Data Structures
CSCI H343, Fall 2023

Homework 2

Name: _____

This homework has **??** questions, for a total of **??** points.

1. 6 points Prove that the following `insert` function produces a list whose length is one greater than the length of the input `list`. In other words, prove that

   ```
   length(insert(x, list)) == 1 + length(list)
   ```

   Do the proof by induction on the `length` of the `list`. Indicate where you use the induction hypothesis.

   ```
   public static Node<Integer> insert(int x, Node<Integer> list) {
       if (list == null) {
           return new Node<Integer>(x, null);
       } else if (x <= list.data) {
           return new Node<Integer>(x, list);
       } else {
           return new Node<Integer>(list.data, insert(x, list.next));
       }
   }
   ```

   The `length` of a list is defined as follows

   ```
   public static <T> int length(Node<T> node) {
       if (node == null) {
           return 0;
       } else {
           return 1 + length(node.next);
       }
   }
   ```

Solution:

Base Case: (2 points) We have `length(list) == 0` and therefore `list == null`.

```
length(insert(x, null)) == length(new Node(x, null)) == 1 + length(null)
```

Induction Step: Suppose we have a list `M` of length $k+1$. From that we know two things: `M != null` and the length of `M.next` is k. We have two cases to consider regarding whether `x <= M.data`.

Suppose `x <= M.data`. **(2 points)**

```
length(insert(x, M)) == length(new Node(x, M)) == 1 + length(M)
```

Next, suppose that `x > M.data`. **(2 points)**

```
length(insert(x, M))
== length(new Node(M.data, insert(x, M.next)))
== 1 + length(insert(x, M.next))
== 1 + (1 + length(M.next))            // by the induction hypothesis
== 1 + length(M)
```