# Innopolis University
# Essentials of Analytical Geometry and Linear Algebra I
# Final Exam

December 18, 2020.

VARIANT 1

| Full name: | Group: |
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| Task: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Score: | | | | | | | | | | | _____ of 40 pts. |

1. (2 points) Determine whether the vectors are linearly independent. Justify your answer.
$\begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -18 \end{bmatrix}$

2. (4 points) Decompose the vector $p = [2, -3, 2]^\top$ into components parallel and perpendicular to the vector $q = [12, 3, 4]^\top$. Find the lengths of both projections.

3. (5 points) Find the inverse of the matrices $A$ and $B$. $A = \begin{bmatrix} 2 & -4 \\ 1 & -3 \end{bmatrix}$; $B = \begin{bmatrix} 2 & 0 & -4 \\ 1 & 2 & 3 \\ 4 & -4 & -18 \end{bmatrix}$

4. (3 points) Given two bases for $\mathbf{R}^2$ (all coordinates are given in standard basis):
$a_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, $a_2 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$ and $b_1 = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 \\ 16 \end{bmatrix}$
Find the change of coordinates matrix from basis $\mathcal{A} = \{a_1, a_2\}$ to $\mathcal{B} = \{b_1, b_2\}$.

5. (3 points) Find the equation of the line passing through the intersection of $2x + y = 8$ and $3x + 7 = 2y$ and parallel to $4x + y = 11$.

6. (3 points) Find the equation of the line passing through the point $[3, 2, -6]^\top$ and perpendicular to the plane $3x - y - 2z + 2 = 0$.

7. (3 points) Find the eccentricity, foci and the length of the latus rectum of the ellipse:
$3x^2 + 4y^2 - 12x - 8y + 4 = 0$.

8. (5 points) Find the equation of the tangent to the ellipse $x^2 + 2y^2 = 6$ at $(2, -1)$.

9. (6 points) Find the equation to the cone whose vertex is the origin and the base circle $x = a, \quad y^2 + z^2 = b^2$ and show that the section of the cone by a plane parallel to the $xy$-plane is hyperbola.

10. (6 points) Find the equation of the sphere which touches the coordinate axes, whose centre lies in the positive octant and has radius 4.

VARIANT 2

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| Task: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
|---|---|---|---|---|---|---|---|---|---|
| Score: | | | | | | | | | |

1. (2 points) Determine whether the vectors are linearly independent. Justify your answer.
   $\begin{bmatrix} 2 \\ 6 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}$

2. (4 points) Decompose the vector $p = [0, -2, 1]^\top$ into components parallel and perpendicular to the vector $q = [2, -1, 0]^\top$. Find the lengths of both projections.

3. (5 points) Find the inverse of the matrices $A$ and $B$. $A = \begin{bmatrix} 5 & -2 \\ 9 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 2 & 6 & -6 \\ 2 & -3 & -1 \\ 3 & 0 & -4 \end{bmatrix}$

4. (3 points) Given two bases for $\mathbf{R}^2$ (all coordinates are given in standard basis):
   $a_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $a_2 = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$ and $b_1 = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$
   Find the change of coordinates matrix from basis $\mathcal{A} = \{a_1, a_2\}$ to $\mathcal{B} = \{b_1, b_2\}$.

5. (3 points) Find the equation of the line passing through the intersection of $2x - 5 = 4y$ and $x + 3y = 12$ and parallel to $x - 3y = 13$.

6. (3 points) Find the equation of the line passing through the point $[-2, 3, -6]^\top$ and perpendicular to the plane $2x - 3y + z - 5 = 0$.

7. (3 points) Find the eccentricity, foci and the center of the hyperbola:
   $9x^2 - 4y^2 + 18x + 16y - 43 = 0$

8. (5 points) Find the equation of the tangent line to the hyperbola $3x^2 - 2y^2 + 20 = 0$ at $(2, 4)$.

9. (6 points) Find the equation to the cone whose vertex is the origin and the base circle $x = a, \quad y^2 + 3z^2 = b^2 + 4$ and show that the section of the cone by a plane parallel to the $xy$-plane is hyperbola.

10. (6 points) Find the equation of the sphere which touches the coordinate axes, whose centre lies in the positive octant and has radius 5.

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VARIANT 3

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| Task: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Score: | | | | | | | | | | | _____ of 40 pts. |

1. (2 points) Determine whether the vectors are linearly independent. Justify your answer.
$\begin{bmatrix}1\\4\\8\end{bmatrix}, \begin{bmatrix}2\\2\\3\end{bmatrix}, \begin{bmatrix}6\\0\\-4\end{bmatrix}$

2. (4 points) Decompose the vector $p = [1, -5, 2]^\top$ into components parallel and perpendicular to the vector $q = [1, 1, 1]^\top$. Find the lengths of both projections.

3. (5 points) Find the inverse of the matrices $A$ and $B$. $A = \begin{bmatrix}-2 & -1\\3 & 4\end{bmatrix}$; $B = \begin{bmatrix}1 & 4 & 8\\2 & 2 & 3\\4 & -2 & -7\end{bmatrix}$

4. (3 points) Given two bases for $\mathbf{R}^2$ (all coordinates are given in standard basis):
$a_1 = \begin{bmatrix}6\\2\end{bmatrix}$, $a_2 = \begin{bmatrix}4\\-12\end{bmatrix}$ and $b_1 = \begin{bmatrix}-2\\-3\end{bmatrix}$, $b_2 = \begin{bmatrix}-5\\4\end{bmatrix}$
Find the change of coordinates matrix from basis $\mathcal{A} = \{a_1, a_2\}$ to $\mathcal{B} = \{b_1, b_2\}$.

5. (3 points) Find the equation of the line passing through the intersection of $2x + 2y = 8$ and $3x + 7 = 2y$ and parallel to $2x - y = 5$.

6. (3 points) Find the equation of the line passing through the point $[5, 5, 4]^\top$ and perpendicular to the plane $-x - 2y - 5z = 9$.

7. (3 points) Find the eccentricity, foci and the length of the latus rectum of the ellipse:
$16x^2 + 25y^2 - 32x + 50y - 359 = 0$,

8. (5 points) Find the equation of the tangent to the ellipse $\frac{(x-3)^2}{4} + \frac{y^2}{9} = 1$ at the point $(1.4,\ 1.8)$.

9. (6 points) Find the equation to the cone whose vertex is the origin and the base circle $x = a, \quad 2y^2 + z^2 = b^2 - 5$ and show that the section of the cone by a plane parallel to the $xy$-plane is hyperbola.

10. (6 points) Find the equation of the sphere which touches the coordinate axes, whose centre lies in the positive octant and has a radius $4\sqrt{2}$.