

Essentials of Analytical Geometry and Linear Algebra. Lecture 5.

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End of Lecture #4

- Part 1. Change of basis and coordinates
- Part 2. Matrix rank
- Part 3. Matrix inverse

Quiz in class

Go to <http://b.socrative.com>

Type Room: **LINAL**

Answer questions.

Lecture 5. Outline

- Part 1. Matrix inverse recap. General method
- Part 2. Applications
- Part 3. Summary of the block / What is in the next block?

Part 1. Matrix inverse recap. General method

Step by step

Find inverse for a square matrix A

Step 1. Find $\det(A)$

Step 2. Build a matrix with minors: $M = m_{ij}$

Step 3. Build a matrix with ± 1 : $H = h_{ij}$, $h_{ij} = (-1)^{i+j}$

Step 4. Build a cofactor matrix: $C = H \odot M$, $c_{ij} = h_{ij}m_{ij}$

Step 5. Transpose and scale $\frac{1}{\det(A)}C^T$

Example

Here we inverse a 3×3 matrix

Demo

Here we check source code

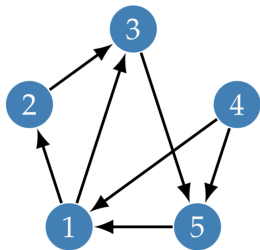
Homework

Implement the method to find inverse for a square matrix A .

Part 2. Applications

Graphs and Matrices

Given a graph you can define its adjacency matrix, A

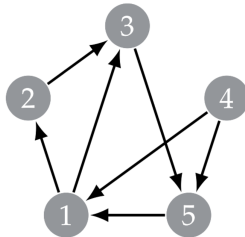


$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Graphs and Matrices: Powers of A

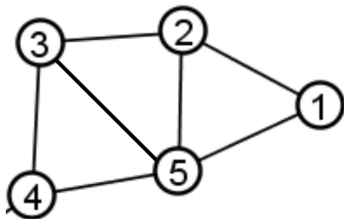
Given an adjacency matrix, A you can find its power ($A^2 = AA$)

$$A^2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$



Graphs and Matrices: Example

Given a graph G



Build its adjacency matrix, A

Find A^3 .

Find the trace of A^3 , $Tr(A^3)$

How can you interpret it?

Systems of equations

[TBA]

Linear models and Matrices

[TBA]

Neural Networks and Matrices

[TBA]

Part 3. Summary / What is in the next block?

End of Lecture #5

Next week:

- Lines in space
- Equations of line
- Finding distances

Useful links

- <https://www.geogebra.org>
- https://youtu.be/fNk_zzaMoSs
- <http://immersivemath.com/ila>