Tutorial 14 : Quadric Surfaces (part 2)

Dr. Mohammad Reza Bahrami

Innopolis University

Course of Essentials of Analytical Geometry and Linear Algebra I

December 4, 2020

Last weeks' topics

- Quadric Surfaces
  - Sphere
  - Ellipsoid

Content

- Quadric Surfaces
  - Paraboloids
  - Hyperboloids

Materials are taken with modification from Thomas' calculus / based on the original work by George B. Thomas, Jr., Massachusetts Institute of Technology; as revised by Maurice D. Weir, Naval Postgraduate School; Joel Hass, University of California, Davis.— Thirteenth edition.

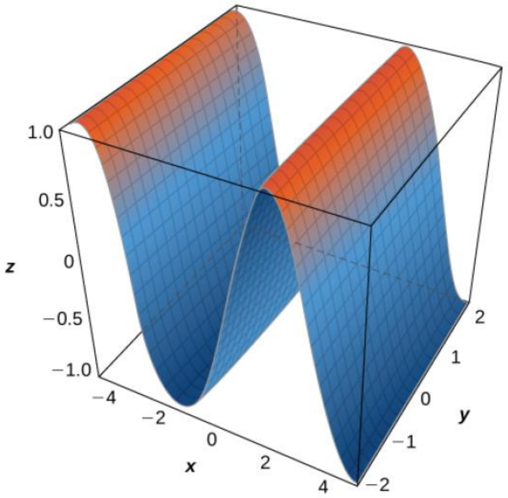
Quadric Surfaces

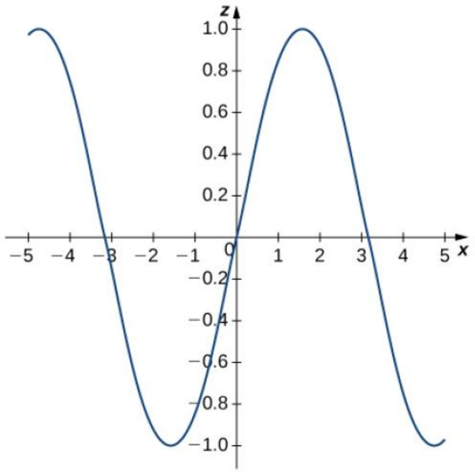
Definition Quadric surfaces are the graphs of equations that can be expressed in the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Jz + K = 0.$$

When a quadric surface intersects a coordinate plane, the trace is a conic section.

Definition
The **traces** of a surface are the cross-sections created when the surface intersects a plane parallel to one of the coordinate planes.





This is one view of the graph of equation $z = \sin x$.
To find the trace of the graph in the *xz*-plane, set $y = 0$. The trace is simply a two-dimensional sine wave.

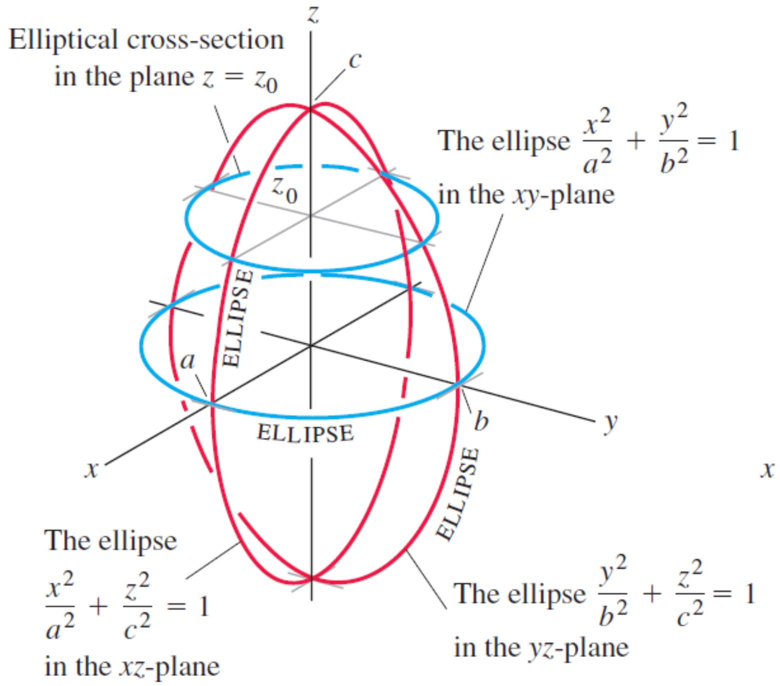
Ellipsoid

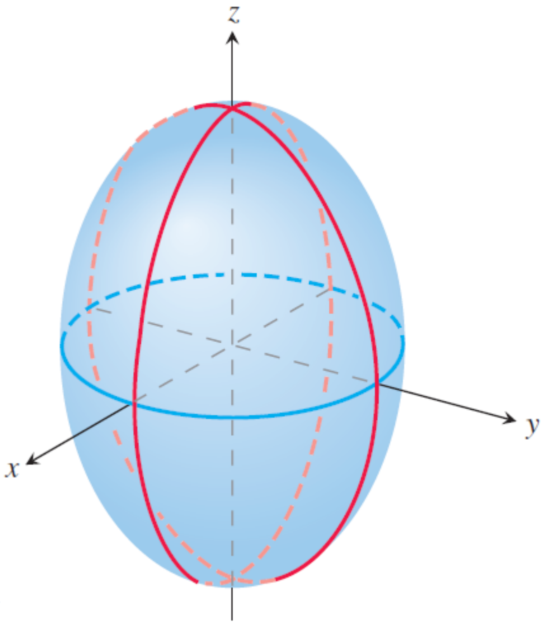
An ellipsoid is a surface described by an equation of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

cuts the coordinate axes at $(\pm a, 0, 0), (0, \pm b, 0),$ and $(0, 0, \pm c)$.

Set $x = 0$ to see the trace of the ellipsoid in the yz-plane. To see the traces in the y- and xz-planes, set $z = 0$ and $y = 0$, respectively. Notice that, if $a = b$, the trace in the xy-plane is a circle. Similarly, if $a = c$, the trace in the xz-plane is a circle and, if $b = c$, then the trace in the yz-plane is a circle. A sphere, then, is an ellipsoid with $a = b = c$.



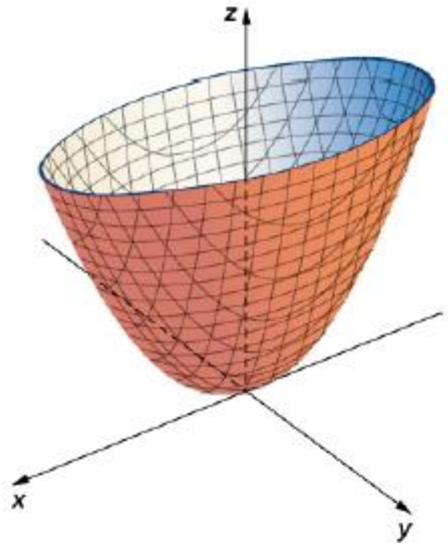


Elliptic Paraboloid

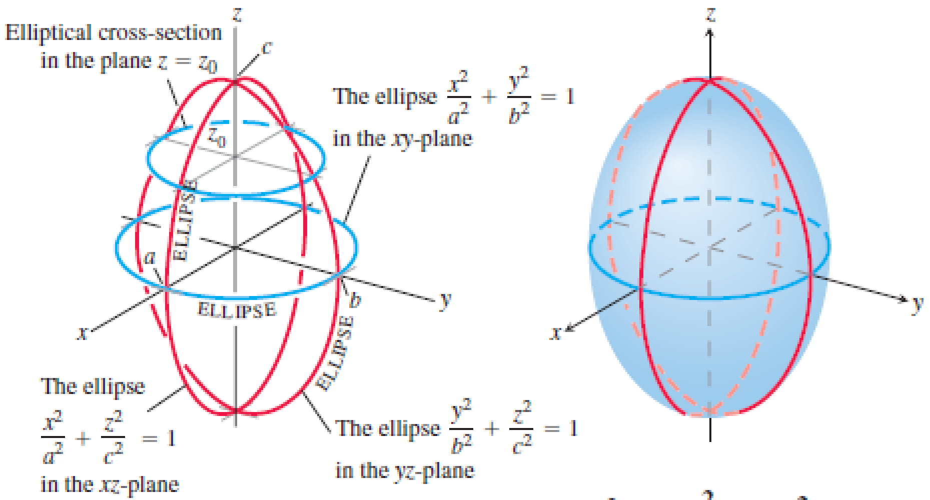
Many quadric surfaces have traces that are different kinds of conic sections, and this is usually indicated by the name of the surface. For example, if a surface can be described by an equation of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

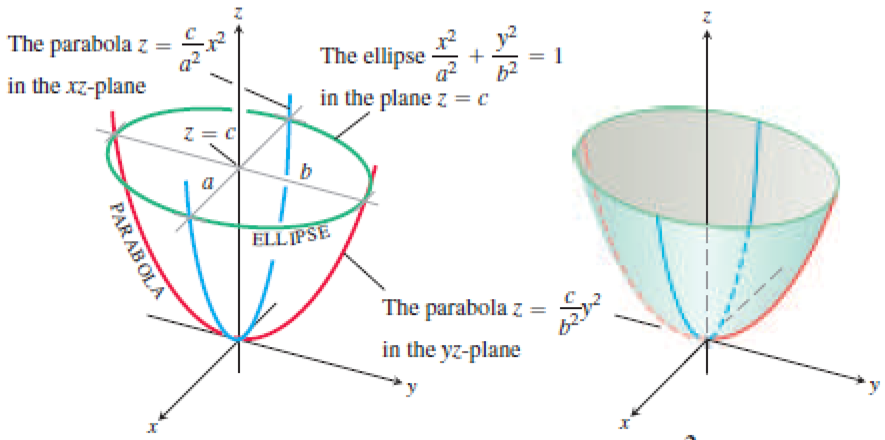
, then we call that surface an **elliptic paraboloid**. The trace in the *xy*-plane is an ellipse, but the traces in the *xz*-plane and *yz*-plane are parabolas (Figure below). Other elliptic paraboloids can have other orientations simply by interchanging the variables to give us a different variable in the linear term of the equation $\frac{x^2}{a^2} + \frac{z^2}{c^2} = \frac{y}{b}$ and $\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{x}{a}$.



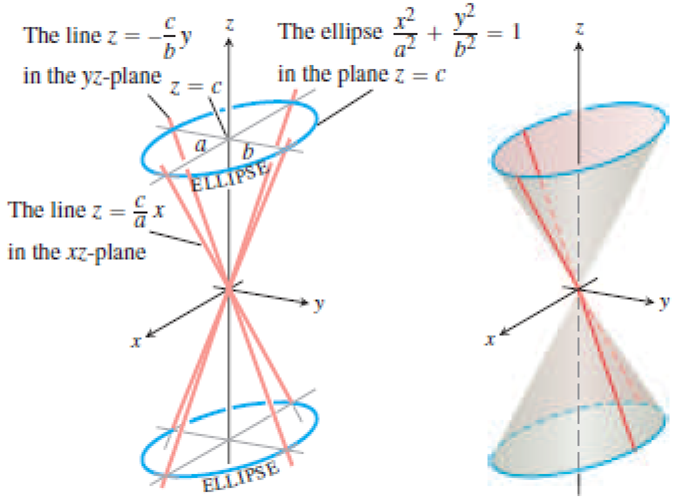
Characteristics of Common Quadratic Surfaces



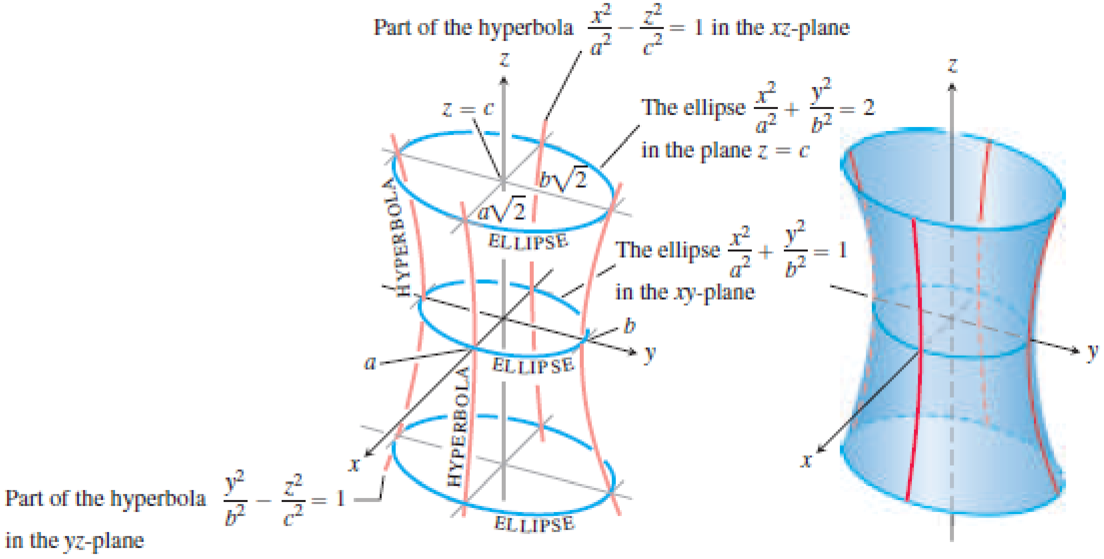
ELLIPSOID $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



ELLIPTICAL PARABOLOID $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

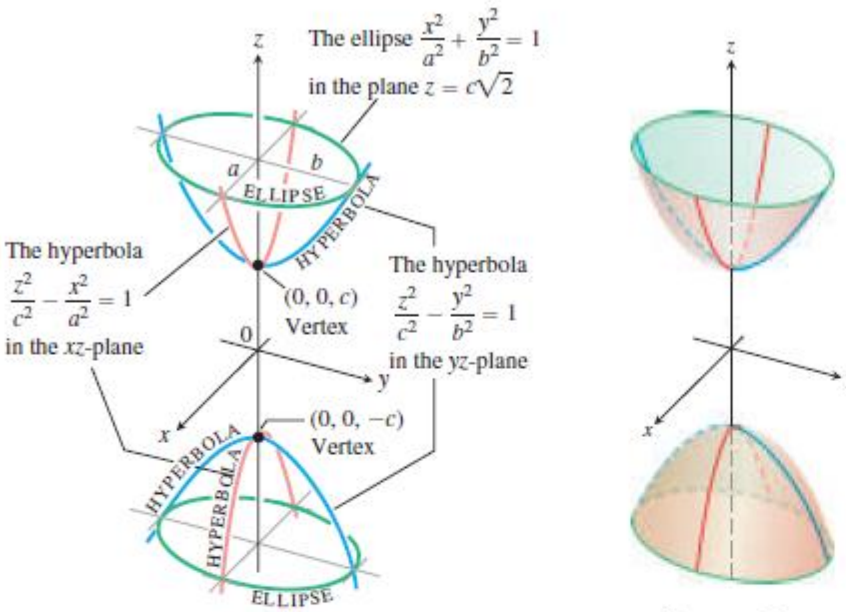


ELLIPTICAL CONE $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$

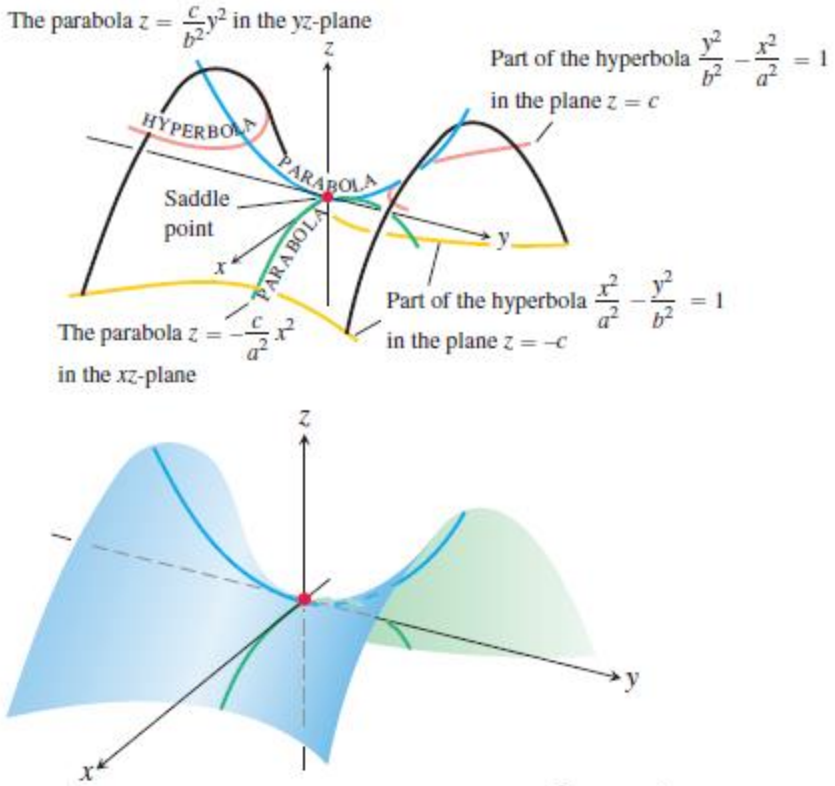


HYPERBOLOID OF ONE SHEET $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Characteristics of Common Quadratic Surfaces



HYPERBOLOID OF TWO SHEETS $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



HYPERBOLIC PARABOLOID $\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, c > 0$

- Sketch the surface given by: $8 - x^2 - y^2 = z$

Solution:

Example 2

- Sketch the surface given by: $4 - 4y^2 - z^2 = x$

Solution:

Example 3

- Sketch the surface given by: $x^2 + y^2 - z^2 = 1$

Solution:

Example 4

- Sketch the surface given by: $-x^2 - y^2 + z^2 = 1$

Solution:

Example 5

- Sketch the surface given by: $x^2 - y^2 = z$

Solution:

Example 6

- Sketch the surface given by: $4x^2 + 4y^2 = z^2$

Solution:

Example 7

➢ Sketch the surface given by: $x^2 + y^2 - z^2 = 4$

Solution:

Example 8

- Sketch the surface given by: $4x^2 = 16y^2 + 9z^2$

Solution:

Example 9

- Identify the surface given by: $9y^2 + z^2 = 16$

Solution:

Example 10

- Identify the surface given by: $-4x^2 - y^2 = z$

Solution:

Example 11

- Identify the surface given by: $x^2 + 4z^2 = y^2$

Solution: