Data Structures and Algorithms Spring 2024 — Problem Sets

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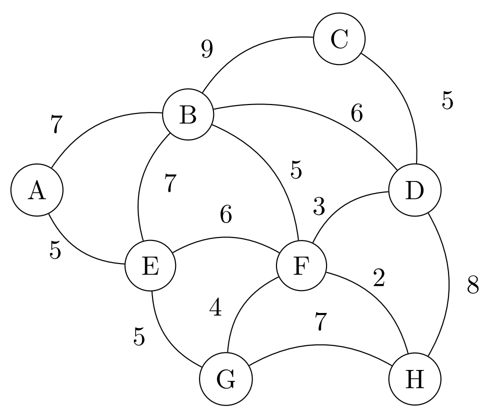
Week 12. Problem set

1. Run Prim-Jarník algorithm [Cormen, §21.2] on the following graph, starting at vertex C. Assuming that the algorithm is using Binary heap implementation [Cormen, §6] of a priority queue, show the state of the Binary heap after each iteration of the algorithm (i.e. after adding each new vertex to the MST). The graph contains 8 vertices, which means that your solution must provide 8 states of the Binary heap. Each heap state must be represented as an array. No justification is required.



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Answer (on next page):

Current vertex	Possible vertices	Weights
C	B, D	9, 5
D	B, F, H	6, 3, 8
F	B, E, G, H	5, 6, 4, 2
H	B, E, G	5, 6, 4
G	B, E	5, 5
B	A, E	7, 5
E	A	5
A		

2. Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Kruskal's algorithm run by modifying it somehow? What if the edge weights are integers in the range from 1 to W for some constant W? Justify your answer in at most two paragraphs.

   Answer:
   Kruskal's algorithm sorts edges in nondecreasing order by weight. If the edge weights are integers in the range 1 to V, we can use Counting-Sort to sort the edges in $(V + E)$ time. Then Kruskal's algorithm will run in $O(V+E+V\log V) = O(E+V\log V)$ time. If the edge weights are integers in the range from 1 to W for some constant W, we can use Counting-Sort to sort the edges in $(W + E)$ time and Kruskal's algorithm will run in $O(W + E + V\log V)$ time.

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. Introduction to Algorithms, Fourth Edition. The MIT Press 2022