THE FOUNDATIONS OF STEERING AND MANOEUVRING

by

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Layout of paper

- Introduction
- The early days
- The first equations of motion
- Forces and moments in potential flow
- Linear equations of motion
- Non-linear forces and moments
- Full equations of motion
- Gain and time constant equations

Layout of paper (continued)

- Non-linear yaw rate equations
- A closer look at the gain and time constants
- Linear derivatives
- Ship rudder control
- Simple autopilot tuning
- Concluding remarks

The early days

New Testament

The Epistle General of St. James

Chapter 3, Verse 4.

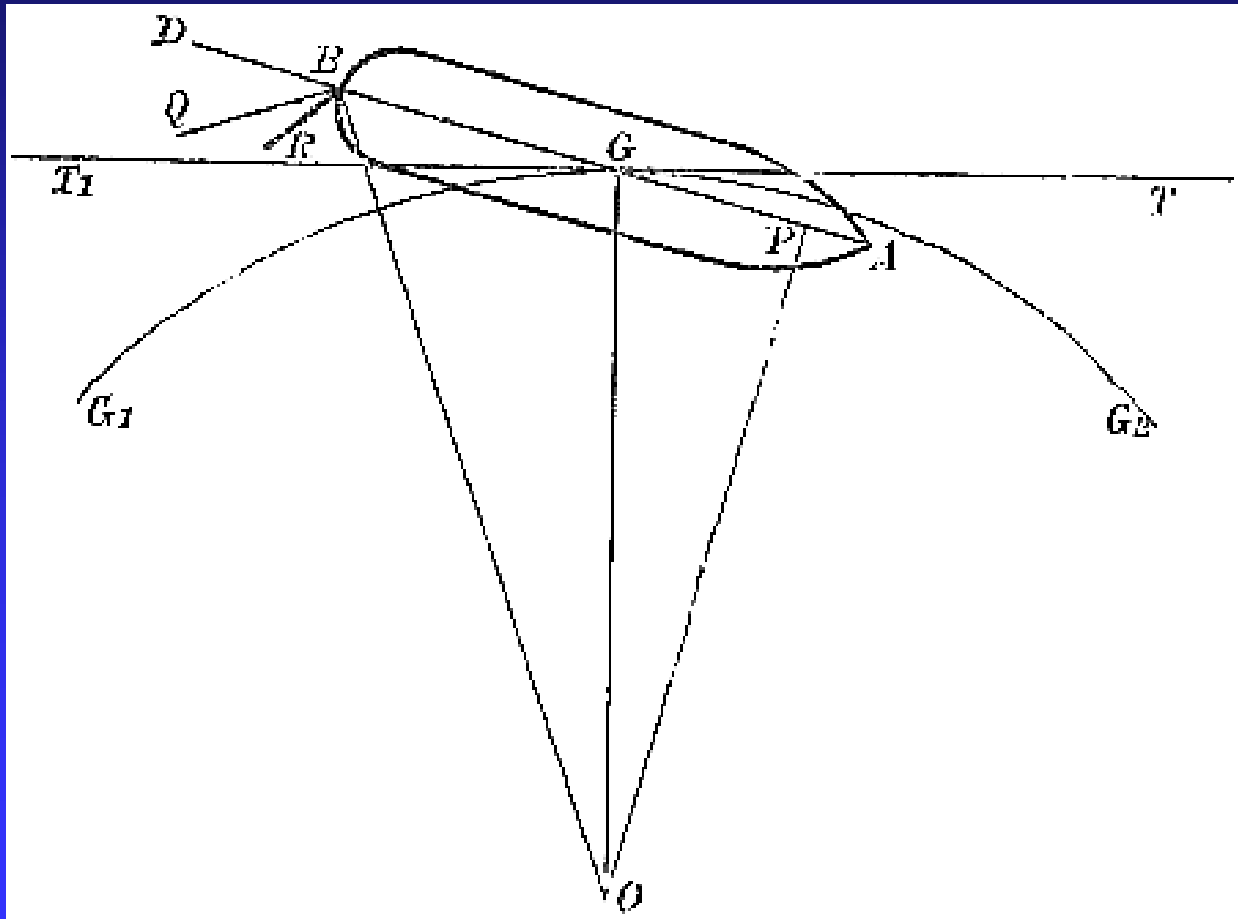
"...or look at ships: though they are so large that it takes strong winds to drive them, yet they are guided by a very small rudder wherever the will of the pilot directs."

The early days

- Turning ability of warships
- Model tests on rudders
- Zig-zag manoeuvres
- No attempts at mathematical modelling

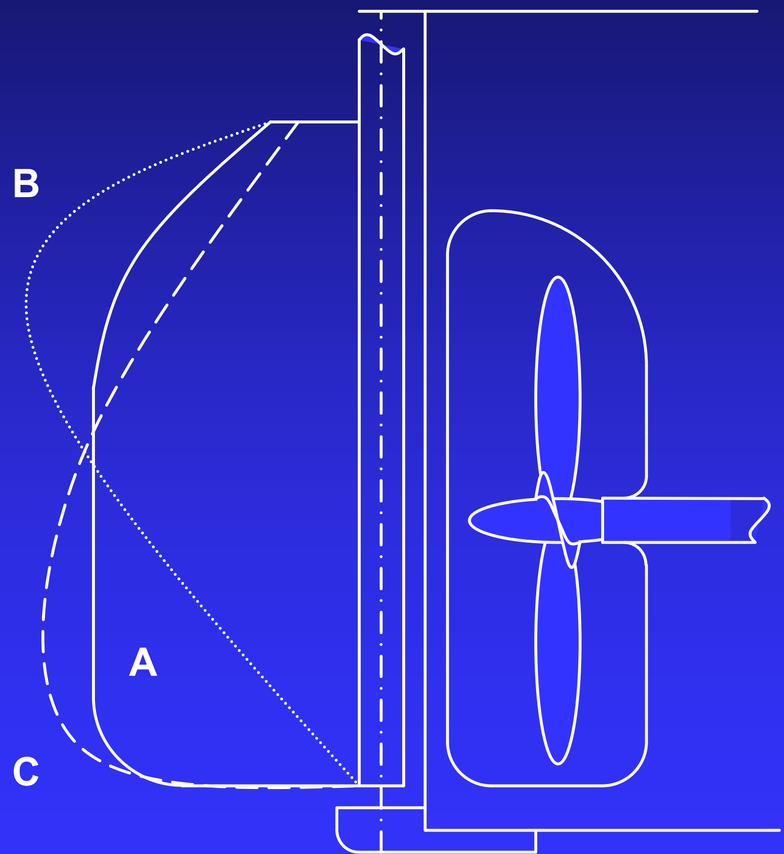
The early days

Figure 1. Ship turning, from White (1900)



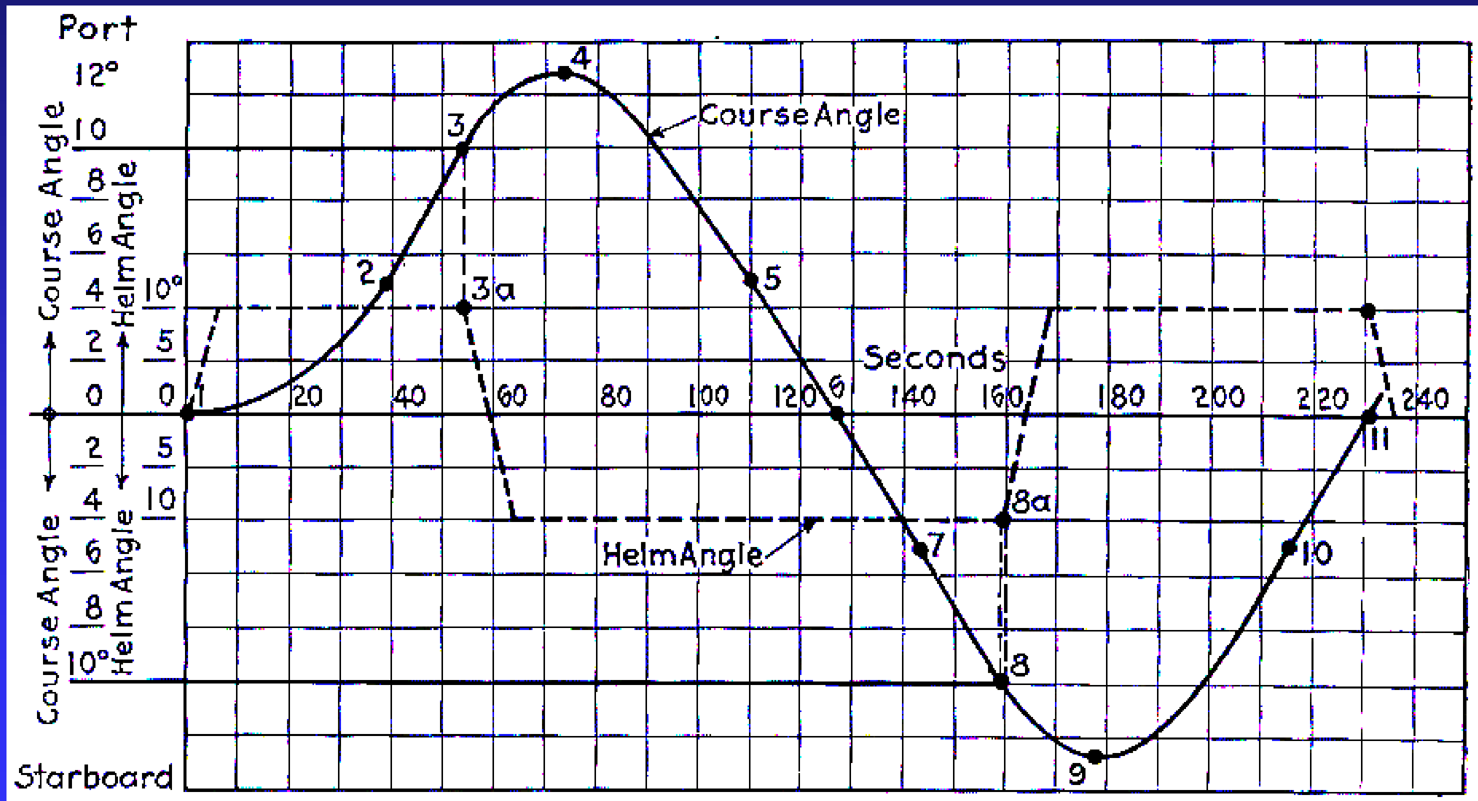
The early days

Figure 2. Ship's rudders, from Baker & Bottomley (1922-31)



The early days

Figure 3. Zig-zag manoeuvre, from Kempf (1932)

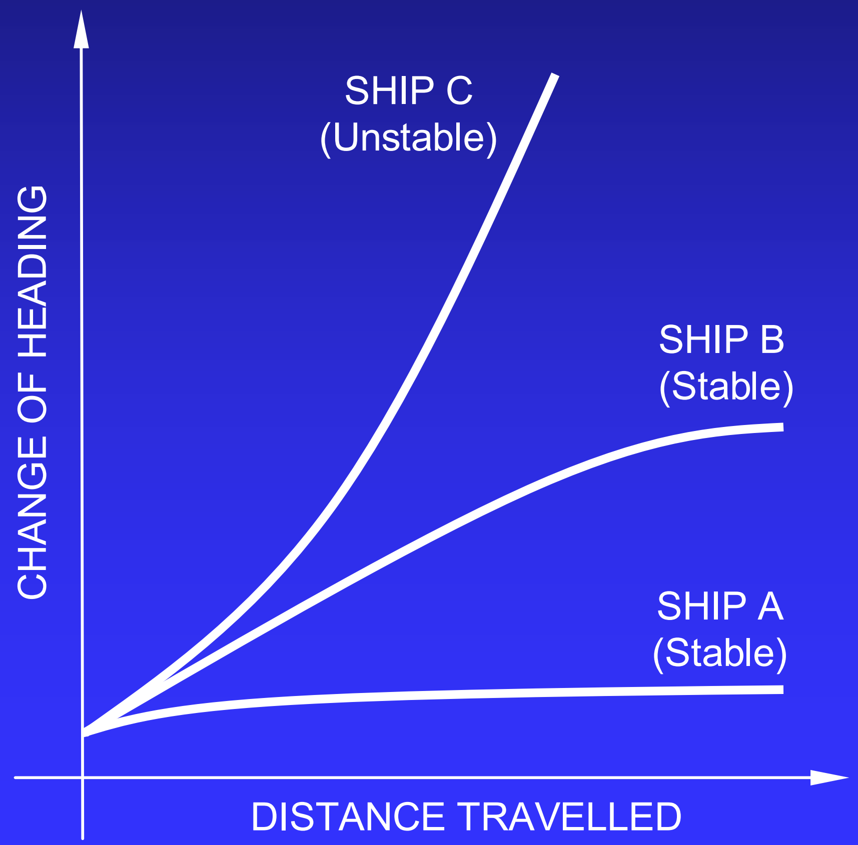


The first equations of motion

- Early studies of rudder forces which ignored the hull entirely
- Three degrees of freedom
- Followed aircraft stability approach
- First attempts by Davidson & Schiff (1946)
- Full-scale ship trials for dynamic stability by Dieudonné (1953) and Bech (1966)

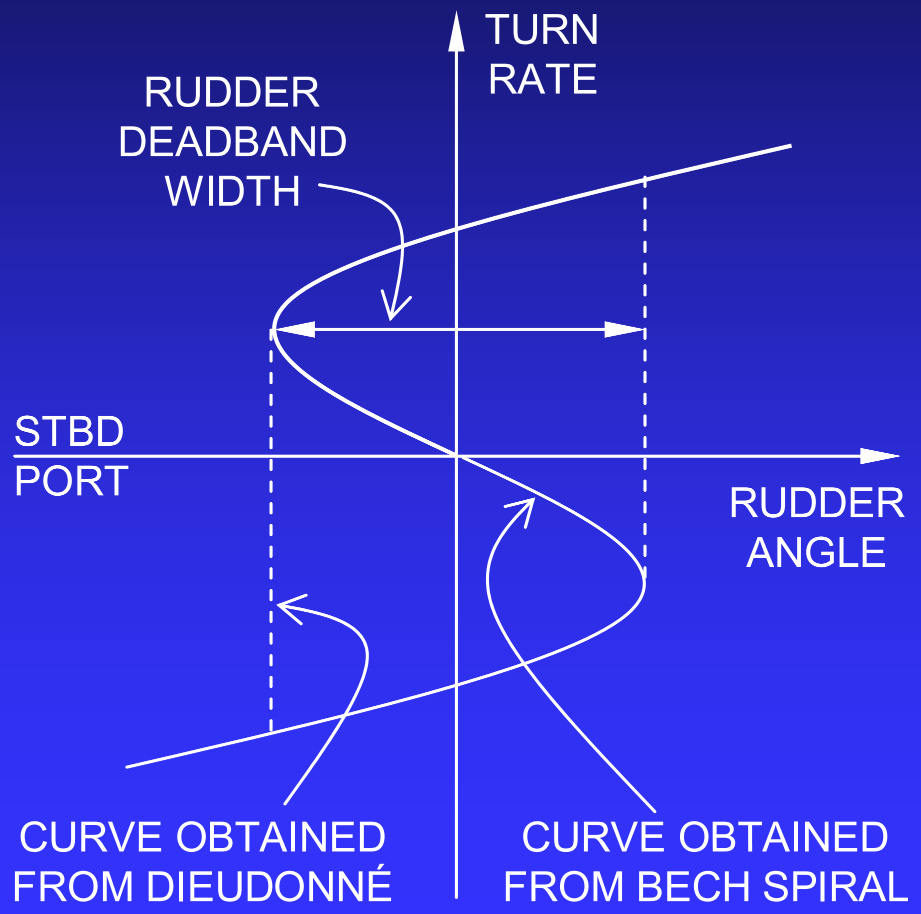
The first equations of motion

Figure 4. Course changing trajectories,
from Davidson & Schiff (1946)



The first equations of motion

Figure 5. Spiral curves, from Dieudonné (1953) and Bech (1966)

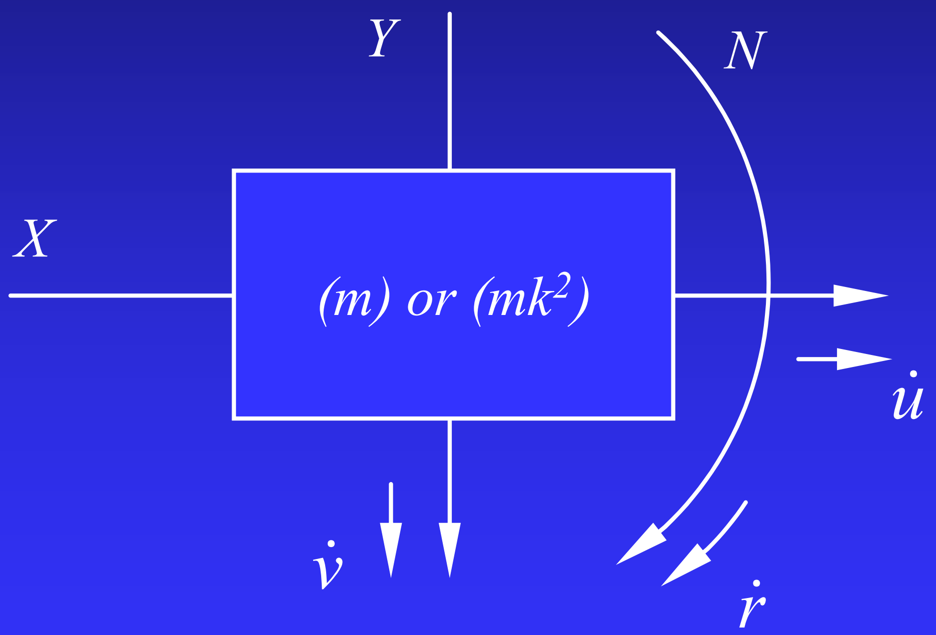


The first equations of motion

- Three degrees of freedom
- Perturbation equations
- Newton’s second law of motion
- Force = mass x acceleration
- Surge, sway and yaw
- Earth fixed co-ordinates

The first equations of motion

Figure 6. Motion with earth fixed axes



$$X = m\,\dot{u}$$

$$Y = m\,\dot{v}$$

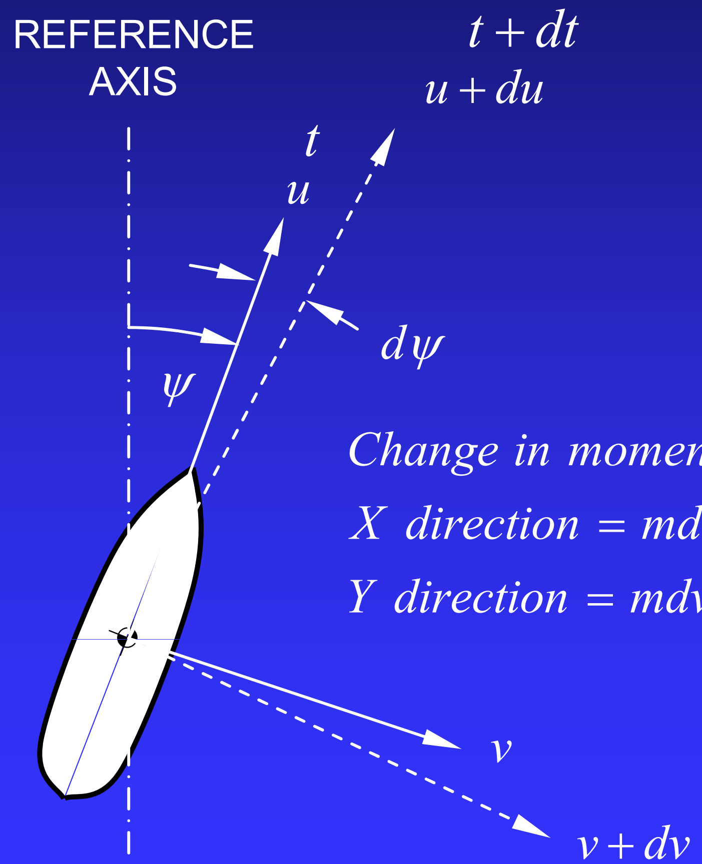
$$N = I_z\,\dot{r}$$

The first equations of motion

- Change to ship fixed moving co-ordinates
- Gives rise to extra terms in equations
- Components of centrifugal force along axes due to motion about instantaneous centre

The first equations of motion

Figure 7. Motion with ship fixed axes



Change in momentum in dt

X *direction* $= mdu + mv\, d\psi$

Y *direction* $= mdv + mu\, d\psi$

$$X = m(\dot{u} - rv)$$

$$Y = m(\dot{v} + ru)$$

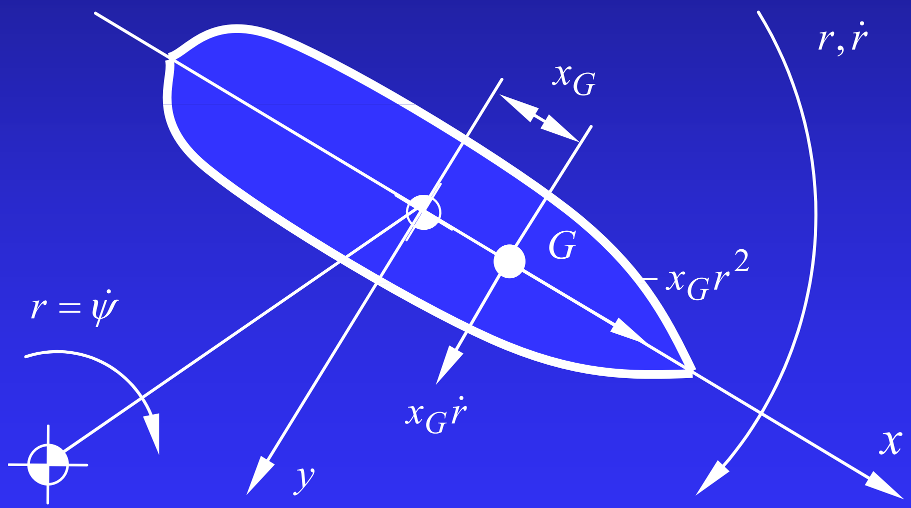
$$N = I_z\, \dot{r}$$

The first equations of motion

- Change to origin at amidships rather than at centre of gravity
- Gives rise to extra terms in equations
- Extra forces due to the centre of gravity rotating about the midship point

The first equations of motion

Figure 8. Rotation of centre of gravity about amidships



$$X = m(\dot{u} - rv - x_G r^2)$$
$$Y = m(\dot{v} + ru + x_G \dot{r})$$
$$N = I_z \dot{r} + m x_G(\dot{v} + ru)$$

Forces and moments in potential flow

- First classical analysis by Lamb (1879)
- Forces and moments and mainly dependent on perturbation accelerations only
- Forces and moments do not arise from perturbation velocities in classical potential flow
- This is achieved by certain techniques in slender body theory.

Forces and moments in potential flow

Equation (4), (5) and (6)

- Non dimensionalising factors $\frac{1}{2}\rho U^2 L^2$ $\frac{1}{2}\rho U^2 L^3$
- Use prime notation in all equations
- Forces and moments from potential flow

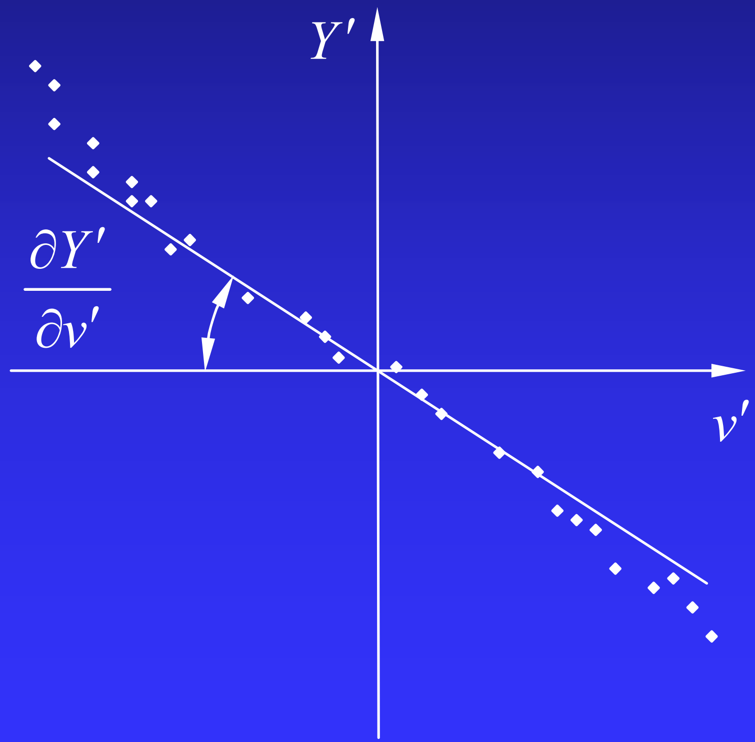
$$X' = X'_{\dot{u}}\dot{u}' - Y'_{\dot{v}}v'r' - Y'_{\dot{r}}r'^2$$
$$Y' = Y'_{\dot{v}}\dot{v}' + X'_{\dot{u}}u'r' + Y'_{\dot{r}}\dot{r}')$$
$$N' = N'_{\dot{r}}\dot{r}' + (Y'_{\dot{v}} - X'_{\dot{u}})u'v' + Y'_{\dot{r}}(\dot{v}' + u'r')$$

- Forces and moments for slender body theory

$$X' = X'_{\dot{u}}\dot{u} + X'_{u}(u' - u'_0)$$
$$Y' = Y'_{\dot{v}}\dot{v}' + Y'_{\dot{r}}\dot{r}' + Y'_{v}v' + Y'_{r}r' + Y'_{\delta}\delta$$
$$N' = N'_{\dot{v}}\dot{v}' + N'_{\dot{r}}\dot{r}' + N'_{v}v' + N'_{r}r' + N'_{\delta}\delta$$

Forces and moments in potential flow

Fig. 9. Sway force versus sway velocity, from oblique towing



Derivative Y_v' is the slope of this line

Other derivatives are found in a similar manner

OPTIPOD Cruise ship at HSVA

OPTIPOD Ropax at CTO

OPTIPOD Cargo ship at SSPA

Linear equations of motion

Equation (7)

First order linear equations in two motion variables v' and r' and one control variable δ. All the other derivative, mass and inertia terms are constants for a particular ship and must be found by experiment or calculation

$$(Y'_{\dot{v}} - m')\,\dot{v}' + Y'_v\,v' + (Y'_{\dot{r}} - m'x'_G)\,\dot{r}' + (Y'_r - m')\,r' + Y'_\delta\,\delta = 0$$

$$(N'_{\dot{v}} - m'x'_G)\,\dot{v}' + N'_v\,v' + (N'_{\dot{r}} - I'_z)\,\dot{r}' + (N'_r - m'x'_G)\,r' + N'_\delta\,\delta = 0$$

Non-linear forces and moments

Equation (8). Taylor series cubic curves.

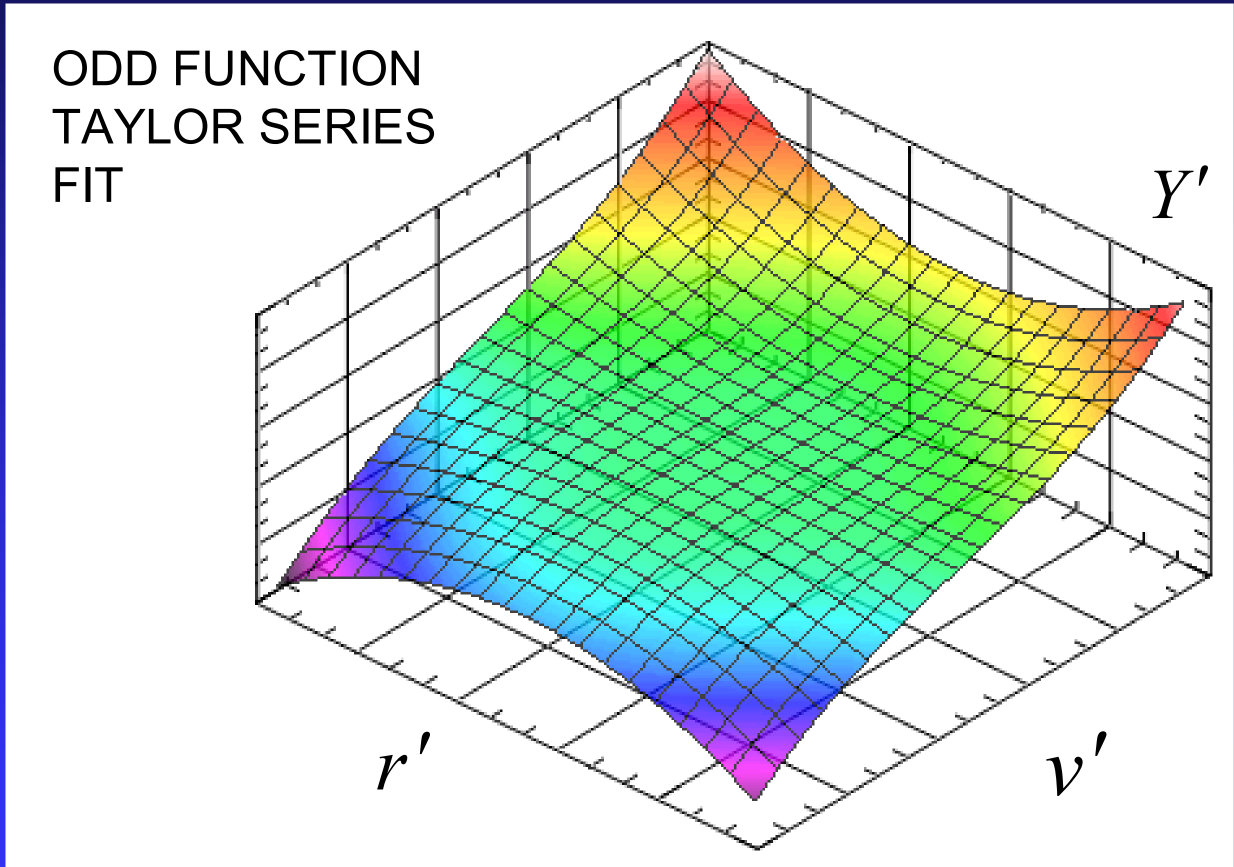
Hydrodynamic force Y' and moment N' are really non-linear functions v' and r'. Here only first and third order terms are used to provide an odd function

$$Y' = Y'_v v' + Y'_r r' + Y'_{vvv} v'^3 + Y'_{vvr} v'^2 r' + Y'_{vrr} v' r'^2 + Y'_{rrr} r'^3$$

$$N' = N'_v v' + N'_r r' + N'_{vvv} v'^3 + N'_{vvr} v'^2 r' + N'_{vrr} v' r'^2 + N'_{rrr} r'^3$$

Non-linear forces and moments

Figure 10. Taylor series cubic curve fit.



This form of non-linearity gives a smooth surface

Non-linear forces and moments

Equation (9). Second order modulus curves.

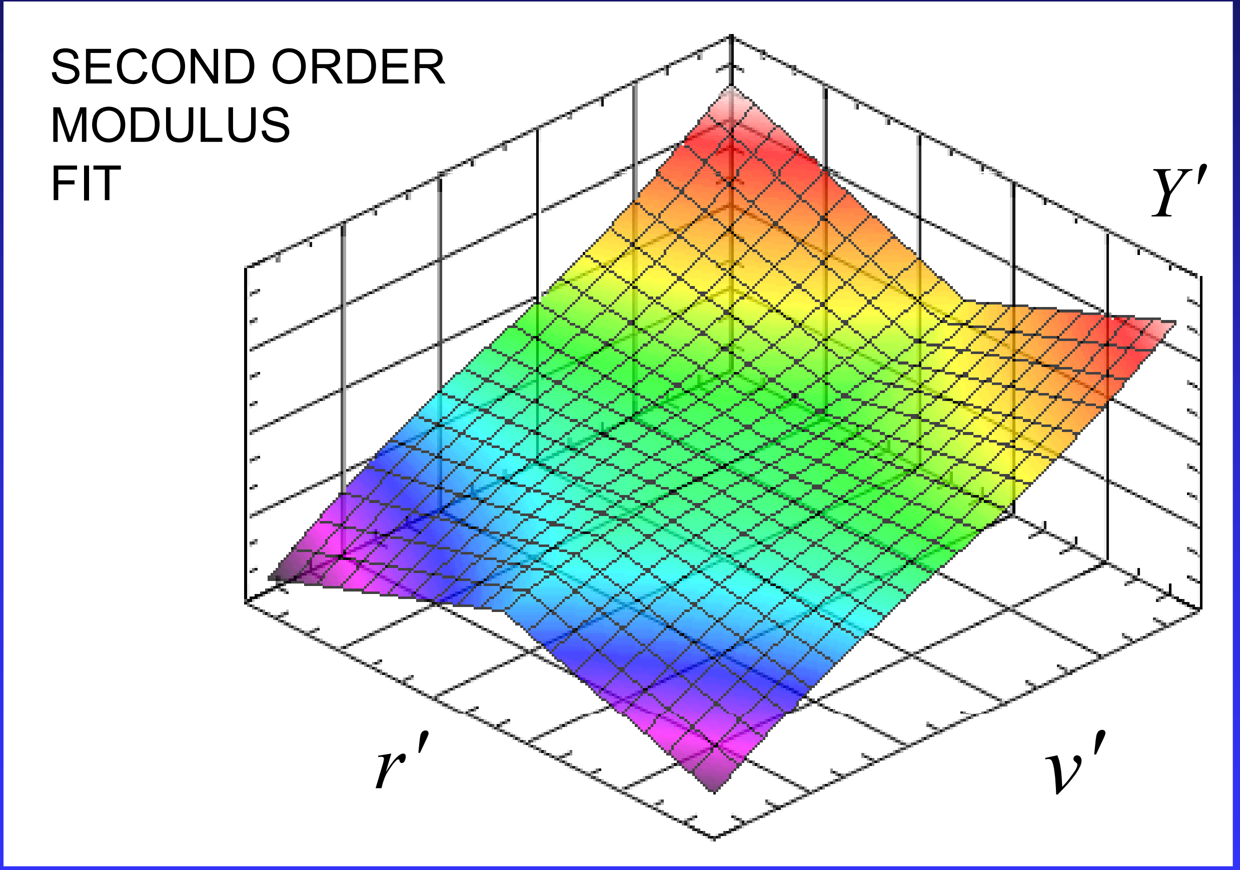
Hydrodynamic force Y' and moment N' are really non-linear functions v' and r'. Here only first and second order terms with modulus function are used to provide an odd function

$$Y' = Y'_v v' + Y'_r r' + Y'_{v|v|} v'|v'| + Y'_{v|r|} v'|r'|$$
$$+ Y'_{|v|r} |v'| r' + Y'_{r|r|} r'|r'|$$

$$N' = N'_v v' + N'_r r' + N'_{v|v|} v'|v'| + N'_{v|r|} v'|r'|$$
$$+ N'_{|v|r} |v'| r' + N'_{r|r|} r'|r'|$$

Non-linear forces and moments

Figure 11. Second order modulus curve fit.

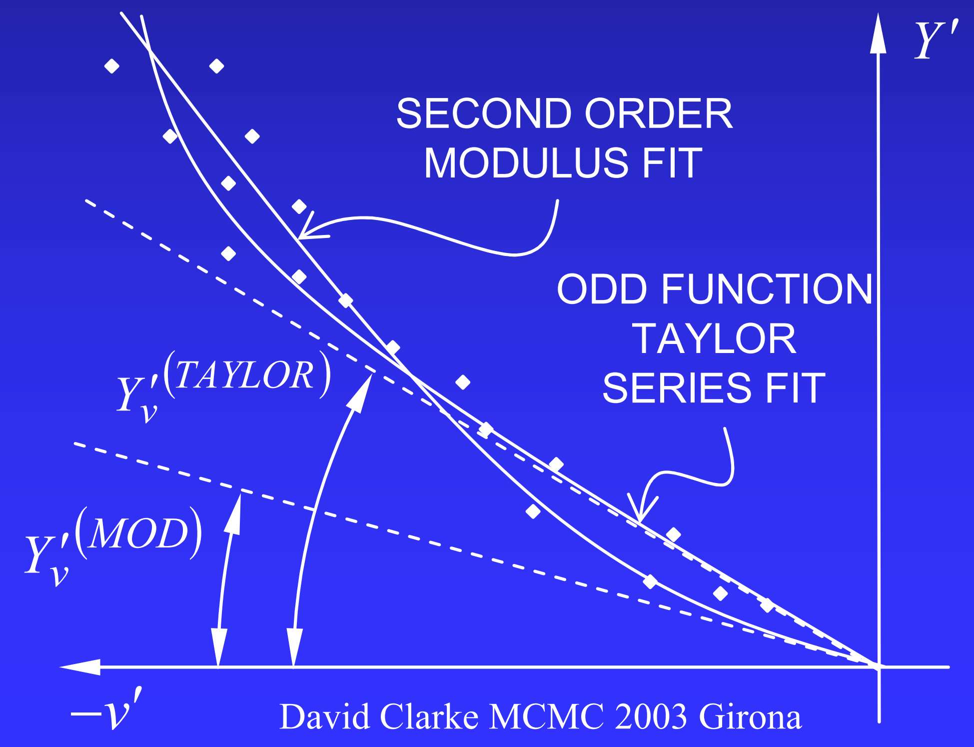


This form of non-linearity gives a flatter facetted surface

Non-linear forces and moments

Figure 12. Different derivative values of from cubic and second order modulus curve fit.

This is a big problem when comparing derivatives from different experimental facilities



Full equations of motion

- Needed for full mission simulators.
- Slow speed, transverse and rotational. motion, as well as the effects of thrusters.
- Use of tugs, operation in shallow water, bank effects and passing ships.
- To avoid complicated equations, often force and moment coefficient look up tables are used in practice.

Gain and time constant equations

Equation (10) & (11)

First order linear equations, from Equation (7), in two motion variables v' and r' and one control variable δ, have been changed into two second order equations, one in the variable v' and the other in r' .

Equation Form, often referred to as the Nomoto Equations

$$T_1'T_2'\ddot{r}' + (T_1' + T_2')\dot{r}' + r' = K'\delta' + K'T_3'\dot{\delta}'$$

$$T_1'T_2'\ddot{v}' + (T_1' + T_2')\dot{v}' + v' = K_v'\delta' + K_v'T_4'\dot{\delta}'$$

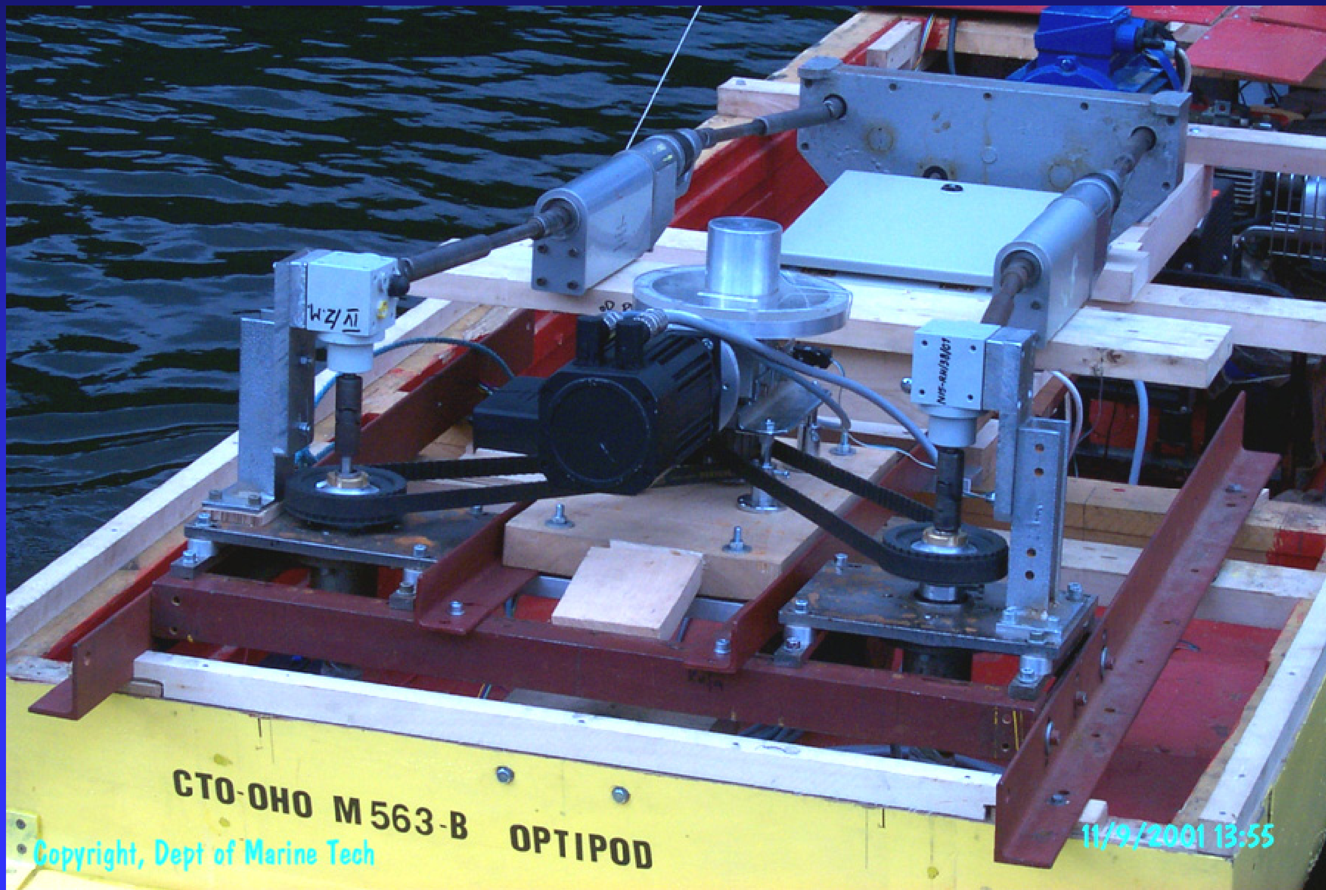
Only the r' version is used and it often appears in its Laplace form

$$\frac{r'}{-\delta}(s) = \frac{K'(1+T_3's)}{(1+T_1's)(1+T_2's)}$$

OPTIPOD Ropax at CTO

OPTIPOD Ropax at CTO

OPTIPOD Ropax at CTO



Gain and time constant equations

Equation (12). The relationship between the derivatives and the time constants is easily achieved by algebra.

$$T'_1 T'_2 = \frac{\begin{bmatrix} (Y'_{\dot v} - m')(N'_{\dot r} - I'_z) \\ \quad - (Y'_{\dot r} - m'x'_G)(N'_{\dot v} - m'x'_G) \end{bmatrix}}{C'}$$

$$T_3 = \frac{(N'_{\dot v} - m'x'_G)Y'_\delta - (Y'_{\dot v} - m')N'_\delta}{N'_v Y'_\delta - Y'_v N'_\delta}$$

$$T'_1 + T'_2 = \frac{\begin{bmatrix} (Y'_{\dot v} - m')(N'_r - m'x'_G) \\ + (N'_{\dot r} - I'_z)Y'_v - (Y'_{\dot r} - m'x'_G)N'_v \\ \quad - (N'_{\dot v} - m'x'_G)(Y'_r - m') \end{bmatrix}}{C'}$$

$$K'_v = \frac{(N'_r - m'x'_G)Y'_\delta - (Y'_r - m')N'_\delta}{C'}$$

$$T'_4 = \frac{(N'_{\dot r} - I'_z)Y'_\delta - (Y'_{\dot r} - m'x'_G)N'_\delta}{(N'_r - m'x'_G)Y'_\delta - (Y'_r - m')\,N'_\delta}$$

$$K' = \frac{N'_v Y'_\delta - Y'_v N'_\delta}{C'}$$

$$C' = (N'_r - m'x'_G)\,Y'_v - (Y'_r - m')\,N'_v$$

Ten derivatives, one mass and one mass moment of inertia versus one gain and three time constants

Gain and time constant equations

Equation (13)

Second order equation was simplified by Nomoto to an approximate first order equation, either by ignoring the smaller time constant from the second order equation, giving rise to the following

$$T'\dot{r}' + r' = K'\delta$$

Alternatively, assuming the smaller pole and the numerator zero are of similar magnitude and cancel each other out, giving the Laplace form as shown

$$\frac{r'}{-\delta}(s) = \frac{K'}{(1 + T's)}$$

The ship is now described by only one gain and one time constant, but BEWARE.

Gain and time constant equations

Two heading versus time solutions

Second order solution for initial turning, giving heading versus time

$$\psi(t) = K'\delta \begin{bmatrix} t' \\ -\left(T_1' + T_2' - T_3'\right) + t_r'/2 \\ +\dfrac{T_1' - T_3'}{T_1' - T_2'}\dfrac{T_1'^2}{t_r'}\left(e^{t_r'/T_1'} - 1\right)e^{-t'/T_1'} \\ -\dfrac{T_2' - T_3'}{T_1' - T_2'}\dfrac{T_2'^2}{t_r'}\left(e^{t_r'/T_2'} - 1\right)e^{-t'/T_2'} \end{bmatrix}$$

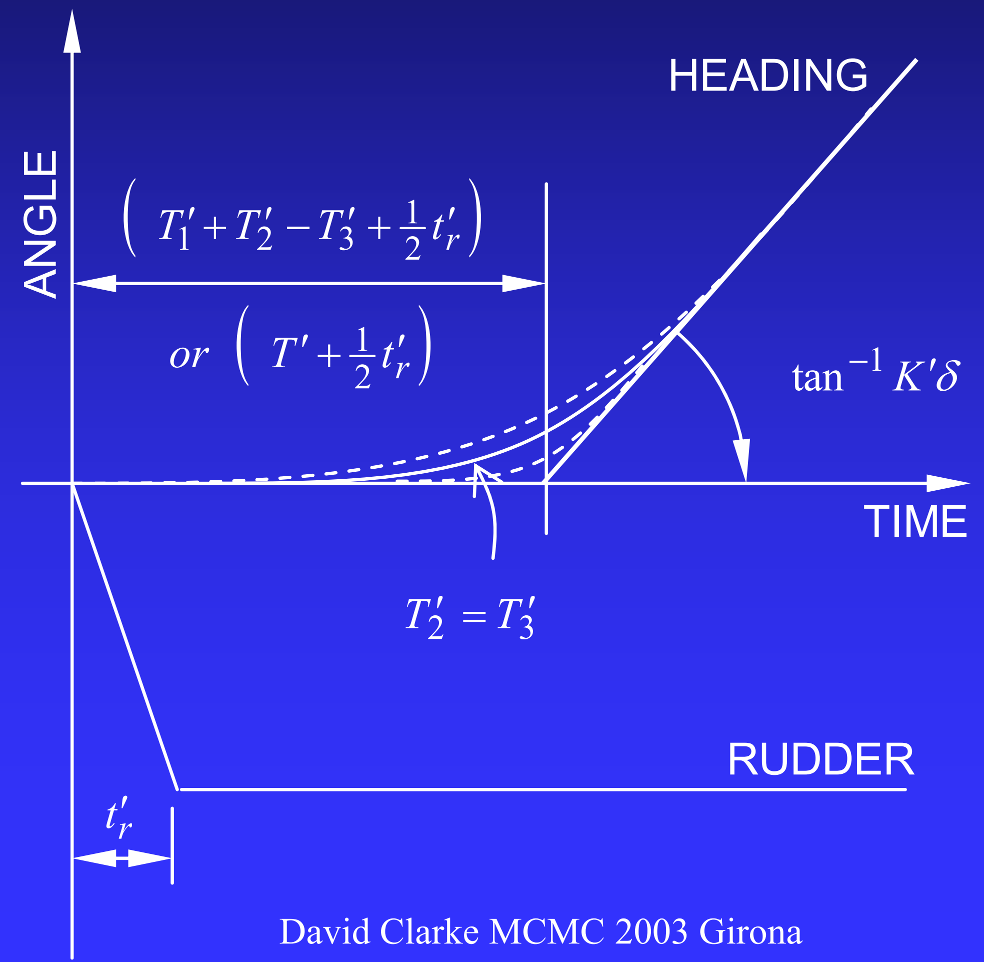
First order solution, giving heading versus time

$$\psi(t) = K'\delta \begin{bmatrix} t' - T' + t_r'/2 \\ +\dfrac{T'^2}{t_r'}\left(e^{t_r'/T'} - 1\right)e^{-t'/T'} \end{bmatrix}$$

Gain and time constant equations

Figure 13. Two heading versus time curves, showing first and

second order solutions



Paradise

Gain and time constant equations

Figs. 14 & 15. Derivation of *K'* from zig-zag manoeuvre

Nomoto first order equation

$$T'\dot{r}' + r' = K'\delta$$

Integrate wrt t'

$$T'\int_0^{t'} \dot{r}'dt' + \int_0^{t'} r'dt' = K'\int_0^{t'} \delta dt'$$

$$T'\left[r'\right]_0^{t'} + \left[\psi\right]_0^{t'} = K'\int_0^{t'} \delta dt'$$

Re-arrange, when $r' = 0$ or when $\psi = 0$

$$K' = -\frac{\psi_1 - \psi_2}{\int_{t'_1}^{t'_2} \delta \, dt'}$$

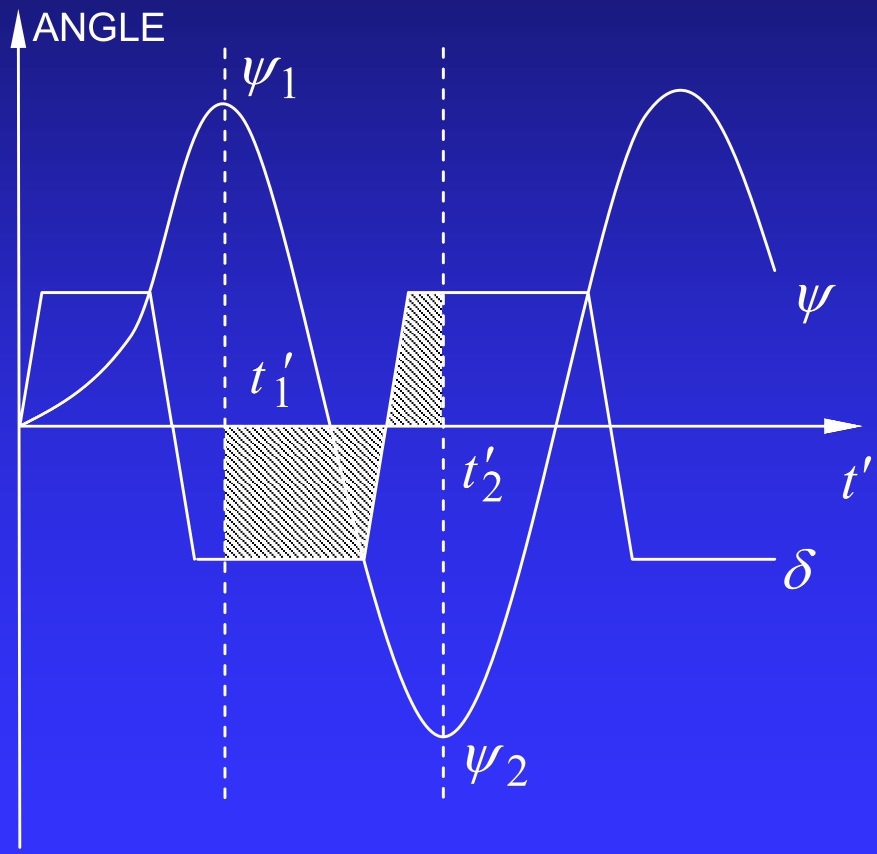
$$\frac{K'}{T'} = -\frac{r'_3 - r'_4}{\int_{t'_3}^{t'_4} \delta \, dt'}$$

Gain and time constant equations

Fig. 14. Derivation of K' from zig-zag manoeuvre

Equation (19)

$$K' = -\frac{\psi_1 - \psi_2}{\int_{t'_1}^{t'_2} \delta \, dt'}$$

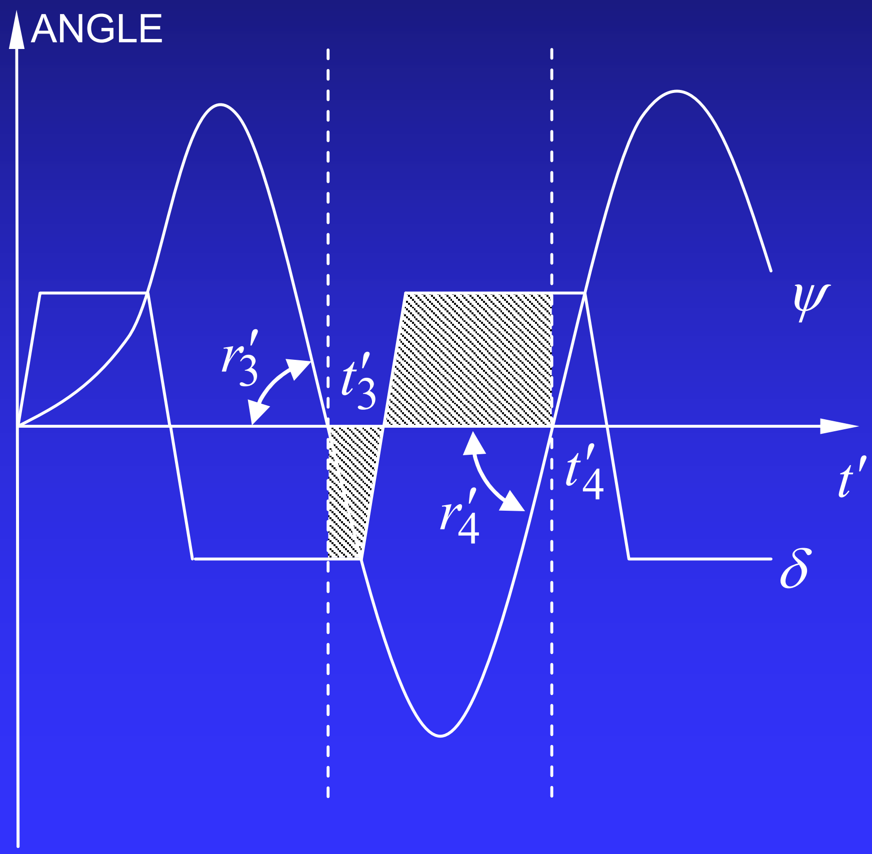


Numerator integral is shown by shaded area

Gain and time constant equations

Fig. 15. Derivation of T' from zig-zag manoeuvre

$$\frac{K'}{T'} = -\frac{r_3' - r_4'}{\int_{t_3'}^{t_4'} \delta \, dt'}$$



Numerator integral is shown by shaded area

Non-linear yaw rate equations

Approximate separation of variables v' and r' in non-linear

parts of equations of motion, using the simple relationship

$$v' = k\,r'$$

This gives linear behaviour, correct results in steady state and gives rise to the following analytically derived equation

$$A_0\ddot{r}' + (A_1 + A_2 r' + A_3 r'^2)\,\dot{r}' + (A_4 + A_5 r' + A_6 r'^2)\,r'$$

$$+ (A_7 + A_8\delta^2)\,\dot{\delta}' + (A_9 + A_{10}\delta^2)\,\delta = 0$$

This may be compared with the empirical equation of Nomoto

$$T_1' T_2' \ddot{r}' + (T_1' + T_2')\dot{r}' + r' + \alpha r'^3 = K'\delta + K' T_3' \dot{\delta}'$$

where

$$A_0 / A_4 = T_1' T_2'$$
$$A_1 / A_4 = T_1' + T_2'$$
$$A_7 / A_4 = K' T_3'$$
$$A_9 / A_4 = K'$$

and

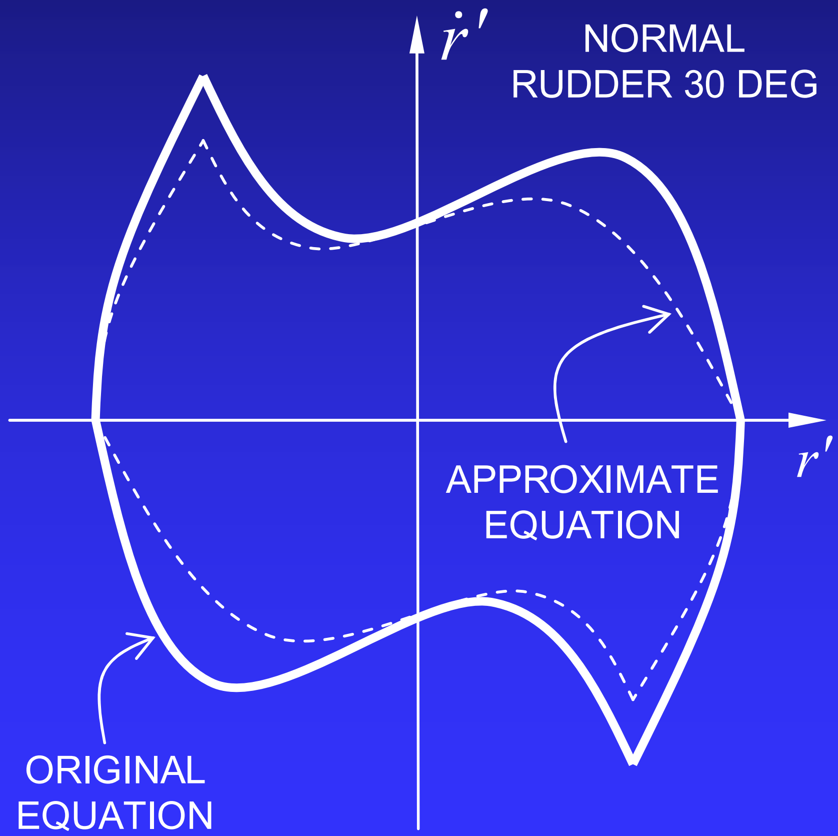
$$A_6 / A_4 = \alpha$$

Phase portrait for zig-zag manoeuvre

Figure 16.

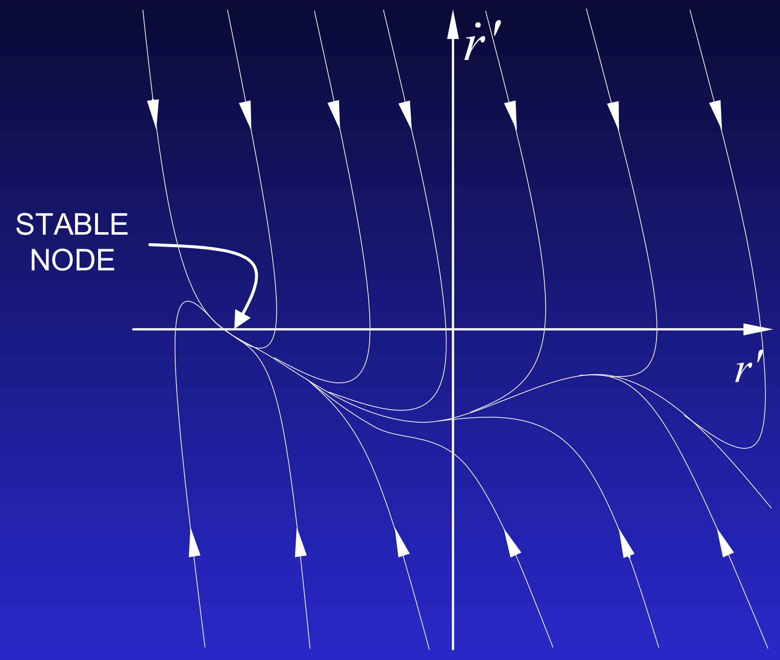
The previous equations showed that a great deal of information has been neglected from the equations of motion

For a zig-zag manoeuvre, when the angular acceleration plotted is against angular velocity it shows how non-linear ship response can be. The lines leading to the nodes should be straight for a linear system



Phase portrait

Fig. 17. Phase portrait for zero rudder angle. Shows three roots for cubic steady state rate of turn curve, two nodes and one saddle point.



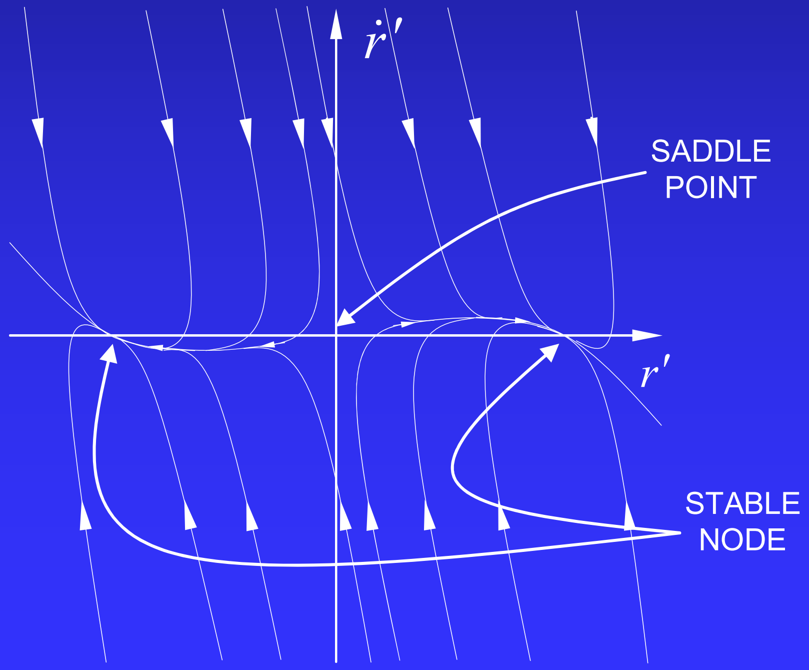


Fig. 18. Phase portrait for rudder angle outside the spiral loop. Only one real root now, a stable node the other two roots form a complex pair

A closer look at the gain and the time constants

Re-arrange Equation (12) in terms of derivative ratios, which are moments divided by forces. These are levers and centres of pressure.

$$K' = \frac{Y'_\delta}{Y'_r - m'}\left[\frac{\left(\frac{N'_v}{Y'_v}\right) - \left(\frac{N'_\delta}{Y'_\delta}\right)}{\left(\frac{N'_r - m'x'_G}{Y'_r - m'}\right) - \left(\frac{N'_v}{Y'_v}\right)}\right]$$

For rudder at stern:-

$$K' = \frac{Y'_\delta}{Y'_r - m'}\left[\frac{0.3-(-0.5)}{0.4-0.3}\right] = \frac{Y'_\delta}{Y'_r - m'}[+8.0]$$

Or rudder at bow:-

$$K' = \frac{Y'_\delta}{Y'_r - m'}\left[\frac{0.3-(+0.5)}{0.4-0.3}\right] = \frac{Y'_\delta}{Y'_r - m'}[-2.0]$$

A closer look at the gain and the time constants

Re-arranging Eq. (12), for the time constant T'_3 in the numerator of the ship yaw rate transfer function, to test for non-minimum phase.

$$T'_3 = \frac{Y'_{\dot{v}} - m'}{Y'_v}\left[\frac{\left(\dfrac{N'_{\dot{v}} - m'x'_G}{Y'_{\dot{v}} - m'}\right) - \left(\dfrac{N'_\delta}{Y'_\delta}\right)}{\left(\dfrac{N'_v}{Y'_v}\right) - \left(\dfrac{N'_\delta}{Y'_\delta}\right)}\right]$$

The value of T'_3 must become negative, test for numerator equal to zero.

$$\left(\frac{N'_v}{Y'_v}\right) - \left(\frac{N'_\delta}{Y'_\delta}\right) = 0$$

$$(+0.35) - (-0.50) = 0$$

Is unlikely to be negative!

A closer look at the gain and the time constants

Re-arranging Eq. (12), for the time constant T'_4 in the numerator of the ship sway velocity transfer function, to test for non-minimum phase.

$$T'_4 = \frac{Y'_{\dot r} - m'x'_G}{Y'_r - m'} \left[\frac{\left(\frac{N'_{\dot r} - I'_z}{Y'_{\dot r} - m'x'_G}\right) - \left(\frac{N'_\delta}{Y'_\delta}\right)}{\left(\frac{N'_r - m'x'_G}{Y'_r - m'}\right) - \left(\frac{N'_\delta}{Y'_\delta}\right)} \right]$$

$$\left(\frac{N'_r - m'x'_G}{Y'_r - m'}\right) - \left(\frac{N'_\delta}{Y'_\delta}\right) = 0$$

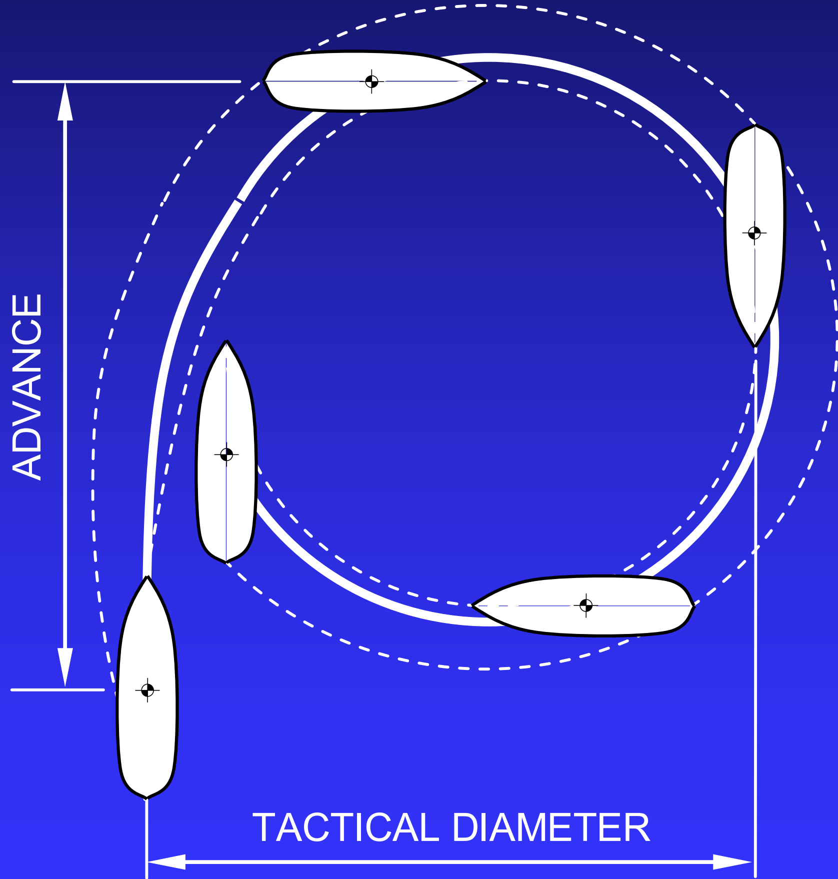
The value of T'_4 must become negative, test for numerator equal to zero.

$$(+0.40) - (-0.50) = 0$$

Is unlikely to be negative!

A closer look at the gain and the time constants

Non-minimum phase cannot occur with normal ships, either in yaw rate or sway velocity.
This contradicts popular belief, that a ship moves the wrong way, as a first response to the rudder.
Of course this refers to the midship point.
The stern may move in the opposite direction.



Linear derivatives

- Derivatives are the constants which characterise the ship's manoeuvring behaviour.
- May be found by model experiments, using a rotating arm facility or a planar motion mechanism.
- Calculation using slender body theory or CFD methods is still unreliable.
- System identification has yielded limited results.
- Model test results used for establishing a data base.
- Regression analysis of data base gives empirical equations
- Only reliable within confines of data base.
- Cannot forecast derivatives for new hull types not in data base

Ship rudder control

Simple autopilot equation plus steering engine.

$$a\psi + br' = \delta + T'_E \dot{\delta}'$$

This equation is substituted into the linear equations of motion

$$(Y'_{\dot{v}} - m')\dot{v}' + Y'_v v' + (Y'_{\dot{r}} - m'x'_G)\dot{r}' + (Y'_r - m')r' + Y'_\delta \delta = 0$$

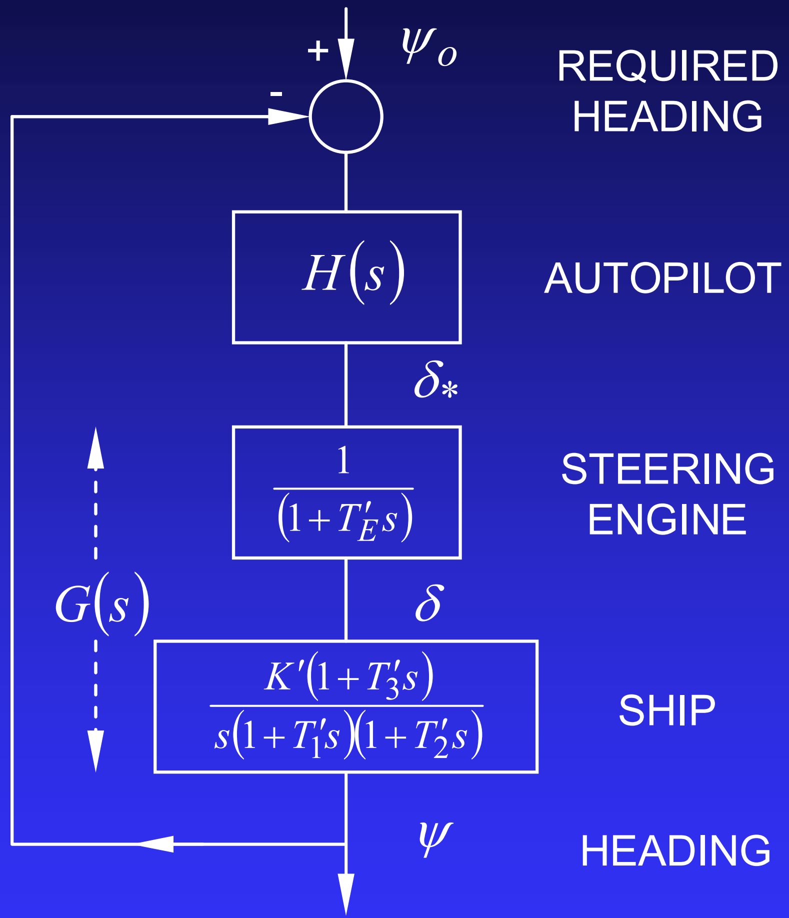
$$(N'_{\dot{v}} - m'x'_G)\dot{v}' + N'_v v' + (N'_{\dot{r}} - I'_z)\dot{r}' + (N'_r - m'x'_G)r' + N'_\delta \delta = 0$$

This give rise to a new stability criterion including the autopilot settings.

$$C' = (N'_r - m'x'_G + bN'_\delta)\ Y'_v - (Y'_r - m' + bY'_\delta)\ N'_v$$

Ship rudder control

The normal closed loop system includes autopilot, steering engine and ship. Since the usual equations of motion are for yaw rate, a free integrator must be added into the forward path transfer function.



$$\frac{\psi}{-\delta}(s) = \frac{K'(1+T'_3 s)}{s(1+T'_1 s)(1+T'_2 s)(1+T'_E s)}$$

Ship rudder control

Forward path transfer function.

$$\frac{\psi}{-\delta}(s) = \frac{K'(1+T_3's)}{s(1+T_1's)(1+T_2's)(1+T_E's)}$$

Log magnitude form

$$\begin{aligned} 20\ log\ G(\omega)[db] &= 20\ log\ K' - 20\, log\ \omega' \\ &+ 10\ log\left[1+(T_3'\omega')^2\right] - 10\ log\left[1+(T_1'\omega')^2\right] \\ &- 10\ log\left[1+(T_2'\omega')^2\right] - 10\ log\left[1+(T_E'\omega')^2\right] \end{aligned}$$

Phase for stable ship

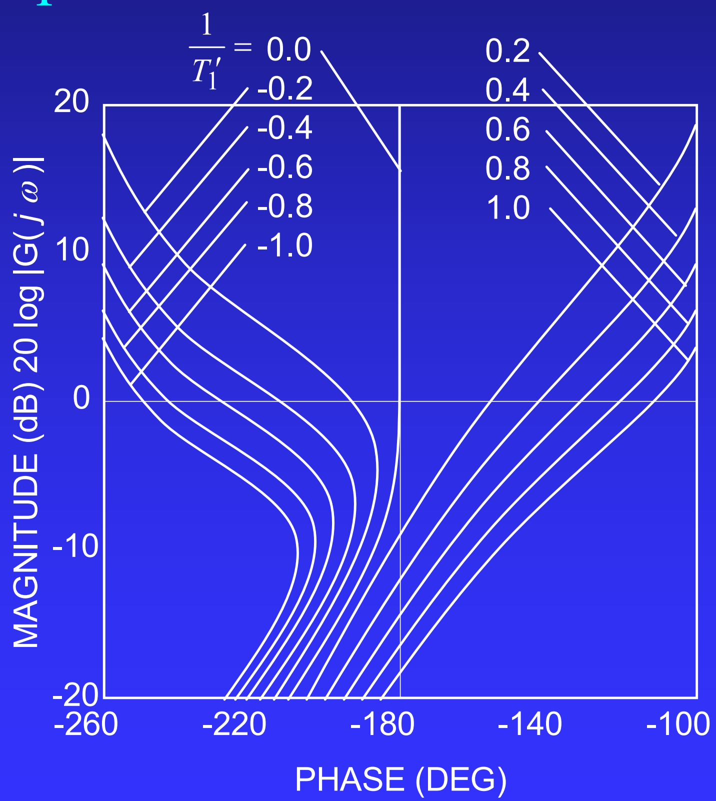
$$\begin{aligned} Phase\ \phi\ [deg] &= -90 - tan^{-1}(-\omega' T_1') - tan^{-1}(-\omega' T_2') \\ &+ tan^{-1}(-\omega' T_3') - tan^{-1}(-\omega' T_E') \end{aligned}$$

Phase for unstable ship

$$\begin{aligned} Phase\ \phi\ [deg] &= -270 + tan^{-1}(-\omega' T_1') - tan^{-1}(-\omega' T_2') \\ &+ tan^{-1}(-\omega' T_3') - tan^{-1}(-\omega' T_E') \end{aligned}$$

Ship rudder control

Nichols Chart for a range of stable and unstable ships. For stability the trajectory must pass to the right of the zero gain and the –180deg phase point. The amount of phase shift required from the compensator can be estimated from this diagram.



$$\frac{K'}{T_1'} = 0.5$$

$$T_2' = 0.35$$

$$T_3' = 0.6$$

$$T_E' = 0.025$$

$$\frac{L}{U} = 10\,\text{sec}$$

Ship rudder control

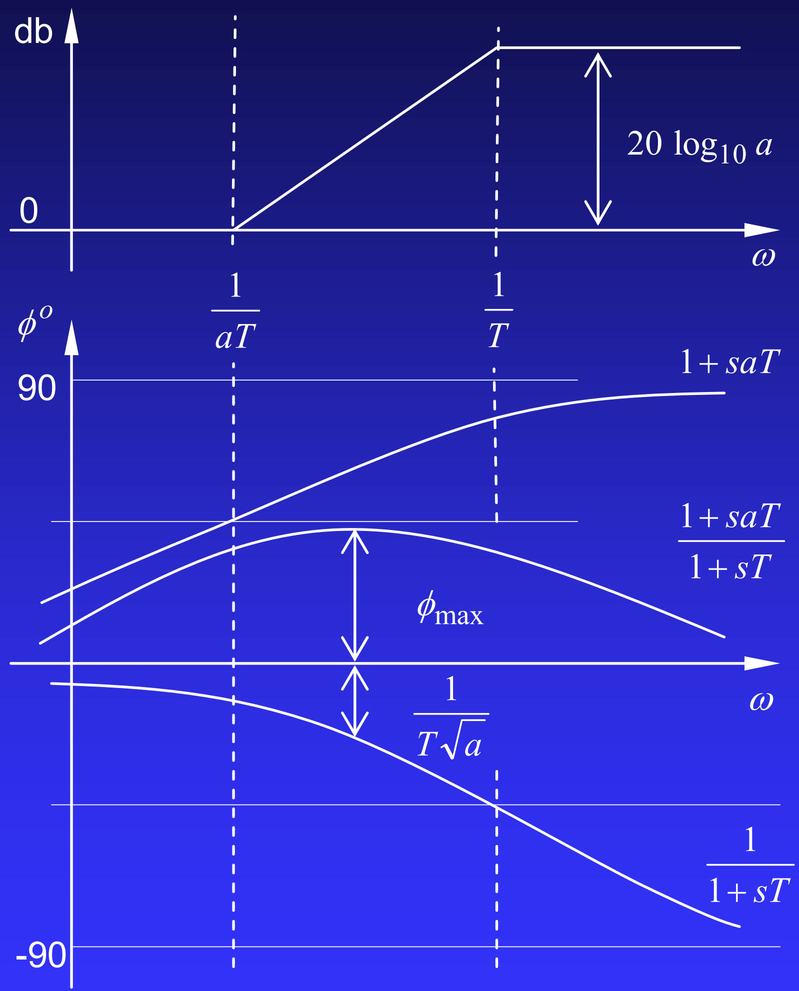
Lead-lag network is simplest compensator. This is the derivative action required to stabilise a ship.

$$G(s) = \frac{1 + aT's}{1 + T's} \qquad where \quad a > 1$$

The phase advance possible from this network is

$$\phi_{max} = \sin^{-1}\left[\frac{a-1}{a+1}\right]$$

A maximum value of a is usually 8.0, so that ϕ_{max}= 51.06 deg.



Simple auto-pilot tuning

Substitute the simple controller $\delta = K_p \psi + K_d \dot{\psi}$
into the yaw rate equation to get the following characteristic equation

$$T_1 T_2 s^3 + (T_1 + T_2 - KT^3 K_d)s^2 + (1 - KT_3 K_p - KK_d)s - KK_p = 0$$

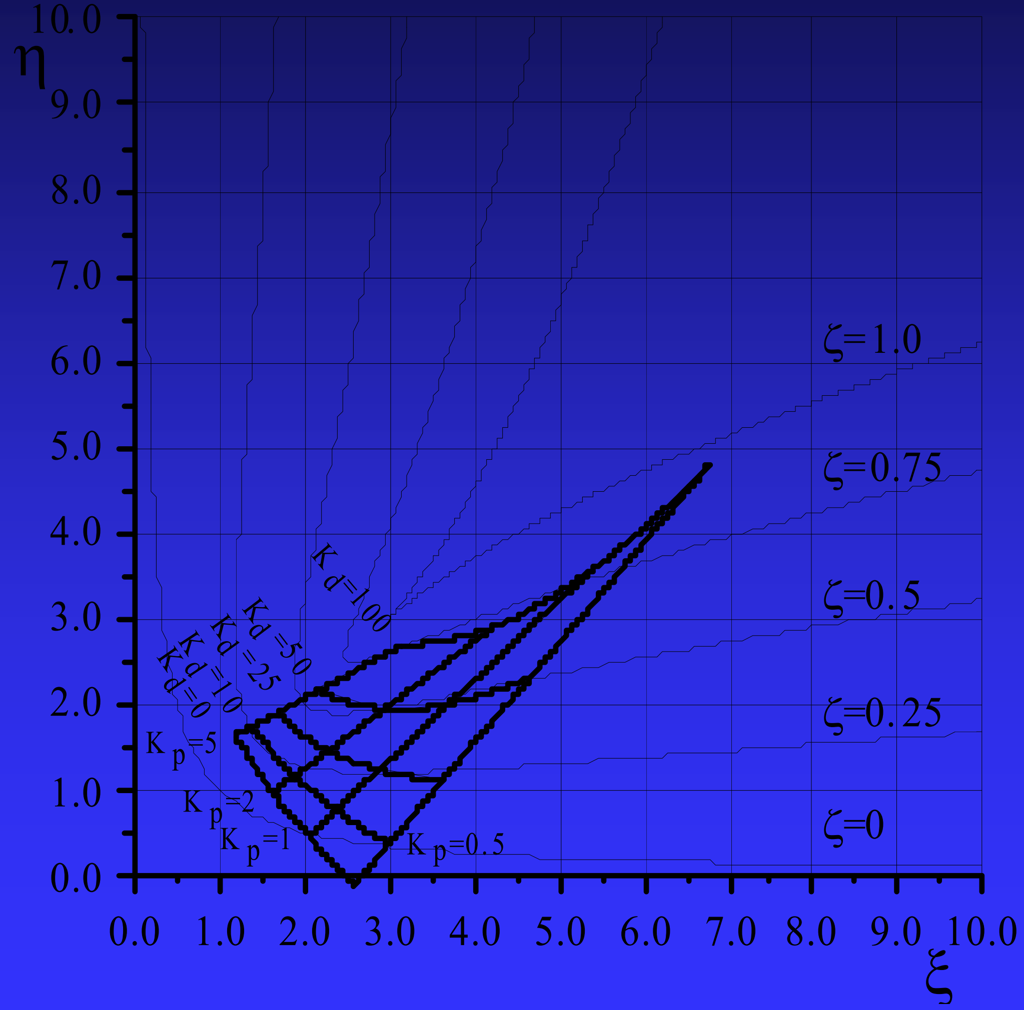
This must be tranformed into the Vishnegradskii-Aizerman polynomial $p^3 + \xi p^2 + \eta p + 1 = 0$

Which gives rise to the following Xi-Eta coordinates

$$\xi = \left[\frac{-KK_p}{T_1 T_2}\right]^{-\frac{1}{3}} \left[\frac{T_1 + T_2 - KT_3 K_d}{T_1 T_2}\right] \qquad \eta = \left[\frac{-KK_p}{T_1 T_2}\right]^{-\frac{2}{3}} \left[\frac{1 - KT_3 K_p - KK_d}{T_1 T_2}\right]$$

Simple auto-pilot tuning

Vishnegradskii-Aizerman diagram showing stability hyperbola and lines of constant damping factor.
Xi-Eta coordinates for a tanker are plotted on diagram.
Shows instability area and optimum damping factor.



Concluding remarks

- Steering and manoeuvring appears to be complicated but it is not so.
- Only simple second order differential equations with a few non-linearities.
- The nomenclature and symbols cause the biggest problem for newcomers.
- Next time look at derivatives.
- Maybe these explanations have been helpful.

Questions and answers