Class 1

Introduction
Vectors
Operations
Basis
Subspace

Essentials of Analytical Geometry and Linear Algebra 1

Lab requirements

- Attendance — remind me, if I'll forget
- Responsibility — do not cheat
- Activity — ask me questions, ANY questions
- Participation — more participation → less time on HW

Lab organization

- All the tasks beforehand, after the lecture
  - Try to solve it at weekends
  - Prepare your questions
- Weekly quizzes of different types (NOT graded)
- QA sessions
- Videos / examples
- Homeworks

Quiz time !

Pick column and give definitions

Vector

Span

Linear combination

Linear independence

Length

Subspace

Q & A

Task 1

Points $\mathbf{A}(3, -2)$ and $\mathbf{B}(1,4)$ are given. The $\mathbf{M}$ point is on the line $\mathbf{AB}$ in the way that $|\mathbf{AM}| = 3|\mathbf{AB}|$. Find coordinates of the $\mathbf{M}$ point, if:

1. The points $\mathbf{M}$ and $\mathbf{B}$ are from the same side from $\mathbf{A}$.

2. The points $\mathbf{M}$ and $\mathbf{B}$ are from the different sides from $\mathbf{A}$.

Main solution steps

- Find the distance between points $\mathbf{A}$ and $\mathbf{B}$
- Find the equation for the line $|\mathbf{AB}|$
- Find 2 points on the line with distance $3|\mathbf{AB}|$ from A.

Task 2

Check if the result of each of the following operations is a vector or not. Explain your answer.

1. $\mathbf{a} + \mathbf{b}$, if $\mathbf{a}$ and$\mathbf{b}$ are vectors

2. $\mathbf{a} - \mathbf{a}$, if $\mathbf{a}$ is a vector

3. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

4. $\begin{bmatrix} 2x + 15 - 4y \\ y - x \end{bmatrix}$, if x and y are integer numbers

5. $\begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix} - \begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix}$, i if x and y are real numbers

First things first

What is a vector?

Vectors as lists of numbers

Column vectors. Examples

$\begin{bmatrix}1\\2\end{bmatrix}$, $\begin{bmatrix}1\\2\\3\end{bmatrix}$, $\begin{bmatrix}a\\b\\c\end{bmatrix}$ — we will use **this notation**! We represent vectors as **columns**!

And now the Task 2 !

Check if the result of each of the following operations is a vector or not. Explain your answer.

1. $\mathbf{a} + \mathbf{b}$, if $\mathbf{a}$ and$\mathbf{b}$ are vectors
2. $\mathbf{a} - \mathbf{a}$, if $\mathbf{a}$ is a vector
3. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
4. $\begin{bmatrix} 2x + 15 - 4y \\ y - x \end{bmatrix}$, if x and y are integer numbers
5. $\begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix} - \begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix}$, i if x and y are real numbers

Task 3

Three vectors are given $\mathbf{a}\begin{bmatrix}1\\2\end{bmatrix}, \mathbf{b}\begin{bmatrix}-5\\-1\end{bmatrix}, \mathbf{c}\begin{bmatrix}-1\\3\end{bmatrix}$.

Find the vectors $2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$ and $16\mathbf{a} + 5\mathbf{b} - 9\mathbf{c}$.

Task 3 – solution tip

$$2\mathbf{a} + 3\mathbf{b} - \mathbf{c} = 2\begin{bmatrix}1\\2\end{bmatrix} + 3\begin{bmatrix}-5\\-1\end{bmatrix} - \begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}2\cdot 1\\2\cdot 2\end{bmatrix} + \begin{bmatrix}3\cdot(-5)\\3\cdot(-1)\end{bmatrix} - \begin{bmatrix}-1\\3\end{bmatrix}$$

$$16\mathbf{a} + 5\mathbf{b} - 9\mathbf{c} = 16\begin{bmatrix}1\\2\end{bmatrix} + 5\begin{bmatrix}-5\\-1\end{bmatrix} - 9\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}16\cdot 1\\16\cdot 2\end{bmatrix} + \begin{bmatrix}5\cdot(-5)\\5\cdot(-1)\end{bmatrix} - \begin{bmatrix}9\cdot(-1)\\9\cdot 3\end{bmatrix}$$

Task 4

Check for each case if the following set of vectors is a basis or not.
Explain your answer.

1. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

2. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

4. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

5. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

6. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

First things first

What is a basis?

Basis in $\mathbb{R}^2$

Basis in $\mathbb{R}^2$

A **set** of vectors is a *basis* of $\mathbb{R}^2$ if it spans $\mathbb{R}^2$ and this set is **linearly independent**.

Standard basis in $\mathbb{R}^2$

$\{\hat{\mathbf{i}}, \hat{\mathbf{j}}\} = \{(1,0),(0,1)\}$ is a basis of $\mathbb{R}^2$. They are the standard basis in $\mathbb{R}^2$.

Standard basis in $\mathbb{R}^3$

$\{\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}\} = \{(1,0,0),(0,1,0),(0,0,1)\}$ is a basis of $\mathbb{R}^2$. They are the standard (canonical) basis in $\mathbb{R}^3$.

What does it means ‘span’?

Span

Span

Let $S = \{\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_n}\} \subset V$.

$$span(S) \equiv \left\{ \mathbf{w} \in V : \mathbf{w} = \sum_{k=1}^{n} c_k \mathbf{v_k}, \quad \forall c_k \in \mathbb{R} \right\}$$

In words, $W = span(S)$ is the set of all (possible) linear combinations of the vectors $\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_n}$.
Note that W is a subspace of V.

What does it means ‘linearly imdependent’?

Linear independence in $\mathbb{R}^2$ and in $\mathbb{R}^3$

Linearly independent vectors in $\mathbb{R}^2$

Two vectors $\mathbf{a}$ and $\mathbf{b}$ are *linearly independent* if for $\alpha_1, \alpha_2 \in \mathbb{R}$, $\alpha_1\mathbf{a} + \alpha_2\mathbf{b} = \mathbf{0}$ if and only if $\alpha_1 = \alpha_2 = 0$.

Linearly independent vectors in $\mathbb{R}^3$

Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are *linearly independent* if for $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$, $\alpha_1\mathbf{a} + \alpha_2\mathbf{b} + \alpha_3\mathbf{c} = \mathbf{0}$ if and only if $\alpha_1 = \alpha_2 = \alpha_3 = 0$.

And now the Task 4

Check if the set of vectors is a basis or not.

1. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Let's think…

1. What is this?
2. Basis of what?

And now the Task 4

Check if the set of vectors is a basis or not.

2. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Let’s think…

1. What is this?
2. Basis of what?

And now the Task 4

Check if the set of vectors is a basis or not.

3. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Let's think…

1. What is this?
2. Basis of what?

And now the Task 4

Check if the set of vectors is a basis or not.

4. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

Let’s think…

1. What is this?
2. Basis of what?

And now the Task 4

Check if the set of vectors is a basis or not.

5. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

Let's think…

1. What is this?
2. Basis of what?

And now the Task 4

Check if the set of vectors is a basis or not.

6. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Let's think…

1. What is this?
2. Basis of what?

Task 5

Check for each case if the following set of vectors is a subspace or not. Explain your answer.

1. Part of the plane $x > 0$
2. Entire plane
3. Part of the plane $y < 0$
4. Part of the plane $x > 0,\, y > 0$
5. Inner circle with the radius $r = 5$

First things first

What is a subspace?

Subspace

Definition

W is a subspace of V if

a) $W \subset V$ (subset)

b) $\mathbf{u}, \mathbf{v} \in W \Rightarrow \mathbf{u} + \mathbf{v} \in W$ (closure under addition)

c) $\mathbf{u} \in W, \lambda \in \mathbb{R} \Rightarrow \lambda\mathbf{u} \in W$ (closure under scalar multiplication)

And now the Task 5

Check for each case if the following set of vectors is a subspace or not. Explain your answer.

1. Part of the plane $x > 0$
2. Entire plane
3. Part of the plane $y < 0$
4. Part of the plane $x > 0,\, y > 0$
5. Inner circle with the radius $r = 5$

Kahoot time !

Task 7

Check for each case if the following set of vectors is a coplanar or not.
Explain your answer.

1. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$
2. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$
3. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

First things first

What is a coplanar vectors?

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coplanar vertors

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Coplanar vectors are the **vectors** which lie on the same plane, in a three-dimensional space. These are **vectors** which are parallel to the same plane. We can always find in a plane any two random **vectors**, which are **coplanar**.

And now the Task 7

Check for each case if the following set of vectors is a coplanar or not.
Explain your answer.

1. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

And now the Task 7

Check for each case if the following set of vectors is a coplanar or not. Explain your answer.

2. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$

And now the Task 7

Check for each case if the following set of vectors is a coplanar or not. Explain your answer.

3. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

Task 6

Find the coordinates of the gravity center of a triangular plate $\mathbf{ABC}$ with vertices in points $\mathbf{A}(3,1), \mathbf{B}(6,3), \mathbf{C}(0,2)$.

Task 8

In the plane of the triangle $\mathbf{ABC}$ find the point $\mathbf{O}$ such that

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \mathbf{0}$. Are there such points outside of the triangle?

Note: $\mathbf{0}$ is a zero-vector.

Quiz task

- What is the length of each following vector?
- Are they linearly dependent?
- What is their span?
- Give an example of linear combination of this vectors

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$$