Hodgkin-Huxley Neuron Model Equations (with Extensions)

1 Hodgkin-Huxley Neuron Model

The voltage dynamics for a single-compartment neuron is described by:

$$C_m \frac{dV}{dt} = -I_{\mathrm{L}} - \sum_{x \in \mathbb{X}} I_x + I_{\mathrm{ext}} + I_{\mathrm{noise}} \tag{1}$$

where C_m is the membrane capacitance, I_{ext} is external current, and I_{noise} is noise current. Here I_x is the ionic current for each ion channel type x in the set $\mathbb{X}$, which includes sodium (Na), potassium (K), L-type and T-type calcium (CaL, CaT), hyperpolarization-activated (H), slow potassium (Ks), after-hyperpolarization (AHP), and calcium-activated non-specific (CAN) channels (i.e., $\mathbb{X} = \{Na, K, CaL, CaT, H, Ks, AHP, CAN\}$). Each ionic current follows Ohm's with activation and inactivation dynamics:

$$I_x = g_x m^a h^b (V - V_x) \tag{2}$$

where g_x is the maximal conductances for ion type x, m and h are activation and inactivation gating variables, respectively, and V_x is the reversal potential for ion type x. The parameters a and b depend on the specific channel type describing the on/off kinetics of the channel provided in the table. All gating variables evolve according to first-order kinetics:

$$\tau_m \frac{dx}{dt} = (m_\infty - m) \tag{3}$$

Here the steady-state value m_∞ is a function of the membrane potential V, and τ_m is the voltage-dependent time constant for the gating variable m. The same applies to inactivation variables h.

2 Ionic Currents

Belows are equations for various ionic currents in the neuron model:

$$I_{\mathrm{L}} = g_{\mathrm{L}}(V - V_{\mathrm{L}}) \tag{4}$$

$$I_{\mathrm{Na}} = g_{\mathrm{Na}}\, m_{\mathrm{Na}}^3\, h_{\mathrm{Na}}\, (V - V_{\mathrm{Na}}) \tag{5}$$

$$I_{\mathrm{K}} = g_{\mathrm{K}}\, n_{\mathrm{K}}^4\, (V - V_{\mathrm{K}}) \tag{6}$$

$$I_{\mathrm{CaL}} = g_{\mathrm{CaL}}\, m_{\mathrm{CaL}}^2 (V - V_{\mathrm{CaL}}) \tag{7}$$

$$I_{\mathrm{CaT}} = g_{\mathrm{CaT}}\, (m_{\mathrm{CaT}}^{\infty})^2\, h_{\mathrm{CaT}} (V - V_{\mathrm{CaT}}) \tag{8}$$

$$I_{\mathrm{AHP}} = g_{\mathrm{AHP}}\, x_{\mathrm{AHP}} (V - V_{\mathrm{AHP}}) \tag{9}$$

$$I_{\mathrm{CAN}} = g_{\mathrm{CAN}}\, x_{\mathrm{CAN}} (V - V_{\mathrm{CAN}}) \tag{10}$$

$$I_{\mathrm{Ks}} = g_{\mathrm{Ks}}\, m_{\mathrm{Ks}}\, h_{\mathrm{Ks}} (V - V_{\mathrm{Ks}}) \tag{11}$$

$$I_{\mathrm{H}} = g_{\mathrm{H}}\, m_{\mathrm{H}}\, (V - V_{\mathrm{H}}) \tag{12}$$

Gating and Activation Variables

Sodium Channel (Na)

$$m_{Na}(V) = \frac{1}{1 + \exp\left(-\frac{V+30}{9.5}\right)} \tag{13}$$

$$h_{Na,\infty}(V) = \frac{1}{1 + \exp\left(\frac{V+53}{7}\right)} \tag{14}$$

$$\tau_h(V) = 0.37 + 2.78 \cdot \frac{1}{1 + \exp\left(\frac{V+40.5}{6}\right)} \tag{15}$$

Potassium Channel (K)

$$n_{K,\infty}(V) = \frac{1}{1 + \exp\left(-\frac{V+30}{10}\right)} \tag{16}$$

$$\tau_n(V) = 0.37 + 1.85 \cdot \frac{1}{1 + \exp\left(\frac{V+27}{15}\right)} \tag{17}$$

L-type Calcium Channel (CaL)

$$m_{CaL,\infty}(V) = \frac{1}{1 + \exp\left(-\frac{V+12}{7}\right)} \tag{18}$$

$$\tau_{CaL}(V) = 10^{(0.6-0.02\cdot V)} \tag{19}$$

T-type Calcium Channel (CaT)

$$m_{CaT,\infty}(V) = \frac{1}{1 + \exp\left(-\frac{V+57}{6.2}\right)} \tag{20}$$

$$h_{CaT,\infty}(V) = \frac{1}{1 + \exp\left(\frac{V+81}{4}\right)} \tag{21}$$

$$\tau_{mT}(V) = 0.612 + \frac{1}{\exp\left(\frac{V+132}{-16.7}\right) + \exp\left(\frac{V+16.8}{18.2}\right)} \tag{22}$$

$$\tau_{hT}(V) = \begin{cases} \exp\left(\frac{V+467}{66.6}\right) & \text{if } V < -80 \\ \exp\left(-\frac{V+22}{10.5}\right) + 28 & \text{otherwise} \end{cases} \tag{23}$$

Hyperpolarization-Activated Channel (H)

$$m_{H,\infty}(V) = \frac{1}{1 + \exp\left(\frac{V-V_{\tau,peak}}{k_\tau}\right)} \tag{24}$$

$$\tau_{mH}(V) = \tau_{min} + \frac{\tau_{diff}}{\exp\left(\frac{V-V_{\tau,peak}}{k_\tau}\right) + \exp\left(-\frac{V-V_{\tau,peak}}{k_\tau}\right)} \tag{25}$$

CAN Channel

$$x_{CAN,\infty}([Ca^{2+}]) = \frac{a_{CAN}\cdot[Ca^{2+}]}{a_{CAN}\cdot[Ca^{2+}] + b_{CAN}} \tag{26}$$

$$\tau_{xCAN}([Ca^{2+}]) = \frac{1}{a_{CAN}\cdot[Ca^{2+}] + b_{CAN}} \tag{27}$$

AHP Channel

$$x_{AHP,\infty}([Ca^{2+}]) = \frac{a_{AHP}\cdot[Ca^{2+}]}{a_{AHP}\cdot[Ca^{2+}] + b_{AHP}} \tag{28}$$

$$\tau_{xAHP}([Ca^{2+}]) = \frac{1}{a_{AHP}\cdot[Ca^{2+}] + b_{AHP}} \tag{29}$$

Slow Potassium Channel (Ks)

$$m_{Ks,\infty}(V) = \frac{1}{1 + \exp\left(-\frac{V+44}{5}\right)} \tag{30}$$

$$h_{Ks,\infty}(V) = \frac{1}{1 + \exp\left(\frac{V+74}{9.3}\right)} \tag{31}$$

$$\tau_{hKs}(V) = 200 + \frac{4800}{1 + \exp\left(-\frac{V+50}{9.3}\right)} \tag{32}$$

General Form

All gating variables follow first-order kinetics:

$$\tau_x \frac{dx}{dt} = (x_\infty - x) \tag{33}$$

where x represents the gating variable, x_∞ is its steady-state value, and τ_x is its voltage-dependent or calcium-dependent time constant.