Tutorial 13 : Quadric Surfaces (part 1)

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Course of Essentials of Analytical Geometry and Linear Algebra I

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Last weeks' topics

- Quadratic Curves
  - Parabolas
  - Circles
  - Ellipses
  - Hyperbolas
  - Rotation of axes

Content

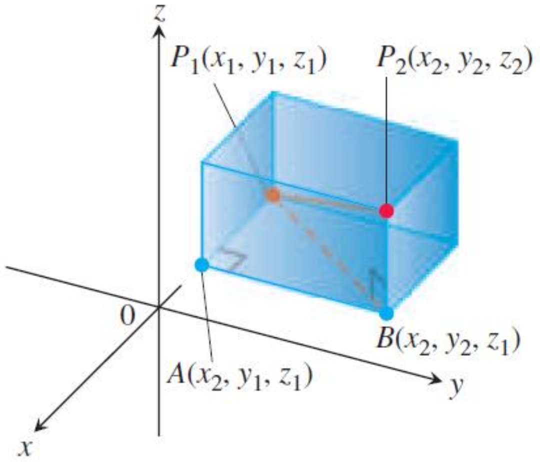
- Quadric Surfaces
  - Sphere
  - Ellipsoid

Materials are taken with modification from Thomas' calculus / based on the original work by George B. Thomas, Jr., Massachusetts Institute of Technology; as revised by Maurice D. Weir, Naval Postgraduate School; Joel Hass, University of California, Davis.— Thirteenth edition.

Distance and Spheres in Space

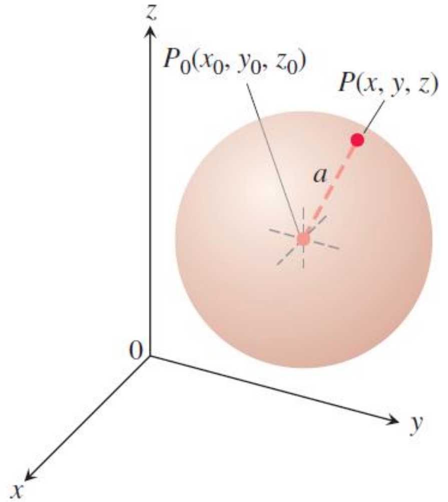
The formula for the distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ in space.

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



We can use the distance formula to write equations for spheres in space. A point $P(x, y, z)$ lies on the sphere of radius a centered at $P_0(x_0, y_0, z_0)$ precisely when $|P_0P| = a$ or

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$



Example 1

- Find the centers and radii of the spheres given by: $2x^2 + 2y^2 + 2z^2 + x + y + z = 9$

Solution:

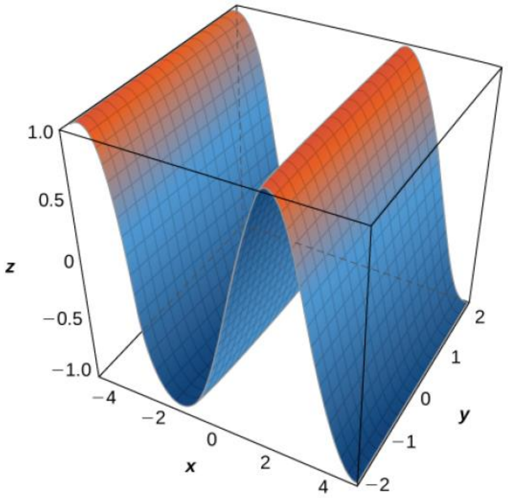
Quadric Surfaces

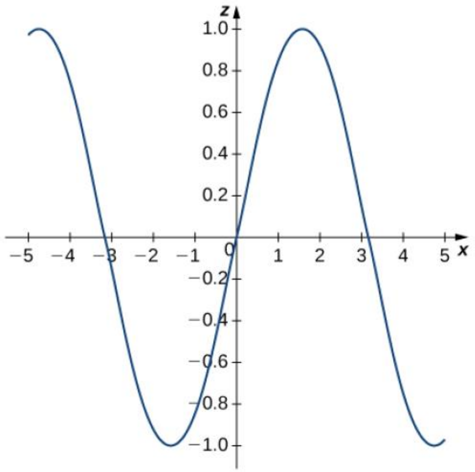
Definition Quadric surfaces are the graphs of equations that can be expressed in the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Jz + K = 0.$$

When a quadric surface intersects a coordinate plane, the trace is a conic section.

Definition
The **traces** of a surface are the cross-sections created when the surface intersects a plane parallel to one of the coordinate planes.





This is one view of the graph of equation $z = \sin x$.
To find the trace of the graph in the xz-plane, set $y = 0$. The trace is simply a two-dimensional sine wave.

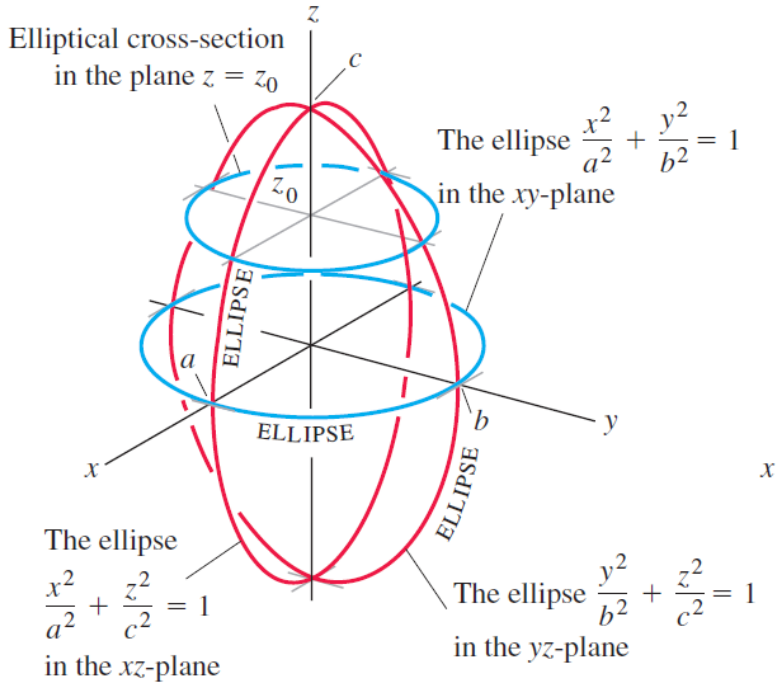
Ellipsoid

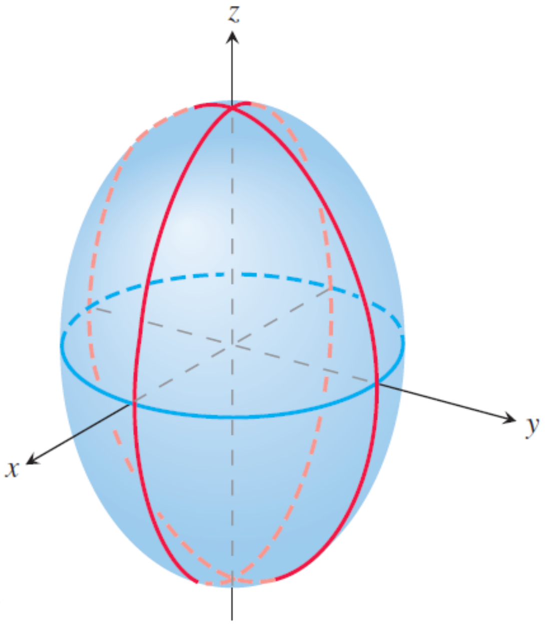
An ellipsoid is a surface described by an equation of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

cuts the coordinate axes at $(\pm a, 0, 0), (0, \pm b, 0),$ and $(0, 0, \pm c)$.

Set $x = 0$ to see the trace of the ellipsoid in the yz-plane. To see the traces in the y- and xz-planes, set $z = 0$ and $y = 0$, respectively. Notice that, if $a = b$, the trace in the xy-plane is a circle. Similarly, if $a = c$, the trace in the xz-plane is a circle and, if $b = c$, then the trace in the yz-plane is a circle. A sphere, then, is an ellipsoid with $a = b = c$.





Example 2

- Sketch the ellipsoid $\frac{x^2}{4}+\frac{y^2}{16}+\frac{z^2}{9}=1$

Solution:

Example 3

- For the surface $4x^2 + y^2 - z = 0$, classify the indicated trace as an ellipse, hyperbola, or parabola.

a) $x = 0$ (b) $y = 0$ (c) $z = 1$

Solution:

Example 4

- Sketch the surface $9x^2 + 4y^2 + 36\, z^2 = 36$.

Solution:

Example 5

- Identify the quadric surface.

a) $16x^2 + 9y^2 + 16z^2 = 144$

b) $9x^2 - 18x + 4y^2 + 16y - 36z + 25 = 0$

Solution:

Example 6

- Rewrite the given equation of the quadric surface in standard form. Identify the surface: is it an ellipsoid?

$$x^2 + 5y^2 + 3z^2 - 15 = 0$$

Solution:

Example 7

- Rewrite the given equation of the quadric surface in standard form. Identify the surface: is it an ellipsoid?

$$-3x^2 + 5y^2 - z^2 - 10 = 0$$

Solution:

Example 8

- Write the standard form of the equation of the ellipsoid centered at the origin that passes through points $A(2, 0, 0), B(0, 0, 1)$, and $C(\frac{1}{2}, \sqrt{11}, \frac{1}{2})$

Solution:

Example 9

- Write the standard form of the equation of the ellipsoid centered at point $P(1, 1, 0)$ that passes through points $A(6, 1, 0)$, $B(4, 2, 0)$ and $C(1, 2, 1)$.

Solution:

- Helpful Links
  - https://math.libretexts.org/Bookshelves/Calculus/Book%3A_Calculus_(OpenStax)/12%3A_Vectors_in_Space/12.6%3A_Quadric_Surfaces

- Next Week Topics
  - Quadric Surfaces (to be continued)