Dynamics Modeling of a Delta-type Parallel Robot (ISR 2013)

Shi Baek Park[1], Han Sung Kim[2], Changyong Song, Kyunghwan Kim[3]
[1]Department of Mechanical Engineering, Yonsei University, Republic of Korea, amuro1988@yonsei.ac.kr
[2]School of Mechanical Engineering of Kyungnam University, Republic of Korea, hkim@kyungnam.ac.kr
[3]NT Research, Republic of Korea, kimk@ntresearch.net
(Tel: 82-55-249-2627, Fax: 82-55-249-2617)

***Abstract*-- This paper presents a simplified dynamics modeling and hardware implementation of a Delta-type parallel manipulator. Due to complex kinematics, the Lagrangian equations of the first type is employed to derive the inverse dynamic equations. Commercial Delta parallel robot can achieve more than 10m/s and 15g. Accuracy and fast calculation of dynamics are very essential typically in computed torque control of a Delta-type parallel manipulator for high speed applications. It is presented that the simplified dynamics equation matches very well with ADAMS modeling and the calculation time of the inverse kinematics and dynamics is less than 0.04msec.**

***Index Terms*— Delta parallel robot, Dynamics modeling, Lagrangian equation, computed torque control, ADAMS**

I. INTRODUCTION

In many sectors such as electronics, packing, food, pharmacy, and other light industries, Cartesian-type or SCARA-type serial-kinematic manipulators with 3-DOF or 4-DOF are mainly used. However, serial-kinematic structures suffer from large moving inertia and small ratio of payload to weight. In order to overcome the above shortcomings, parallel manipulators have been investigated. Since heavy actuators locate near or at the fixed base and payload is distributed to several serial chains, parallel manipulators can generate high speed, high stiffness and high accuracy.

Recently, many researchers have focused on the Delta-type TPMs for high-speed applications in place of SCARA-type serial manipulators. Delta parallel robot [1~3] has very small moving inertia and the fastest commercial Delta robot can achieve more than 10m/s and 15g.

Accuracy and fast calculation of dynamics are very essential typically in computed torque control of a Delta-type parallel manipulator for high speed applications. In this paper, a simplified dynamic modeling of a Delta-type parallel manipulator is presented. First, the position, velocity, and acceleration analyses are performed. Then, the dynamics modeling is derived by using the Lagrangian equation of the first type due to the complex kinematics. Finally, the derived simplified inverse dynamic equations are compared with full ADAMS modeling and the calculation time is measured in the real-time controller.

II. KINEMATIC ANALYSIS

As shown in Fig. 1, the Delta parallel robot consists of three R-Pa (Revolute-spatial Parallelogram) legs connecting in parallel from the base platform to the moving platform, which allows three translational motions to the moving platform. Since two S-S (Spherical-Spherical) chains consisting of each Pa chain are subject to only axial load, Delta parallel robot has small moving inertia.

Figure 2 shows the vector-loop diagram with the kinematic parameters; a and b denote the radii of the fixed and moving platforms, and l_1 and l_2 are the actuating and connecting link lengths. The actuated joint angles are denoted by θ_{1i} and passive joint angles are denoted by θ_{2i} and θ_{3i}. The vector-loop equation of each leg is given by

$$\overline{A_iM_i} + \overline{M_iB_i} = \overline{OP} + \overline{PB_i} - \overline{OA_i} \tag{1}$$

For simplicity of expression, the local frame for the i^{th} leg ($A_i - x_iy_iz_i$) is introduced and the leading superscript denotes the frame in which a vector is expressed. The vector-loop equation is expressed in the i^{th} local frame by

$$l_1\,{}^i\mathbf{u}_{1i} + l_2\,{}^i\mathbf{u}_{2i} = {}^i(\mathbf{p} + \mathbf{b} - \mathbf{a}) \tag{2}$$

where

$${}^i\mathbf{u}_{1i} = \begin{bmatrix} c\theta_{1i} \\ 0 \\ s\theta_{1i} \end{bmatrix},\ {}^i\mathbf{u}_{2i} = \begin{bmatrix} s\theta_{3i}c(\theta_{1i}+\theta_{2i}) \\ c\theta_{3i} \\ s\theta_{3i}s(\theta_{1i}+\theta_{2i}) \end{bmatrix},\ \mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix},$$

$${}^i\mathbf{b} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix},\ {}^i\mathbf{a} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix},\ {}^i\mathbf{p} = R^T\mathbf{p},\ R = \begin{bmatrix} c\phi_i & -s\phi_i & 0 \\ s\phi_i & c\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where $\phi_1 = +\pi/3,\ \phi_2 = \pi,\ \phi_3 = -\pi/3$ are defined as shown in Fig. 3. Solving Eq. (2) for given $\mathbf{p}$, two passive joint angles (θ_{3i}, θ_{2i}) and one active joint angle (θ_{1i}) of the i^{th} leg are obtained by

$$\theta_{3i} = \cos^{-1}\frac{b_{2i}}{l_2} \ \text{ and } \ \theta_{2i} = \cos^{-1}\kappa \tag{4}$$

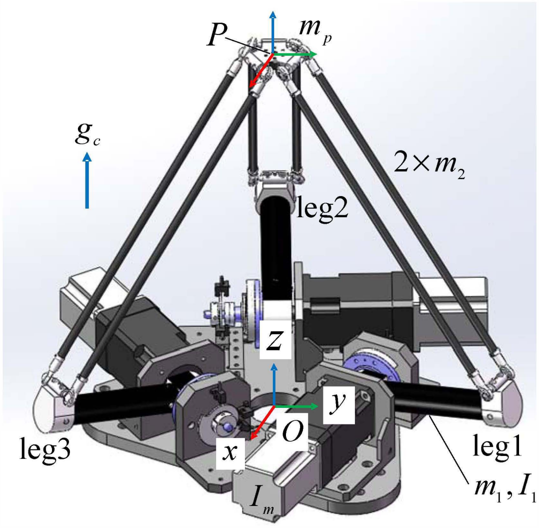


Fig. 1. Configuration of a Delta parallel robot

where $\kappa = (b_{1i}^2 + b_{2i}^2 + b_{3i}^2 - l_1^2 - l_2^2)/(2l_1 l_2 s\theta_{3i})$, $b_{3i} = p_3$, $b_{1i} = c\phi_i p_1 + s\phi_i p_2 + (b-a)$, $b_{2i} = -s\phi_i p_1 + c\phi_i p_2$.

$$\theta_{1i} = \mathrm{Atan2}(-g_{2i}b_{1i} + g_{1i}b_{3i},\ g_{1i}b_{1i} + g_{2i}b_{3i}) \quad (5)$$

where $g_{1i} = l_1 + l_2 c\theta_{2i} s\theta_{3i}$, $g_{2i} = l_2 s\theta_{2i} s\theta_{3i}$.

The linear velocity of the moving platform is obtained by taking derivatives of Eq. (2) with respect to time.

$${}^i\dot{\mathbf{p}} = l_1({}^i\boldsymbol{\omega}_{1i} \times {}^i\mathbf{u}_{1i}) + l_2({}^i\boldsymbol{\omega}_{2i} \times {}^i\mathbf{u}_{2i}) + l_2({}^i\boldsymbol{\omega}_{3i} \times {}^i\mathbf{u}_{2i}) \quad (6)$$

where ${}^i\boldsymbol{\omega}_{1i} = \dot\theta_{1i}[0,-1,0]^T$, ${}^i\boldsymbol{\omega}_{2i} = (\dot\theta_{1i} + \dot\theta_{2i})[0,-1,0]^T$, ${}^i\boldsymbol{\omega}_{3i} = \dot\theta_{3i}[s(\theta_{1i}+\theta_{2i}),0,-c(\theta_{1i}+\theta_{2i})]^T$. In order to eliminate passive joint rates, $\boldsymbol{\omega}_{2i}$ and $\boldsymbol{\omega}_{3i}$, taking dot-product at the both sides of Eq. (6) yields,

$${}^i\mathbf{u}_{2i} \cdot R^T\dot{\mathbf{p}} = l_1\, {}^i\boldsymbol{\omega}_{1i} \cdot ({}^i\mathbf{u}_{1i} \times {}^i\mathbf{u}_{2i}) \quad (7)$$

Writing Eq. (7) for i=1,2,3 gives the velocity relation by

$$J_x\dot{\mathbf{p}} = J_q\dot{\boldsymbol{\theta}} \quad \text{or} \quad \dot{\boldsymbol{\theta}} = J\dot{\mathbf{p}} \quad (8)$$

where $\dot{\boldsymbol{\theta}} = [\dot\theta_1, \dot\theta_2, \dot\theta_3]^T \equiv [\dot\theta_{11}, \dot\theta_{12}, \dot\theta_{13}]^T$ for simple expression, $J = J_q^{-1}J_x$, $J_q = l_1\,\mathrm{diag}(s\theta_{21}s\theta_{31}, s\theta_{22}s\theta_{32}, s\theta_{23}s\theta_{33})$,

$$J_x = \begin{bmatrix} j_{1x} & j_{1y} & j_{1z} \\ j_{2x} & j_{2y} & j_{2z} \\ j_{3x} & j_{3y} & j_{3z} \end{bmatrix}, \quad \begin{aligned} j_{ix} &= c(\theta_{1i}+\theta_{2i})s\theta_{3i}c\phi_i - c\theta_{3i}s\phi_i \\ j_{iy} &= c(\theta_{1i}+\theta_{2i})s\theta_{3i}s\phi_i + c\theta_{3i}c\phi_i \\ j_{iz} &= s(\theta_{1i}+\theta_{2i})s\theta_{3i} \end{aligned}.$$

Once $\dot\theta_i \equiv \dot\theta_{1i}$ is solved, the two passive joint rates are obtained from Eq. (6),

$$\dot\theta_{3i} = -{}^i\dot p_2/(l_2 s\theta_{3i}),$$
$$\dot\theta_{2i} = (-{}^i\dot p_1 c\theta_{1i} - {}^i\dot p_3 s\theta_{1i} + l_2 c\theta_{2i} c\theta_{3i}\dot\theta_{3i})/(l_2 s\theta_{2i} s\theta_{3i}) - \dot\theta_i \quad (9)$$

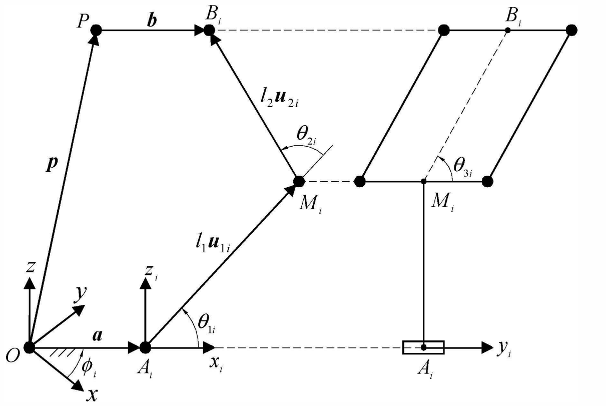
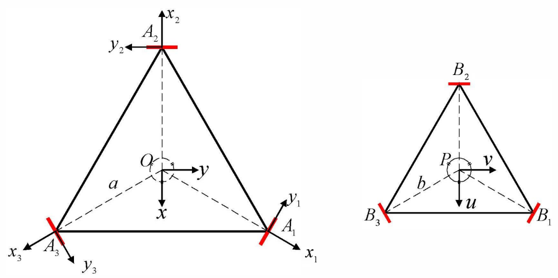


Fig. 2. Vector-loop diagram of the i^{th} leg.



(a) Fixed frame and i^{th} local frame (b) Moving frame

Fig. 3. Frame definitions.

Using the principle of virtual works, the statics relation between force at the moving platform and actuator force can be expressed as

$$\mathbf{F} = J^T\boldsymbol{\tau} \quad (10)$$

where $\mathbf{F} = [f_1, f_2, f_3]^T$ is the applied force vector at the moving platform and $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^T$ is the actuator torque.

The acceleration relation can be derived by taking derivatives of Eq. (8) with respect to time.

$$\dot J_x\dot{\mathbf{p}} + J_x\ddot{\mathbf{p}} = \dot J_q\dot{\boldsymbol{\theta}} + J_q\ddot{\boldsymbol{\theta}},\quad \ddot{\boldsymbol{\theta}} = J_q^{-1}(\dot J_x\dot{\mathbf{p}} + J_x\ddot{\mathbf{p}} - \dot J_q\dot{\boldsymbol{\theta}}) \quad (11)$$

The derivation of the time derivatives of J_x and J_q is omitted.

III. Dynamics Analysis

The dynamic analysis can be performed by using only three generalized coordinates since it is a 3-DOF manipulator. However, due to complex kinematics, three redundant coordinates (p_x, p_y, p_z) are included. Thus the generalized coordinates become

$$\mathbf{q} = [p_1, p_2, p_3, \theta_1, \theta_2, \theta_3]^T \quad (12)$$

To simplify the analysis, it is assumed that the mass of each connecting rod, m_2, in a parallelogram is distributed evenly and concentrated at the two endpoints M_i and B_i. Also, the acceleration of gravity points in the +Z direction since the robot is installed in the downward.

Using the Lagrange equations of the first type, the closed-form dynamics equations can be derived by [4]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=Q_j+\sum_{i=1}^{k}\lambda_i\frac{\partial \Gamma_i}{\partial q_j} \quad \text{for } j=1,\cdots,n \tag{13}$$

where $n=6$ is the number of generalized coordinates, $k=3$ is the number of constraint functions, $n-k=3$ is the number of actuated joint variables, Γ_i denotes the i^{th} constraint function, and λ_i is the Lagragian multiplier.

The first set of equations related to constraints can be written in the form

$$\sum_{i=1}^{k}\lambda_i\frac{\partial \Gamma_i}{\partial q_j}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}-\hat{Q}_j \quad \text{for } j=1,2,3 \tag{14}$$

where $\hat{Q}_j$ denotes the generalized force contributed by an externally applied force. Once the Lagrangian multipliers are found from Eq. (14), the second set of equations related to actuation forces can be written as

$$Q_j=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}-\sum_{i=1}^{k}\lambda_i\frac{\partial \Gamma_i}{\partial q_j} \quad \text{for } j=4,5,6 \tag{15}$$

where Q_j is the actuator torque.

The constraint functions are given by

$$\begin{aligned}\Gamma_i&=\overline{M_iB_i}^2-l_2^2\\&=(p_1+bc\phi_i-ac\phi_i-l_1c\phi_ic\theta_i)^2\\&\quad+(p_2+bs\phi_i-as\phi_i-l_1s\phi_ic\theta_i)^2\\&\quad+(p_3-l_1s\theta_i)^2-l_2^2 \quad \text{for } i=1,2,3\end{aligned} \tag{16}$$

The total kinetic energies can be obtained by

$$K=K_p+\sum_{i=1}^{3}(K_{1i}+K_{2i}) \tag{17}$$

where $K_p=\frac{1}{2}m_p\sum_{i=1}^{3}\dot{p}_i^2$, $K_{1i}=\frac{1}{2}(\gamma^2I_m+I_1)\dot{\theta}_i^2$, $K_{2i}=\frac{1}{2}m_2\sum_{i=1}^{3}\dot{p}_i^2+\frac{1}{2}m_2l_1^2\dot{\theta}_i^2$ where γ and I_m denote gear ratio and its moment of inertia.

And total potential energies can be obtained by

$$U=U_p+\sum_{i=1}^{3}(U_{1i}+U_{2i}) \tag{18}$$

where $U_p=-m_pg_cp_3$, $U_{1i}=-m_1g_cl_{1c}s\theta_i$, and $U_{2i}=-m_2g_c(p_3+l_{1c}s\theta_i)$.

The Lagrangian function can be reduced to

$$\begin{aligned}L&=K-U\\&=\frac{1}{2}(m_p+3m_2)\sum_{i=1}^{3}\dot{p}_i^2+\frac{1}{2}(\gamma^2I_m+I_1+m_1l_1^2)\sum_{i=1}^{3}\dot{\theta}_i^2\\&\quad+(m_1l_{1c}+m_2l_1)g_c\sum_{i=1}^{3}s\theta_i+(m_p+3m_2)g_cp_3\end{aligned} \tag{19}$$

Using Eq. (14), the Lagrangian multipliers are determined by

$$\begin{aligned}&2\sum_{i=1}^{3}\lambda_i\left(p_1+bc\phi_i-ac\phi_i-l_1c\phi_ic\theta_i\right)=\left(m_p+3m_2\right)\ddot{p}_1-f_1\\&2\sum_{i=1}^{3}\lambda_i\left(p_2+bs\phi_i-as\phi_i-l_1s\phi_ic\theta_i\right)=\left(m_p+3m_2\right)\ddot{p}_2-f_2\\&2\sum_{i=1}^{3}\lambda_i\left(p_3-l_1s\theta_i\right)=\left(m_p+3m_2\right)\left(\ddot{p}_3-g_c\right)-f_3\end{aligned} \tag{20}$$

Once the Lagrangian multipliers are obtained, the joint torques are calculated by

$$\begin{aligned}\tau_1&=(\gamma^2I_m+I_1+m_2l_1^2)\ddot{\theta}_1-(m_1l_{1c}+m_2l_1)g_cc\theta_1\\&\quad-2l_1\lambda_1[(p_1c\phi_1+p_2s\phi_1+b-a)s\theta_1-p_3c\theta_1]\\\tau_2&=(\gamma^2I_m+I_1+m_2l_1^2)\ddot{\theta}_2-(m_1l_{1c}+m_2l_1)g_cc\theta_2\\&\quad-2l_1\lambda_2[(p_1c\phi_2+p_2s\phi_2+b-a)s\theta_2-p_3c\theta_2]\\\tau_3&=(\gamma^2I_m+I_1+m_2l_1^2)\ddot{\theta}_3-(m_1l_{1c}+m_2l_1)g_cc\theta_3\\&\quad-2l_1\lambda_3[(p_1c\phi_3+p_2s\phi_3+b-a)s\theta_3-p_3c\theta_3]\end{aligned} \tag{21}$$

When the trajectories of the moving platform ($\mathbf{p},\dot{\mathbf{p}},\ddot{\mathbf{p}}$) are given, the procedure to calculate actuator torques ($\boldsymbol{\tau}$) can be summarized as follows:

(a) $\theta_{3i},\theta_{2i},\theta_{1i}$: inverse kinematics (Eqs. (4) and (5))

(b) $\dot{\theta}_i,\dot{\theta}_{3i},\dot{\theta}_{2i}$: velocity analysis (Eqs. (8) and (9))

(c) $\ddot{\theta}_i$: acceleration analysis (Eq. (11))

(d) Lagrangian multiplier (λ_i): Eq. (20)

(e) Actuator torque (τ_i): inverse dynamics (Eq. (21))

IV. Dynamics Simulation and Implementation

The kinematic parameters of the prototype Delta parallel robot are given by Table I and from CAD data, the mass properties are calculated by Table II. Line trajectories with 7 m/sec and 100 m/sec^2 along the x- and y-axes are generated as shown in Fig. 4.

Figure 5 shows the joint torques for the trajectory and payload 0.5kg. It is noted that the simplified dynamics equation matches very well with ADAMS modeling. The small difference comes from the assumption on link 2 for simple dynamics equation and thus fast calculation.

TABLE I

KINEMATIC PARAMETERS OF A DELTA PARALLEL MANIPULATOR

Parameters	Values
Radius of the fixed base (a)	100 mm
Radius of the moving platform (b)	22.5 mm
Length of link1 (l_1)	171 mm
Length of link2 (l_2)	396 mm

TABLE II

MASS PROPERTIES OF A DELTA PARALLEL MANIPULATOR

Parameters	Values
Mass of link1 (m_1)	426 g
Half of mass of link2 (m_2)	69 g
Mass of moving platform (m_p)+payload	96 g + 500 g
Length of link1's mass center (l_{1c})	61.84 mm
Link1's mass moment of inertia (I_1)	39.8 x 10^{-4} kg·m^2
Motor inertia with gear ratio ($\gamma^2 I_m$)	25^2 x 0.24 x 10^{-4} kg·m^2

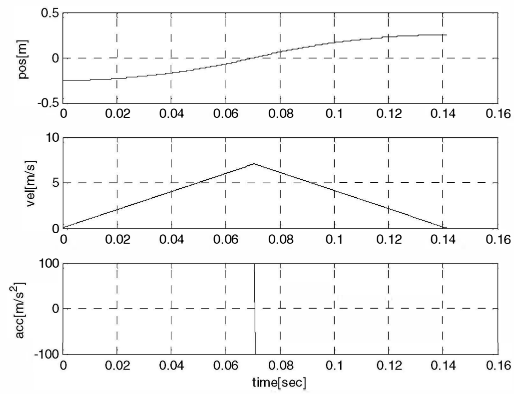


Fig. 4. Line trajectory along the x- and y-axes

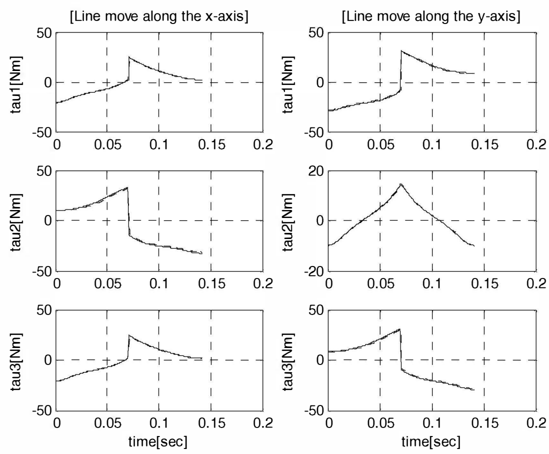


Fig. 5. Inverse dynamics simulation results
(solid: dynamics equation, dotted: ADAMS modeling)

As shown in Fig. 6, the control hardware consists of host PC, real-time target PC with DAQs and AC servo drivers. The host PC running on Windows OS plays a role in Simulink programming, user interface, etc. The target PC running on xPC target provides the real-time control of AC servo motors. Figure 7 shows the control program made by Simulink including PID joint controller and dynamics compensation. The PID joint control is performed in every 0.2msec. The inverse kinematics and dynamics compensation are calculated in every 1.0 msec. Figure 8 shows the TET (Task Execution Time). It is shown that the PID control takes about 0.03msec and the dynamics compensation takes about 0.04msec on Intel CPU (2.16 GHz dual core).

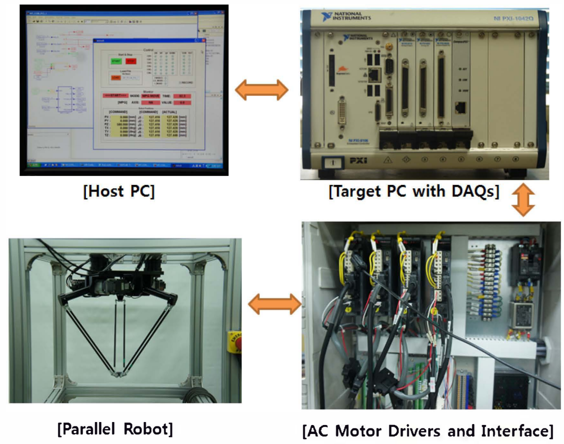


Fig. 6. Configuration of control hardware with xPC Target

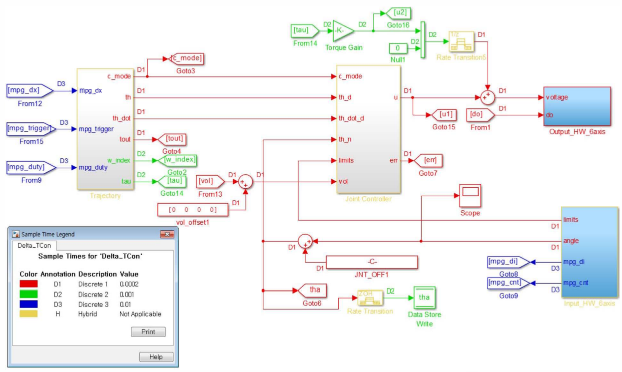


Fig. 7. Control program with Simulink and xPC Target

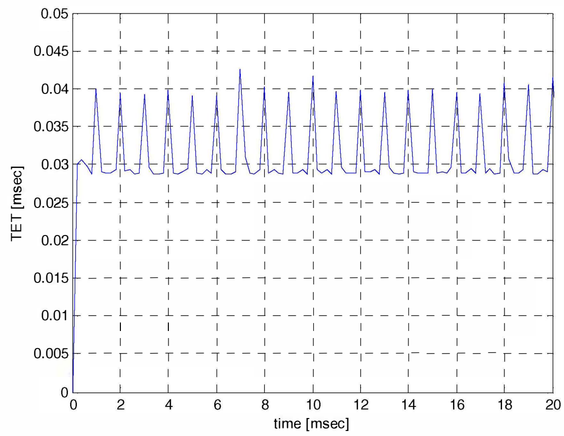


Fig. 8. TET of the Simulink control program

V. CONCLUSIONS

In this paper, a simplified dynamics modeling and hardware implementation of a Delta-type parallel manipulator are presented. For real-time computed torque control, the derivation of the closed form solution of inverse dynamics is essential. The position, velocity, and acceleration analyses are performed. Due to complex kinematics, the inverse dynamics is derived by using Lagrangian equation of the first type. The numerical simulations with ADAMS are performed to verify the accuracy of the analytical dynamic modeling. It is shown that numerical and analytical results match very well. The real-time controller is developed with xPC Target and it is shown that the calculation time of the inverse kinematics and dynamics is less than 0.04msec on Intel CPU (2.16 GHz dual core).

ACKNOWLEDGMENT

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