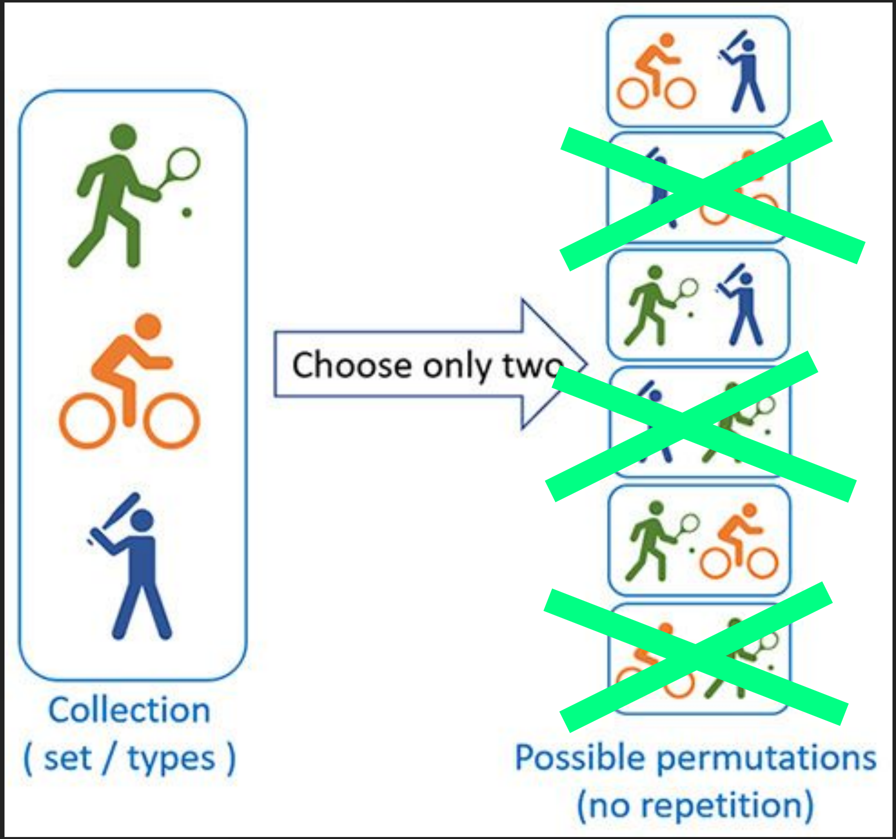
/ There are several ways in which we could select *r* sports our of *n* possibilities.

/ Repetitions are not allowed

/ Order **does not matter**

/ Clues: group, sample, selection

$$C(n,r) = \frac{n!}{r!(n-r)!}$$



/ Q&A

What are your doubts?

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