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NMPC-based Trajectory Tracking and Collison Avoidance of Underactuated Vessels with Elliptical Ship Domain [⋆]

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Abstract: This paper presents a Nonlinear Model Predictive Control (NMPC) for position and velocity tracking of underactuated surface vessels of elliptical ship domains with a collision avoidance scheme. A constrained nonlinear optimization problem is formulated to minimize vessel states' deviation from a time varying reference generated by a virtual vessel over a finite horizon. Linear constraints is imposed on input force and moment to be within the physical limits. Collision avoidance is represented as separation of each pair of the elliptic disks, representing the ship domain of our vessel and each encountered one, and is formulated as nonlinear time-varying constraint over the prediction horizon. A real-time C-code is generated using the ACADO toolkit and `qpOASES` solver with multiple shooting technique for discretization and Gauss-Newton iteration algorithm. This algorithm is computationally efficient, thus enabling real-time implementation of the proposed technique. MATLAB simulations are used to assess the validity of the proposed technique.

Keywords: Nonlinear model predictive control (NMPC), Collision avoidance, Trajectory tracking, Underactuated vessel, Autonomous vessels.

1. INTRODUCTION

Although there are many collision avoidance algorithms for autonomous surface vessels, most of them consider it as a planning problem independent from the motion controller; ignoring vessel dynamics. This might lead a high collision risk specially in dense traffic areas where vessels closely encounter each other. Integrating collision avoidance schemes into trajectory tracking controller ensures that the vessel tracks a time-parameterized reference trajectory efficiently while doing necessary accurate maneuvers to avoid colliding with nearby vessels or obstacles. Following the International Regulations for Preventing Collisions at Sea (COLREGs), published by the International Maritime Organization (IMO), leads to non contrary actions among the vessels. Considering the ship domain, defined as the area around the ship that should be free from other ships and obstacles, enhances the accuracy of the maneuvering especially for close range encounters.

Tam et al. (2009) stated in their review paper that all the reported studies on collision avoidance have either disregarded the regulations, employed specific databases or used different safety domain geometries to emulate COLREGS; and assumed a highly simplified version of the ship dynamic model. There are many techniques, developed afterwards, used for solving collision avoidance problem from the planning point of view and handle COLREGs. In Szlapczynski (2012), evolutionary approaches are used to find a safe and optimum trajectory of surface vessels employing the kinematic model. In Zhuo (2014), A fuzzy logic approach is used, where the collision avoidance is formulated as an optimization problem and solved using particle swarm algorithm. A^* searching algorithm is used in Ari et al. (2013) to find the shortest path between two given coordinates in the presence of obstacles. Although the aforementioned techniques demonstrate collision avoidance and handle COLREGs, they ignore the dynamics of the vessel.

Recently, integrating collision avoidance into the the control design has attracted many researchers. In Wang and Ding (2014), linear Model Predictive Control (MPC) is used for the tracking and formation problem of multiagent linear systems with collision avoidance as a constraint for the optimization problem. To make use of linear MPC well-established theory, Alrifaee et al. (2014) have presented collision avoidance scheme for networked vehicles by successively linearizing the nonlinear prediction model using Taylor series. In the robotics domain, Wang et al. (2015) have used MPC for obstacle avoidance of a space robot. However, they have either considered linear agents, simplified the nonlinearities, or handled only static obstacles.

Compared to different ship domains presented in Szlapczynski and Szlapczynska (2015), elliptic disk is the best polygon that represent the vessel safety envelop and facilitate mathematical formulation for collision avoidance purposes. As discussed in Do (2012), representing vessels' ship domain by circular disks results in a problem of the large conservative area defined as the difference between

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the areas enclosed by the circle and the ellipse. Other shapes are complex mathematically for NMPC formulation.

The main contribution of this paper is to remove the drawbacks of considering collision avoidance as a planning problem and ignoring the dynamics, and to consider the vessels' ship domain in the controller design. An NMPC-based trajectory tracking scheme of three-degree-of-freedom (3-DOF) nonlinear dynamic model of underactuated vessels, with embedded collision avoidance, is presented. Collision avoidance is formulated as nonlinear time-varying constraints to the NMPC optimization problem by reformulating the separation conditions of two moving elliptic disks. Constraint prioritization is used to follow COLREGs rules. This will lead to accurate reference tracking and reduce the collision risk between vessels.

2. PROBLEM FORMULATION

The surface vessel model has 6-DOF: surge, sway, yaw, heave, roll, and pitch, which can be simplified to motion in surge, sway, and yaw under the following assumptions mentioned in Chwa (2011):

(1) The heave, roll, and pitch modes induced by wind and currents are negligible.
(2) The inertia, added mass, and hydrodynamic damping matrices are diagonal.
(3) The available control variables are surge force and yaw moment.

Based on that, the 3-DOF model, as presented in Fossen (2011), will be:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g\boldsymbol{u} \tag{1}$$

Here, $\boldsymbol{x} = [x_p\ y_p\ \psi\ u\ \upsilon\ r]^T \in \Re^6$ is the state vector, $\boldsymbol{u} = [\tau_u\ \ \tau_r]^T \in \Re^2$ is the input vector, x_p and y_p are the positions, and ψ is the heading angle of the ship with respect to the earth-fixed frame, u and υ are longitudinal and transverse linear velocities in surge (body-fixed x_B) and sway (body-fixed y_B) directions respectively, r is the angular velocity in yaw around body-fixed z axis (see Fig. 1), $f(\cdot)$, and $g(\cdot)$ are continuous nonlinear functions in $\boldsymbol{u}$ and $f(\cdot)$ is locally Lipschitz in $\boldsymbol{x}$ that satisfies $f(0) = 0$,

$$f(\boldsymbol{x}) = \begin{bmatrix} u\cos(\psi) - \upsilon\sin(\psi) \\ u\sin(\psi) + \upsilon\cos(\psi) \\ r \\ \dfrac{m_2}{m_1}\upsilon r - \dfrac{d_1}{m_1}u \\ \dfrac{m_1}{m_2}ur - \dfrac{d_2}{m_2}\upsilon \\ \dfrac{(m_1 - m_2)}{m_3}u\upsilon - \dfrac{d_3}{m_3}r \end{bmatrix} \in \Re^6, \text{ and}$$

$$g_(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 & 0 & \dfrac{1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{m_3} \end{bmatrix}^T \in \Re^6 \times \Re^2.$$

The parameters m_1, m_2, m_3 are the ship inertia including added mass effects, and d_1, d_2, d_3 are the hydrodynamic damping coefficients. A time-varying reference trajectory is generated by a virtual ship with the same dynamics as (1):

$$\dot{\boldsymbol{x}}_r = f(\boldsymbol{x}_r) + g\boldsymbol{\tau}_r \tag{2}$$

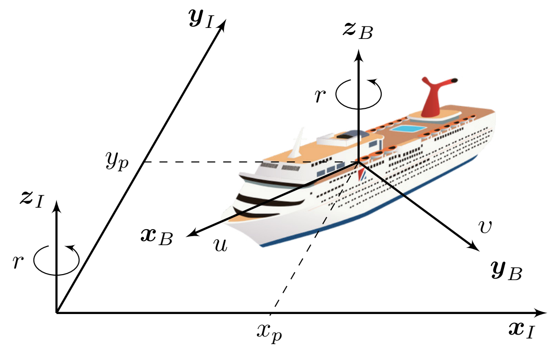

Fig. 1. Earth-fixed (x_I, y_I, z_I) and body-fixed (x_B, y_B, z_B) frames.

where $\boldsymbol{x}_r = [x_{pr}\ y_{pr}\ \psi_r\ u_r\ \upsilon_r\ r_r]^T$ denotes the generated reference states. The same assumptions as in Yan and Wang (2012) are adopted throughout this paper:

Assumption 1: All ship state variables (position, orientation, and velocities) are measurable or can be accurately estimated.

Assumption 2: The reference velocities and positions are smooth over time.

Hence, the control objective is to steer the vessel states $(x_p,\ y_p,\ \psi,\ u,\ \upsilon,\ r)$ to follow the reference states $(x_{pr},\ y_{pr},\ \psi_r,\ u_r,\ \upsilon_r,\ r_r)$ while satisfying control input and collision avoidance constraints considering the ship domain of the vessel by elliptic representation.

3. PRELIMINARIES

3.1 Separation Condition between Two Elliptic Disks

This subsection presents the conditions for two elliptic discs to be separated, that will be reformulated later for embedding into NMPC synthesis as collision avoidance constraint. Elliptic disks are the closed type of conic section results from the intersection of a cone by a plane, and is expressed in the plane with respect to inertial frame as (see Fig. 2):

$$\bar{\mathcal{A}} \equiv \{(x,y) \mid Ax^2 - 2Bxy + Cy^2 + (2By_p - 2Ax_p)x + (2Bx_p - 2Cy_p)y + (Ax_p^2 - 2Bx_py_p + Cy_p^2 - 1) \le 0\} \tag{3}$$

where $A = \left(\frac{cos(\psi)^2}{a^2} + \frac{sin(\psi)^2}{b^2}\right)$, $B = \frac{sin(2\psi)}{2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$, $C = \left(\frac{sin(\psi)^2}{a^2} + \frac{cos(\psi)^2}{b^2}\right)$, a and b are the radii of the ellipse, x_p and y_p are the position of the center of the ellipse, and ψ is the heading angle of the disk,. The elliptic disk can be represented by a 3×3 matrix $\mathcal{A} = [a_{i,j}]$ as:

$$\bar{\mathcal{A}} \equiv \{X \mid X^T\mathcal{A}X \le 0\} \tag{4}$$

where $X = [x\ y\ 1]^T$ is the 3-D column vector containing the homogeneous coordinates and

$$\mathcal{A} = \begin{bmatrix} A & -B & By_p - Ax_p \\ -B & C & Bx_p - Cy_p \\ By_p - Ax_p & Bx_p - Cy_p & Ax_p^2 - 2Bx_py_p + Cy_p^2 - 1 \end{bmatrix}.$$

By elementary math, the matrix $\mathcal{A}$ satisfies the condition that $det(\mathcal{A}) < 0$.

Theorem 1. Given two elliptic disks $(\bar{\mathcal{A}}, \bar{\mathcal{B}})$represented by the matrices $\mathcal{A} = [a_{i,j}]$ and $\mathcal{B} = [b_{i,j}]$ respectively:

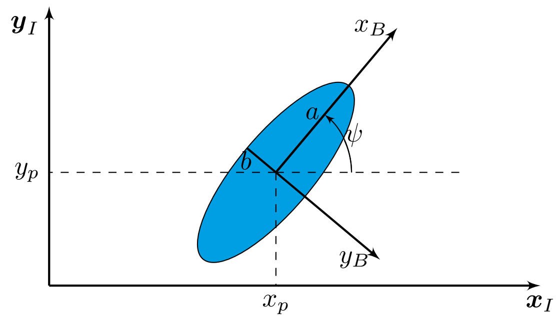

Fig. 2. Elliptic Disk.

(1) $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$ touch externally iff $\mathcal{P}(\lambda) = det(\lambda\mathcal{A} - \mathcal{B}) = 0$ has two repeated real negative roots.
(2) $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$ are separate iff $\mathcal{P}(\lambda) = 0$ has two distinct real negative roots.

Proof. see reference Choi et al. (2006)

3.2 Cubic Polynomial

In this subsection we present the result about the nature of the cubic polynomial roots without solving it.

Theorem 2. Consider a cubic polynomial $\mathcal{P}(\lambda) = C_3\lambda^3 + C_2\lambda^2 + C_1\lambda + C_0$. Let $\Delta = 18C_3C_2C_1C_0 - 4{C_2}^3C_0 + {C_2}^2{C_1}^2 - 4C_3{C_1}^3 - 27{C_3}^2{C_0}^2$ be the discriminant of $\mathcal{P}(\lambda)$. Δ can be positive, negative, or zero depending on the roots nature as follow:

(1) If $\Delta > 0$, then $\mathcal{P}(\lambda)$ has three distinct real roots;
(2) if $\Delta = 0$, then $\mathcal{P}(\lambda)$ has a multiple root and all its roots are real;
(3) if $\Delta < 0$, then $\mathcal{P}(\lambda)$ has one real root and two complex conjugate roots.

Proof. See chapter 10 in Axler et al. (2004).

4. CONTROLLER SYNTHESIS

4.1 Nonlinear model predictive control

Consider a continuous time-invariant nonlinear state space model of the vessel in the form of (1) subject to the constraints:

$$\begin{aligned} \boldsymbol{x}(t) &\in \mathcal{X}, \\ \boldsymbol{u}(t) &\in \mathcal{U},\ t \geq 0, \end{aligned} \tag{5}$$

where $\boldsymbol{x} \in \Re^n$ is the state vector; $\boldsymbol{u} \in \Re^m$ is the control vector; $\mathcal{X} \subset \Re^n$ and $\mathcal{U} \subset \Re^m$ are compact sets and contain the origin in their interior points. In general, the scheme of MPC is to predict the future states over finite prediction horizon using a nominal model, as in (1), for the system and the last available measurement or accurate estimation of the states to get the optimum control action over a control horizon (Control Vector). The control horizon is less than or equal to the prediction horizon. This control vector steers the system states to follow a time-varying reference generated by a model of the form (2) then, the first element of the control vector is applied and the whole process is repeated at the next sample.

Hence, NMPC is considered as a nonlinear state feedback $u(t) = \mathcal{K}(x(t))$ obtained online from an optimal control problem that minimizes a least squares (LS) objective function by penalizing the deviation of the system inputs and states from the reference trajectories. It takes the form:

$$\min_{\boldsymbol{x}(t),\boldsymbol{u}(t)} J(\boldsymbol{x},\boldsymbol{u}) = \int_{t=T_0}^{T_p+T_0} \ell(\boldsymbol{x}(t),\boldsymbol{u}(t))dt + F(\boldsymbol{x}(T_p)) \quad \text{s.t. } (5). \tag{6}$$

where $\ell(\boldsymbol{x}(t),\boldsymbol{u}(t))$ is the stage cost function and must satisfy the following conditions:

- $\ell(\boldsymbol{x}_r,\boldsymbol{u}_r) = 0$
- $\ell(\boldsymbol{x}(t),\boldsymbol{u}(t)) > 0, \forall \boldsymbol{x}(t) \in \mathcal{X}, \boldsymbol{u}(t) \in \mathcal{U}, \boldsymbol{x}(t) \neq \boldsymbol{x}_r(t),$

$F(\boldsymbol{x}(T_p))$ is the terminal cost function, and $T_p > 0$ is the length of both the prediction and control horizons. As discussed in Yang and Zheng (2014), the stage and terminal cost functions are usually defined as weighted norm of the states and control input errors :

$$\ell(\boldsymbol{x}(t),\boldsymbol{u}(t)) = \|\boldsymbol{x}(t) - \boldsymbol{x}_r(t)\|_Q + \|\boldsymbol{u}(t) - \boldsymbol{u}_r(t)\|_R \tag{7}$$

$$F(x(T_p))) = \|\boldsymbol{x}(T_p) - \boldsymbol{x}_r(T_p)\|_P \tag{8}$$

Here Q, R, P are positive semidefinite weighing matrices. Control action is penalized because that makes the optimization problem easier and avoids control values of high energy, see Grüne and Pannek (2011).

4.2 Collision Avoidance

This subsection presents the formulation of the collision avoidance between vessels, represented by elliptic disks as ship domains, as time-varying nonlinear constraints for the NMPC optimization problem (6) over the prediction horizon T_p. Suppose that our vessel, represented by the moving elliptic disk $\bar{\mathcal{A}}(t)$, encounters some other vessels indexed by the superscript $[i]$ and represented by the moving elliptic disks $\bar{\mathcal{B}}^{[i]}(t)$, then the motion of our vessel is collision-free if every pair of elliptic disks $\{\bar{\mathcal{A}}(t), \bar{\mathcal{B}}^{[i]}(t)\}$ is separate for all $t \in [T_0, T_p]$. Otherwise, one of these pairs collides;, i.e. $\bar{\mathcal{A}}(t)$ and $\bar{\mathcal{B}}^{[i]}(t)$ are touching or overlapping for some $t \in [T_0, T_p]$. The characteristic polynomial between every pair $\{\bar{\mathcal{A}}(t), \bar{\mathcal{B}}^{[i]}(t)\}$ is

$$\mathcal{P}^{[i]}(\lambda, t) = det(\lambda\mathcal{A}(t) - \mathcal{B}^{[i]}(t)) \tag{9}$$

and can be expanded as

$$\mathcal{P}^{[i]}(\lambda, t) = C_3^{[i]}(t)\lambda^3 + C_2^{[i]}(t)\lambda^2 + C_1^{[i]}(t)\lambda + C_0^{[i]}(t) = 0, \tag{10}$$

where:

$$\begin{aligned}
C_3^{[i]}(t) &= a_{11}M^{[i]}{}_{11} - a_{12}M^{[i]}{}_{21} + a_{13}M^{[i]}{}_{31}, \\
C_2^{[i]}(t) &= a_{11}M^{[i]}{}_{12} - a_{12}M^{[i]}{}_{22} + a_{13}M^{[i]}{}_{32} - b^{[i]}{}_{11}M^{[i]}{}_{11} \\
&\quad + b^{[i]}{}_{12}M^{[i]}{}_{21} - b^{[i]}{}_{13}M^{[i]}{}_{31}, \\
C_1^{[i]}(t) &= a_{11}M^{[i]}{}_{13} - a_{12}M^{[i]}{}_{23} + a_{13}M^{[i]}{}_{33} - b^{[i]}{}_{11}M^{[i]}{}_{12} \\
&\quad + b^{[i]}{}_{12}M^{[i]}{}_{22} - b^{[i]}{}_{13}M^{[i]}{}_{32}, \\
C_0^{[i]}(t) &= -b^{[i]}{}_{11}M^{[i]}{}_{13} + b^{[i]}{}_{12}M^{[i]}{}_{23} - b^{[i]}{}_{13}M^{[i]}{}_{33}, \\
M^{[i]}{}_{11} &= a_{22}a_{33} - a_{23}a_{32}, \\
M^{[i]}{}_{12} &= a_{32}b^{[i]}{}_{23} + b^{[i]}{}_{32}a_{23} - b^{[i]}{}_{22}a_{33} - a_{22}b^{[i]}{}_{33}, \\
M^{[i]}{}_{13} &= b^{[i]}{}_{22}b^{[i]}{}_{33} - b^{[i]}{}_{23}b^{[i]}{}_{32}, \\
M^{[i]}{}_{21} &= a_{21}a_{33} - a_{23}a_{31}, \\
M^{[i]}{}_{22} &= a_{31}b^{[i]}{}_{23} + b^{[i]}{}_{31}a_{23} - a_{21}b^{[i]}{}_{33} - b^{[i]}{}_{21}a_{33},
\end{aligned}$$

$M^{[i]}{}_{23} = \mathscr{b}^{[i]}{}_{21}\mathscr{b}^{[i]}{}_{33} - \mathscr{b}^{[i]}{}_{23}\mathscr{b}^{[i]}{}_{31},$
$M^{[i]}{}_{31} = \mathscr{a}_{21}\mathscr{a}_{32} - \mathscr{a}_{22}\mathscr{a}_{31},$
$M^{[i]}{}_{32} = \mathscr{a}_{31}\mathscr{b}^{[i]}{}_{22} + \mathscr{b}^{[i]}{}_{31}\mathscr{a}_{22} - \mathscr{a}_{32}\mathscr{b}^{[i]}{}_{21} - \mathscr{b}^{[i]}{}_{32}\mathscr{a}_{21},$
$M^{[i]}{}_{33} = \mathscr{b}^{[i]}{}_{21}\mathscr{b}^{[i]}{}_{32} - \mathscr{b}^{[i]}{}_{22}\mathscr{b}^{[i]}{}_{31}.$

The discriminant of $\mathcal{P}^{[i]}(\lambda, t)$ with respect to λ, as a function of t, is

$$\begin{aligned}\Delta^{[i]}(t) =& 18C_3^{[i]}C_2^{[i]}C_1^{[i]}C_0^{[i]} - 4{C_2^{[i]}}^3C_0^{[i]} + {C_2^{[i]}}^2{C_1^{[i]}}^2 \\ & - 4C_3^{[i]}{C_1^{[i]}}^3 - 27{C_3^{[i]}}^2{C_0^{[i]}}^2\end{aligned} \tag{11}$$

Proposition 3. Consider a pair of moving elliptic disks $\{\bar{\mathcal{A}}(t), \bar{\mathcal{B}}^{[i]}(t)\}$ in the Euclidean plane $\boldsymbol{E}^2$ represented by the matrices $\mathcal{A}$ and $\mathcal{B}^{[i]}(t)$ respectively. Let $\mathcal{P}^{[i]}(\lambda, t) = C_3^{[i]}(t)\lambda^3 + C_2^{[i]}(t)\lambda^2 + C_1^{[i]}(t)\lambda + C_0^{[i]}(t)$ be their characteristic polynomial and $\Delta^{[i]}(t)$ denotes the discriminant of $\mathcal{P}^{[i]}(\lambda, t)$ with respect to λ. Suppose that every pair $\{\bar{\mathcal{A}}(T_0), \bar{\mathcal{B}}^{[i]}(T_0)\}$ is separate, then the motions of them are collision-free for all $t \in [T_0, T_p]$ if

(1) $C_2^{[i]}(t) < 0$
(2) $\Delta^{[i]}(t) > 0$, $\forall t \in [T_0, T_p]$.

Proof. we will show that the conditions $C_2^{[i]}(t) < 0$ and $\Delta^{[i]}(t) > 0$ are sufficient for the condition in Theorem 1 to be satisfied.
First, we provide the condition for $\mathcal{P}^{[i]}(\lambda, t) = 0$ to be real. From Theorem 2, the third order characteristic polynomial has three real roots if the discriminant $\Delta^{[i]}(t) > 0$ for all $t \in [T_0, T_p]$.
Second, we develop a condition for negative roots. From the elliptic disk matrix properties, it is simple to show that $C_3^{[i]} = det(\mathcal{A}) < 0$ and $C_0^{[i]} = -det(\mathcal{B}^{[i]}) > 0$. By constructing the following RouthâĂŞHurwitz table for $\mathcal{P}^{[i]}(\lambda, t)$

$$\begin{array}{c|cc} \lambda^3 & C_3^{[i]} & C_1^{[i]} \\ \lambda^2 & C_2^{[i]} & C_0^{[i]} \\ \lambda & \frac{C_2^{[i]}C_1^{[i]} - C_3^{[i]}C_0^{[i]}}{C_2^{[i]}} & \\ \lambda^0 & C_0^{[i]} & \end{array},$$

if we constrain $C_2^{[i]}(t)$ to be less than zero, we have only one sign change for the first column elements of Routh table regardless of the undetermined sign of the third element $\frac{C_2^{[i]}C_1^{[i]} - C_3^{[i]}C_0^{[i]}}{C_2^{[i]}}$. That results in at least two negative roots.

4.3 COLREGs Constraints

In case of collision risk, surface vessels should follow COLREGs rules while maneuvering to avoid conflicting actions. We will focus on the three situations; head-on, overtaking, and crossing. In a head-on situation between two vessels, the vessel must turn to starboard so that they pass on the port side of the other one. In an overtaking situation, the overtaking vessel must turn to starboard so that they pass on the port side of the overtaken one. When two vessels are crossing each other, the vessel which has the other on the starboard side must give way and avoid crossing ahead of her. We here conclude that the vessel must maneuver by turning to starboard side at all three situations. Therefore, we will reformulate optimization problem by adding a soft constraint on the rate of change of yaw moment to prioritize the ship maneuvering to the starboard side. The final form of the optimization problem used is:

$$\min_{\boldsymbol{x}(t), \boldsymbol{u}(t)} J(\boldsymbol{x}, \boldsymbol{u}) = \int_{t=T_0}^{T_p+T_0} \ell(\boldsymbol{x}(t), \boldsymbol{u}(t))dt + F(\boldsymbol{x}(T_p)) \tag{12a}$$

subject to :

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}) + g\boldsymbol{u}, \tag{12b}$$
$$\tau_{u,min} < \tau_u < \tau_{u,max} \tag{12c}$$
$$\tau_{r,min} < \tau_r < \tau_{r,max} \tag{12d}$$
$$\dot{\tau_r} - slk < 0 \tag{12e}$$
$$slk > 0 \tag{12f}$$
$$C_2^{[i]}(t) < 0 \tag{12g}$$
$$\Delta^{[i]}(t) > 0. \tag{12h}$$

where slk is a positive slack variable used for softening the constraint (12e) to give priority to negative yaw moment, i.e. turning to starboard, equations (12g) and (12h) are the elliptic collision avoidance constraints for the $[i]$ encounter vessel as in prop.3, $\tau_{u,min}$ and $\tau_{u,max}$ are the minimum and maximum limits for surge force, $\tau_{r,min}$ and $\tau_{r,max}$ are the minimum and maximum limits for sway moment, and the stage cost function will be modified to add penalty on the slack variable as follows:

$$\begin{aligned}\ell(\boldsymbol{x}(t), \boldsymbol{u}(t)) = \|\boldsymbol{x}(t) - \boldsymbol{x}_r(t)\|_Q + \|\boldsymbol{u}(t) - \boldsymbol{u}_r(t)\|_R \\ + \|slk(t)\|_S.\end{aligned} \tag{13}$$

The slack variable weight S should be carefully tuned to avoid possible violation of the COLREGs rules by turning to the port side as the solver might find a minimum there.

4.4 Stability of NMPC

To guarantee asymptotic stability by using the control law $u(k) = \mathcal{K}(x(k))$, it is desirable to use infinite prediction and control horizons, i.e., set $N = \infty$ in (6), but it is infeasible to get the solution of the infinite horizon non-linear optimization problem as discussed in Magni et al. (2001). On the other hand, stability can be guaranteed for finite horizon problems by suitably choosing a terminal cost F and terminal attractive region Ω. This result has been studied in Magni et al. (2001) and conditions required for that have been presented. Although there are clear conditions for the asymptotic stability, designing the terminal cost F and the attractive region ω is still an open problem and may make the online optimizations more difficult and time consuming to solve; see Jadbabaie and Hauser (2005) for more details. In Grüne and Pannek (2011), it was shown that asymptomatic stability can be guaranteed just by tuning N, Q, and R. In Jadbabaie and Hauser (2005), closed loop stability is achieved for relatively long horizons without the need to use terminal cost or terminal constraint . Based on that, the cost function (12) with ℓ selected as in (13) will be used in this paper and without a terminal constraint.

Table 1. Surface Vessel Parameters

Para.	Value	Unit	Para.	Value	Unit
m_1	120.0×10^3	kg	d_1	215.0×10^2	kg$\cdot s^{-1}$
m_2	172.9×10^3	kg	d_2	97.0×10^3	kg$\cdot s^{-1}$
m_3	636.0×10^5	kg	d_3	802.0×10^4	kg$\cdot m^2 \cdot s^{-1}$

Table 2. States and Control variables weights

Variable	x	y	χ	u	v	r	τ_u	τ_r	slk
Weight	5	5	5	1	1	1	0.001	0.001	0.0001

4.5 Discretization

In order to solve the optimization problem (6), a direct multiple shooting technique, introduced in Bock and Plitt (1984), is used. It is derived from the solution of boundary value problems of differential equations. The concept of this technique is based on uniform discretization of the horizon into N smaller intervals such that $t_0 = T_0 < t_1 < \cdots < T_N = Tp$, solving an initial value problem in each of the smaller intervals $[t_j, t_{j+1}]$, and imposing additional matching conditions to form a solution on the whole horizon. The problem constraints are evaluated on the grid nodes t_j and the control vector has been parametrized as piecewise constant. The result of this technique is a least-squares Nonlinear Programming (NLP) with fixed dimensions that can be solved employing Gauss-Newton method.

5. SIMULATION RESULTS

In this section, simulation results are presented to demonstrate the validity and assess the performance of the proposed NMPC scheme for tracking of the underactuated ship (1) with collision avoidance formulated as elliptic disks separation condition. Simulation is done on the MATLAB 2014b with the mex files exported using ACADO toolkit and `qpOASES`. These results have been obtained on a 3.3 GHz core i5 CPU with 8 GB RAM.

The ship chosen for simulation is a monohull ship with a length of $L = 32m$, a width of about $W = 9\ m$, a mass of $118 \times 10^3\ kg$ and other parameters calculated by using VEssel RESponse (VERES)—a program that calculates the added mass and damping matrices for surface ships, see Fathi (2004). The vessel parameters are taken from Do et al. (2002) and are presented in Table 1. The prediction horizon length is selected to be $T_p = 200\ s$ and the sampling interval is selected to be $T_s = 5\ s$, which both lead to a discrete horizon of $N = 40$ samples. The matrices Q and R are chosen to be diagonal to weigh each state and control law independently with the values given in Table 2. It is assumed throughout this simulation that the vessel is equipped with a radar system or LiDAR that provides the controller with the position of nearby vessels. The elliptic disk parameters of the vessel are chosen to be $a = 3L = 96\ m$ and $b = 1.5W = 13.5\ m$ to account for a safety distance between vessels while maneuvering for collision avoidance. The encountered vessels are assumed to have the same dynamics and parameters.

Three different situations are used for assessment; head-on, overtaking, and crossing.

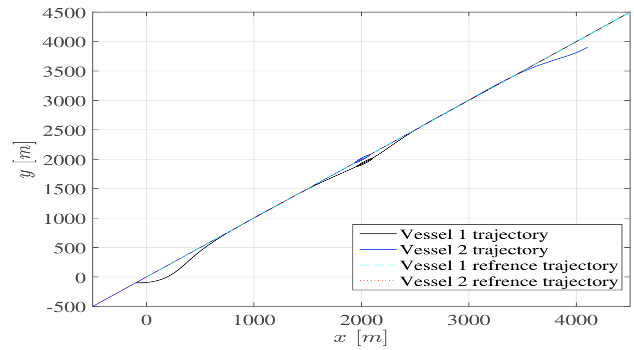

Fig. 3. Trajectory of both vessels at head-on situation

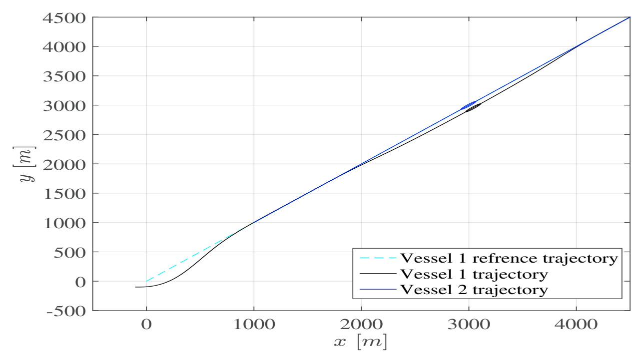

Fig. 4. Trajectory of both vessels at overtaking situation

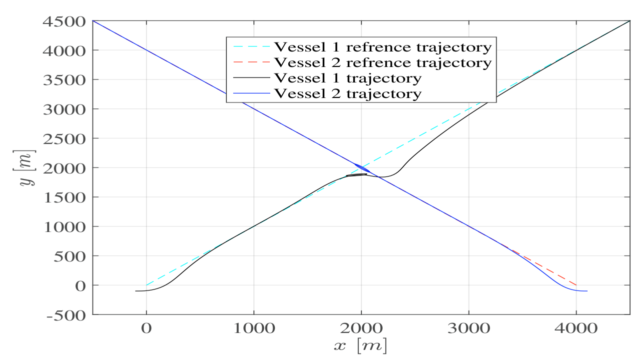

Fig. 5. Trajectory of both vessels at crossing situation

Fig. 3 shows the trajectory when one vessel heads on towards our vessel. Our vessel uses NMPC with elliptic based collision avoidance constraint, and the other one uses NMPC without any collision avoidance schemes. The initial conditions of both vessels are $\boldsymbol{x}(T_0) = [-100\ -100\ 0\ 5\ 0\ 0]^T$ and $\boldsymbol{x}^{[1]}(T_0) = [4100\ 4100\ \frac{5\pi}{4}\ 5\ 0\ 0]^T$. The initial condition of both reference vessels are $\boldsymbol{x_r}(T_0) = [0\ 0\ \frac{\pi}{4}\ 5\ 0\ 0]^T$ and $\boldsymbol{x}^{[1]}{}_r(T_0) = [4000\ 4000\ \frac{5\pi}{4}\ 5\ 0\ 0]^T$. It is clear from the two elliptic disks, drawn at the closes point of approach, that our vessel effectively avoids colliding with the other one and then returns back to its trajectory. For the overtaking situation, our vessel, with the same initial condition and reference trajectory as in the heading-on situation, is overtaking a slower vessel with the following initial condition $\boldsymbol{x}^{[1]}(T_0) = [1000\ 1000\ \frac{\pi}{4}\ 3\ 0\ 0]^T$ that maintains the same initial course and speed. It is shown in Fig. 4 that our vessel maneuvers to the starboard side before approaching the other vessel and then tracks back to its reference trajectory after successfully overtak-

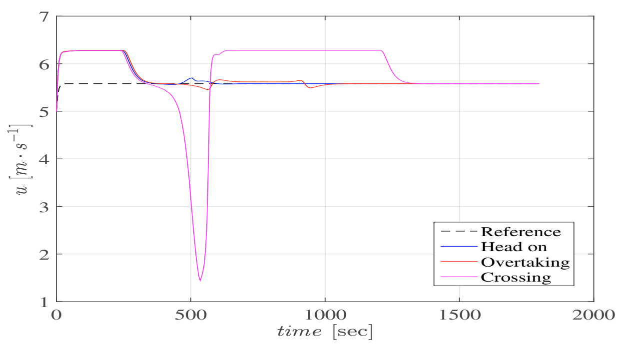

Fig. 6. Surge speed for the three situations

ing it. The crossing situation trajectories are presented in Fig. 5. A vessel, with the initial condition of $\boldsymbol{x}^{[1]}(T_0) = [4100\ \ -100\ \pi\ 5\ 0\ 0]^T$ and the reference initial condition of $\boldsymbol{x}^{[1]}{}_r(T_0) = [4000\ 0\ \frac{3\pi}{4}\ 5\ 0\ 0]^T$, crosses our vessel's path from the right. According to COLREG rules, our vessel maneuvers to the broadside to avoid this possible collision. The surge speed for each the three situations are presented in Fig. 6. During the first 400 *sec*, the vessel accelerates and violates the reference surge velocity to compensate for different initial conditions between reference and actual vessel. It also violates the surge speed while collision avoidance maneuvering.

6. CONCLUSION

An NMPC scheme is presented for trajectory tracking of underactuated surface vessels with controller-embedded collisions avoidance technique based on elliptical ship domain representation. A 3-DOF model is used with only two control variables; surge force and yaw moment. Employing a quadratic cost function, real-time efficient C code is generated using the ACADO toolkit and `qpOASES` solver with collision avoidance as inequality constraints for the optimization problem. The collision avoidance constraints are formulated based on the separation conditions between elliptic disks. For the sake of COLREGs, the optimization problem is reformulated to add priority to maneuvering to the starboard direction in case of collision risk. Simulation results show the ability of the proposed scheme to track desired trajectory while satisfying control law constraints, and achieving collision avoidance for head-on, crossing, and overtaking situations. The maximum execution time of the implemented algorithm is less than 16 ms, which amounts to appr. 0.32% of the sampling interval.
Future research will aim at taking into consideration both uncertainty and time delay of measurement devices used for detecting the position of nearby vessels or obstacles.

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