Theoretical Computer Science

Tutorial - week 3

February 4, 2021

Innopolis University

Agenda

- Recap
- A bit of theory ... again
- Examples of Finite State Automaton

Recap

- What is ...

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- What is ... FSA (FSM)?

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- Why is F?

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- What else is finite?
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- What is complete FSA?
- What is extended transition?
- Automata and Automaton?

Finite State Automata

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Let's see an example.

FSA - Example (intuition)

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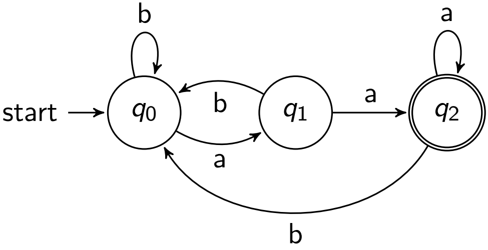
Does the following FSA accepts all and only the strings represented by the language L_1?



Example (informally)

String $x = ababaa$ belongs to L_1. Meaning, $x \in \Sigma^*$ and it ends with aa. Let's see if the FSA accepts the string x

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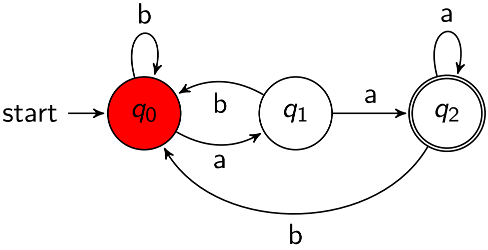
q_0 is the starting point (it is graphically denoted by *start*). So the FSA starts in state q_0

$x = ababaa$



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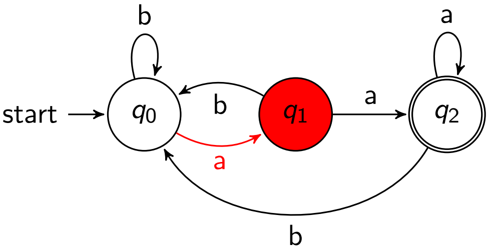
$x = \mathbf{a}babaa$

From state q_0 and label a, we reach state q_1



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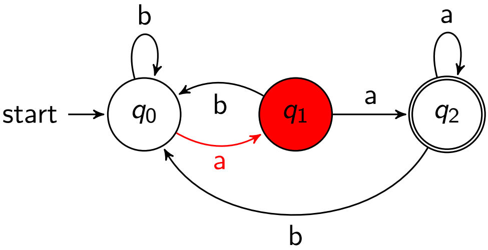
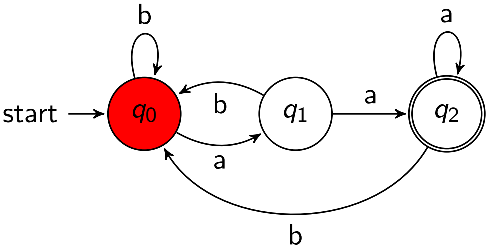


We repeat the process for all characters in x. If at the end we reach a final state (graphically denoted by the double circle state), we say the FSA accepts the string x.

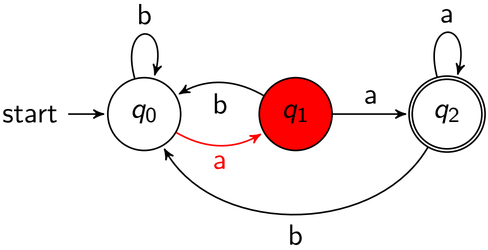
Example (informally) (1)

$x = ababaa$



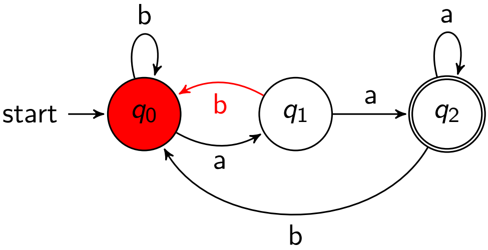
Example (informally) (2)

$x = \mathbf{a}babaa$



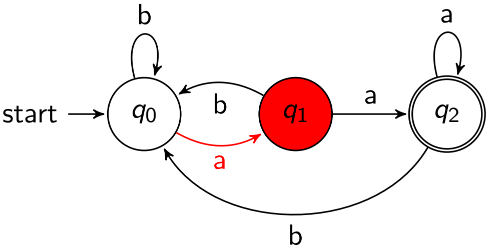
Example (informally) (3)

$x = a\mathbf{b}abaa$



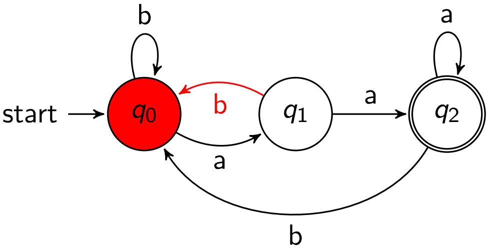
Example (informally) (4)

$x = ab\mathbf{a}baa$



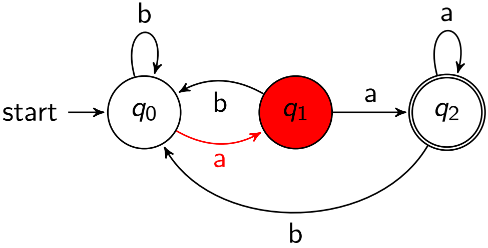
Example (informally) (5)

$x = aba\mathbf{b}aa$



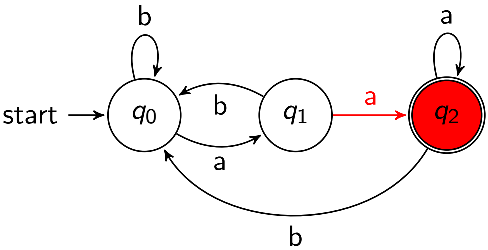
Example (informally) (6)

$x = abab\mathbf{a}a$



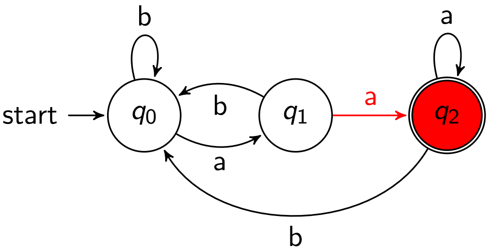
Example (informally) (7)

$x = ababa\mathbf{a}$



Example (informally) (7)

$x = ababa\mathbf{a}$



We went through all characters of x and ended up in a final state: string x belongs to language L_1.

FSA (Formal definition)

A complete Finite State Automaton

A complete Finite State Automaton is a tuple $< Q, \Sigma, q_0, A, \delta >$, where

Q is a finite set of *states*;
Σ is a finite *input alphabet*;
$q_0 \in Q$ is the *initial* state;
$A \subseteq Q$ is the set of *accepting* states;
$\delta : Q \times \Sigma \rightarrow Q$ is a total *transition* function.

For any element q of Q and any symbol $\sigma \in \Sigma$, we interpret $\delta(q, \sigma)$ as the state to which the FSA moves, if it is in state q and receives the input σ.

The extended transition δ^*

A move sequence starts from an initial state and is *accepting* if it reaches one of the final states (informally explained with the previous example).
Formally, this transition is defined recursively:

the extended transition δ^*

Let $M =< Q, \Sigma, q_0, A, \delta >$ be a complete finite state automaton. We define the extended transition function

$$\delta^* : Q \times \Sigma^* \to Q$$

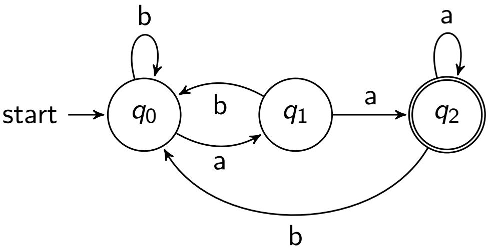
as follows:

1. For every $q \in Q$, $\delta^*(q, \epsilon) = q$
2. For every $q \in Q$, every $y \in \Sigma^*$, and every $\sigma \in \Sigma$,

$$\delta^*(q, y\sigma) = \delta(\delta^*(q, y), \sigma)$$

The extended transition (Example)

The complete FSA M contains the following transitions



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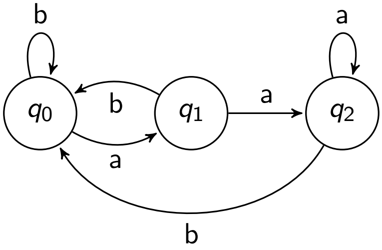

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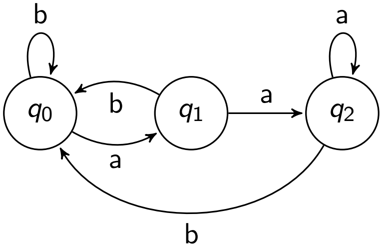

$$
\begin{aligned}
\delta^*(q_1, bab) &= \delta(\delta^*(q_1, ba), b) \\
&= \delta(\delta(\delta^*(q_1, b), a), b) \\
&= \delta(\delta(\delta(\delta^*(q_1, \epsilon), b), a), b) \\
&= \delta(\delta(\delta(q_1, b), a), b) \\
&= \delta(\delta(q_0, a), b) \\
&= \delta(q_1, b) \\
&= q_0
\end{aligned}
$$

Acceptance by a FSA

The extended transition function is used to determine what it means for a FSA to accept (or reject) a string or a language.

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Acceptance by a FSA

Let $M =< Q, \Sigma, q_0, A, \delta >$ be a complete FSA, and let $x \in \Sigma^*$. The string x is accepted by M if

$$\delta^*(q_0, x) \in A$$

and it is rejected by M otherwise. The language accepted by M is the set

$$L(M) = \{x \in \Sigma^* \mid x \text{ is accepted by } M\}$$

If L is a language over Σ, L is accepted by M iff $L = L(M)$

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- Properties of objects: ex. - number of sides
- Functions: ex. - $2 + 2$

Example 2

Build[1] a complete FSA accepting a language L which comprises of all binary representations of integers divisible by 3.

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Let's consider 2 solutions:

1. L includes ϵ (for simplicity)
2. L **does not** include ϵ

[1] using a tool available here or here

Wrap up

- What have you learnt today?
- Why problems are about accepting strings?