Theoretical computer science

Ttutorial - week 8

March 11, 2021

Agenda

- Operations on DPDA

Recap - Operations

Recap - Operations

- Union

Recap - Operations

- Union
- Intersection

Recap - Operations

- Union
- Intersection
- Difference

Recap - Operations

- Union
- Intersection
- Difference
- Complement

Operations

Suppose L_1 and L_2 are both languages over the alphabet A. If $x \in A^*$, then knowing whether $x \in L_1$ and whether $x \in L_2$ is enough to determine whether $x \in L_1 \cup L_2$.

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- $L_1 \cap L_2$?

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- $L_1 \cup L_2$?
- $L_1 \cap L_2$?
- $L_1 \backslash L_2$?

Example 1

Suppose A and B are both languages over the alphabet $\Sigma = \{a, b, c\}$:

$$AnBn = \{a^n b^n | n \geq 1\}$$

$$AnB2n = \{a^n b^{2n} | n \geq 1\}$$

What is the language recognized by $AnBn \cup AnB2n$?

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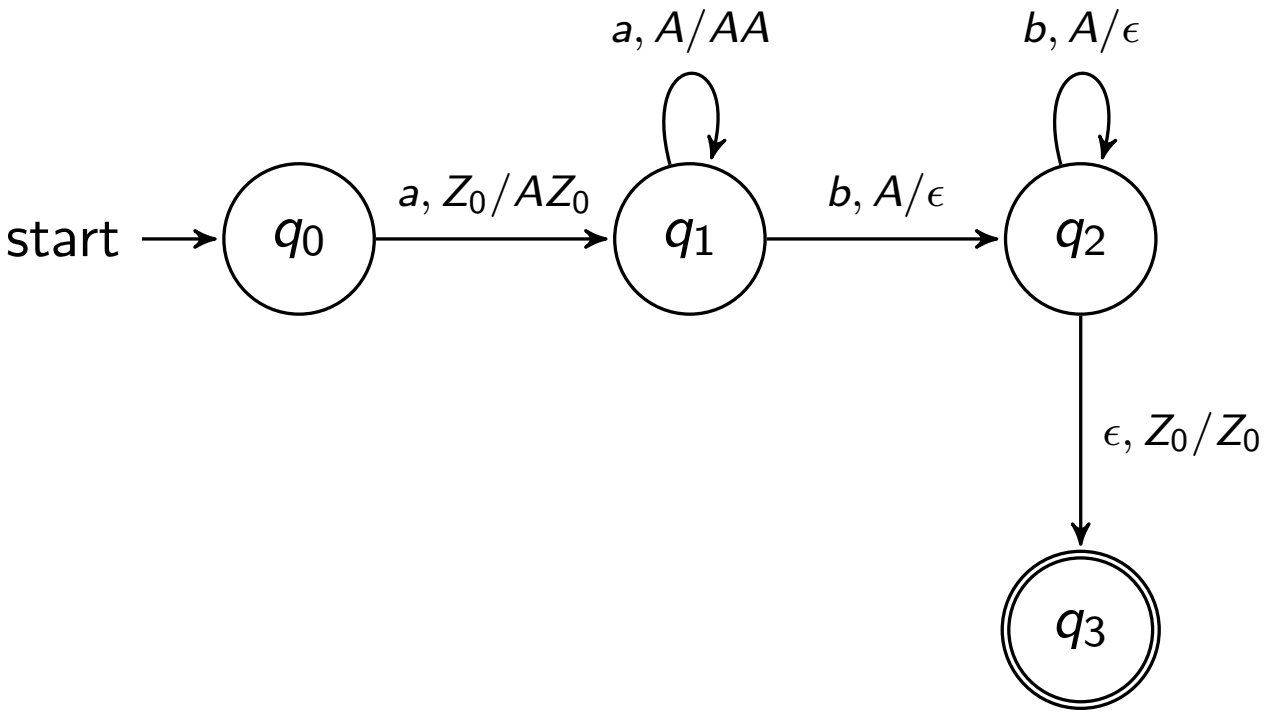
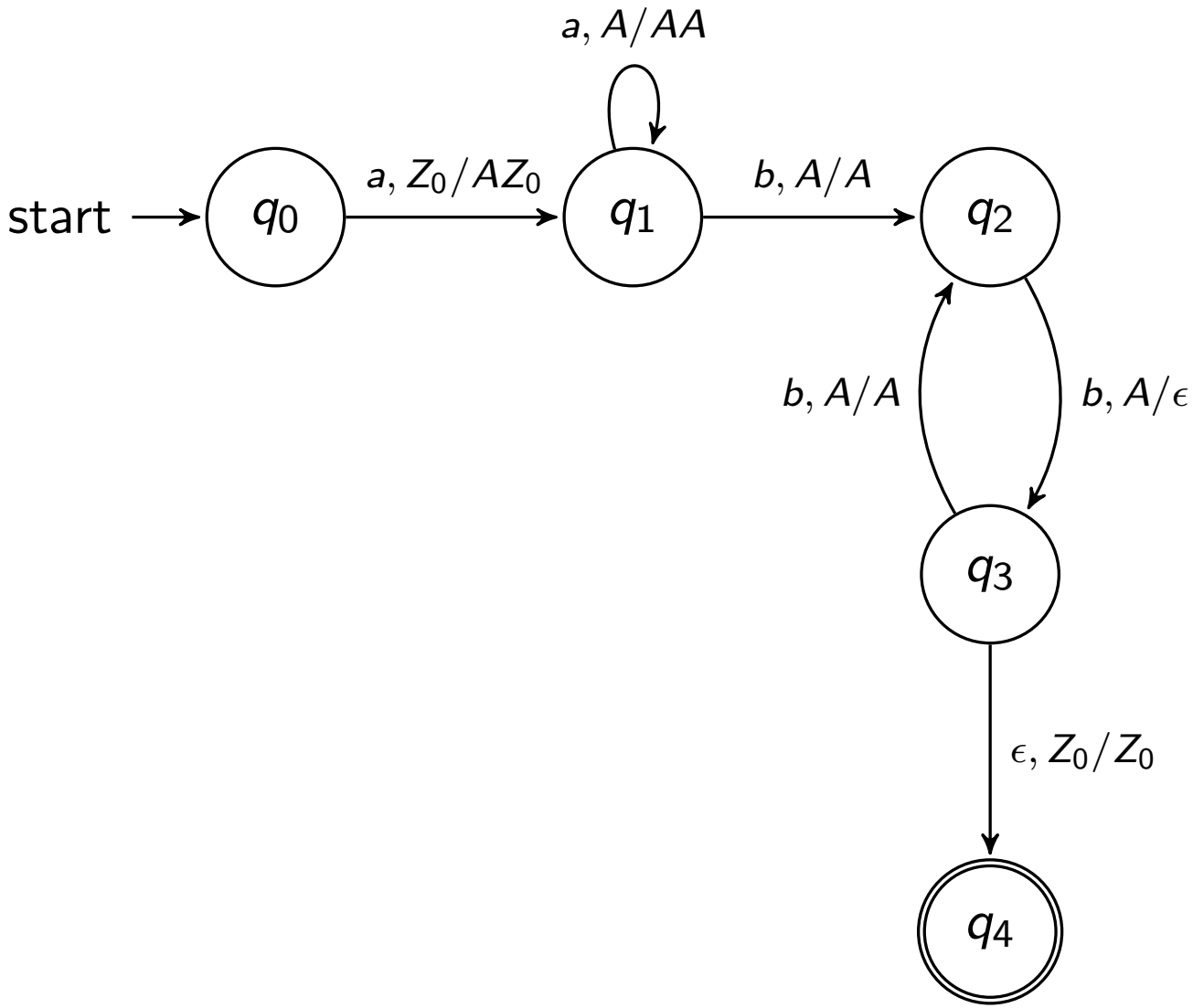
$$AnBn \cup AnB2n = \{a^n b^n | n \geq 1\} \cup \{a^n b^{2n} | n \geq 1\}$$

This language cannot be recognized by DPDA, thus the class of languages recognized by PDA is not closed under intersection

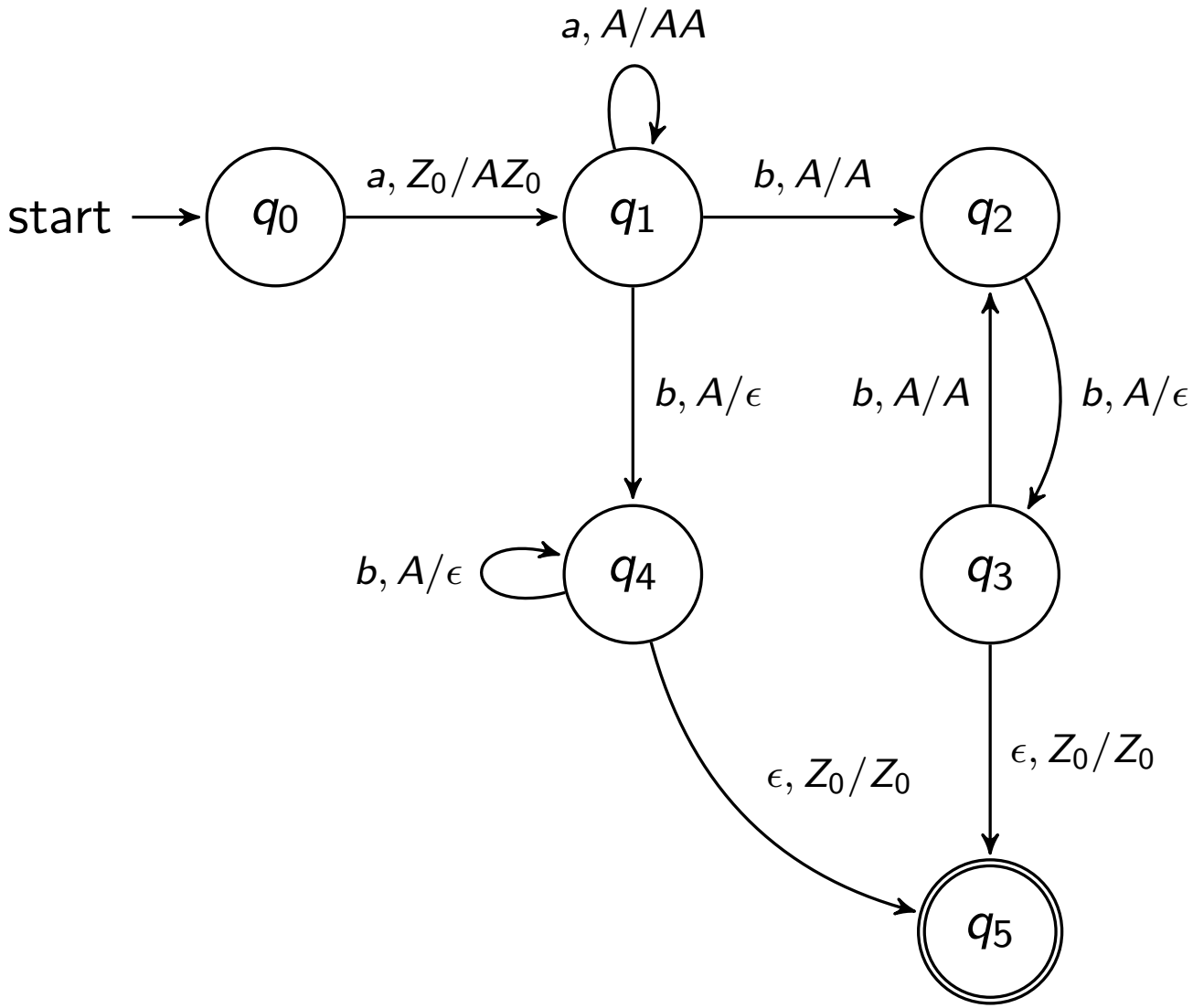
AnBn


start
q_0
q_1
q_2
q_3
$a, Z_0/AZ_0$
$a, A/AA$
$b, A/\epsilon$
$b, A/\epsilon$
$\epsilon, Z_0/Z_0$

AnB2n


start
q_0
$a, Z_0/AZ_0$
q_1
$a, A/AA$
$b, A/A$
q_2
$b, A/A$
$b, A/\epsilon$
q_3
$\epsilon, Z_0/Z_0$
q_4

AnBn ∪ AnB2n (non-deterministic)



Example 2

Suppose A and B are both languages over the alphabet $\Sigma = \{a, b, c\}$:

$$A = \{a^n b^n c^m | n, m \geq 0\}$$

$$B = \{a^m b^n c^n | n, m \geq 0\}$$

What is the language recognized by $A \cap B$?

Example 2

Suppose A and B are both languages over the alphabet $\Sigma = \{a, b, c\}$:

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$$B = \{a^m b^n c^n | n, m \geq 0\}$$

What is the language recognized by $A \cap B$?

$$A \cap B = \{a^n b^n c^n | n \geq 0\}$$

This language cannot be recognized by DPDA, thus the class of languages recognized by PDA is not closed under intersection

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For complement, note that $A \cap B = \overline{\overline{A} \cup \overline{B}}$

- If context-free languages were closed under complement, they would also be closed under intersection
- Therefore context-free languages are not closed under complementation because they are not closed under intersection

Recap - Operations

Languages accepted by Deterministic PDA have closure under ...?

- Union -

Recap - Operations

Languages accepted by Deterministic PDA have closure under ...?

- Union - not closed
- Intersection -

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Wrap up

- What have you learnt today?

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- What have you learnt today?
- What for this could be useful?