# Data Structures and Algorithms Spring 2023 — Problem Sets

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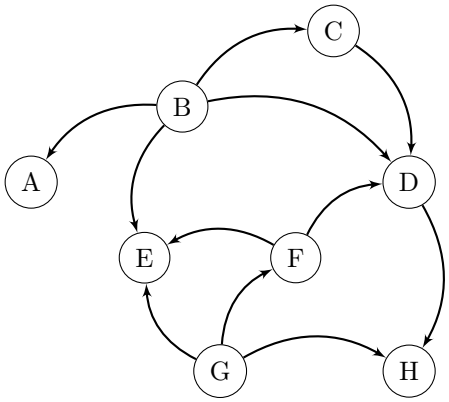
## Week 11. Problem set

1. Write down all possible topological sortings for the nodes of the following directed graph:




**Answer:**

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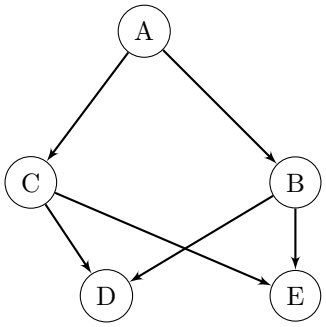
2. Give an example of a directed graph $G = (V, E)$, a source vertex $s$, and a set of edges $T \subseteq E$ such that

- $T$ forms a tree and

- for each vertex $v \in V$, the unique simple path in the graph $(V, T)$ from $s$ to $v$ is a shortest path in $G$, yet
- the set of edges $T$ cannot be produced by running BFS on $G$, no matter how the vertices are ordered in the adjacency lists.

**Answer:**

Graph $G$:




Set of edges $T \subseteq E$: