Essentials of Analytical Geometry and Linear Algebra I, Class #11

Innopolis University, September 2020

1 Hyperbola, polar coordinates

1. Find the equations of directrices and asymptotes of a hyperbola:
   (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$;
   (b) $xy = -2$
2. Find the equation of a hyperbola if it's known that it has a canonical form and:
   (a) the eccentricity of a hyperbola is equal to 1.4, and the distance from its focus to the nearest vertex is 2;
   (b) the angle between the asymptotes that contains a focus is equal to $60°$ and the distance from the vertex to the nearest directrix is equal to $3 - \frac{3\sqrt{3}}{2}$.
3. Show that the locus of midpoints of normal chords of the hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^2 = 4a^2xy$.
4. Fin the equation of the line joining the points $\begin{bmatrix} 2 \\ \frac{\pi}{3} \end{bmatrix}$ and $\begin{bmatrix} 3 \\ \frac{\pi}{6} \end{bmatrix}$. It should deduce that this line also passes through the point $\begin{bmatrix} \frac{6}{3\sqrt{3}-2} \\ \frac{\pi}{2} \end{bmatrix}$.
5. Show that the feet of the perpendiculars from the origin on the sides of the triangle formed by the points with vectorial angles α, β, γ and which lie on the circle $r = 2acos(\theta)$ lie on the straight line $2a\ cos(\alpha)cos(\beta)cos(\gamma) = r\ cos(\pi - \alpha - \beta - \gamma)$.
6. A focal chord SP of an ellipse is inclined at an angle α to the major axis. Prove that the perpendicular from the focus on the tangent at P makes with the axis an angle $atan(\frac{sin(\alpha)}{e+cos(\alpha)})$

Essentials of Analytical Geometry and Linear Algebra I, HW #3

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2 Hyperbola, polar coordinates

1. Find the equation of the hyperbola whose focus is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, directrix $2x+y=1$ and eccentricity $\sqrt{3}$.

   Ans: $7x^2 + 12xy - 2y^2 - 2x + 14y - 22 = 0$

2. Find the equation of the hyperbola whose foci are $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} -4 & 4 \end{bmatrix}$ and eccentricity 2.

   Ans: $\frac{4(x-1)^2}{25} - \frac{4(y-4)^2}{75} = 1$

3. Find the equation of the asymptotes of the hyperbola $9y^2 - 4x^2 = 36$. Obtain the product of the perpendicular distance of any point on the hyperbola from the asymptotes.

   Ans: asymptotes $ay^2 - 4x^2 = 0$, distance $\frac{36}{13}$

4. Fint the equation of the line perpendicular to $\frac{l}{r} = cos(\theta - \alpha) + e\ cos(\theta)$ and passing through the point $\begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$.

   Ans: $\frac{r_1 sin(\theta_1 - \alpha) + e\ sin(\theta_1)}{r} = sin(\theta - \alpha) + e\ sin(\theta)$

5. A circle passes through the point $\begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$ and touches the initial line at a distance c from the pole. Show that its polar equation is $\frac{r^2 - 2cr\ cos(\theta) + c^2}{r\ sin(\theta)} = \frac{r_1^2 - 2cr_1\ cos(\theta) + c^2}{r_1\ sin(\theta_1)}$

6. If the tangent and normal at any point P of a conic meet the transverse axis of T and G, respectively, and if S be the focus then prove that $\frac{1}{SG} - \frac{1}{-ST}$ is a constant.