Dynamics of Parallel Robots

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Synonyms

Dynamic modelling of Parallel Kinematic Manipulators

Definition

A parallel robot is a closed-loop multi-body system controlling the motion of its end-effector (moving platform) by means of parallel kinematic chains going from its base to the end-effector. The most important problems in the dynamics study are the calculation of the inverse and direct dynamic models.

Extended Definition

The Inverse Dynamic Model (IDM) is used in the control applications, it calculates the input joint efforts (torques and forces) to achieve a set of prescribed joint accelerations. The Direct Dynamic Model (DDM) is used in simulation applications, it calculates the joint accelerations resulting from a set of input joint efforts.

nghịch

thuận

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Theory & Application

1 Introduction

Dynamic modelling is essential for design specifications and advanced control of parallel robots. Many works have been devoted for this topics using different mechanical formalisms. For example Lee and Shah (1988), Geng *et al.* (1992), Lebret *et al.* (1993), Bhattacharya *et al.* (1998), Liu *et al.* (2000), Miller (2004) and Abdellatif and Heimann (2009) used Lagrange-Euler formalism. The principle of virtual work has been used by Codourey and Burdet (1997) and Tsai (2000). On the other hand, Newton-Euler equations have been used by Reboulet and Berthomieu (1991), Ji (1993), Gosselin (1993), Dasgupta and Choudhury (1999). However, recently, Fu *et al.* (2007), Vakil *et al.* (2008), Carricato and Gosselin (2009) and Afroun *et al.* (2012) have pointed out common errors in many methods related to the kinematic behavior of the legs. These errors may cause kinematic and dynamic miscalculation.

The aim of this article is to present a systematic procedure that provides the full dynamics of any parallel robot, taking into account the whole dynamics of the legs and the platform. This article is based on the works (Briot and Arakelian, 2008; Briot and Khalil, 2015; Khalil and Guegan, 2004; Khalil and Ibrahim, 2007).

As an application in this article, the proposed method is used to calculate the IDM of the Gough-Stewart (GS) robot.

tính toán nghịch của Stewart

2 Inverse dynamic modeling of parallel robots

In what follows, the number of legs is denoted by m and the number of degrees of freedom (dof) of the platform is denoted by n. This paper deals with non-redundant rigid-link robots. Thus the number of active joints is also equal to n. Each leg is considered to be made with a serial architecture (for dealing with more complex legs or with redundant robots, the reader is referred to (Briot and Khalil, 2015)). The frame $\mathscr{F}_p$, with origin O_p, is defined to be fixed to the platform and the frame $\mathscr{F}_0$, with origin O_0, to the base. The state of the manipulator (position and velocity) can be described using either

- $\mathbf{q}_a$, $\dot{\mathbf{q}}_a$: vectors of active joint positions and velocities, respectively, or
- ${}^0\mathbf{T}_P$, $\mathbb{V}_P$: the homogeneous transformation matrix of the frame $\mathscr{F}_p$ into $\mathscr{F}_0$, and the platform twist, respectively. $\mathbb{V}_P$ groups the frame $\mathscr{F}_p$ rotational velocity $\boldsymbol{\omega}_P$ and the translational velocity of its origin $\mathbf{v}_P$, i.e. $\mathbb{V}_P = [\mathbf{v}_P^T \ \boldsymbol{\omega}_P^T]^T$. Usually these quantities are expressed either in the world frame or the platform frame.

Note that, for robots whose number of dof is lower than six ($n < 6$), it is sufficient to describe the platform velocity by using n independent coordinates of the vector $\mathbb{V}_P$ grouped into a vector $\mathbb{V}_r$, defined by

$$\mathbb{V}_P = \boldsymbol{\Psi} \mathbb{V}_r \tag{1}$$

where $\boldsymbol{\Psi}$ is a $(6 \times n)$ matrix typically composed of 0 and 1 only.

Usually, the IDM is defined as calculating, at a given state, the input efforts (torques or forces) of the actuated joints, denoted as $\boldsymbol{\tau}_a$, corresponding to given actuated joint accelerations $\ddot{\mathbf{q}}_a$. In other words, it computes the function:

$$\boldsymbol{\tau}_a = \mathbf{idm_q}(\mathbf{q}_a, \dot{\mathbf{q}}_a, \ddot{\mathbf{q}}_a) \tag{2}$$

For parallel robots, it may be more convenient to compute the IDM as a function of a desired accelerations of the platform, denoted as $\dot{\mathbb{V}}_P$:

$$\boldsymbol{\tau}_a = \mathbf{idm}({}^0\mathbf{T}_P, \mathbb{V}_P, \dot{\mathbb{V}}_P) \tag{3}$$

sẽ không được cover

Here, we assume that there is no external wrench applied on the robot. This is out of the scope of this article. For introducing them into the models, the reader is referred to (Briot and Khalil, 2015).

Classically the IDM of a closed-loop mechanical system can be computed by first calculating the IDM of a (virtual) tree structure obtained by opening all the closed loops and virtually actuating all joints. Then the input joint efforts of the real system are obtained by taking into account the loop-closure constraint equations (Featherstone and Orin, 2016; Khalil and Dombre, 2002).

More explicitly explained

To exploit the special structure of parallel robots, the solution proposed in this article is based on virtually separating the platform from the legs in order to obtain two sub-systems: (i) the free platform and (ii) a tree structure composed of the base and the legs in which all joints are considered to be virtually actuated. The dynamics of the platform is computed using the Newton-Euler equation in terms of its Cartesian (operational) coordinates $({}^0\mathbf{T}_P, \mathbb{V}_P, \dot{\mathbb{V}}_P)$, whereas the dynamics of the legs is expressed in terms of the joint coordinates of the legs denoted as $(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)$ $(i = 1, \ldots, m)$. Then, the joint efforts of the real system are obtained using the (geometric and kinematic) loop-closure equations and the principle of virtual powers or Lagrange equations with multipliers.

The relations between all parallel robot coordinates are found by writing the closed-loop kinematic constraints equations on both sides of the opened joints connecting the platform with the legs, by using the following relations:

$$\mathbb{V}_i = \mathbf{J}_i \, \dot{\mathbf{q}}_i \tag{4}$$

$$\mathbb{V}_i = \mathbf{J}_{vi} \, \mathbb{V}_P = \mathbf{J}_{vi} \, \boldsymbol{\Psi} \, \mathbb{V}_r \tag{5}$$

$$\mathbb{V}_r = \mathbf{J}_r \, \dot{\mathbf{q}}_a \tag{6}$$

where

- $\mathbb{V}_i$ is the reduced twist of the frame attached to the last link of the leg i and it represents the velocity components transmitted from the leg to the platform (this leg being composed of n_i joints, $\mathbb{V}_i$ is of size n_i),
- $\mathbf{J}_i$ is the $(n_i \times n_i)$ leg i kinematic Jacobian matrix,

- $\mathbf{J}_{vi}$ is the $(n_i \times 6)$ kinematic Jacobian matrix linking the reduced twist $\mathbb{V}_i$ to the platform twist $\mathbb{V}_P$ through the rigid body velocity relation,
- $\mathbf{J}_r$ is the $(n \times n)$ robot kinematic Jacobian matrix.

Introducing (1) into (6) leads to:

$$\mathbb{V}_P = \boldsymbol{\Psi}\, \mathbf{J}_r\, \dot{\mathbf{q}}_a = \mathbf{J}_P\, \dot{\mathbf{q}}_a \tag{7}$$

while introducing (5) and (6) into (4) brings

$$\dot{\mathbf{q}}_i = \mathbf{J}_i^{-1}\, \mathbf{J}_{vi}\, \boldsymbol{\Psi}\, \mathbf{J}_r\, \dot{\mathbf{q}}_a = \mathbf{J}_i^{-1}\, \mathbf{J}_{vi}\, \mathbf{J}_P\, \dot{\mathbf{q}}_a = \mathbf{G}_i\, \dot{\mathbf{q}}_a \tag{8}$$

It is to be noted that for most parallel robots the calculation of the inverse of $\mathbf{J}_r$, denoted by $\mathbf{J}_r^{-1}$, is easy to obtain symbolically, while $\mathbf{J}_r$ is obtained numerically by inverting $\mathbf{J}_r^{-1}$ (Briot and Khalil, 2015; Tsai, 2000). In the following, the matrix $\mathbf{J}_r^{-1}$ is denoted as $\mathbf{J}_{inv}$. Consequently $\mathbf{J}_r$ is equal to $\mathbf{J}_{inv}^{-1}$.

Using the principle of virtual powers, or also the Lagrange equations with multipliers, the dynamics of the platform can be projected on the active joint space by multiplying it by the transpose of the robot Jacobian matrix $\mathbf{J}_P$. Similarly, in order to project the legs dynamics on the active joint space, it is necessary to use the Jacobian between these two spaces, i.e. the matrix $\mathbf{G}_i$. Thus the dynamic model of the parallel structure is given by the following equation:

$$\boldsymbol{\tau}_a = \mathbf{J}_P^T\, \mathbf{w}_P + \sum_{i=1}^{m} \mathbf{G}_i^T\, \boldsymbol{\tau}_i \tag{9}$$

with:

- $\mathbf{w}_P$ is the total wrench on the free platform,
- $\boldsymbol{\tau}_i = \mathbf{idm_q}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)$ is the IDM of leg i considered virtually fully actuated and separated from the robot platform.

Lưu ý:
a - là chỉ input vào UpperLeg
i - là chỉ torque / position ảo tại điểm trên của LowerLeg

=> Giải nghĩa cho Fully Actuated

The platform wrench (forces and moments) $\mathbf{w}_P$ can be calculated using Newton-Euler equation (Khalil and Dombre, 2002):

$$\mathbf{w}_P = \bar{\mathbf{I}}_P \begin{bmatrix} \dot{\mathbf{v}}_P - \mathbf{g} \\ \dot{\boldsymbol{\omega}}_P \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}_P \times (\boldsymbol{\omega}_P \times \mathbf{ms}_P) \\ \boldsymbol{\omega}_P \times (\mathbf{I}_P\, \boldsymbol{\omega}_P) \end{bmatrix} \tag{10}$$

where:

- $\dot{\mathbf{v}}_P$ is the translational acceleration of the origin of the frame $\mathscr{F}_P$ and $\dot{\boldsymbol{\omega}}_P$ is the rotational acceleration of this frame,
- $\mathbf{g}$ is the acceleration of gravity,
- $\bar{\mathbf{I}}_P$ is the (6×6) generalized inertia matrix of the platform:

$$\bar{\mathbf{I}}_P = \begin{bmatrix} m_P \mathbf{1}_3 & -\widehat{\mathbf{ms}}_P \\ \widehat{\mathbf{ms}}_P & \mathbf{I}_P \end{bmatrix} \tag{11}$$

- m_P is the mass of the platform,

- $\mathbf{1}_3$ is the (3×3) identity matrix,
- $\mathbf{ms}_P$ is the (3×1) vector of first moments of the platform around O_p, the origin of $\mathscr{F}_P$; $\mathbf{ms}_P = [mx_P \; my_P \; mz_P]^T$,
- $\widehat{\mathbf{ms}}_P$ is the (3×3) vector-product skew matrix associated with the vector $\mathbf{ms}_P$,
- $\mathbf{I}_P$ is the (3×3) inertia matrix of the platform around O_p,

Finally, using (9) and (8), the IDM of the robot is given by the following compact forms:

$$\tau_a = \mathbf{J}_P^T \left[\mathbf{w}_P + \sum_{i=1}^{m} \mathbf{J}_{vi}^T \mathbf{J}_i^{-T} \tau_i \right] \tag{12}$$

or also

$$\tau_a = \mathbf{J}_r^T \boldsymbol{\Psi}^T \left[\mathbf{w}_P + \sum_{i=1}^{m} \mathbf{J}_{vi}^T \mathbf{J}_i^{-T} \tau_i \right] \tag{13}$$

It will be denoted by:

$$\tau_a = \mathbf{idm}({}^0\mathbf{T}_P, \mathbb{V}_P, \dot{\mathbb{V}}_P) \tag{14}$$

and can be obtained from (13) by replacing the variables $(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)$ in the expression of $\tau_i = \mathbf{idm_q}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)$ as follows:

- the expression of $\mathbf{q}_i$ as a function of ${}^0\mathbf{T}_P$ can be found by using the inverse geometric model of the leg i, considered as a serial leg connected to the platform (Khalil and Dombre, 2002),
- by using Eqs. (4) and (5), we can find that $\dot{\mathbf{q}}_i = \mathbf{J}_i^{-1} \mathbf{J}_{vi} \mathbb{V}_P$, while $\ddot{\mathbf{q}}_i$ can be found by derivation of the previous expression with respect to time.

The effects of friction and of rotor inertia for actuators terms can be approximated by simple functions provided in (Briot and Khalil, 2015; Khalil and Dombre, 2002).

Refer về Friction và Rotor Inertia

The Cartesian dynamic model of the robot can be obtained from (13) as:

$$\mathbf{J}_{inv}^T \tau_a = \boldsymbol{\Psi}^T \left[\mathbf{w}_P + \sum_{i=1}^{m} \mathbf{J}_{vi}^T \mathbf{J}_i^{-T} \tau_i \right] \tag{15}$$

It will be denoted by:

$$\mathbf{J}_{inv}^T \tau_a = \mathbf{idm_x}({}^0\mathbf{T}_P, \mathbb{V}_P, \dot{\mathbb{V}}_P) \tag{16}$$

Many methods can be used to calculate τ_i representing the IDM of a serial rigid bodies structure (Angeles, 2003; Featherstone, 2008; Khalil and Dombre, 2002). To reduce the computational cost, the recursive Newton-Euler algorithm (Luh *et al.*, 1980) and customized symbolic methods can be used (Featherstone, 2008; Khalil and Dombre, 2002; Khalil and Kleinfinger, 1987).

Refer về cách tính \tau_i

+

Eq.(13) sẽ ra được phương trình Động lực học nghịch, từ các input là q_i, q'_i và q''_i là các tham số trong \tau_i, ta tìm được torque cần actuated là \tau_a

3 Direct dynamic model of parallel robots

The DDM of the robot gives the platform Cartesian acceleration as a function of the state variables and the input of the motorized joint efforts:

$$\dot{\mathbb{V}}_r = \mathbf{ddm}({}^0\mathbf{T}_P, \mathbb{V}_r, \boldsymbol{\tau}_a) \tag{17}$$

In a simulation algorithm, $\dot{\mathbb{V}}_P$ can be obtained from $\dot{\mathbb{V}}_r$ through the formula

$$\dot{\mathbb{V}}_P = \boldsymbol{\Psi}\dot{\mathbb{V}}_r + \dot{\boldsymbol{\Psi}}\mathbb{V}_r \tag{18}$$

obtained by differentiating (1) with respect to time. Then by integration, $\mathbb{V}_P$ and ${}^0\mathbf{T}_P$ can be obtained.

The DDM can be derived from (15) by substituting $\boldsymbol{\tau}_i$ by its Lagrangian form $\boldsymbol{\tau}_i = \mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{c}_i$, in which $\mathbf{M}_i$ is the generalized inertia matrix of the leg i and $\mathbf{c}_i$ its vector of Coriolis, centrifugal, gravitational effects and friction terms. Then, substituting $\ddot{\mathbf{q}}_i$ in terms of $\dot{\mathbb{V}}_r$ by using the time derivative of the expressions (4) and (5), the final result will be given as:

$$\mathbf{J}_{inv}^T\boldsymbol{\tau}_a = \mathbf{M}_{rob}\dot{\mathbb{V}}_r + \mathbf{c}_{rob} \tag{19}$$

Thus the desired Cartesian acceleration is given by:

$$\dot{\mathbb{V}}_r = \mathbf{M}_{rob}^{-1}(\mathbf{J}_{inv}^T\boldsymbol{\tau}_a - \mathbf{c}_{rob}) \tag{20}$$

where:

$$\mathbf{M}_{rob} = \boldsymbol{\Psi}^T\bar{\mathbf{I}}_P\boldsymbol{\Psi} + \sum_{i=1}^{m}\mathbf{J}_{vi}^T\mathbf{M}_{xi}\mathbf{J}_{vi} \tag{21}$$

in which:

- $\mathbf{M}_{rob}$ is the generalized inertia matrix of the robot in the Cartesian space,
- $\mathbf{M}_{xi}$ is the generalized inertia matrix of leg i referred to the Cartesian space of the terminal frame of leg i; it is equal to $\mathbf{J}_i^{-T}\mathbf{M}_i\mathbf{J}_i^{-1}$,
- $\mathbf{c}_{rob}$ is the wrench of Coriolis, centrifuge and gravity effects.

The expression of $\mathbf{c}_{rob}$ is complicated to use and will not be given here. However identifying equations (16) and (19), it can be deduced that $\mathbf{c}_{rob}$ can be calculated using the Cartesian IDM after setting in it $\dot{\mathbb{V}}_r = \mathbf{0}$, thus

$$\mathbf{c}_{rob} = \mathbf{idm_x}({}^0\mathbf{T}_P, \mathbb{V}_r, \dot{\mathbb{V}}_r = \mathbf{0}) \tag{22}$$

This procedure of calculation of $\mathbf{c}_{rob}$ is similar to what was proposed for serial robots by Walker and Orin (1982). Similarly, the matrix $\mathbf{M}_{rob}$ can also be calculated column per column by using the Cartesian IDM.

Tổng kết lại:
Các kết quả cần được tính toán để rút ra được phương trình hoàn chỉnh cho cả Inverse và Direct:
+ J_r : Robot kinematic Jacobian matrix
+ J_i : Leg i kinematic Jacobian matrix
+ J_{vi} : Linking V_i to V_p kinematic Jacobian matrix

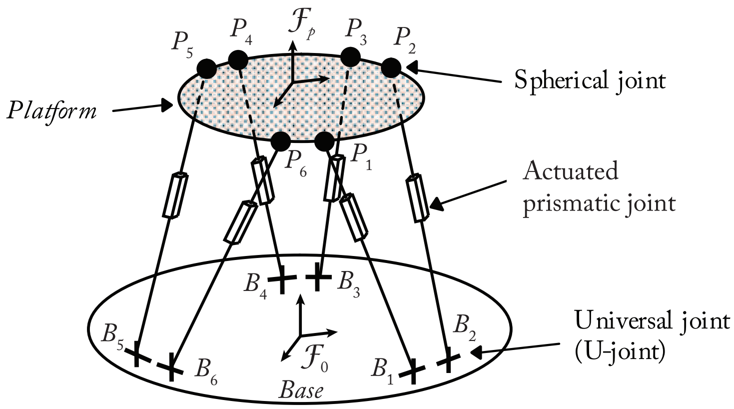

Fig. 1 Gough-Stewart platform

4 Inverse dynamic model of the Gough-Stewart parallel robot

The 6-dof Gough-Stewart platform (Fig. 1) is composed of a moving platform connected to a fixed base by six extendable legs (Merlet, 2006). The extremities of each leg are fitted with a 2-dof passive universal joint (U) at the base and a 3-dof passive spherical joint (S) at the platform. The lengths of the legs are actuated using prismatic joints ($\underline{\text{P}}$). The Gough-Stewart platform is thus said to be a 6-U$\underline{\text{P}}$S robot. For this robot, $\mathbf{\Psi} = \mathbf{1}_6$, thus $\mathbf{J}_P = \mathbf{J}_r$.

4.1 *Description of the robot*

Let us assume that B_i is the point connecting leg i to the base and P_i is the point connecting leg i to the platform. The frame $\mathscr{F}_0$ is fixed with respect to the base, and the frame $\mathscr{F}_p$ is fixed with respect to the mobile platform. In this example, their respective origins O_0 and O_p can be arbitrarily placed.

các quy ước

The notations of Khalil and Kleinfinger (1986) are used to describe the kinematics of the tree structure composed of the base and the legs after separating the platform. The definition of the local link frames of leg i are given in Fig. 2, while the corresponding geometric parameters are given in Table 1; $p(j)$ denotes the frame precedent to frame $\mathscr{F}_j$, σ_j defines the type of joint, where $\sigma_j = 1$ if joint j is prismatic and $\sigma_j = 0$ if it is revolute.

ý nghĩa các ký hiệu trong bảng 1

The geometric parameters $(\gamma_j, b_j, \alpha_j, d_j, \theta_j, r_j)$ are used to determine the (4×4) homogeneous transformation matrix ${}^{p(j)}\mathbf{T}_j$ giving the location of frame $\mathscr{F}_j$ with respect to the frame $\mathscr{F}_{p(j)}$ of the body $p(j)$, preceding the body j, see (Khalil and Dombre, 2002; Khalil and Kleinfinger, 1986).

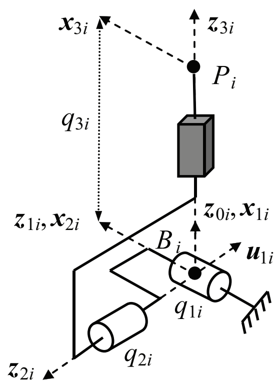

Fig. 2 Link frames of the leg i

Table 1 Geometric parameters of the legs frames for $i = 1, \ldots, 6$.

ji	p_{ji}	σ_{ji}	γ_{ji}	b_{ji}	α_{ji}	d_{ji}	θ_{ji}	r_{ji}
$1i$	0	0	γ_{1i}	b_{1i}	α_{1i}	d_{1i}	q_{1i}	0
$2i$	$1i$	0	0	0	$\pi/2$	0	q_{2i}	0
$3i$	$2i$	1	0	0	$\pi/2$	0	0	q_{3i}

4.2 Calculation of the Jacobian matrices

The following notations are used:

- $\mathbf{q}_a$: vector of the active joint variables,

universal joint 2 dof

$$\mathbf{q}_a = \left[q_{31}\ q_{32}\ q_{33}\ q_{34}\ q_{35}\ q_{36}\right]^T \tag{23}$$

spherical joint 3dof -> 3 thành phần

  in which q_{ji} denotes the position of joint j of leg i,
- $\mathbf{q}_i$: vector of the joint positions of leg i; it does not contain the variables of the spherical joint between the leg and the platform, i.e.

$$\mathbf{q}_i = \left[q_{1i}\ q_{2i}\ q_{3i}\right]^T \tag{24}$$

The Jacobian matrices required to calculate the dynamic models are calculated in what follows.

4.2.1 Calculation of the matrix $\mathbf{J}_i$

The direct kinematic model of a leg corresponds to one of the RRP serial structure, i.e.:

$$\mathbf{v}_i = \mathbf{J}_i\,\mathbf{q}_i \tag{25}$$

in which $\mathbf{v}_i$ is the velocity of point P_i.

The Jacobian matrix $\mathbf{J}_i$ of leg i is calculated with respect to frame $\mathscr{F}_{3i}$ as follows (Khalil and Dombre, 2002):

$$^{3i}\mathbf{J}_i = \begin{bmatrix} ^{3i}\mathbf{a}_{1i} \times {}^{3i}\overrightarrow{B_iP_i} & ^{3i}\mathbf{a}_{2i} \times {}^{3i}\overrightarrow{B_iP_i} & ^{3i}\mathbf{a}_{3i} \end{bmatrix} = \begin{bmatrix} 0 & q_{3i} & 0 \\ -q_{3i}S_{2i} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$

with:

- $\mathbf{a}_{ji}$ the unit vector along the joint axis j of leg i, corresponding to the z_{ji} axis of local frame,
- $\overrightarrow{B_iP_i}$ position vector from B_i to P_i,
- C_α and S_α represent respectively $\cos(q_\alpha)$ and $\sin(q_\alpha)$ functions.

The matrix $^{3i}\mathbf{J}_i^{-1}$ is the inverse of the (3×3) Jacobian matrix of leg i. Its expression is

$$^{3i}\mathbf{J}_i^{-1} = \begin{bmatrix} 0 & -1/(q_{3i}S_{2i}) & 0 \\ 1/q_{3i} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{27}$$

Note that $^{0}\mathbf{J}_i$ and $^{p}\mathbf{J}_i$ can be calculated by using the expressions $^{0}\mathbf{R}_{3i}{}^{3i}\mathbf{J}_i$ and $^{p}\mathbf{R}_{3i}{}^{3i}\mathbf{J}_i$, where $^{k}\mathbf{R}_l$ is the (3×3) rotation matrix between frames $\mathscr{F}_k$ and $\mathscr{F}_l$.

The singular configurations of equation (27) occur when $q_{3i} = 0$ or $\sin(q_{2i}) = 0$, which are outside the operating space of the robot.

4.2.2 Calculation of the matrix $\mathbf{J}_{vi}$

The terminal velocity of leg i, denoted $\mathbf{v}_i$, which is also the linear velocity of point P_i, is calculated in terms of the platform velocity as follows:

$$\mathbf{v}_i = \mathbf{v}_P + \boldsymbol{\omega}_P \times \overrightarrow{O_PP_i} = \begin{bmatrix} \mathbf{1}_3 & -\widehat{\overrightarrow{O_PP_i}} \end{bmatrix} \mathbb{V}_P \tag{28}$$

Thus:

$$\mathbf{J}_{vi} = \begin{bmatrix} \dfrac{\partial \mathbf{v}_i}{\partial \mathbb{V}_P} \end{bmatrix} = \begin{bmatrix} \mathbf{1}_3 & -\widehat{\overrightarrow{O_PP_i}} \end{bmatrix} \tag{29}$$

where $\widehat{\overrightarrow{O_PP_i}}$ designates the (3×3) skew matrix associated with the position vector $\overrightarrow{O_PP_i}$.

The joint velocities of leg i are obtained as:

$$\dot{\mathbf{q}}_i = \mathbf{J}_i^{-1}\mathbf{J}_{vi}\mathbb{V}_P \tag{30}$$

4.2.3 Inverse kinematic model of the robot

The inverse kinematic model of the robot is given by

$$\dot{\mathbf{q}}_a = \mathbf{J}_{inv} \mathbb{V}_P \tag{31}$$

in which the ith row of $\mathbf{J}_{inv}$ is given by (Merlet, 2006)

$$\mathbf{J}_{inv}(i,:) = \begin{bmatrix} \mathbf{a}_{3i}^T & -\mathbf{a}_{3i}^T \widehat{\overrightarrow{O_P P_i}} \end{bmatrix} \tag{32}$$

Using the components of the vectors appearing in $\mathbf{J}_{inv}$ and $\mathbf{J}_{vi}$ expressed in frame $\mathscr{F}_p$ or $\mathscr{F}_0$ allows to give the expressions of these matrices in frame $\mathscr{F}_p$ or $\mathscr{F}_0$ respectively.

4.3 *Inverse dynamic model of the Gough-Stewart platform*

The dynamic model is obtained by applying (12). Expressing all the elements in frame $\mathscr{F}_p$ gives:

$$\boldsymbol{\tau}_a = {}^p\mathbf{J}_{inv}^{-T} \left[{}^p\mathbf{w}_P + \sum_{i=1}^{6} \begin{bmatrix} \mathbf{1}_3 \\ {}^p\widehat{\overrightarrow{O_P P_i}} \end{bmatrix} {}^p\mathbf{J}_i^{-T} \boldsymbol{\tau}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i) \right] \tag{33}$$

$\boldsymbol{\tau}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)$ is the IDM of leg i. Well-known methods and techniques, which have already been applied to serial robots can be used for calculating it (Angeles, 2003; Featherstone and Orin, 2016; Khalil and Dombre, 2002). One of the most efficient methods for it is the recursive Newton-Euler algorithm (Luh *et al.*, 1980). Giving general geometric parameters to the first frame of each leg, see Table 1, makes it possible to use a unique subroutine to calculate this model for all the legs.

5 Conclusion and further readings

This article has presented the calculation of the IDM and DDM of parallel robots. The models are expressed in terms of the dynamic model of the legs and the dynamics of the platform and some Jacobian matrices. The method is applied on a Gough-Stewart platform. These models represent the most important problems in parallel robots dynamics, however there are many topics in dynamics of parallel robots that have not been mentioned in this article. For further readings the following subjects are suggested.

5.1 Dynamics of flexible parallel robots

In the present article, the joints are supposed to be perfect and the links are supposed to be rigid. However, some structures may contain flexibility in the joints or in the links that must be taken into account in order to obtain models with acceptable accuracy approaching the real response of the system. For instance, for the Gough-Stewart robot, the effects of leg fexibility are examined in (Mahboubkhah *et al.*, 2009; Mukherjee *et al.*, 2007). In general the joint flexibility is modelled using lumped elasticity (Khalil and Gautier, 2000; Kruszewski *et al.*, 1975; Wittbrodt *et al.*, 2006). The link flexibility can be approximated by finite number of lumped springs as done in (Stachera and Schumacher, 2008) where the calculation of the IDM and DDM of parallel robots has been derived using Lagrange formulation and the principle of virtual works on the flexible system. However, in order to obtain a correct model accuracy, a higher number of elements may be required, thus increasing the complexity of the computation.

To have good accuracy, the link distributed flexibility are treated using finite elements techniques either with Lagrange equations (De Luca and Siciliano, 1996) or with a Generalized Newton–Euler formulation as proposed in (Boyer and Khalil, 1998; Shabana, 1990; Sharf and Damaren, 1992) for serial robots. The work (Briot and Khalil, 2014a) used this technique to compute the IDM and DDM of parallel robots with leg flexibilities. The computation of the natural frequencies is proposed in the work (Briot and Khalil, 2014b). The work of Long *et al.* (2014) presented the dynamic models of GS robots with platform flexibility modelled using finite elements method.

5.2 Identification of the dynamic parameters

To use the dynamic models in a real application, the numerical values of the inertial parameters of the links (legs and platform) of the robot are needed. These values can be obtained by identification techniques by collecting the state variables and the input efforts of the joints along selected trajectories. The identification model is obtained by expressing the IDM as a linear function of the inertial parameters. Then the techniques developed for the identification of dynamic parameters of serial structures (Hollerbach *et al.*, 2016) can be generalized on this linear system. Diaz-Rodriguez *et al.* (2010); Grotjahn *et al.* (2004); Guegan *et al.* (2003) show first work on such applications. More advanced results are developed in (Briot and Gautier, 2015) where the authors present the global identification of all robot dynamic parameters, including joint drive gains.

5.3 Singularities of parallel robot dynamics

Parallel robots encounter several types of singularities. The most known are probably the Type 1 (or also serial) singularities for which the platform loses some dof or the Type 2 (or also parallel) singularities (Gosselin and Angeles, 1990) for which the robot inverse kinematic Jacobian matrix $\mathbf{J}_{inv}$ is singular and cannot be inverted, leading to an uncontrollable motion of the end-effector.

Type 2 singularities also impact the dynamic model: near these configurations, the joint reactions may increase with the possibility of mechanism break. Also the controllers may send very high commands to the robot leading to controller instability, higher tracking errors and impossibility of crossing these singularities. However, it was shown in (Briot and Arakelian, 2008) that their crossing was possible if and only if the trajectory for crossing respects a criterion based on the analysis of the degeneracy of the dynamic model. Dedicated controllers for crossing the singularities can also be found in (Six *et al.*, 2017).

Cross-references

[Copy Editor: all cross references are marked in the source text with macro `\xref`. The form 'A ⇒ B' means that the phrase that has been marked is A, but the article to which the reader is referred is B.]

- parallel robot ⇒ Parallel Mechanisms
- homogeneous transformation matrix ⇒ Homogeneous Transforms
- geometric parameters ⇒ Kinematics Equations (DH Convention)
- twist ⇒ Kinematics
- inverse geometric model ⇒ Inverse Kinematics
- mechanical formalisms ⇒ Dynamics calculation methods
- simulation algorithm ⇒ Dynamics simulation
- closed-loop multi-body system ⇒ Closed-loop Dynamics
- symbolic methods ⇒ Symbolic dynamics
- The Recursive Newton-Euler Algorithm
- Inverse Dynamic Model ⇒ Inverse Dynamics

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