Naïve Bayes Classifier

Course: Data Mining

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Subject: Classification Problem and Solution

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Question

A company wants to classify incoming emails as **Spam** or **Not Spam** based on the occurrence of the words *offer* and *discount*. You are given the following information:

Data:

- **Training Emails:**
  - 6 emails classified as **Spam**.
  - 4 emails classified as **Not Spam**.
- **Occurrences of words:**
  - In **Spam emails**:
    - *offer* appears in 4 emails.
    - *discount* appears in 3 emails.
  - In **Not Spam emails**:
    - *offer* appears in 1 email.
    - *discount* appears in 2 emails.

Task: Using Naïve Bayes, calculate whether an email containing the words *offer* and *discount* is classified as **Spam** or **Not Spam**.

Solution

Step 1: Calculate Prior Probabilities

$$P(\text{Spam}) = \frac{\text{Number of Spam emails}}{\text{Total emails}} = \frac{6}{10} = 0.6$$

$$P(\text{Not Spam}) = \frac{\text{Number of Not Spam emails}}{\text{Total emails}} = \frac{4}{10} = 0.4$$

Step 2: Calculate Likelihoods

For Spam:

$$P(\text{''offer''}|\text{Spam}) = \frac{\text{Number of Spam emails with ''offer''}}{\text{Total Spam emails}} = \frac{4}{6} = 0.67$$

$$P(\text{''discount''}|\text{Spam}) = \frac{\text{Number of Spam emails with ''discount''}}{\text{Total Spam emails}} = \frac{3}{6} = 0.5$$

For Not Spam:

$$P(\text{''offer''}|\text{Not Spam}) = \frac{\text{Number of Not Spam emails with ''offer''}}{\text{Total Not Spam emails}} = \frac{1}{4} = 0.25$$

$$P(\text{''discount''}|\text{Not Spam}) = \frac{\text{Number of Not Spam emails with ''discount''}}{\text{Total Not Spam emails}} = \frac{2}{4} = 0.5$$

Step 3: Combine Probabilities Using Independence Assumption

For Spam:

$$P(\text{''offer and discount''}|\text{Spam}) = P(\text{''offer''}|\text{Spam}) \times P(\text{''discount''}|\text{Spam}) = 0.67 \times 0.5 = 0.335$$

For Not Spam:

$$P(\text{''offer and discount''}|\text{Not Spam}) = P(\text{''offer''}|\text{Not Spam}) \times P(\text{''discount''}|\text{Not Spam}) = 0.25 \times 0.5 = 0.1$$

Step 4: Calculate Posterior Probabilities

For Spam:

$$P(\text{Spam}|\text{''offer and discount''}) \propto P(\text{Spam}) \times P(\text{''offer and discount''}|\text{Spam}) = 0.6 \times 0.335 = 0.201$$

For Not Spam:

$$P(\text{Not Spam}|\text{''offer and discount''}) \propto P(\text{Not Spam}) \times P(\text{''offer and discount''}|\text{Not Spam}) = 0.4 \times 0.125 =$$

Step 5: Compare Probabilities

$$P(\text{Spam}|\text{''offer and discount''}) > P(\text{Not Spam}|\text{''offer and discount''})$$

Conclusion: The email is classified as **Spam**.