Essentials of Analytical Geometry and Linear Algebra I, Class #3

Innopolis University, September 2020

1 Inverse Matrix

1. Find inverse matrices for the following matrices:

   (a) $\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}$;

   (b) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$;

2. Solve matrix equations:

   (a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$;

   (b) $X \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$;

2 Matrix Rank

1. Calculate the ranks of the following matrices:

   (a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$.

2. Determine the ranks of the following matrices for all real values of parameter α:

   (a) $\begin{bmatrix} 1 & \alpha & -1 & 2 \\ 2 & -1 & \alpha & 5 \\ 1 & 10 & -6 & 1 \end{bmatrix}$;

   (b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$;

3 Changing Basis and Coordinates

1. Two bases are given in the plane: $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}'_1$, $\mathbf{e}'_2$. The vectors of the second basis have coordinates $(-1; 3)$ and $(2; -7)$ in the second basis.

   (a) Compose transition matrices from the old basis to the new and vice versa.

   (b) Find the coordinates of a vector in the old basis given that it has coordinates α'_1, α'_2 in the new basis.

(c) Find the coordinates of a vector in the new basis given that it has coordinates α_1, α_2 in the old basis.

2. Find the coordinates of a vector in a basis:
   $\mathbf{e}_1 = \begin{bmatrix}1\\3\\2\end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix}-1\\1\\0\end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix}2\\-1\\1\end{bmatrix}$ given that its coordinates in a basis $\mathbf{e}'_1 = \begin{bmatrix}-1\\0\\2\end{bmatrix}$, $\mathbf{e}'_2 = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $\mathbf{e}'_3 = \begin{bmatrix}4\\3\\-1\end{bmatrix}$ are equal to α'_1, α'_2, α'_3.

3. If vectors $\mathbf{a}$ and $\mathbf{b}$ form a basis (you should check it), it is needed to find coordinates $\mathbf{c}$ and $\mathbf{d}$ in the basis.
   $\mathbf{a} = \begin{bmatrix}-5\\-1\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}-1\\3\end{bmatrix}$, $\mathbf{c} = \begin{bmatrix}-1\\2\end{bmatrix}$, $\mathbf{d} = \begin{bmatrix}2\\-6\end{bmatrix}$.

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4 Inverse Matrix

1. Find inverse matrices for the following matrices:

   (a) $\begin{bmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$;

   (b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix}$;

   (c) $\begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}$;

2. It is known that $A^2 + A + I = O$ (O is a zero matrix) for a square matrix A. Is it true that matrix A is invertible? If it is so, how can we find the inverse matrix?

3. Solve matrix equations:

   (a) $X \begin{bmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 2 \\ 5 & 8 & -1 \end{bmatrix}$.

5 Matrix Rank

1. Calculate the ranks of the following matrices:

   (a) $\begin{bmatrix} 13 & 16 & 16 \\ -5 & -7 & -6 \\ -6 & -8 & -7 \end{bmatrix}$;

2. Determine the rank of $A - \lambda I$ for all values of λ if

   (a) $A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 6 & 4 \\ -2 & -3 & -1 \end{bmatrix}$;

   (b) $A = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$.

6 Changing Basis and Coordinates

1. There are two different coordinate systems in space: O, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ and O', $\mathbf{e}'_1$, $\mathbf{e}'_2$, $\mathbf{e}'_3$. It is known that the old coordinates x, y, z are expressed through the new coordinates x', y', z' with the following formulas:

$$x = x' + y' + z' - 1;\ y = -x' + z' + 3;\ z = -x' - y' - 2.$$

   (a) Find the transition matrix from the new basis to the old one and the transition matrix from the old basis to the new one.

   (b) Find the coordinates of O, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ in the new coordinate system.

   (c) Find the coordinates of O', $\mathbf{e}'_1$, $\mathbf{e}'_2$, $\mathbf{e}'_3$ in the old coordinate system.

2. Let us consider two coordinate systems in the plane: O, $\mathbf{e}_1$, $\mathbf{e}_2$ and O', $\mathbf{e}'_1$, $\mathbf{e}'_2$. Point O' has coordinates $(7; -2)$ in the old coordinate system, and vectors $\mathbf{e}'_1$, $\mathbf{e}'_2$ can be obtained from vectors $\mathbf{e}_1$, $\mathbf{e}_2$ by rotating them 60° (a) clockwise; (b) counterclockwise. Find the old coordinates of a point x, y given its new coordinates x', y'.