CSCI 3302 HW1

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1. There is only one mechanical degrees of freedom

   environmental degrees of freedom : x, y, yaw

   Because we can force the different side of wheels to make it change direction.

2. x, y, Yaw

3. (a) $(\cos 45, -\sin 45, 0)^T$ and $(\sin 45, \cos 45, 0)^T$

$$\cos\theta = \frac{(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0) \cdot (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)}{\sqrt{\frac{1}{2}+\frac{1}{2}} \cdot \sqrt{\frac{1}{2}+\frac{1}{2}}} = 0$$

So $\theta = 90^0$

(b) $(0, 0, 1)$

4. (a) ${}^A_B R = \begin{bmatrix} X_B X_A & Y_B X_A & Z_B X_A \\ X_B Y_A & Y_B Y_A & Z_B Y_A \\ X_B Z_A & Y_B Z_A & Z_B Z_A \end{bmatrix}$

   (b) $X^B = \begin{bmatrix} Y_B X_A \\ Y_B Y_A \\ Y_B Z_A \end{bmatrix} + {}^A P$

   (c) ${}^B_A R$ is the inverse of ${}^A_B R$. $\begin{bmatrix} X_B X_A & Y_B Y_A & X_B Z_A \\ Y_B X_A & Y_B Y_A & Y_B Z_A \\ Z_B X_A & Z_B Y_A & Z_B Z_A \end{bmatrix}$

5. $\begin{pmatrix} x \\ y \\ \phi \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \omega r \\ 0 \\ \frac{\omega r}{l} \end{bmatrix}$

6. ${}^A Q = \left[\begin{array}{c|c} {}^A_B R & {}^A P \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \begin{bmatrix} {}^B Q \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos(135) & -\sin 135 & 0 & 6 \\ \cos 45 & \cos 135 & 0 & 10 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ -4 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 2\sqrt{2} \\ 10 + 6\sqrt{2} \\ 9 \\ 1 \end{bmatrix}$$

cited: some ideas come from my friend XinYang Yuan