Theoretical Computer Science

Tutorial - week 4

February 11, 2021

Agenda

- Recap
- State Transition Table
- Operations on FSA
  - Intersection
  - Union
  - Difference
  - Complement
- Examples

Recap

- What is syntax?

Recap

- What is syntax?
- What is semantics?

Recap

- What is syntax?
- What is semantics?
- What is closure?

Recap

- What is syntax?
- What is semantics?
- What is closure? Examples

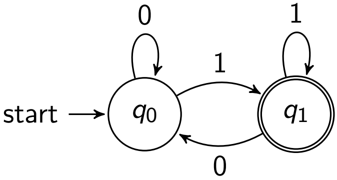
FSA: formally

Example of a complete FSA

$M = \langle$	
$\{q_0, q_1\},$	set of states
$\{0, 1\},$	input alphabet
$\{((q_0, 0), q_0), ((q_0, 1), q_1),$	
$((q_1, 0), q_0), ((q_1, 1), q_1)\},$	total transition function
$q_0,$	initial state
$\{q_1\}$	set of final states
$\rangle$	

Another representation of a complete FSA

Graphical Representation — State Transition Diagram



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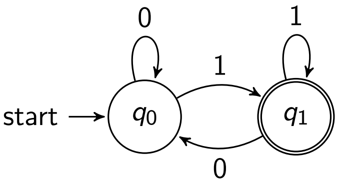
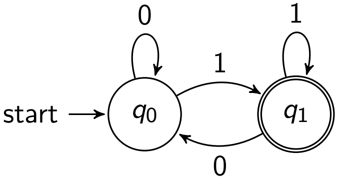


Graphical Representation — State Transition Table

	δ(q,0)	δ(q,1)
$\rightarrow q_0$	q_0	q_1
$^*q_1$	q_0	q_1

Another representation of a complete FSA

Graphical Representation — State Transition Diagram

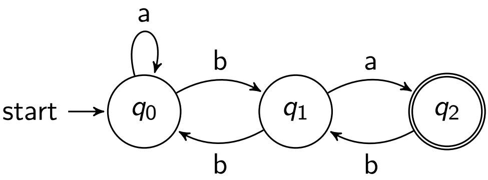


Graphical Representation — State Transition Table

	0	1
$\rightarrow q_0$	q_0	q_1
$^*q_1$	q_0	q_1

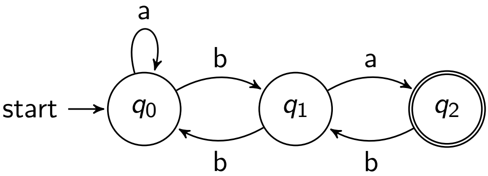
State Transition Table: Example

Given an FSA as a State Transition Diagram, build a State Transition Table



State Transition Table: Example

Given an FSA as a State Transition Diagram, build a State Transition Table

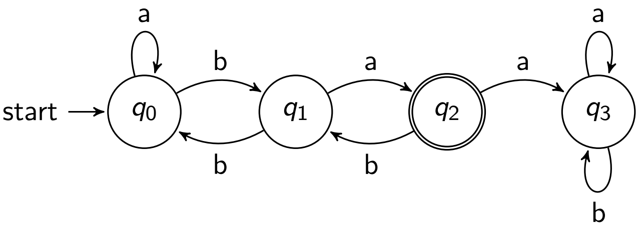


State Transition Table

	$\delta(q, a)$	$\delta(q, b)$
q_0		
q_1		
q_2		

State Transition Table: Example cont.

Given a complete FSA ...



Operations

Suppose L_1 and L_2 are both languages over the alphabet A. If $x \in A^*$, then knowing whether $x \in L_1$ and whether $x \in L_2$ is enough to determine whether $x \in L_1 \cup L_2$.

Operations

Suppose L_1 and L_2 are both languages over the alphabet A. If $x \in A^*$, then knowing whether $x \in L_1$ and whether $x \in L_2$ is enough to determine whether $x \in L_1 \cup L_2$.

If we have one algorithm to accept L_1 and another to accept L_2, how can we formulate an algorithm to accept $L_1 \cup L_2$? (similarly for $L_1 \cap L_2$ and $L_1 \backslash L_2$).

Intersection (Formally)

Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively. Let M be the complete FSA $M = (Q, A, \delta, q_0, F)$, where

$$Q = Q^1 \times Q^2$$
$$q_0 = (q_0^1, q_0^2)$$

the transition function δ is defined by the formula

$$\delta((q,p),a) = (\delta^1(q,a), \delta^2(p,a))$$

for every $q \in Q^1$, every $p \in Q^2$, and every $a \in A$. And the set of final states is defined as

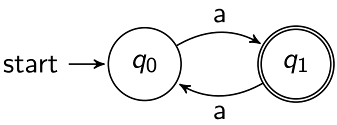
$$F = \{(q,p) \mid q \in F^1 \wedge p \in F^2\}$$

M accepts the language $L_1 \cap L_2$.

Intersection: Example 1

Let M^1 be a complete FSA defined as

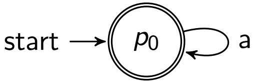
$$\begin{aligned} M^1 = \langle & \{q_0, q_1\}, \{a\}, \\ & \{((q_0, a), q_1), ((q_1, a), q_0)\}, \\ & q_0, \{q_1\}\rangle \end{aligned}$$



Intersection: Example 1

... and M^2 be a complete FSA defined as

$$M^2 = \langle \{p_0\}, \{a\}, \\ \{((p_0, a), p_0)\}, \\ p_0, \{p_0\} \rangle$$



Intersection: Example 1

Let M^1 be a complete FSA defined as

$$\begin{aligned} M^1 = \langle & \{q_0, q_1\}, \{a\}, \\ & \{((q_0, a), q_1), ((q_1, a), q_0)\}, \\ & q_0, \{q_1\}\rangle \end{aligned}$$

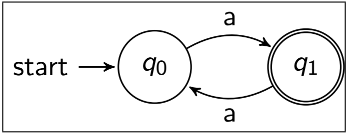
and M^2 be a complete FSA defined as

$$\begin{aligned} M^2 = \langle & \{p_0\}, \{a\}, \\ & \{((p_0, a), p_0)\}, \\ & p_0, \{p_0\}\rangle \end{aligned}$$

then

$$\begin{aligned} (M^1 \cap M^2) = \langle & \{(q_0, p_0), (q_1, p_0)\}, \{a\}, \\ & \Big\{\Big(((q_0, p_0), a), (q_1, p_0)\Big), \Big(((q_1, p_0), a), (q_0, p_0)\Big)\Big\}, \\ & (q_0, p_0), \{(q_1, p_0)\}\rangle \end{aligned}$$

Intersection: Example 1



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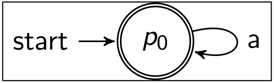

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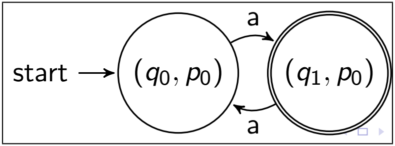
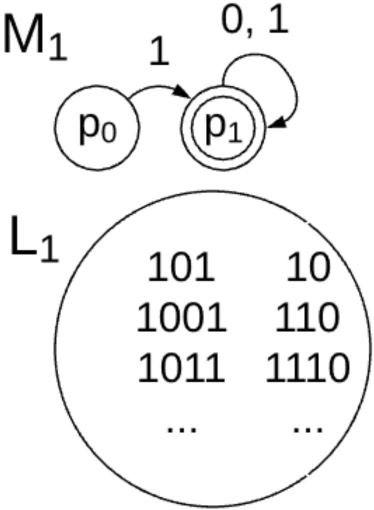
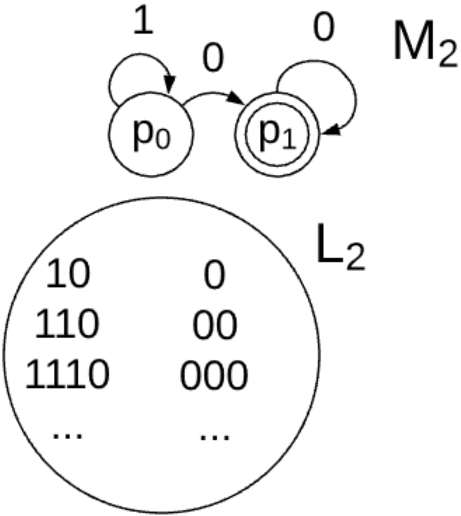


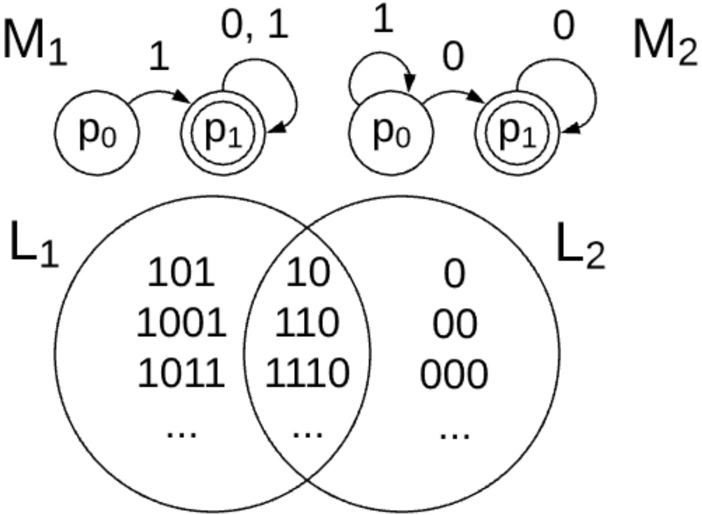
Intersection: Example 2



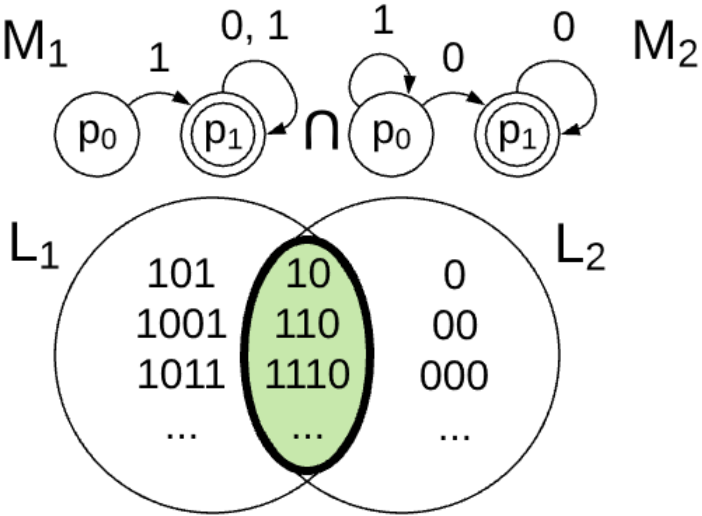
Intersection: Example 2



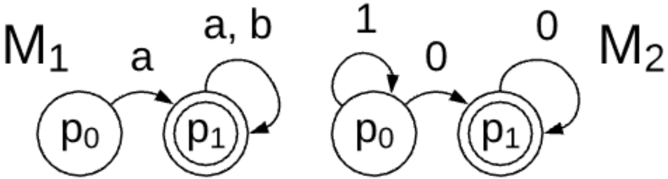
Intersection: Example 2



Intersection: Example 2



Intersection: What if alphabets differ?



Union (Formally)

Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively. Let M be the complete FSA $M = (Q, A, \delta, q_0, F)$, where

$$Q = Q^1 \times Q^2$$
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the transition function δ is defined by the formula

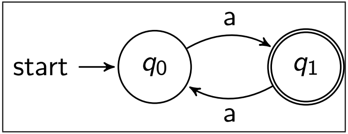
$$\delta((q,p),a) = (\delta^1(q,a), \delta^2(p,a))$$

for every $q \in Q^1$, every $p \in Q^2$, and every $a \in A$. And the set of final states is defined as

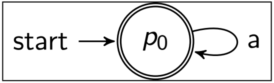
$$F = \{(q,p) \mid q \in F_1 \vee p \in F_2\}$$

M accepts the language $L_1 \cup L_2$.

Union: Example 1



$$\cup$$



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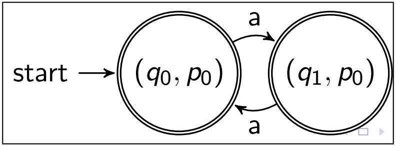
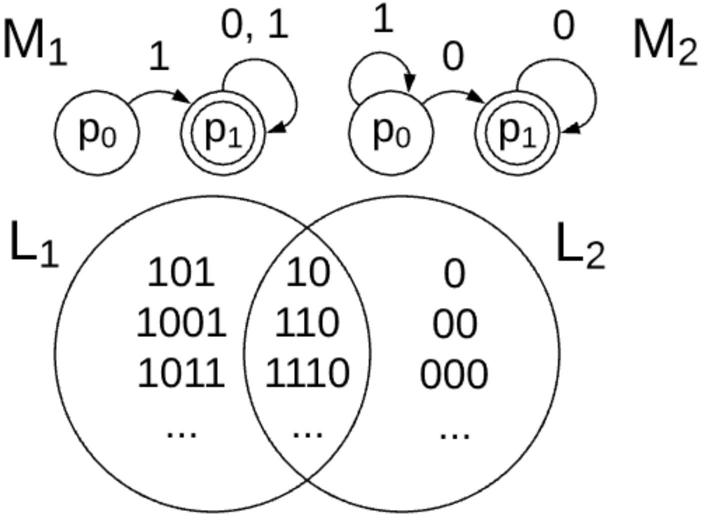
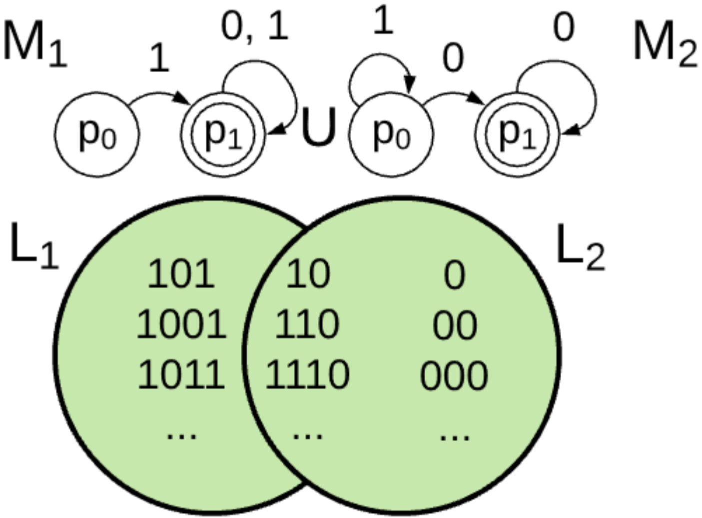


Union: Example 2



Union: Example 2



Difference (Formally)

Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively. Let M be the FSA $M = (Q, A, \delta, q_0, F)$, where

$$Q = Q^1 \times Q^2$$
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the transition function δ is defined by the formula

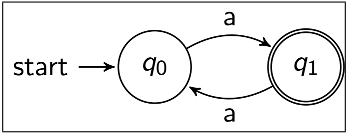
$$\delta((q, p), a) = (\delta^1(q, a), \delta^2(p, a))$$

for every $q \in Q^1$, every $p \in Q^2$, and every $a \in A$. And the set of final states is defined as

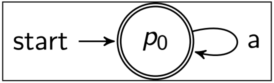
$$F = \{(q, p) \mid q \in F_1 \wedge p \notin F_2\}$$

M accepts the language $L_1 \setminus L_2$.

Difference (Example 1 $L_1 \setminus L_2$)



$$\setminus$$



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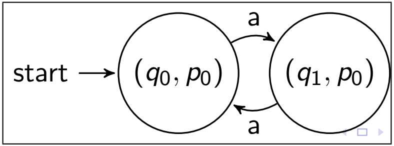
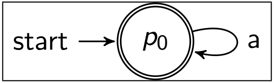
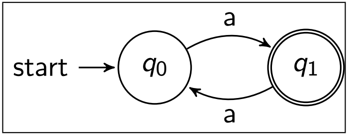


Difference (Example 1 $L_2 \setminus L_1$)



$$\setminus$$



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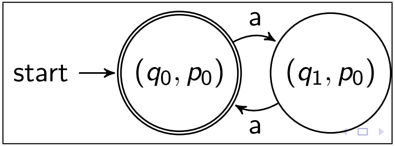
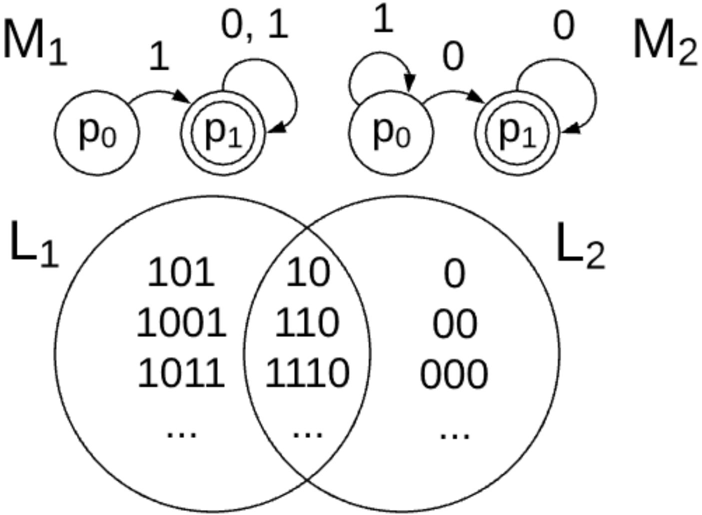
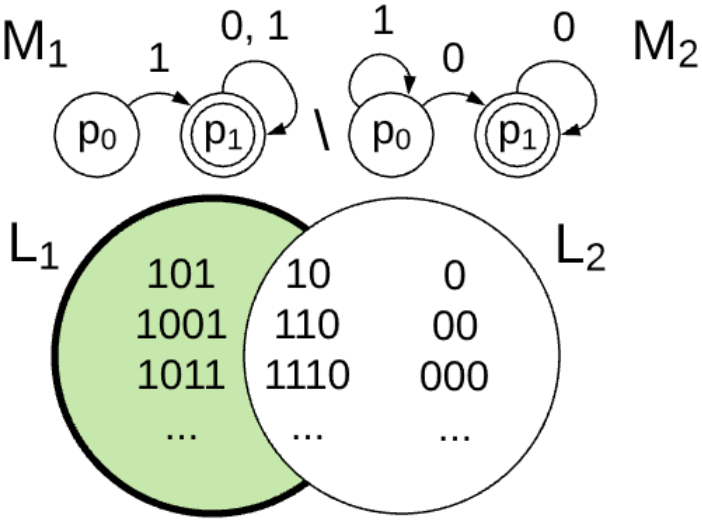


Difference (Example 2)



Difference (Example 2)



Complement

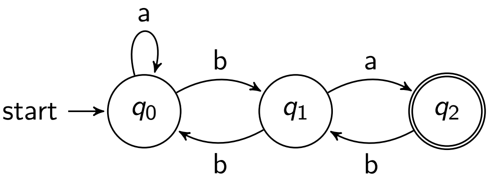
Suppose $M = (Q, A, \delta, q_0, F)$ is a **complete** finite automaton accepting L. A complement M^c is a complete FSA $M^c = (Q, A, \delta, q_0, F^c)$, where the set of final states is defined as

$$F^c = Q \setminus F$$

M^c accepts the language L^c.

Complement: Example

Let M be an FSA represented graphically as follows:



What would be the complement M^c?

Complement: Example (M)

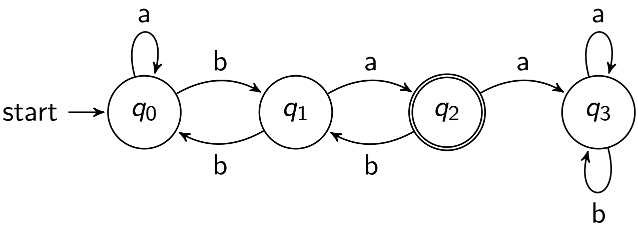


Table representation of M

	$\delta(q, a)$	$\delta(q, b)$
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_0
$^*q_2$	q_3	q_1
q_3	q_3	q_3

Complement: Example (M^c)

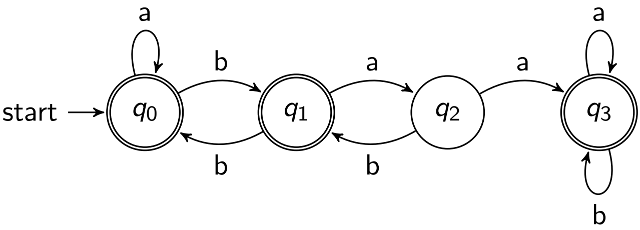
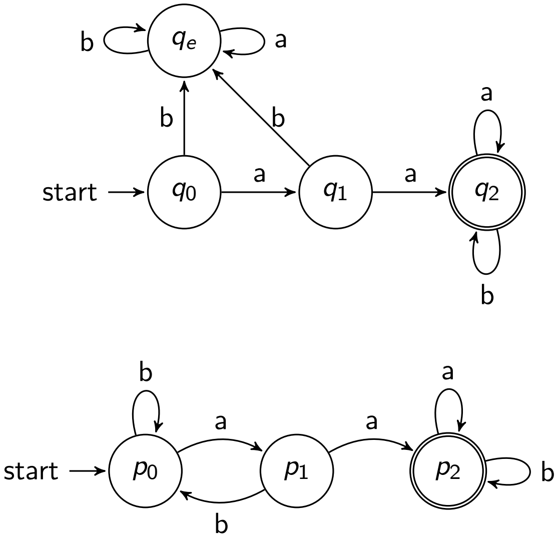


Table representation of M^c

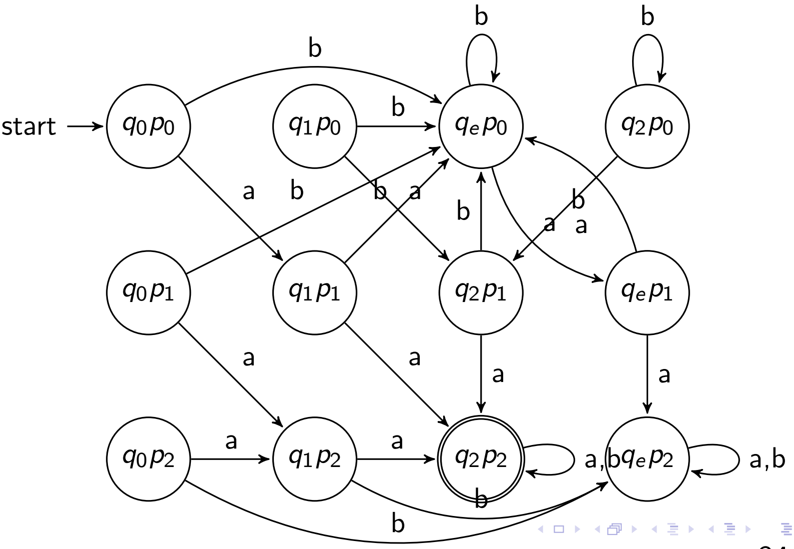
	$\delta(q, a)$	$\delta(q, b)$
$\rightarrow^* q_0$	q_0	q_1
$^* q_1$	q_2	q_0
q_2	q_3	q_1
$^* q_3$	q_3	q_3

Hands-on example: M_1 and M_2



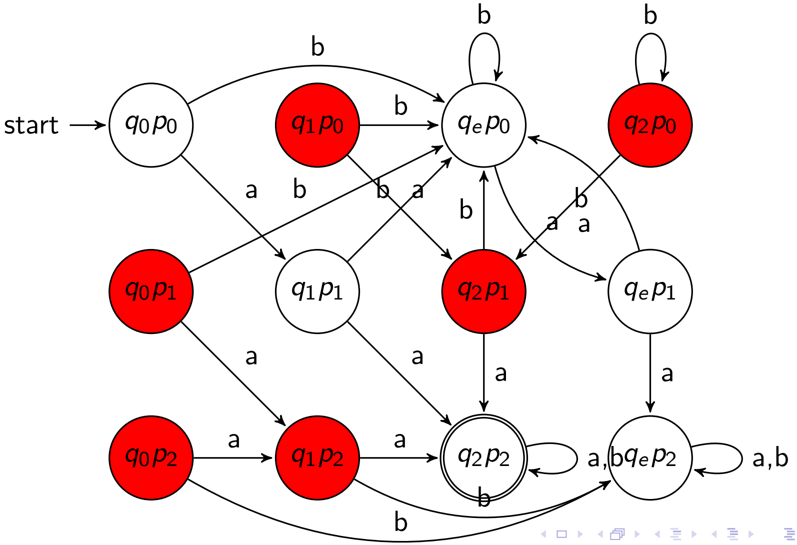
Hands-on example: solution graphically

All possible transitions are depicted



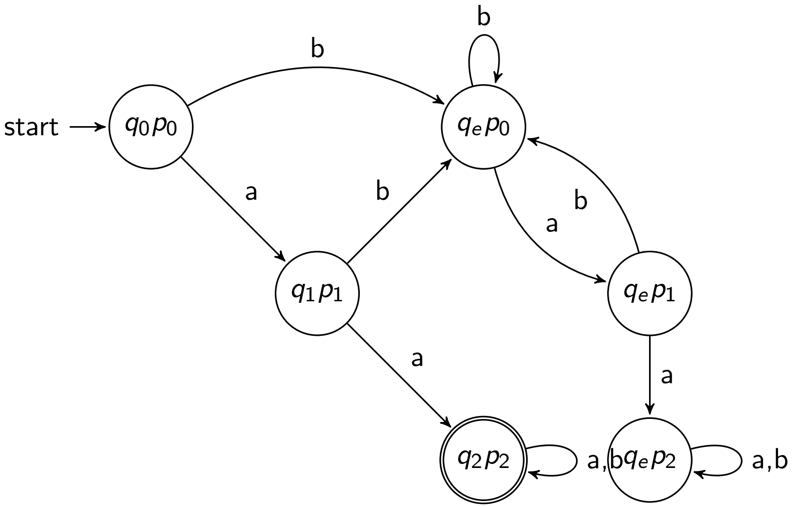
Hands-on example: solution graphically

Let us remove all the unreachable states...

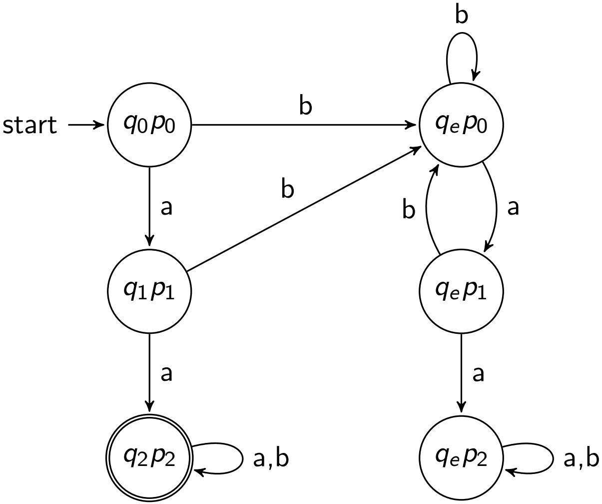


Hands-on example: solution graphically

... and relocate the states to make the diagram more readable



Hands-on example: solution graphically



Hands-on example

Table representation of M_1

	$\delta(q, a)$	$\delta(q, b)$
$\rightarrow q_0$	q_1	q_e
q_1	q_2	q_e
$^*q_2$	q_2	q_2
q_e	q_e	q_e

Table representation of M_2

	$\delta(p, a)$	$\delta(p, b)$
$\rightarrow p_0$	p_1	p_0
p_1	p_2	p_0
$^*p_2$	p_2	p_2

Hands-on example: solution as a State Transition Table

Table representation of $M_1 \cap M_2$

	$\delta((qp), a)$	$\delta((qp), b)$
$\rightarrow (q_0 p_0)$	$(q_1 p_1)$	$(q_e p_0)$
$(q_1 p_0)$	$(q_2 p_1)$	$(q_e p_0)$
$(q_2 p_0)$	$(q_2 p_1)$	$(q_2 p_0)$
$(q_e p_0)$	$(q_e p_1)$	$(q_e p_0)$
$(q_0 p_1)$	$(q_1 p_2)$	$(q_e p_0)$
$(q_1 p_1)$	$(q_2 p_2)$	$(q_e p_0)$
$(q_2 p_1)$	$(q_2 p_2)$	$(q_2 p_0)$
$(q_e p_1)$	$(q_e p_2)$	$(q_e p_0)$
$(q_0 p_2)$	$(q_1 p_2)$	$(q_e p_2)$
$(q_1 p_2)$	$(q_2 p_2)$	$(q_e p_2)$
$^*(q_2 p_2)$	$(q_2 p_2)$	$(q_2 p_2)$
$(q_e p_2)$	$(q_e p_2)$	$(q_e p_2)$

Let us remove unreachable states

Hands-on example: solution as a State Transition Table

Table representation of $M_1 \cap M_2$

	$\delta((qp), a)$	$\delta((qp), b)$
$\rightarrow (q_0 p_0)$	$(q_1 p_1)$	$(q_e p_0)$
$(q_e p_0)$	$(q_e p_1)$	$(q_e p_0)$
$(q_1 p_1)$	$(q_2 p_2)$	$(q_e p_0)$
$(q_e p_1)$	$(q_e p_2)$	$(q_e p_0)$
$^*(q_2 p_2)$	$(q_2 p_2)$	$(q_2 p_2)$
$(q_e p_2)$	$(q_e p_2)$	$(q_e p_2)$

Hands-on example: solution as a State Transition Table

Table representation of $M_1 \cup M_2$

	$\delta((qp), a)$	$\delta((qp), b)$
$\rightarrow (q_0 p_0)$	$(q_1 p_1)$	$(q_e p_0)$
$(q_e p_0)$	$(q_e p_1)$	$(q_e p_0)$
$(q_1 p_1)$	$(q_2 p_2)$	$(q_e p_0)$
$(q_e p_1)$	$(q_e p_2)$	$(q_e p_0)$
$^*(q_2 p_2)$	$(q_2 p_2)$	$(q_2 p_2)$
$^*(q_e p_2)$	$(q_e p_2)$	$(q_e p_2)$

Hands-on example: solution as a State Transition Table

Table representation of $M_1 \setminus M_2$

	$\delta((qp), a)$	$\delta((qp), b)$
$\rightarrow (q_0 p_0)$	$(q_1 p_1)$	$(q_e p_0)$
$(q_e p_0)$	$(q_e p_1)$	$(q_e p_0)$
$(q_1 p_1)$	$(q_2 p_2)$	$(q_e p_0)$
$(q_e p_1)$	$(q_e p_2)$	$(q_e p_0)$
$(q_2 p_2)$	$(q_2 p_2)$	$(q_2 p_2)$
$(q_e p_2)$	$(q_e p_2)$	$(q_e p_2)$

Hands-on example: solution as a State Transition Table

Table representation of $M_2 \setminus M_1$

	$\delta((qp), a)$	$\delta((qp), b)$
$\rightarrow (q_0 p_0)$	$(q_1 p_1)$	$(q_e p_0)$
$(q_e p_0)$	$(q_e p_1)$	$(q_e p_0)$
$(q_1 p_1)$	$(q_2 p_2)$	$(q_e p_0)$
$(q_e p_1)$	$(q_e p_2)$	$(q_e p_0)$
$(q_2 p_2)$	$(q_2 p_2)$	$(q_2 p_2)$
$^*(q_e p_2)$	$(q_e p_2)$	$(q_e p_2)$

Wrap up

- What have you learnt today?

Wrap up

- What have you learnt today?
- What for this could be useful?