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Yingcai Hu

May 9, 2023

1 Introduction

2 Methods

2.1 Microbial Consumer-Resource Model (MiCRM)

Here we adopt the framework from Marsland, 2019: the energy per unit time or, The metabolic rate, v_{ij}^{in} of species i using resource j depends on the concentration of resource j, R_j and species preference on a particular resource type p_{ij}:

$$v_{ij}^{in} = \sigma(p_{ij}R_j) = \frac{v_{max}p_{ij}R_{ij}}{R_h + R_{ij}} \tag{1}$$

where the function $\sigma(x) = x_{\max}\frac{x}{k+x}$ is the Monod function that maps the resource availabe to the resources taken by bacteria. The total mass, v_i^{grow}, used for growth of biomass will be defined by setting a fraction (l_j) of mass returned to the environment:

$$v_i^{grow} = \sum_{j=1}^{M}(1 - l_j)v_{ij}^{in} \tag{2}$$

Therefore, assuming there are N species and M types of resouces, the dynamics of biomass abundance C_i of species i^{th} and resource concentration R_j of type j^{th} is

$$\frac{dC_i}{dt} = \mu C_i(v_i^{grow} - \phi_i) \tag{3}$$

$$\frac{dR_j}{dt} = \rho_j - k_m \sum_{i=1}^{N} C_i(v_{ij}^{out} - v_{ij}^{in}) \tag{4}$$

Here, U_{ij} is uptake rate of resource j by speceis i, l_{ij} is the leakage of resource j to the environment in the form of resource k. m_i is the maintainence required for species i. The terms are explained in Table 1.

Table 1: Definition and Units of Parametres

Symbols	Definition	Units
C	Biomass content	$mass$
R	Resouces content	$mass$
U	Uptake rate	$time^{-1}$
l	Fraction of leakage	$None$
m	Maintainence coefficient	$time^{-1}$
ρ	External resource supply	$mass/time$
μ	Constant that scales the resource uptake to growth	$mass^{-1}$
k_m	Constant that scales the maintanence	$mass^{-1}$
k_{ab}	Constant that scales the body mass and resource intake	$mass^{-1}$

Here v_{ij}, v^{grow} and ϕ_i are scaled by biomass for investigating the effect of biomass on population dynamics within the bacterial community.

2.2 Body mass Parametrisation on MiCRM

2.3 Linear Stability Analysis

Let $x = [C_1, C_2, \ldots, C_N, R_1, R_2, \ldots, R_M] \in S$, then $x^* \in S$ is the steady state (i.e. $f'_n(x^*) = 0$ and $f'_{N+m}(x^*) = 0$ for $n = 1, 2, 3, \ldots, N, m = 1, 2, 3, \ldots, N$) . The jocobian matrix of the MiRCM will be defined as followed:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial C_1} & \frac{\partial f_1}{\partial C_2} & \cdots & \frac{\partial f_1}{\partial C_N} & \frac{\partial f_1}{\partial R_1} & \frac{\partial f_1}{\partial R_2} & \cdots & \frac{\partial f_1}{\partial R_M} \\ \frac{\partial f_2}{\partial C_1} & \frac{\partial f_2}{\partial C_2} & \cdots & \frac{\partial f_2}{\partial C_N} & \frac{\partial f_2}{\partial R_1} & \frac{\partial f_2}{\partial R_2} & \cdots & \frac{\partial f_2}{\partial R_M} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_N}{\partial C_1} & \frac{\partial f_N}{\partial C_2} & \cdots & \frac{\partial f_N}{\partial C_N} & \frac{\partial f_N}{\partial R_1} & \frac{\partial f_N}{\partial R_2} & \cdots & \frac{\partial f_N}{\partial R_M} \\ \frac{\partial f_{N+1}}{\partial C_1} & \frac{\partial f_{N+1}}{\partial C_2} & \cdots & \frac{\partial f_{N+1}}{\partial C_N} & \frac{\partial f_{N+1}}{\partial R_1} & \frac{\partial f_{N+1}}{\partial R_2} & \cdots & \frac{\partial f_{N+1}}{\partial R_M} \\ \frac{\partial f_{N+2}}{\partial C_1} & \frac{\partial f_{N+2}}{\partial C_2} & \cdots & \frac{\partial f_{N+2}}{\partial C_N} & \frac{\partial f_{N+2}}{\partial R_1} & \frac{\partial f_{N+2}}{\partial R_2} & \cdots & \frac{\partial f_{N+2}}{\partial R_M} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{N+M}}{\partial C_1} & \frac{\partial f_{N+M}}{\partial C_2} & \cdots & \frac{\partial f_{N+M}}{\partial C_N} & \frac{\partial f_{N+M}}{\partial R_1} & \frac{\partial f_{N+M}}{\partial R_2} & \cdots & \frac{\partial f_{N+M}}{\partial R_M} \end{bmatrix}$$

The f_n and f_{N+m} are vector-valued function that returns Equation 3 and Equation 4, therefore the elements in J are defined as:

$$\frac{\partial f_n}{\partial C_n} = \mu(v_n^{grow} - \phi_n) \tag{5}$$

$$\frac{\partial f_n}{\partial R_m} = \mu C_n(1 - l_m)\frac{v_{nm}^{max} p_{nm} R_m}{(R_h + p_{nm} R_m)^2} \tag{6}$$

$$\frac{\partial f_{N+m}}{\partial C_n} = k_m C_n(v_{nm}^{out} - v_{nm}^{in}) \tag{7}$$

$$\frac{\partial f_{N+m}}{\partial R_m} = k_m \sum_{i=1}^{N} C_i (D_{mm} l_m - 1) \frac{v_{nm}^{max} p_{nm} R_m}{(R_h + p_{nm} R_m)^2} \tag{8}$$

To investigate the stability of the system on steady state x^*, eigenvalue decomposition will be performed on the jocobian evaluated at x^*, i.e., $J_{|x=x^*} = P^{-1} \Lambda P$, using numpy library.