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DYNAMIC ANALYSIS OF CLAVEL' S DELTA PARALLEL ROBOT

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Some iterative matrix relations for the geometric, kinematic and dynamic analysis of a Delta parallel robot are established in this paper. The prototype of this manipulator is a three degree of freedom spatial mechanism, which consists of a system of parallel chains. Supposing that the position and the translation motion of the platform are known, an inverse dynamic problem is solved using the virtual powers method. Finally, some recursive matrix relations and some graphs for the moments and the powers of the three active couples are determined.

Key words: robotics, manipulator, platform, matrix, dynamics.

1. Introduction

The parallel robots are spatial mechanical structures that consist of kinematic closed chains. Generally, a parallel manipulator, have two platforms. One of them is attached to the fix reference frame. The other one can have arbitrary motions in its workspace. Three mobile legs, made up as serial robots, connect the effector, which is attached to the moving platform, to the fixed platform. The elements of the robot are connected one to the other by spherical joints, revolute joints or prismatic joints.

The parallel manipulators have some special characteristics with respect to the serial robots such as: more rigid structure, high orientation accuracy, stabile functioning, control on the limits of velocities and accelerations, suitable position of the acting systems and a good positional repetitivity. The parallel robots are equipped with hydraulic or pneumatic actuators. They have a robust construction and they can move bodies of considerable masses and dimensions with high speeds. This is why the mechanisms, which produce a translation or spherical motion to a platform, are based on the concept of parallel manipulator.

The most known application is the flight simulator with six degree of freedom, which is in fact the Gough-Stewart platform [Stewart, 1965; Merlet, 1997]. The parallel manipulator Star [Hervé and Sparacino, 1992; Tremblay and Baron, 1999] and the parallel Delta robot [Clavel, 1988; Zsombor-Murray, 2001] equipped with three engines, which have a parallel setting, train on the effector in a three degree of freedom general translation motion, used in quick operations of *pick and place*. Angeles (1997), Wang and Gosselin (2001) developed the direct kinematic analysis of a prototype of spherical manipulator Agile Wrist, which has three concurrent rotations.

This paper establishes some recursive matrix relations used for positional, kinematic and dynamic analysis for a three degree of freedom Delta robot. In 1988, R. Clavel developed the prototype of this robot at the Lausanne Federal Polytechnic Institute.

2. Inverse geometric model

The following elements are the elements of the topological structure of one of the three kinematic closed chains of the manipulator, respectively: an engine, an active revolute joint, an intermediary mechanism with four revolute links that connect four bars, which are parallel two and two, and finally a passive revolute link connected to the moving platform (fig.1).

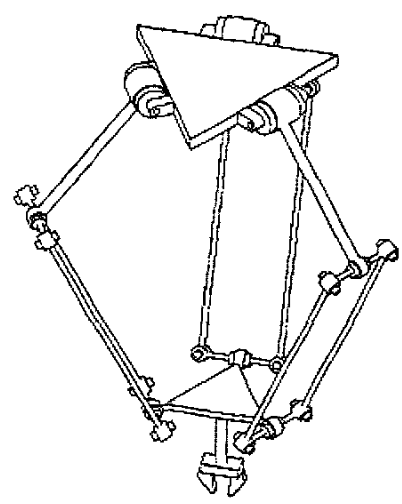

Fig. 1. The Clavel's Delta Robot

Let $Ox_0y_0z_0$ (T_0) be a fix cartesian frame. A three degrees of freedom Delta manipulator is moving with respect to this reference frame. The manipulator has three legs. The elements of these legs have known dimensions and masses. One of the three active elements of the robot is the first body of the leg A. This is a homogenous crank, which rotates about the axis $A_1z_1^A$ with the angular velocity ω_{10}^A and the angular acceleration ε_{10}^A. It has the length $A_1A_2 = l_1^A$, the mass m_1^A and the tensor of inertia $\hat{J}_1^A$. The transmission bar $A_3A_6 = l_2^A$ is connected to the $A_2x_2^Ay_2^Az_2^A$ (T_2^A) frame and it has a relative rotation with the angle φ_{21}^A, so that $\omega_{21}^A = \dot{\varphi}_{21}^A$ and $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$. It has the mass m_2^A and the tensor of inertia $\hat{J}_2^A$.

Further on, two identical and parallel bars with same length $l_3^A = l_6^A$, rotate about the T_2^A frame with the angle $\varphi_{32}^A = \varphi_{62}^A$. They have also the same mass $m_3^A = m_6^A$ and the same tensor of inertia $\hat{J}_3^A = \hat{J}_6^A$. The parallelogram is closed by an element T_4^A, which has the same length and mass with T_2^A. Its tensor of inertia is $\hat{J}_4^A$. This element rotates with the relative angle $\varphi_{43}^A = \varphi_{32}^A$.

The platform of the robot is an equilateral triangle. The relation $l = \sqrt{3}\left(l_0^A - l_3^A \sin\beta_A\right)$ gives the side dimension of this triangle, which has the mass m_5^A. Let us denote with $\omega_{54}^A = \dot{\varphi}_{54}^A$ (fig.2), the angular velocity of the platform with respect to the nearby body T_4^A. The following angles give the initial position of the manipulator:

$$\alpha_A = \frac{\pi}{3}, \alpha_B = \pi, \alpha_C = -\frac{\pi}{3}, \beta_A = \beta_B = \beta_C = \frac{\pi}{6}. \quad (1)$$

Let us consider the rotation angles φ_{10}^A, φ_{10}^B, φ_{10}^C, of the three actuators A_1, B_1, C_1, the parameters which give the position of the mechanism. In the inverse geometric problem, one can consider that the coordinates of the mass centre of the platform, x_0^G, y_0^G, z_0^G, give the position of the mechanism.

Pursuing the leg A in the $OA_1A_2A_3A_4A_5$ way, one obtains the following passing matrices:

$$a_{10} = a_{10}^{\varphi}\theta_1\theta_2 a_{\alpha_A},\ a_{21} = a_{21}^{\varphi} a_{\beta_A}\theta_3,\ a_{32} = a_{32}^{\varphi}\theta_1\theta_2,$$
$$a_{43} = a_{43}^{\varphi}\theta_3,\ a_{54} = a_{54}^{\varphi} a_{\beta_A}\theta_1\theta_4,\ a_{62} = a_{32}, \quad (2)$$

quay quanh trục z -pi/2

where one denoted [Staicu, 1998]:

quay quanh trục y -pi/2

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \theta_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

quay quanh trục y pi

quay quanh trục z pi

$$\theta_4 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, a_{\alpha_A} = \begin{bmatrix} \cos\alpha_A & \sin\alpha_A & 0 \\ -\sin\alpha_A & \cos\alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$a_{\beta_A} = \begin{bmatrix} \cos\beta_A & \sin\beta_A & 0 \\ -\sin\beta_A & \cos\beta_A & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

$$a_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^A & \sin\varphi_{k,k-1}^A & 0 \\ -\sin\varphi_{k,k-1}^A & \cos\varphi_{k,k-1}^A & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$a_{k0} = \prod_{j=1}^{k} a_{k-j+1,k-j} \ (k = 1,2,\ldots,5).$$

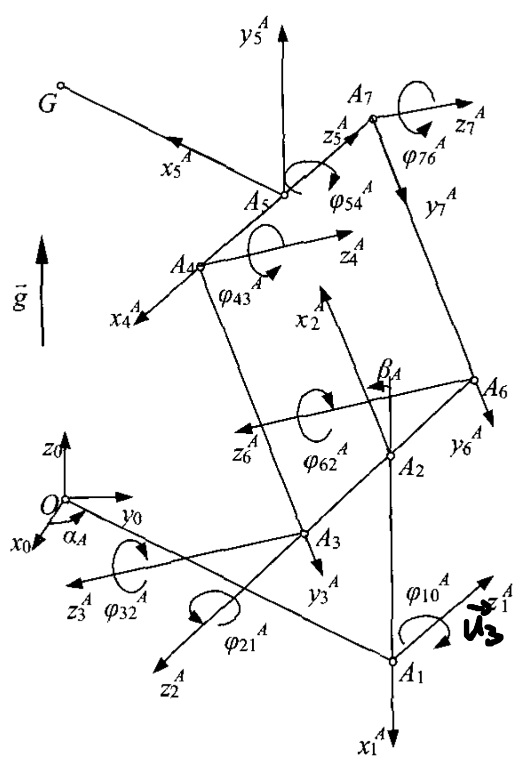

Fig. 2. The kinematic schema of the mechanism

If the other legs, B and C, of the mechanism are pursued, some analogous relations can be written.

The translation conditions of the platform are given by the following identities

$$a_{50}^{\circ T} a_{50} = b_{50}^{\circ T} b_{50} = c_{50}^{\circ T} c_{50} = I \quad (4)$$

and by the following matrices

$$a_{50}^{\circ} = \frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} & 0 \\ 0 & 0 & 2 \\ -\sqrt{3} & 1 & 0 \end{bmatrix}, b_{50}^{\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad (5)$$

$$c_{50}^{\circ}=\frac{1}{2}\begin{bmatrix}-1 & \sqrt{3} & 0\\ 0 & 0 & 2\\ \sqrt{3} & 1 & 0\end{bmatrix}.$$

From this relations, one obtains the following relations between angles:

$$\begin{aligned}&\varphi_{54}^{A}=-\varphi_{20}^{A}=\varphi_{21}^{A}-\varphi_{10}^{A}\,,\\ &\varphi_{54}^{B}=-\varphi_{20}^{B}=\varphi_{21}^{B}-\varphi_{10}^{B}\,,\\ &\varphi_{54}^{C}=-\varphi_{20}^{C}=\varphi_{21}^{C}-\varphi_{10}^{C}\,.\end{aligned}\tag{6}$$

Supposing that the motion of the mass centre of the platform along an ellipses is given by the relations

$$\begin{aligned}&\vec{r}_0^{\,G}=\begin{bmatrix}x_0^G & y_0^G & z_0^G\end{bmatrix}^T\\ &x_0^G(t)=x_0^{G*}\sin\left(\frac{\pi}{3}t\right)\\ &y_0^G(t)=y_0^{G*}\left[1-\cos\left(\frac{\pi}{3}t\right)\right]\\ &z_0^G(t)=l_1^A+l_3^A\cos\beta_A-z_0^{G*}\left[1-\cos\left(\frac{\pi}{3}t\right)\right],\end{aligned}\tag{7}$$

the angles φ_{10}^{A}, φ_{21}^{A}, φ_{32}^{A}, φ_{10}^{B}, φ_{21}^{B}, φ_{32}^{B}, φ_{10}^{C}, φ_{21}^{C}, φ_{32}^{C} are given by the following geometric conditions:

$$\begin{aligned}&\vec{r}_{10}^{\,A}+\sum_{k=1}^{4}a_{k0}^T\vec{r}_{k+1,k}^{\,A}+a_{50}^T\vec{r}_5^{\,GA}=\\ &=\vec{r}_{10}^{\,B}+\sum_{k=1}^{4}b_{k0}^T\vec{r}_{k+1,k}^{\,B}+b_{50}^T\vec{r}_5^{\,GB}=\\ &=\vec{r}_{10}^{\,C}+\sum_{k=1}^{4}c_{k0}^T\vec{r}_{k+1,k}^{\,C}+c_{50}^T\vec{r}_5^{\,GC}=\vec{r}_0^{\,G}\,,\end{aligned}\tag{8}$$

where one denoted:

$$\begin{aligned}&\vec{u}_1=\begin{bmatrix}1\\0\\0\end{bmatrix},\ \vec{u}_2=\begin{bmatrix}0\\1\\0\end{bmatrix},\ \vec{u}_3=\begin{bmatrix}0\\0\\1\end{bmatrix},\ \tilde{u}_3=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\\ &\vec{r}_{10}^{\,A}=l_0^A a_{\alpha_A}^T\vec{u}_1\,,\ \vec{r}_{21}^{\,A}=-l_1^A\vec{u}_1\\ &\vec{r}_{32}^{\,A}=\frac{l_2^A}{2}\vec{u}_3\,,\ \vec{r}_{43}^{\,A}=-l_3^A\vec{u}_2\\ &\vec{r}_{54}^{\,A}=-\frac{l_2^A}{2}\vec{u}_1\,,\ \vec{r}_5^{\,GA}=\left(l_0^A-l_3^A\sin\beta_A\right)\vec{u}_1\,.\end{aligned}\tag{9}$$

3. Velocities and accelerations

The motions of the component elements of each leg (for example the leg A) are characterised by the following skew symmetric matrices [Staicu and Carp-Ciocardia, 2001]

$$\tilde{\omega}_{k0}^{A}=a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^T+\omega_{k,k-1}^{A}\tilde{u}_3\,,\tag{10}$$

associated to the absolute angular velocities given by the recurrence relations

$$\vec{\omega}_{k0}^{A}=a_{k,k-1}\vec{\omega}_{k-1,0}^{A}+\omega_{k,k-1}^{A}\vec{u}_3\,,\ \omega_{k,k-1}^{A}=\dot{\varphi}_{k,k-1}^{A}\,.\tag{11}$$

The velocity $\vec{v}_{k0}^{\,A}$ of the joint A_k is given by the relation

$$\begin{aligned}&\vec{v}_{k0}^{\,A}=a_{k,k-1}\left\{\vec{v}_{k-1,0}^{\,A}+\tilde{\omega}_{k-1,0}^{A}\vec{r}_{k,k-1}^{\,A}\right\},\\ &\vec{v}_{k,k-1}=\vec{0}\,,\ (k=1,2,\ldots,5).\end{aligned}\tag{12}$$

The following matrix relations give the kinematic constraints:

$$\begin{aligned}&\omega_{10}^{A}\vec{u}_i^{\,T}a_{10}^T\vec{u}_3+\omega_{21}^{A}\vec{u}_i^{\,T}a_{20}^T\vec{u}_3+\omega_{54}^{A}\vec{u}_i^{\,T}a_{50}^T\vec{u}_3=0\\ &\omega_{10}^{A}\left(l_1^A\vec{u}_i^{\,T}a_{10}^T\tilde{u}_3\vec{u}_1+l_3^A\vec{u}_i^{\,T}a_{10}^T\tilde{u}_3a_{21}^Ta_{32}^T\vec{u}_2\right)+\\ &+\omega_{21}^{A}l_3^A\vec{u}_i^{\,T}a_{20}^T\tilde{u}_3a_{32}^T\vec{u}_2+\omega_{32}^{A}l_3^A\vec{u}_i^{\,T}a_{30}^T\tilde{u}_3\vec{u}_2=\\ &=-\vec{u}_i^{\,T}\dot{\vec{r}}_0^{\,G}\quad,\quad(i=1,2,3).\end{aligned}\tag{13}$$

The relations (13) give the Jacobi matrix of the mechanism. This matrix is an essential element for the analysis of the robot workspace.

Also, the relations (13) represent *the connectivity conditions of the relative angular velocities.* These relations give the angular velocities ω_{10}^{A}, ω_{21}^{A}, ω_{32}^{A}, $\omega_{54}^{A}=\omega_{21}^{A}-\omega_{10}^{A}$ as a function of the translation velocity of the platform.

Let us assume that the robot has a virtual motion determined by the angular velocities $\omega_{10a}^{Av}=1$, $\omega_{10a}^{Bv}=0$, $\omega_{10a}^{Cv}=0$. The characteristic virtual velocities expressed as functions of the position of the robot are given by the connectivity conditions of the relative velocities of the loops A-B and B-C:

$$\begin{aligned}&\vec{u}_i^{\,T}a_{50}^T\vec{v}_{50a}^{\,Av}=\vec{u}_i^{\,T}b_{50}^T\vec{v}_{50a}^{\,Bv}=\vec{u}_i^{\,T}c_{50}^T\vec{v}_{50a}^{\,Cv}\,,\ (i=1,2,3)\\ &\omega_{21}^{Av}=1+\omega_{54}^{Av}\,,\ \omega_{21}^{Bv}=\omega_{54}^{Bv}\,,\ \omega_{21}^{Cv}=\omega_{54}^{Cv}\,.\end{aligned}\tag{14}$$

Some other compatibility relations can be obtained if one considers successively that $\omega_{10a}^{Bv}=1$ and $\omega_{10a}^{Cv}=1$.

The angular accelerations, ε_{10}^{A}, ε_{21}^{A}, ε_{32}^{A}, ε_{54}^{A}, of the elements of the robot are given by some new *connectivity conditions*, obtained by deriving the relations (13). The following relations result:

$$\begin{aligned}&\varepsilon_{10}^{A}\vec{u}_i^{\,T}a_{10}^T\vec{u}_3+\varepsilon_{21}^{A}\vec{u}_i^{\,T}a_{20}^T\vec{u}_3+\varepsilon_{54}^{A}\vec{u}_i^{\,T}a_{50}^T\vec{u}_3=0\\ &\varepsilon_{10}^{A}\left(l_1^A\vec{u}_i^{\,T}a_{10}^T\tilde{u}_3\vec{u}_1+l_3^A\vec{u}_i^{\,T}a_{10}^T\tilde{u}_3a_{21}^Ta_{32}^T\vec{u}_2\right)+\\ &+\varepsilon_{21}^{A}l_3^A\vec{u}_i^{\,T}a_{20}^T\tilde{u}_3a_{32}^T\vec{u}_2+\varepsilon_{32}^{A}l_3^A\vec{u}_i^{\,T}a_{30}^T\tilde{u}_3\vec{u}_2=\\ &=-\vec{u}_i^{\,T}\ddot{\vec{r}}_0^{\,G}-\omega_{10}^{A}\omega_{10}^{A}\left(l_1^A\vec{u}_i^{\,T}a_{10}^T\tilde{u}_3\tilde{u}_3\vec{u}_1+\right.\\ &\left.+l_3^A\vec{u}_i^{\,T}a_{10}^T\tilde{u}_3\tilde{u}_3a_{21}^Ta_{32}^T\vec{u}_2\right)-\\ &-\omega_{21}^{A}\omega_{21}^{A}l_3^A\vec{u}_i^{\,T}a_{20}^T\tilde{u}_3\tilde{u}_3a_{32}^T\vec{u}_2-\\ &-\omega_{32}^{A}\omega_{32}^{A}l_3^A\vec{u}_i^{\,T}a_{30}^T\tilde{u}_3\tilde{u}_3\vec{u}_2-\\ &-2\omega_{10}^{A}\omega_{21}^{A}l_3^A\vec{u}_i^{\,T}a_{10}^T\tilde{u}_3a_{21}^T\tilde{u}_3a_{32}^T\vec{u}_2-\end{aligned}\tag{15}$$

$$-2\omega_{10}^A\omega_{32}^A l_3^A \vec{u}_i^T a_{10}^T \tilde{u}_3 a_{21}^T a_{32}^T \tilde{u}_3 \tilde{u}_2 -$$
$$-2\omega_{21}^A\omega_{32}^A l_3^A \vec{u}_i^T a_{20}^T \tilde{u}_3 a_{32}^T \tilde{u}_3 \tilde{u}_2 \text{ , } (i = 1,2,3).$$

If the other two kinematic chains of the manipulator are pursued analogous relations can be easily obtained.

The following recurrence relations give the angular accelerations $\vec{\varepsilon}_{k0}^A$ and the accelerations $\vec{\gamma}_{k0}^A$ of joints

$$
\begin{aligned}
\vec{\varepsilon}_{k0}^A &= a_{k,k-1}\vec{\varepsilon}_{k-1,0}^A + \varepsilon_{k,k-1}^A \vec{u}_3 + \\
&\quad + \omega_{k,k-1}^A a_{k,k-1}\tilde{\omega}_{k-1,0}^A a_{k,k-1}^T \vec{u}_3 \\
\tilde{\omega}_{k0}^A\tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A &= \\
&= a_{k,k-1}\left(\tilde{\omega}_{k-1,0}^A\tilde{\omega}_{k-1,0}^A + \tilde{\varepsilon}_{k-1,0}^A\right)a_{k,k-1}^T + \\
&\quad + \omega_{k,k-1}^A\omega_{k,k-1}^A\tilde{u}_3\tilde{u}_3 + \varepsilon_{k,k-1}^A\tilde{u}_3 + \\
&\quad + 2\omega_{k,k-1}^A a_{k,k-1}\tilde{\omega}_{k-1,0}^A a_{k,k-1}^T\tilde{u}_3 \\
\vec{\gamma}_{k0}^A &= a_{k,k-1}\left[\vec{\gamma}_{k-1,0}^A + \left(\tilde{\omega}_{k-1,0}^A\tilde{\omega}_{k-1,0}^A + \tilde{\varepsilon}_{k-1,0}^A\right)\vec{r}_{k,k-1}^A\right].
\end{aligned}
\tag{16}
$$

The relations (13), (15) represent the *inverse kinematic model* of the Delta robot.

4. Equations of motion

Three electric engines, A, B, C, that generate three couples of moments $\vec{m}_{10}^A = m_{10}^A\vec{u}_3$, $\vec{m}_{10}^B = m_{10}^B\vec{u}_3$, and $\vec{m}_{10}^C = m_{10}^C\vec{u}_3$, which have the directions of the axes $A_1z_1^A$, $B_1z_1^B$, $C_1z_1^C$, control the motion of the legs of the manipulator. The force of inertia and the resultant moment of the forces of inertia of the rigid body T_k are determined with respect to the centre of the joint O_k. On the other hand, the characteristic vectors $\vec{f}_k^*$ and $\vec{m}_k^*$ evaluate the influence of the action of the weight $m_k\vec{g}$ and of other external and internal forces applied to the same element T_k of the robot.

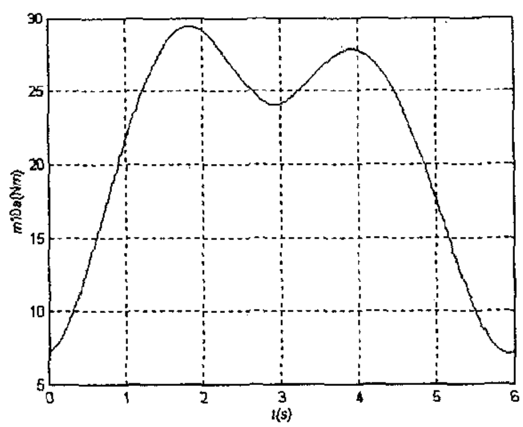

Fig. 3. The moment m_{10}^A

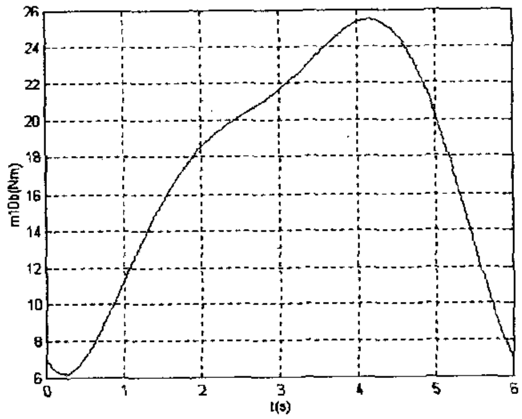

Fig. 4. The moment m_{10}^B

Let us consider that the motion of the platform is known. In these conditions, one determines first the position, the velocity and the acceleration of each joint. Then the forces and the moment that are acting each body are determined. Finally, one calculates the moments of the active couples. There are three methods, which can provide the same results concerning these moments. The first one is using the Newton-Euler classic procedure, the second one applies the Lagrange' equations and multipliers formalism and the third one is based on the virtual work principle.

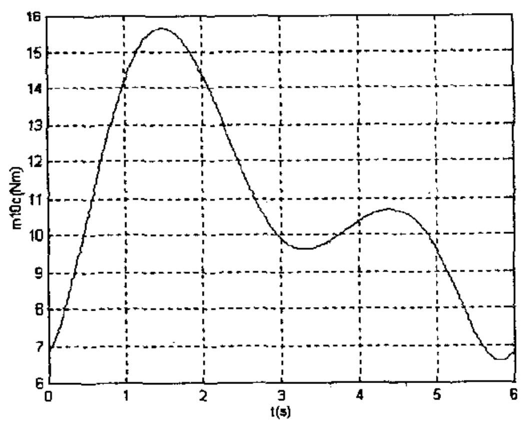

Fig. 5. The moment m_{10}^C

Kane and Levinson (1985) obtained some vectorial recursive relations concerning the equilibrium of the generalized forces that are applied to a serial robot arm.

In the inverse dynamic problem, in this paper, one applies the virtual powers method in order to establish some recursive matrix relations for the moments and the powers of the three active couples. Some graphs of these moments and powers are also obtained.

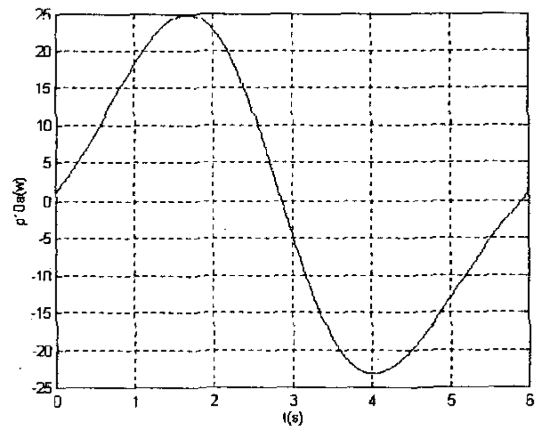

Fig. 6. The power of the first actuator

As the virtual velocities method shows, the dynamic equilibrium condition of the mechanism is that the virtual power of the external, internal and inertia forces, which is developed during a general virtual displacement, must be null. Applying *the fundamental equations of the parallel robots dynamics* obtained by St. Staicu (2000), the following matrix relation results

$$\boxed{\begin{aligned} m_{10}^{A} = \vec{u}_3^{T} \Big[& \vec{M}_1^{A} + \omega_{54a}^{Av} \vec{M}_5^{A} + \\ & + \omega_{21a}^{Av} \vec{M}_2^{A} + \omega_{32a}^{Av} \left(\vec{M}_3^{A} + \vec{M}_4^{A} + \vec{M}_6^{A} \right) + \\ & + \omega_{21a}^{Bv} \vec{M}_2^{B} + \omega_{32a}^{Bv} \left(\vec{M}_3^{B} + \vec{M}_4^{B} + \vec{M}_6^{B} \right) + \\ & + \omega_{21a}^{Cv} \vec{M}_2^{C} + \omega_{32a}^{Cv} \left(\vec{M}_3^{C} + \vec{M}_4^{C} + \vec{M}_6^{C} \right) \Big], \end{aligned}} \tag{17}$$

where one denoted:

$$\begin{aligned} & \vec{F}_{k0}^{A} = m_k^{A} \left[\vec{\gamma}_{k0}^{A} + \left(\tilde{\omega}_{k0}^{A} \tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A} \right) \vec{r}_k^{CA} \right] - \vec{f}_k^{*A} \\ & \vec{M}_{k0}^{A} = m_k^{A} \tilde{r}_k^{CA} \vec{\gamma}_{k0}^{A} + \hat{J}_k^{A} \vec{\varepsilon}_{k0}^{A} + \tilde{\omega}_{k0}^{A} \hat{J}_k^{A} \vec{\omega}_{k0}^{A} - \vec{m}_k^{*A} \\ & \vec{F}_k^{A} = \vec{F}_{k0}^{A} + a_{k+1,k}^{T} \vec{F}_{k+1} \\ & \vec{M}_k^{A} = \vec{M}_{k0}^{A} + a_{k+1,k}^{T} \vec{M}_{k+1} + \tilde{r}_{k+1,k}^{T} a_{k+1,k}^{T} \vec{F}_{k+1}, \\ & (k = 1,2,\ldots,6). \end{aligned} \tag{18}$$

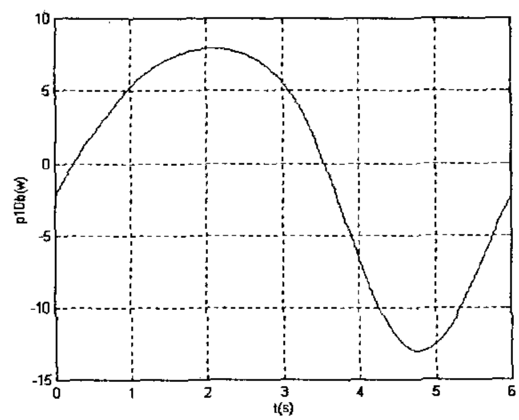

Fig. 7. The power of the second actuator

The relations (17) and (18) represent the *inverse dynamic model* of the parallel Delta robot.

As application let us consider a robot which has the following characteristics:

$$x_0^{*} = 0.03m, \; y_0^{*} = 0.05m, \; z_0^{*} = 0.07m,$$

$$l_0^{A} = l_3^{A} = 0.35m, \; l_1^{A} = 0.1m, \; l_2^{A} = 0.05m,$$

$$m_1^{A} = 2.5kg, \; m_2^{A} = m_4^{A} = 2kg$$

$$m_3^{A} = m_6^{A} = 5kg, \; m_5^{A} = 15kg$$

$$\hat{J}_1^{A} = \begin{bmatrix} 0.05 & & \\ & 0.1 & \\ & & 0.1 \end{bmatrix}, \hat{J}_2^{A} = \begin{bmatrix} 0.02 & & \\ & 0.02 & \\ & & 0.01 \end{bmatrix}$$

$$\hat{J}_3^{A} = \begin{bmatrix} 0.4 & & \\ & 0.2 & \\ & & 0.4 \end{bmatrix}, \hat{J}_4^{A} = \begin{bmatrix} 0.04 & & \\ & 0.08 & \\ & & 0.08 \end{bmatrix}.$$

Finally, one obtains the graphs of the moments m_{10}^{A} (fig.3), m_{10}^{B} (fig.4), m_{10}^{C} (fig.5) and of the powers p_{10}^{A} (fig.6), p_{10}^{B} (fig.7), p_{10}^{C} (fig.8) given by the active couples of the three actuators.

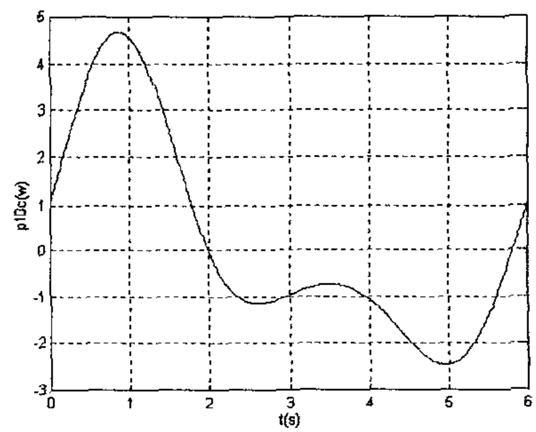

Fig. 8. The power of the third actuator

5. Conclusions

1. Within the inverse positional analysis some exact relations that give in real time the position, the velocity and the acceleration of each element of the parallel robot have been established.
2. Using the Newton-Euler classic method, which takes into account each separate body of the mechanism, an one hundred and five equations system, that must be solved, would result. Finally, the moments of the active couples could be obtained.

The analytical calculi involved in the Lagrange's equations and multipliers formalism are too long and they have risk of making errors. Also, the time for numerical calculus grows with the number of the bodies of the mechanism.

3. The new approach based on the virtual work principle establishes a direct recursive determination of the variation in real time of the moments and the powers of the active couples. The iterative matrix relations, (17) and (18), of the theoretical model of dynamic simulation, can be transformed in a model for the automatic command of the parallel Delta robot.

6. References

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