Macroeconomic Theory

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Education

- Ph.D. ECNU and UA 2018
- M.A. PKU 2014
- B.A. SWJTU 2008

Careers

- Postdoctoral Teaching Fellow, FUDAN 2018.12-now
- Research Assistent, SHINE, SJTU 2009.11-2014.06
- Project Supervisor, ICED, SJTU 2009.06-2009.11

Academic activities

- Deng et al., Economic Rsearch Journal. Under review;
- Deng et al. (2017, Management World);
- A forum hosted by Management World etc., 2018.12;
- A forum hosted by Economic Rsearch Journal etc., 2016.07.

Macroeconomics

- Better called aggregate economics
- Focus on dynamic and intertemporal nature of economic decision-making
- Micro data for macro models: “macro” just studies issues at aggregated (country) level (first part of my teaching plan)

Key questions: economy growth and economy fluctuation

- What is the role of money?
- Why does the economy grow over time? Why are some countries rich and others poor? Why do economies experience recessions?
- What's the role of government?
- The global finance crisis; A lost ten years in Japan; The zero lower bound.

Logistics

- Office hours (12:30am-1:30pm)
- Textbooks (more than 5)
- Assignments (3)
- Grading (10%+15%+25%+50%)
- Course outline (1st-16th)

Computer science

- Matlab
- Dynare
- Stata
- Python
- Coding (information theory)

Mathematics

- Math courses for NYU students
- Math courses for Stanford students

Two kinds of variables

- Exogenous: taken as given, determined outside of a model
- Endogenous: determined inside of a model

Timing notation

- Time is discreet. t is the present. t-1 is one period in the past. t+1 is one period in the future. (e.g., C_t, N_{t+1}, Y_{t-1})
- Parameter: fixed value governing mathematical relationships in a model; time-varying value. (e.g., $\alpha, \beta; \alpha_t, \beta_t$)

GDP

- Current dollar value of all final goods and services produced within a country during a particular period of time
- A measure of production and a flow concept
- Production=Income=Expenditure
- Income approach:
  $GDP_t = W_t + IR_t(wokingcapital) + R_t(physicalcapital) + Pro_t$
- Expenditure approach: $GDP_t = C_t + I_t + G_t + NE_t$

Nominal vs. Real

- GDP is defined in terms of current dollar prices
- Prices are weights reflecting relative valuations of different goods, but makes comparisons across time difficult
- Want a "real" or "inflation-adjusted" measure of GDP. How to do this?
- In a single good world, something real is denominated in quantities of goods, whereas nominal is measured in units of money.
- So suppose you produce 10 cans of beer valued at $2 per can. Real quantity is 10 cans, nominal value is $20.

Cont'd

- Not so obvious how to do this with many different goods (the real world)
- Solution: "constant dollar" GDP. Value quantities of goods at different points in time using fixed prices (base year prices). So real GDP actually denominated in units of money, but facilitates comparisons over time.
- Can "back out" a measure of aggregate prices via the implicit price index: ratio of nominal (current dollar) GDP to real (constant dollar) GDP.
- Inflation: rate of growth of price index.

Cycle vs. Trend

- Y_t: real GDP (also output, income, production)
- C_t: consumption; I_t: investment; G_t: government spending; NX_t: net exports.
- P_t: price level; $P_t Y_t = GDP$ (nominal)
- N_t: labor hous (also labor input); K_t: capital stock; r_t: real interest rate.
- w_t: real wage; i_t: nominal interest rate; W_t: nominal wage; π_t: inflation rate.
- $\hat{y}$ output after detrend; $\tilde{y}$: output gap.

Basically three related models of inquiry

- Retrospective: trying to understand what happened in the past and why it happened.
- Counterfactuals: trying to understand what would have happened under some alternative scenario or policy regime.
- Policy advice: trying to advise policymakers on what to do in the future.
- Ultimately out objective is the 3rd point, but to do so need to conquer the 1st and 2nd analysis.

Models

- For better or worse, the real world is messy.
- It isn't always easy t do retrospective analysis (e.g., why did the Great Recession happen?)
- It's hard to do counterfactual analysis (e.g., what would have happened had the CB not done QE?)
- It's even harder to give policy advice about the future (e.g., should the CB raise interest rates?)

Cont'd

- Economics tries to be scientific. In an ideal world, we would like to run experiments.
- What happens when the CB raises or lowers interest rates? Run an experiment: the treatment group vs. the control group. Compare differences across groups to get the "treatment effect".
- For most macro questions, this kind of experiment is impossible.

Art and Science

- Given a model, we try do “real science”: run experiments, and use the outcomes from those experiments to inform policy
- But building the model itself is as much art as it is science.

The framework

A model makes predictions about endogenous variables:

Exogenous variables$\longrightarrow$ Model (decision rules coming from optimization s.t. scarcity; equilibrium/market-clearing conditions; governed by mathematical relationships and parameters)$\longrightarrow$ Endogenous Variables

Characteristics of a good model

- Makes good predictions: stronger test (makes good predictions about things which it wasn't designed to explain, i.e., over-identification)
- Is as simple as possible: abstract from things which are not relevant; the simpler it is, the easier it is to understand the mechanisms.
- Makes reasonable assumptions

How to judge/build a model

- No firm criteria. This is the “art” part.
- In macro, we do a lot of abstraction (e.g., the representive)

- Long run (decades): abstract from endogenous labor input and many sources of shocks, focus on capital accumulation and productivity growth, e.g., the Solow model.
- Medium run (several years): abstract from capital accumulation and productivity growth; abstract from nominal price and wage rigidity, e.g., the neoclassical model.
- Short run (months to several years): abstract from capital accumulation and productivity growth, allow for price and/or wage stickiness, e.g., the new Keynesian model.

Economic Analysis

- **Static analysis**: we mean the analysis of events assumed to occur at a point in time. In effect, statics stuties the alternative point-in-time or **momentary equilibrium values** for a set of endogenous variables associated with alternative possible settings for the exogenous variables at the particular point in time under consideration.
- **dynamic analysis**: it is to study the **time paths of the endogenous variables** associated with alternative possible time paths of the exogenous variables.
- **stationary states analysis**: it is **a limiting form of dynamic analysis**, and is directed toward establishing the ultimate tendencies of certain engoenous variables, e.g.,$\frac{K}{Y}$.

Static analysis vs. Dynamic analysis

The distinguishing feature of a static analysis: it is capable of determining alternative values of the EnV, taking as given only the values of the ExV at that point in time, which may include values of En and Ex variables that were determined in the past and are thus given or predetermined at the present momoent.

Some models for which a dynamic analysis is possible simply cannot be subjected to static analysis. In order to perform static experiments it is necessary partly to divorce current events from future evens so that what happens in the future does not affect what happens now. This requires restricting the way in which peopele are assumed to form expectations about the future, and in particular requires that people not possess perfect foresight.

Structural equations

Gnerally, our model will consist of n structural equations in n endogenous variables $y_i(t), i = 1, \dots, n$, and m exogenous variables $x_i(t), i = 1, \dots, m$:

$$g_i(y_1(t), y_2(t), \dots, y_n(t), x_1(t), \dots, x_m(t)) = 0, \qquad i = 1, \dots, n. \tag{1}$$

A structural equation summarizes behavior, an equilibrium condition, or an accounting identity, and constitutes a building block of the model.

Cont'd

- In general, more than one and possibly all n endogenous variables can appear in any given structural equation. The system of equations (1) will be thought of as holding at each point in time t. Time itself will be regarded as passing continuously, so that t may be regarded as taking all values along the (extended) real line.
- The exogenous variables $x_i(t), i = 1, \ldots, m$, are assumed to be right-continuous functions of time, and furthermore are assumed to possess right-hand time derivatives of at least first and sometimes higher order at all points in time.

Cont'd

By right-continuity of the functions $x_i(t)$ we mean $\lim_{t \to \bar{t}, t > \bar{t}} = x_i(\bar{t})$, so that $x_i(t)$ approaches $x_i(\bar{t})$ as t approaches $\bar{t}$ from above, i.e., from the future. However, the function $x_i(t)$ can jump at $\bar{t}$, so that we do not require $\lim_{t \to \bar{t}, t < \bar{t}} = x_i(\bar{t})$. For example, consider the function:

$$x_i(t) = \begin{cases} 0, & t < \bar{t} \\ 1, & t \geq \bar{t}, \end{cases}$$

which is graphed in Figure 1. It is right-continuous everywhere even though it jumps, i.e., it discontinuous, at $\bar{t}$.

Cont'd

The right-hand time derivative of $x_i(t)$, which is assumed to exist everywhere, is defined as $\frac{d}{dt}x_i(\bar{t}) \equiv \lim_{t \to \bar{t}, t > \bar{t}} \frac{x_i(t) - x_i(\bar{t})}{t - \bar{t}}$.

For the function graphed in Fig1, the right-hand derivative is zero everywhere, even though the fucntion jumps and here is not differentiable at $t = \bar{t}$.

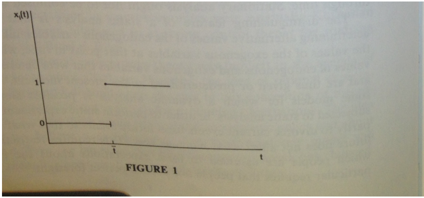


FIGURE 1

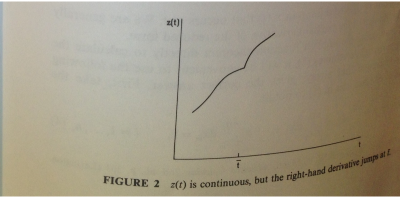


FIGURE 2 $z(t)$ is continuous, but the right-hand derivative jumps at $\bar{t}$.

Cont'd

- A model is said to be in **static equilibrium** at a particular moment if the endogenous variables assume values that assure that eqs (1) are all satisfied.
- Notice that it is **not** an implication of this definition of equlibrium that the values of the endogenous variables are **unchanging** through time.
- On the contrary, since the values of the exogenous variables will in general be changing at some nonzero rates per unit time, the endogenous variables will also be changing over time.

Cont'd

- **Static analysis** is directed towad answering questions of the following form. Suppose that one of the exogenous variables $x_i(t)$ takes a (small) jump at time $\bar{t}$, so that

$$\lim_{t \to \bar{t}, t < \bar{t}} x_i(t) \neq x_i(\bar{t}).$$

- Then the question is to determine the responses of the endogenous variables at $\bar{t}$. The **distinguishing characteristic of endogenous variables** is that each of them is assumed to be able to **jump discontinuously** (a Jump Discontinuity?) at any momoent in time in order to **guarantee** that system (1) remains satisfied in the face of jumps in the $x_i(t)$.

Cont'd

- Thus, to be endogenous from the point of view of statics, a variable must be able to change instantaneously. Notice that it is possible for the right-hand time derivative of a variable to be endogenous, i.e., to be capable of jumping discontinuously, even though the variable itself must change continuously through time (Fig2 gives an example).
- One way to view the difference between the classical and Keynesian models is that in the former the money wage is an endogenous variable in static experiments, while in the latter the level of the money wage is exogenous but the right-hand time derivative of the money wage is endogenous.

Cont'd

To answer the typical question addressed in statics the reduced form equations corresponding to the system (1) must be found. The reduced form equations are a set of equations, each expressing one $y_i(t)$ as a function of only the $x_i(t)$:

$$y_i(t) = h_i(x_1(t), x_2(t), \ldots, x_m(t)), \qquad i = 1, \ldots, n. \qquad (2)$$

Cont'd

We shall generally assume that the functions $g_i(\ )$ in the structural eqs (1) are continuously differentiable in all directions, that the n structural equations were satisfied at all moments immediately preceding the moment we are studying, and that the Jacobian determinant

$$\left|\frac{\partial g}{\partial y}\right| = \begin{vmatrix} \frac{\partial g_1}{\partial y_1} & \frac{\partial g_1}{\partial y_2} & \cdots & \frac{\partial g_1}{\partial y_n} \\ \frac{\partial g_2}{\partial y_1} & \frac{\partial g_2}{\partial y_2} & \cdots & \frac{\partial g_2}{\partial y_n} \\ \vdots & & & \\ \frac{\partial g_n}{\partial y_1} & \frac{\partial g_n}{\partial y_2} & \cdots & \frac{\partial g_n}{\partial y_n} \end{vmatrix} \neq 0$$

Cont'd

when evaluated at the immediately preceding values of all variables. That is, we shall assume the hypotheses of the implicit function theorem. See, e.g., A. E. Taylor and W. R. Mann, Advanced Calculus, 2nd ed., p. 363 (Lexington, Massachusetts: Xerox, 1972).

Under these hypotheses there exist continuously differentiable functions of the reduced form (2) that hold for the $x_i(t)$ sufficiently close to the initial (prejump) values of the $x_i(t)$. If these eqs (2) are satisfied, we are guaranteed that the structural eqs (1) are satisfied.

Cont'd

For jumps in $x_i(t)$ sufficiently small, i.e., within the neighborhood identified in the implicit function theorem, the eqs (2) hold and can be used to answer the characteristic question posed in static analysis. In particular, the reduced form partial derivative

$$\frac{\partial y_i(t)}{\partial x_j(t)} = \frac{\partial h_i}{\partial x_j(t)}(x_1(t), \ldots, x_m(t)) \tag{3}$$

gives the response of $y_i(t)$ to a jump in $x_j(t)$ that occurs at t. We are generally interested in the sign of the partial derivative of the reduced form.

Cont'd

Rather than using the implicit function theorem directly to calculate the reduced form partial derivatives (3), it will be convenient to use the following alternative technique that always gives the correct answer. First, take the differential of all equations in (1) to obtain

$$\frac{\partial g_i}{\partial y_1}dy_1+\cdots+\frac{\partial g_i}{\partial y_n}dy_n+\frac{\partial g_i}{\partial x_1}dx_1+\cdots+\frac{\partial g_i}{\partial x_m}dx_m=0, \qquad i=1,\ldots,n, \tag{4}$$

all partial derivatives being evaluated at the initial values of the x_i and y_i.

Cont'd

Then by successive substitution eliminate $dy_2, \ldots, dy_n$ from the above system (4) of linear equations to obtain an equation of the form

$$dy_1 = f_1^1 dx_1 + f_2^1 dx_2 + \cdots + f_m^1 dx_m, \tag{5}$$

where the f_j^1 are functions of the partial derivatives that appear in (4). Now eq.(5) is the total differential of the reduced form for y_1 since dy_1 is a function of only $dx_1, \ldots, dx_m$.

Cont'd

Taking the differential of the first equation of (2) gives

$$dy_1 = \frac{\partial h_1}{\partial x_1} dx_1 + \cdots + \frac{\partial h_1}{\partial x_m} dx_m. \tag{6}$$

From (6) and (5) it therefore follows that

$$f_j^1 = \frac{\partial h_1}{\partial x_j} \qquad \text{for} \quad j = 1, \ldots, n,$$

so that the f_j^1 are the reduced form partial derivatives.

Cont'd

- Successive substitution in the system (4) will also, of course, yield the differentials of the reduced forms for the other endogenous variables, thereby enabling us to obtain the cooresponding reduced form partial derivatives.
- The reduced form partial derivatives are often called "multipliers" in macroeconomics.

Firms

- Employ physical capital (measured in units of the one good) and labor (measured in number of men) to produce the same single good s.t. the same production function;
- Perfectly competitive;
- Physical capital is fixed both to the economy and to each individual firm.

Assumtions on physical capital

- Rule out once-and-for-all gifts of physical capital from abroad or from heaven and once-and-for-all decreases in the capital stock due to natural or human disasters;
- Rule out the existence of a perfect market in the exsting stock of capital in which individual firms can purchase or sell or rent capital, and so **effect** a **discrete change** in their stock of capital;
- The absence of a market in existing capital might be rationalized by positing that once in place capital becomes completely specialized to each firm. Firms simply have no use for the existing capital of another firm, so that there is no opportunity for making a market in exsiting capital.

Cont'd

- **Ruling out trading of existing stocks of capital** is a fundamental feature of the class of "classical" and "Keynesian" models that we shall be describing. It is the feature that makes flow aggregate demand play such an important role.
- In contradistinction, in Tobin's "dynamic aggregative model"(Macroeconomic Theory, Sargent, 1987, ch.III), there is a perfect market in which firms trade stocks of capital, flow aggregate demand plays no role in determining the level of output at a point in time.

Cont'd

- Alternatively, it is often posited that there are costs of adjusting the capital stock that are internal to the firm and that rise at an increasing rate with increases in the absolute value of the rate of investment. This can give rise to a Keynesian investment demand schedule.(Macroeconomic Theory, Sargent, 1987, ch.VI)
- In effect, **the role of the costs of adjustment** is to **prevent** firms from wanting to make **continuous adjustments** in their stocks of capital.

Cont'd

- While firms cannot trade capital at a point in time, they are assumed to be able to vary employment instantaneously. Firms operate in a competitive labor market in which at any moment they hire all the labor they want at the going money wage w measured in dollars per man per unit of time.
- Firms are perfectly competitive in the output market also, and each can sell output at any rate it wishes at the price of the one good in the model, p measured in dollars per good.

Firms' behavioral equations

Each firm maximizes its profits per unit time w.r.t. N_i, taking K_i as fixed momentarily.

$$Y_i = F_i(K_i, N_i) = F(K_i, N_i), i = 1, 2, ..., n, \tag{7}$$

$$\Pi_i = pF(K_i, N_i) - wN_i - (r + \delta - \pi)pK_i, \tag{8}$$

$$F_{N_i}(K_i, N_i) = w/p, \quad \text{labor demand function} \tag{9}$$

$$Y = F(K, N), \tag{10}$$

$$F_N(K, N) = w/p, \tag{11}$$

where r is the instantaneous rate of interest on government bonds, δ is the instantaneous rate of physical depreciation of capital, and π is the anticipated rate of increase in the price of (newly produced) capital goods.

Cont'd

- In a sense to be defined below $r + \delta - \pi$ is the appropriate cost of capital that should be used to define the firm's profits. Were there a rental market in capital, $(r + \delta - \pi)p$ would be the rental rate, expressed in dollars per unit time.
- For each firm, eq.(9) determinea a capital-labor ratio, which is identical for all firms since all face a common real wage. At any moment the n firms have amounts of capital $K_i, i = 1, \ldots, n$, which might differ across firms. N_i then varies proportionately with K_i across firms.

Cont'd

- Notice that eq.(10) is a valid description of the aggregate production relationship among Y, N, and K only for a certain distribution of the N_i across firms, the one predicted by eq.(9). That distribution is one that maximizes Y for any given N, given the fixed distribution of the K across firms. If some other distribution of the N_i across firms is imposed, one that violates (9), then eq.(10) will not describe the relationship among aggregate output Y, aggregate employment N, and the aggregate capital stock K.
- For our work, however, it is sufficient that (10) hold only in the sense described above.

Cont'd

Our assumptions about the identity of firms' production functions and their profit-maximizing behavior in the face of perfectly competitive markets for output and labor imply that there is a useful sense in which there exists an aggregate production function. The total rate of output of the one good in the economy is Y, defined and deduced by

$$
\begin{aligned}
Y &= \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} F(K_i, N_i) \\
&= \sum_{i=1}^{n} [F_{K_i}(K_i, N_i)K_i + F_{N_i}(K_i, N_i)N_i] = F_{\frac{K_j}{N_j}}(\frac{K_j}{N_j}, 1)\sum_{i=1}^{n} K_i + F_{\frac{N_j}{K_j}}(\frac{N_j}{K_j}, 1)\sum_{i=1}^{n} N_i \\
&= F_{\frac{K}{N}}(\frac{K}{N}, 1)K + F_{\frac{N}{K}}(\frac{N}{K}, 1)N = F_K(K, N)K + F_N(K, N)N.
\end{aligned}
$$

More details

The total rate of output of the one good in the economy is Y, defined by

$$Y = \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} F(K_i, N_i).$$

By Euler's theorem we have

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} [F_{K_i}(K_i, N_i)K_i + F_{N_i}(K_i, N_i)N_i].$$

But since the marginal products of capital and labor depend only on the capital-labor ratio and since that ratio is the same for all firms, the marginal products of capital and of labor, respectively, are the same for all firms.

Thus, we can write

$$\sum_{i=1}^{n} Y_i = F_K(\frac{K_j}{N_j}, 1) \sum_{i=1}^{n} K_i + F_N(\frac{K_j}{N_j}, 1) \sum_{i=1}^{n} N_i.$$

Cont'd

Since the raios $\frac{K_i}{N_i}$ are the same for all n firms, they must be equal to the ratio of capital to employment for the economy $\frac{\sum_{i=1}^{n} K_i}{\sum_{i=1}^{n} N_i}$. Consequently, we have

$$Y = F_K(\frac{K}{N}, 1)K + F_N(\frac{K}{N}, 1)N,$$

where $K = \sum_{i=1}^{n} K_i$ and $N = \sum_{i=1}^{n} N_i$. But by applying Euler's theorem to F the above expression for Y can be written as the aggregate production function

$$Y = F(K, N).$$

Moreover, notice that $\frac{\partial F}{\partial N}$ equals the marginal product of labor for each firm, while $\frac{\partial F}{\partial K}$ equals the marginal product of capital for each firm. This fact makes it legitimate to carry out subsequent analysis solely in terms of the aggregate production function (10) and the equality between the real wage and $\frac{\partial Y}{\partial N}$:

$$F_N(N, K) = \frac{w}{p}.$$

The above equation could be derived by maximizing economy-wide profits w.r.t. N.

Cont'd

- Notes on Assumption (The production fucntion is assumed to be characterized by positive though diminishing marginal products of capital and labor, and by a direct dependence of the marginal product of capital on employment or employment on capital): $F_k, F_N > 0, \quad F_{KK}, F_{NN} < 0, \quad F_{KN} > 0.$
- It remains to describe the behavior of firms w.r.t. the accumulation of capital over time.
- **If there were a perfect market** in which firms could trade capital at each moment, firms would want to purchase capital instantaneously as long as the marginal product of capital exceeded the real cost of capital ($F_K > r + \delta - \pi$), or sell if less than.
- However, such trading of capital has been ruled out.

Cont'd

In its place we **posit a Keynesian investment demand function** on the part of firms. This function **relates** firms' demand to accumulate newly produced capital at some finite rate per unit time directly **to** the gap between the marginal product of capital and the cost of capital:

$$\frac{\mathrm{d}K}{\mathrm{d}t} \equiv I = I\left(\frac{F_K - (r + \delta - \pi)}{r - \pi}\right), \quad I' > 0, \qquad \text{AID schedule} \tag{12}$$

where $\mathrm{d}K/\mathrm{d}t$ is interpreted as a right-hand derivative. According to (12), firms invest at a higher rate the higher is the marginal product of capital and the lower is the real interest rate $r - \pi$.

Cont'd

- Eq.(12) descrbies aggregate investment demand for the economy and is assumed to have been derived from individual firms' investment demand fucntions of the same form.
- It's convenient to write (12) in the compact form (AID: aggregate investment demand)

$$I = I(q - I), \quad I' > 0, \tag{12'}$$

$$q = \frac{F_K - (r + \delta - \pi)}{r - \pi} + 1 \equiv q(K, N, r - \pi, \delta). \tag{13}$$

Cont'd

We shall presently provide an interpretation of q as an important relative price that might plausibly govern firm's demand to accumulate capital. Notice that q is a function of K and N by virtue of the dependence of the marginal product of capital on the labor-capital ratio. (**Keynesian investment schedule**, Tobin 1969)

At instant s firms pay out a flow of dividends (measured in dollars per unit time) $p(s)F(K(s), N(s)) - w(s)N(s) - \delta p(s)K(s) \Rightarrow$ nominal value of firms' equities at t:
$V(t) = \int_t^\infty [p(s)F(K(s), N(s)) - w(s)N(s) - \delta p(s)K(s)]e^{-r(s-t)}ds$. Assume $p(s) = p(t)e^{\pi(s-t)},\ w(s) = w(t)e^{\pi(s-t)}$ and assume that the public expects the real rate of dividends to remain unchanged over time at the current rate. Then:

Cont'd

$$V(t) = [p(t)F(K(t), N(t)) - w(t)N(t) - p(t)\delta K(t)] \int_t^\infty e^{-(r-\pi)(s-t)} ds$$

$$= [p(t)Y(t) - w(t)N(t) - p(t)\delta K(t)] \frac{1}{r-\pi}$$

$$= \frac{p(t)[Y(t) - \frac{w(t)}{p(t)}N(t) - F_K(t)K(t)]}{r-\pi} + \frac{[F_K - (r+\delta-\pi)]p(t)K(t)}{r-\pi} + p(t)K(t).$$

But by the marginal productivity condition for employment and Euler's theorem the first term in this expression equals zero, so that we have the following expression for the nominal value of equities:

$$V(t) = \left(\frac{F_K - (r+\delta-\pi)}{r-\pi} + 1 \right) p(t)K(t).$$

Cont'd

The value of equities varies directly with the gap between the marginal product of capital and the cost of capital. It is interesting to compute the ratio of the nominal value of equities to the nominal value of the capital stock evaluated at the price of newly produced capital *p*:

$$\frac{V(t)}{p(t)K(t)} = \frac{F_K - (r + \delta - \pi)}{r - \pi} + 1 \equiv q,$$

which will be stated again in the next section.

Assets owned by households

- Households own the government's money and bond liabilities and all of the equities of firms.
- Money, the quantity of which is denoted by M, measured in dollars, is a paper asset that is supposed to be used as the medium of exchange. it is issued by the government and bears a nonimal yield that is fixed at zero.
- By **real yield** we mean the yield in percent unit time that can be obtained while setting aside enough resources to **keep the real stock** (i.e., the stock measured in terms of its command over goods) of the asset held **intact** over time.

Cont'd

The nominal yield on money is fixed at zero because holding money gives rise to no payments of interest. However, the real yield on money in general is not zero. The real quantity of money is $\frac{M}{p}$, a quantity measured in units of output.

The time derivative of $\frac{M}{p}$ (set it equal to "0" to keep the $\frac{M}{p}$ intact over time) is

$$\frac{\mathrm{d}(M/p)}{\mathrm{d}t} = \frac{p\dot{M} - M\dot{p}}{p^2} = \frac{\dot{M}}{p} - \frac{M}{p}\frac{\dot{p}}{p} = 0 \quad \Rightarrow \quad \frac{\dot{M}}{M} = \frac{\dot{p}}{p}.$$

Thus to keep real money balances $\frac{M}{p}$ intact, it is necessary to add to nominal money balances at the rate $\frac{\dot{p}}{p}$.

Cont'd

- Dots above variables will be taken to denote derivatives w.r.t time, in general right-hand time derivative
- Consequently, money has a **real yield** of $-\frac{\dot{p}}{p}$. That is, with $\dot{M} = 0$, $\frac{M}{p}$ depreciate at the rate $\frac{\dot{p}}{p}$ per unit time.
- People do not necessarily perceive the rate $\frac{\dot{p}}{p}$ at which prices are depreciating since that would in general require **perfect foresight**.
- We denote the rate per unit time at which people expect the price level to increase as π, which **can differ from** $\frac{\dot{p}}{p}$. The **expected real rate** of yield on money then equals $-\pi$.

Cont'd

- The second asset is a variable-coupon bond that is issued by the government.
- The bond is essentially a savings deposit, changes in the interest rate altering the coupon but leaving the dollar value of bonds outstanding unchanged. We denote the nonimal value of bonds outstanding by B, which is measured in dollars.
- The bonds bear a nominal yield of r in percent per unit time. Thus the bonds throw off a stream of interest payments of rB, which is measured in dollars per unit time.

Cont'd

- The real yield on bonds is defined as r minus the percentage real rate at which households must buy bonds to keep their real value $\frac{B}{p}$ intact.
- Like money, the real value of a fixed nominal quantity of bonds depreciates at the rate $\frac{\dot{p}}{p}$. Thus, the **expected** real rate of return associated with holding bonds is $r - \pi$.

Cont'd

- The third paper asset consists of equities, which are issued by firms in order to finance investment.
- We assume that firms issue no bonds and retain no earnings, so that all investment is financed by issuing equitites. Only households hold, buy, and sell equities.
- By assuming that there is no market in which firms can purchase or sell physical capital we are obliged also to **rule out** the possibility that firms trade equities, which are the financial counterpart of physical capital.

Cont’d

- We assume also that households regard equities and bonds as **perfect substitutes**. This implies that their expected **real yields will be equal**, an equality enforced by investors’ refusing to hold the lower yielding asset should the equality not hold.
- It follows that the nominal bond rate of interest is the pertinent yield for discounting expectations of firms’ net cash flow (which equals expected aggregate dividends) in order to determine the value of firm’s equities.

Cont'd

The value of equities (V) directly with the gap between the marginal product of capital and the cost of capital. It is interesting to compute the ratio of the nominal value of equities to the nominal value of the capital stock evaluated at the price of newly produced capital p:

$$\frac{V(t)}{p(t)K(t)} = \frac{F_K - (r + \delta - \pi)}{r - \pi} + 1 \equiv q,$$

which is the argument that appeas in the aggregate investment schedule, eq.(12).

Cont'd

Thus, our investment demand schedule is one that relates firms' demand for capital accumulation directly to the ratio of the value of equities to the replacement value of the capital stock q. This the way Tobin formulates the Keynesian investment schedule (Tobin, 1969).

Recall that $V(t) = [p(t)Y(t) - w(t)N(t) - p(t)\delta K(t)]\frac{1}{r-\pi}$. Thus the dividend-price ratio, which equals the earning-price ratio, is

$$\frac{pY - wN - p\delta K}{V} = \frac{r-\pi}{1},$$

which is the **expected real rate of interest** on bonds and equities.

Cont'd

If we add the expected rate of appreciation in the nominal value of existing equities π to the earings-price ratio $r - \pi$, we obtain the nominal yield on equities: $r - \pi + \pi = r$.

Recall:

$$V(t) = \frac{p(t)[Y(t) - \frac{w(t)}{p(t)}N(t) - F_K(t)K(t)]}{r - \pi} + \frac{[F_K - (r + \delta - \pi)]p(t)K(t)}{r - \pi} + p(t)K(t).$$

Cont'd

By the marginal productivity condition for employment and Euler's theorem the first term in this expression eauals 0, so the nominal value of equities (V) at t is:

$$V(t) = \left(\frac{F_K - (r + \delta - \pi)}{r - \pi} + 1 \right) p(t)K(t) \equiv qp(t)K(t). \qquad (14)$$

Cont'd

It is interesting also to calculate the time derivative of $V(t)$. We have by logarithmic differentiation w.r.t. t

$$\frac{\dot{V}(t)}{V(t)} = \frac{\dot{p}(t)}{p(t)} + \frac{\dot{K}(t)}{K(t)} + \frac{\dot{q}(t)}{q(t)}.$$

Suppose q is constant over time, so that $\dot{q} = 0$. Then the nominal value of equities changes for two reasons. First, existing equities appreciate in nominal value at the rate $\frac{\dot{p}}{p}$, while investment leads to the issuing of new equities at the rate $\frac{\dot{K}}{K}$.

Cont'd

At each moment in time households allocate their existing wealth between, on the one hand, bonds and equities which they view as perfect substitutes, and on the other hand, money.
Households' total real wealth is denoted W and defined by

$$(V + B + M)/p = W. \tag{15}$$

Cont'd

Househods **desire** a division of their wealth between M^D and $B^D + V^D$ that is described by the pair of asset demand functions:

$$M^D/p = m(r, Y, W), \tag{16}$$

$$(B^D + V^D)/p = b(r, Y, W), \tag{17}$$

Cont'd

The demand schedules are constructed in such a way that at each value of r, Y, and W, total wealth is allocated between M and $B + V$, so that

$$(B^D + V^D + M^D)/p = W, \tag{18}$$

for all r, Y, and W.

Cont'd

This implies that the partial derivatives of (16)-(17) are realted in certain definite way. Tobin (1969) has emphasized this. Thus, take the total differential of eq.(16) and add it to the total differential of eq.(17):

$$\mathrm{d}(M^D + B^D + V^D)/p = (m_r + b_r)\mathrm{d}r + (m_Y + b_Y)\mathrm{d}Y + (m_W + b_W)\mathrm{d}W.$$

Cont'd

Subtract the total differential of eq.(18) from the above expression to obtain

$$0 = (m_r + b_r)\mathrm{d}r + (m_Y + b_Y)\mathrm{d}Y + (m_W + b_W - 1)\mathrm{d}W.$$

This equality can hold for all dr, dY, and dW if and only if

$$m_r + b_r = 0, \quad m_Y + b_Y = 0, \quad and \quad m_W + b_W = 1.$$

We shall assume that these restrictions characterize the asset demand functions (16)-(17).

Cont'd

Portfolio equilibrium requires that households be satisfied with the division of their portfolios between bonds and equities, on the one hand, and money on the other:

$$\frac{M^D}{p} = \frac{M}{p} \quad \text{and} \quad \frac{B^D + V^D}{p} = \frac{B + V}{p}.$$

But notice that (15) and (18) together imply that either one of the above equations is sufficient to describe portfolio equilibrium. For Suppose that $\frac{M}{p} = \frac{M^D}{p}$.

Cont'd

Then substracting this equality from (18) give

$$\frac{B^D + V^D}{p} = W - \frac{M}{p} = \frac{B + V}{p},$$

so that the demand for the stock of bonds and equities equals the supply.

This is an example of **Walras' law**: If demand functions build in balance-sheet constraints and if individulas are content with their holdings of all assets but one, then they must be satisfied with their holdings of that last asset too.

Cont'd

- We choose to characterize portfolio equilibrium by equality between the supply and demand for money balance

$$\frac{M}{p} = \frac{M^D}{p} = m(r, Y, W).$$

- We assume that $m_r < 0, m_Y > 0, m_W = 0$. It follows that $b_r > 0, b_Y < 0, b_W = 1$, so that at given r and Y households desire to hold any increments in real wealth entirely in the form of bonds and equities.

Cont'd

We can write out condition for portfolio equilibrium as

$$\frac{M}{p} = m(r, Y), \qquad m_r < 0, \quad m_Y > 0. \tag{19}$$

Real output Y enters (19) as a proxy for the rate of transacting in the economy. We posit that the higher the rate of transacting, the higher is the demand for real money balances. The nominal interest rate r equals the difference between the real yield on bonds and equitie $r - \pi$ and the real yield on money $-\pi$. We posit that the larger the difference between those real yields, the greater is the incentive to economize on money balances in order to hold one of the higher yielding assets.

Cont'd

We can summarize the portfolios of the three sectors of the economy with the following three balance sheets:

The government		Firms		The household	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
	B	qpK	V	V	
	M			B	
	Net worth			M	Net worth

Firms hold neither bonds nor money, while the government owns neither capital nor equities. Households own only paper assets.

Households

Households decide how to alloacte assets besides how to choose S.

$$C = C(Y_D, r - \pi), 0 < C_1 < 1, C_2 < 0, \quad Y_D = C + S, \tag{20}$$

$$Y_D = Y - \delta K - T - \frac{M + B}{p}\pi + (q - 1)\dot{K} \quad = C + \dot{W}_e, \tag{21}$$

$$Y_D = wp^{-1}N + (Y - wp^{-1}N - \delta K) - T - ((M+B)p^{-1}\pi) + (q\dot{K} + K\dot{q}) - \dot{K}$$

$$\dot{q} = 0 \quad \Rightarrow \quad \dot{W}_e = Y - \delta K - T - ((M + B)/p)\pi + (q - 1)\dot{K} - C$$

Cont'd

- Households make two distinct sets of decisions. First, given their stock of wealth at any point in time, they decide how they wish to allocate it among alternative assets. This decision is described by the portfolio equilibrium condition (19).
- Secondly, the make a distinct decision about how fast they wish their wealth to grow, i.e., they choose a saving rate. (Recall: $\dot{W}_e = S$.) This decision determines how they divide their disposal income between consumption and saving.

Cont'd

- Distinguishing sharply between households' **saving and portfolio decisions** is a **hallmark** of Keynesian macroeconomics. However, that distinction is **not** one that can in general be **derived from microeconomic foundations** with price-taking households. See Merton (1971).
- Under special assumption, as Merton shows, the households's portfolio balance decision does **separate** from its saving decision.

Cont'd

- Household's perceived disposable income represents the rate of income they receive that they expect to be able either to consume or save. Consumption leads to no accumulation of stocks, while saving causes households' wealth in the form of paper assets to grow at some rate per unit time.
- Households' demand for consumption is summarized by a consumption function that relates their intended real consumption C directly to their perceived real disposable income Y_D and inversely to the real interest rate on bonds and equities, as showed by eq. (20).

Cont'd

- As Robert Clower has emphasized, confronting households with a disposable income, which they must then decide to consume or save, is very different from the microeconomic's procedure of confronting households with wages, prices, and interest rates on the basis of which they allocate their labor and consumption over time.
- Clower and Robert Barro and Herschel Grossman have explored setups that might rationalize the consumption. See Clower (1965) and Barro et al. (1971).

Cont'd

Y_D is equal to the real value of wage payments ($\frac{wN}{p}$) plus dividend payments ($Y - \frac{wN}{p} - \delta K$) (which equal economy-wide firms' net cash flow since firms retain no earnings) minus total real tax collections net of government transfer payments minus the perceived rate of capital loss on the real value of the public's net claims on the government ($\frac{\pi(M+B)}{p}$) plus the rate at which the real value of equities is increasing (Firms issue equities at the real rate $\dot{K}$ to finance their investment, while the real value of equities actually increases at the rate $q\dot{K} + K\dot{q}$) minus the real rate at which firms are issuing equities to finance investment. We shall assume that $\dot{q}$ is expected to be 0 (As we shall see, the public's expectation of $\dot{q}$ and $\dot{p}$ must be exogenously set at some values in order to do static analysis), so that the real value of equities is expected to increase at the rate $q\dot{K}$. $\frac{V}{p} = qK$

Cont'd

This concept of disposable income turns out to equal the rate at which society expects that it could consume while leaving its real wealth defined by (15) intact. To show this we differentiate (15) w.r.t. t to obtain

$$\dot{W} = \frac{\dot{M} + \dot{B}}{p} - \frac{M + B}{p}\frac{\dot{p}}{p} + q\dot{K} + \dot{q}K.$$

Next replace $\frac{\dot{p}}{p}$ with the public's expectation of it, and replace $\dot{q}$ with the value the public expects it to be, which we have assumed is 0. We obtain

$$\dot{W}_e = \frac{\dot{M} + \dot{B}}{p} - \frac{M + B}{p}\pi + q\dot{K},$$

where $\dot{W}_e$ is the rate of change of wealth expected by the public.

Cont'd

By virtue of the government's flow budget constraint $\frac{\dot{M}+\dot{B}}{p} = G - T$, while by virtue of the national income identity $G = Y - \dot{K} - \delta K - C$.

So $\dot{W}_e$ can be rewritten $\dot{W}_e = Y - \delta K - T - \frac{M+B}{p}\pi + (q-1)\dot{K} - C$.

Using (21), the above equation can be written as $C + \dot{W}_e = Y_D$, which verifies that the concept of disposable income Y_D corresponds to the rate at which households can consume while expecting that their real wealth is being left intact (i.e., $\dot{W}_e = 0$).

The government

The government collects taxes and purchases goods, issues money and bonds, and conducts open-market operations.

$$G = T + \dot{B}/p + \dot{M}/p, \tag{22}$$

$$dM = -dB, \tag{23}$$

$$T = T_0 - (r - \pi)(B/p), \tag{24}$$

$$G + (r - \pi)(B/p) - T_0 = \dot{B}/p + \dot{M}/p. \tag{25}$$

Cont'd

- The government collects taxes net of transfers at the real rate T per unit time and makes expenditures at the real rate G per unit time. It will be assumed that government purchases share with consumption the characteristic that they lead to no accumulation of stocks. Goods purchases by the government are used up immediately and do not augment the capital stock.
- Real taxes net of transfers T are assumed to be collected in a way that makes T independent of real income and the price level. For alternative and more complicated methods of parameterizing tax collections, see Henderson and Sargent (1973).

Cont'd

- G, T, $\frac{\dot{M}}{p}$, $\frac{\dot{B}}{p}$ are all measured in goods per unit time. Open-market operations are once-and-for-all exchanges of bonds for money at a moment.
- It bears emphasizing that we have defined T as real tax collections net of transfers, and that we include interest payments on the stock of government bonds B as part of those transfer payments.
- Let T_0 be real tax collections net of all transfers except real interest paid on the government debt. Then T and T_0 are linked by eq.(24).
- The difference $G + (r - \pi)(B/p) - T_0$ is called the **gross-of-interest** government deficit, while $G - T_0$ is called the **net-of-interest** government deficit.

Cont'd

- In various experiments, analysts sometimes mean by a "constant fiscal policy" that they are holding the net-of-interest deficit $G - T_0$ constant (Sargent and Wallance, 1981).
- By specifying a constant fiscal policy, other analysts intend to hold constant the gross-of-interest deficit $G - T = G + (r - \pi)(B/p) - T_0$ (McCallum, 1984).
- We always adopts the latter interpretation, some thing important to remember when interpreting our results.

Cont'd

- Our choosing to specify fiscal policy in terms of G and T in this way means that when the government conducts an open market operation s.t. (23) with G and T held fixed, it is being assumed that tax collections T_0 are adjusted by just enough to offset the altered interest payments associated with the altered value of $\frac{B}{p}$ in the hands of the public.
- For example, when the open-market authority contracts the stock of M s.t. (23) thereby increasing the stock of B in the hands of the public and also increasing the flow of real interest payments on that debt due to the public (if $r - \pi > 0$), it is assumed that the government simultaneously raises taxes T_0 by just enough to cover those added interest payments. This assumption makes the open-market authority more powerful vis-a-vis the fiscal authority than was assumed by Sargent and Wallance (1981).

Labor supply

$$N^s = N(w/p), N' > 0, \tag{26}$$
$$N = N(w/p), N' > 0. \tag{27}$$

We have now set down enough equations to determine the six endogenous variable Y, N, C, I, p, and r in our **Keynesian model**, a model that views the money wage rate w as exogenous at a point in time. In our **classical model**, on the other hand, w is a variable that must be determined by the model at each moment; so we stand in need of one more equation. The classical model includes a **labor supply curve** that describes the labor-leisure preferences of workers, as showed in eq.(26).

Cont'd

- N^S denotes the volume of employment offered by workers at instant t. It is postulated that the supply of labor in an increasing function of the real wage.
- The description of the workings of the classical labor market is completed by imposing the condition that actual employment N must eqaul to volume of employment forthcoming at the existing real wage N^S. Substituting actual employment N for N^S in (26) then yields (27).

Cont'd

- In the following complete model, 7 eqs potentially able to determine 7 variables at any moment. As we shall see later, however, equality between the number of eqs and the numbers of variables to be determined is not sufficient to guarantee that the model possesses a unique equilibrium.
- Fow now we will simply assume that π and the expected $\dot{q}$ are both exogenous. We begin our analysis of the classical model by linearizing the system around initial equilibrium values of the variables.

The complete model

- The classical model can now be summarized as consisting of equations (10),(11),(12'),(19),(20),(27), and the identity $Y = C + I + G + \delta K$. Endogenous variables: Y, C, I, N, w, r, p. In the following equations we have assumed that $\mathrm{d}\delta = 0, \mathrm{d}M + \mathrm{d}B = 0$.
- We assume an initial equilibrium exists. Later we shall consider some problems posed by Keynesians that may call into question the existence of an equilibrium. All of the 7 endogenous variables are permitted to **jump discontinuously** as functions of time in order to satisfy the 7 equations at each moment. Were π to depend on $\frac{\dot{p}}{p}$, then would involve eighth variable $\dot{p}$. For similar reasons, assume $E[\dot{q}]$ is exogenous, and more particulary $E[\dot{q}] = 0$.

Cont'd

$$w/p = F_N(K, N), \tag{I}$$

$$N^D = N^S = N = N(w/p), \tag{II}$$

$$Y = F(K, N), \tag{III}$$

$$C = C\left(Y_D = Y - T - \delta K - \frac{M+B}{p}\pi + (q(K, N, r-\pi, \delta) - 1)I, r - \pi\right),$$

$$I = I(q(K, N, r-\pi, \delta) - 1), \tag{V}$$

$$Y = C + I + G + \delta K, \tag{VI}$$

$$M/p = m(r, Y). \tag{VII}$$

Cont'd

$$\mathrm{d}(w/p) = F_{NN}\mathrm{d}N + F_{KK}\mathrm{d}K, \tag{i}$$

$$\mathrm{d}N = N'\mathrm{d}(w/p), \tag{ii}$$

$$\mathrm{d}Y = F_N\mathrm{d}N + F_K\mathrm{d}K, \tag{iii}$$

$$\mathrm{d}C = C_1\mathrm{d}Y - C_1\mathrm{d}T - C_1\delta\mathrm{d}K - C_1\frac{M+B}{p}\mathrm{d}\pi - C_1\pi(\frac{\mathrm{d}M+\mathrm{d}B}{p} - \frac{M+B}{p}\frac{\mathrm{d}p}{p})$$
$$+C_1[(q-1)\mathrm{d}I + Iq_N\mathrm{d}N + Iq_K\mathrm{d}K + Iq_{r-\pi}\mathrm{d}r - Iq_{r-\pi}\mathrm{d}\pi] + C_2\mathrm{d}r - C_2\mathrm{d}\pi,$$

$$\mathrm{d}I = I'q_N\mathrm{d}N + I'q_K\mathrm{d}K + I'q_{r-\pi}\mathrm{d}r - I'q_{r-\pi}\mathrm{d}\pi, \tag{v}$$

$$\mathrm{d}Y = \mathrm{d}C + \mathrm{d}I + \mathrm{d}G + \delta\mathrm{d}K, \tag{vi}$$

$$\frac{\mathrm{d}M}{p} - \frac{M}{p}\frac{\mathrm{d}p}{p} = m_r\mathrm{d}r + m_Y\mathrm{d}Y, \tag{vii}$$

Cont'd

$$
\begin{bmatrix}
1 & -F_{NN} & 0 & 0 & 0 & 0 & 0 \\
-N' & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -F_N & 1 & 0 & 0 & 0 & 0 \\
0 & -C_1 I q_N & -C_1 & 1 & -C_1(q-1) & -(C_1 I q_{r-\pi} + C_2) & -C_1 \pi \frac{M+B}{p^2} \\
0 & -I' q_N & 0 & 0 & 1 & -I' q_{r-\pi} & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & m_Y & 0 & 0 & m_r & M/p^2
\end{bmatrix}
\begin{bmatrix}
\mathrm{d}(w/p) \\ \mathrm{d}N \\ \mathrm{d}Y \\ \mathrm{d}C \\ \mathrm{d}I \\ \mathrm{d}r \\ \mathrm{d}p
\end{bmatrix}
=
\begin{bmatrix}
F_{NK}\mathrm{d}K \\
0 \\
F_K \mathrm{d}K \\
-C_1 \mathrm{d}T - C_1 \delta \mathrm{d}K - C_1 \frac{M+B}{p} \mathrm{d}\pi + I q_K \mathrm{d}K - (C_1 I q_{r-\pi} + C_2)\mathrm{d}\pi \\
I' q_K \mathrm{d}K - I' q_{r-\pi} \mathrm{d}\pi \\
\mathrm{d}G + \delta \mathrm{d}K \\
\mathrm{d}M/p.
\end{bmatrix}
$$

Cont'd

- Inspection of the 7×7 matrix on the left-hand side of the above equation reveals a peculiar characteristic of the classical model, a charateristic that is very important. In particular, note that only two variables appear in the first two eqs: $d\left(\frac{w}{p}\right)$ and dN. All of the other variables have 0 coefficients in the first two eqs.
- As a consequence, these two eqs form an independent subset that determines $d\left(\frac{w}{p}\right)$ and dN, no contribution being made by the remaining variables to the determination of these two.

Cont'd

- Similarly, the first 3 eqs also form an independent subset capable of determining dY as well as $d\left(\frac{w}{p}\right)$ and dN independently of the other four eqs in the model.
- The above system is an example of a "**block recursive**" system of eqs. In such a system interdependence is not general, the system being solvable sequentially since at least one subset of eqs involves an independent subset of the variables. However, once the variables in the subset are determined, they may influence, though not be influenced by, the remaining variables. The fact that the classical model has this property is very important.

Cont'd

Note that the differentials being understook as representing deviations from initial equilibrium values of the variables and note that we solve it by substituing (ii) into (i), which yields eq.(28); then substituting the result into (ii), which yields eq.(29); substituing it into (iii), which yields eq.(30).

$$F_{NK} > 0, F_{NN} < 0, N' > 0 \quad \Rightarrow \quad \mathrm{d}\left(\frac{w}{p}\right) = \frac{F_{NK}}{1 - F_{NN}N'}\mathrm{d}K > 0, \tag{28}$$

$$F_{NK} > 0, F_{NN} < 0, N' > 0 \quad \Rightarrow \quad \mathrm{d}N = N' \frac{F_{NK}}{1 - F_{NN}N'}\mathrm{d}K > 0, \tag{29}$$

$$\mathrm{d}Y = \left(\frac{F_N N' F_{NK}}{1 - F_{NN}N'} + F_K\right)\mathrm{d}K > 0, \tag{30}$$

$$\mathrm{d}r = \frac{C_1}{H}\mathrm{d}T - \frac{1}{H}\mathrm{d}G + \mathrm{d}\pi, \tag{31}$$

$$G + qI = S + T \quad \Rightarrow \quad \frac{\dot{M}}{p} + \frac{\dot{B}}{p} + qI = S, \tag{32}$$

$$\frac{dp}{p} = \frac{\mathrm{d}M}{M} - m_r\frac{p}{M}\mathrm{d}r - m_Y\frac{p}{M}\mathrm{d}Y. \tag{33}$$

Cont'd

- Since $N' > 0$, (29) implies that $K \uparrow$ at a point in time would $N \uparrow$. It would do so by causing an increase in the demand for labor, in turn causing an increase in the real wage, which would increase the quantity of labor supplied.
- Since $F_K, F_N > 0$, it follows that a once-and-for-all increase in capital would produce an increase in the rate of output, both because the marginal product of capital is positive and because the increase in capital would increase both the marginal product of capital and the number of workers employed.

Cont'd

- Since it's commonly assumed that real consumption is independent of the price level, we begin our analysis by tentatively assuming that $M + B = 0$ (In the Classical Model, r bears the entire burden of adjusting AD) initially.
- Then substituing (iv) and (v) into (vi) and setting $\mathrm{d}K = \mathrm{d}Y = \mathrm{d}N = 0$ (we know from solving (i)-(iii) that $\mathrm{d}Y = \mathrm{d}N = 0$ if $\mathrm{d}K = 0$), which yields eq.(31), where

$$H = C_1 I q_{r-\pi} + C_2 + [1 + C_1(q-1)]I' q_{r-\pi}.$$

Cont'd

The derivatives of q w.r.t. N, K, and $r-\pi$, which appear in (iv) and (v), are obtained by differentiating eq.(13):

$$\begin{aligned}
\mathrm{d}q &= \frac{[F_{KN}\mathrm{d}N + F_{KK}\mathrm{d}K - (dr - d\pi)](r-\pi) - [F_K - (r+\delta-\pi)](\mathrm{d}r - \mathrm{d}\pi)}{(r-\pi)^2} \\
&= \frac{1}{r-\pi}[F_{KN}\mathrm{d}N + F_{KK}\mathrm{d}K - q(\mathrm{d}r - \mathrm{d}\pi)].
\end{aligned}$$

$$\Rightarrow \quad q_N = \frac{1}{r-\pi}F_{KN} > 0, \qquad q_K = \frac{1}{r-\pi}F_{KK} < 0, \qquad q_{r-\pi} = \frac{-q}{r-\pi} < 0,$$

so q is an increasing function of N and a decreasing function of K and $r-\pi$.

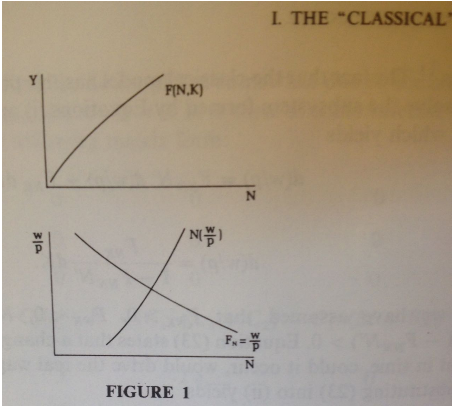
Cont'd

- (28)-(30) show that given the production funciton and the labor supply schedule, only a once-and-for-all change in the capital stock can bring about a once-and-for-all change in the rate of outout: K is the **only** exogenous variable that enters into the determination of the levels of output, employment, and the real wage at a point in time.
- Other exogenous variables **may affect the time rates of change** of those three variables, **but not their level** at a point in time. We shall assume from now on that capital can be accumulated **only** by investing, thus **ruling out** once-and-for-all jumps in the capital stock.

Cont'd

- From this it follows that given the production function and the labor supply schedule, output (Y), employment (N), and the real wage ($\frac{w}{p}$) are constants at a point in time, **independent** of what government policy and the public's expectations are.
- The situation is depicted in Figure 1, which shows the production function, its derivative w.r.t. employment, which is the demand schedule for employment, and the supply schedule for labor. Employment and the real wage are determined at the intersection of the demand and supply schedules for labor, while output is determined by substituting the equilibrium level of employment into the production function.

Cont'd



Cont'd

- Eqs (i)-(iii) may be thought of as determining *AS*. The remaining four eqs play the role of ensuring that *AD* is made equal to *AS*, *AD* doing all of the adjusting.
- In most discussions of the classical model it is only eqs (iv)-(vi) that are involved in equilibrating *AD* and *AS*, the money market equilibrium eq.(vii) playing no role. This requires that *dp* not appear in eq.(iv)-(vi) since only then will those three eqs form an independent subset in $\mathrm{d}C, \mathrm{d}I$, and $\mathrm{d}r$, given the values of $\mathrm{d}Y, \mathrm{d}\left(\frac{w}{p}\right)$, and $\mathrm{d}N$ that emerge from the *AS* subset.

Cont'd

- Alternatively, notice that the coefficients of $\mathrm{d}p$ are all zero, the formal requirements for block recursiveness w.r.t. $\mathrm{d}C, \mathrm{d}I$, and $\mathrm{d}r$ are fullfilled.
- This condition will be met only if it happens that the coefficient of $\mathrm{d}p$ in eq.(iv) happens to equal zero, i.e., $\frac{-C_1\pi(M+B)}{p^2} = 0$. This will be so either if $\pi = 0$ of if $M + B = 0$, B being negative and representing government loas to the public.

Cont'd

- We assume $\frac{\partial Y_D}{\partial r}|_{\mathrm{d}T=\mathrm{d}Y=\mathrm{d}\pi=\mathrm{d}p=0} = Iq_{r-\pi} + (q-1)I'q_{r-\pi} < \frac{-C_2}{C_1}$, which requires that, if it is positive, it not be too large in absolute value. This condition is sufficient (not necessary) to guarantee that $H < 0$.
- As we shall see that H **must be negative if the model is to be "stable."** The magnitude H has a straightforward interpretation that it is the total derivative of the $AD = C + I + G + \delta K$ w.r.t. r.

Cont'd

- Based on eq.(31), we take partial derivatives of r w.r.t. T, G, and π, and obtain: $\frac{\partial r}{T} < 0$, $\frac{\partial r}{G} > 0$, $\frac{\partial r}{\pi} = 1$. If π rises, r rises by the same amount, leaving $r - \pi$ unaltered. This outcome plays an important role in Friedman's (1968) article.
- Substituting eq.(31) into (v)/(iv) respectively, and take partial derivatives of I/C w.r.t. T, G, and π, which determine the effect of changes in the exogenous variables on net investment/consumption. $\frac{\partial I}{T} > 0$, $\frac{\partial I}{G} < 0$, $\frac{\partial I}{\pi} = 0$ (a change in π leaves $r - \pi$ unaltered and so has no effect on I) and $\frac{\partial C}{T} \gtreqless ?0$, $\frac{\partial C}{G} \gtreqless ?0$, $\frac{\partial C}{\pi} = 0$.

Cont'd

- If q is not too much smaller then 1, then $C_2 + C_1 I q_{r-\pi} + C_1(q-1)I' q_{r-\pi} < 0$, and $\frac{\partial C}{\partial G} < 0$ while $\frac{\partial C}{\partial T} < 0$. The expression $I q_{r-\pi} + (q-1)I' q_{r-\pi}$ is the partial derivative of disposable income w.r.t. $r - \pi$. We have assumed that this derivative satisfies $\frac{-C_2}{C_1} > I q_{r-\pi} + (q-1)I' q_{r-\pi}$, which guarantees that $\frac{\partial C}{\partial G} < 0$. It also follows that $\frac{\partial C}{\partial T} < 0$.
- In this version of the classical model ($M + B = 0$ initially), the interest rate bears the entire burden of adjusting the level of AD, so that it equals the AS of output determined by the first 3 eqs. We have already seen that given the capital sock, AS is independent of the other exogenous variables (predetermined).

Cont'd

- If from a position of initial equilibrium, one of the exogenous variables changes in such a manner that it induces an increase in AD at the initial r, the r must rise to diminish desired C and I by enough to restore $AD = AS$. that is why the interest rate must rise, e.g., in response to $AD\uparrow$ by $G\uparrow$ or $T\downarrow$.
- The rise in the interest rate in turn generally induces changes in rates of capital accumulation and consumption. Only in the case of a change in π does the net effect of the change fail to affect I and C. That is because a change in π produces an equivalent change in r which leaves $r - \pi$ unaltered.

Cont'd

- If the economy is to be in equilibrium, $Y_D = Y - T - \delta K + (q-1)I = S + C \quad \Rightarrow$ $G + q(r-\pi)I(q(r-\pi)) = S(= Y_D - C(Y_D, r-\pi)) + T$, which can be interpreted as "injections" $(G + qI)$ must equal "leakages" $(S + T)$ if the economy is to be in equilibrium.
- Eq.(32) which is an alternative form of the equilibrium condition between AD and AS states that in equilibrium real government expenditures plus real investment evaluated at the stock market value of equities must equal saving plus the rate of tax collections.

Cont'd

The role of the interest rate in making $AD = AS$ is shown graphically in Figure 2, which depicts $S + T$ as an increasing function of interest and $qI + G$ as varying inversely with the interest rate.

We know that qI is inversely related to the interest rate since

$$I > -qI' \quad \Rightarrow \quad \frac{\partial q(K, N, r-\pi) I(q(K, N, r-\pi))}{\partial r} = q_{r-\pi} I + qI' q_{r-\pi} < 0.$$

Cont’d

The derivative of saving with respect to the interest rate is

$$\frac{\partial S}{\partial r} = \frac{\partial Y_D}{\partial r} - C_1 \frac{\partial Y_D}{\partial r} - C_2 = \frac{\partial Y_D}{\partial r}(1 - C_1) - C_2.$$

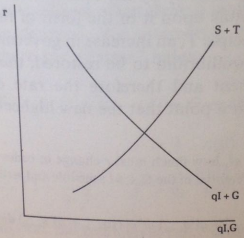


FIGURE 2

Cont'd

We have depicted saving plus exogenous tax collections as varing directly with the interest rate, which need not be true. For $S = Y_D - C(Y_D, r-\pi)$, and notice that $\frac{\partial S}{\partial r} = \frac{\partial Y_D}{\partial r} - C_1 \frac{\partial Y_D}{\partial r} - C_2 = \frac{\partial Y_D}{\partial r}(1 - C_1) - C_2$.

$$\frac{\partial Y_D}{\partial r} < \left.\frac{-C_2}{C_1}\right|_{\text{assume}} \quad \Rightarrow \quad \frac{\partial S}{\partial r} < -\frac{C_2}{C_1}(1 - C_1) - C_2 \quad \text{or} \quad \frac{\partial S}{\partial r} < -\frac{C_2}{C_1}.$$

So our assumptions don't rule out a saving schedule that depends inversely on the rate of interest.

Cont'd

- Both $S+T$ and $G+qI$ also depend on output since I varies with employment and S with output. But output is predetermined from the point of view of interest rate determination, and it will not vary in response to variations in G or T. The equilibrium interest rate is determined at the intersection showed in Figure 2.
- Using Figure 2, it's easy to verify that increase in G drive r upward, while T drive it downward. $\pi\uparrow$ is easily verified to shift both the two curves upward by the amount of the increase in π. The result is that r rises by the increase in π, the equilibrium $G+qI$ being left unchanged.

Cont'd

- The pair of schedule also depend on Y since I varies with N and S with Y. But Y is predetermined from the point of veiw of interest rate determination, and it will not vary in response to variations in G or T.
- E.g., Given $G + qI$, how much msut r change in order to stay on the $G + qI$ curve when π changes? Taking the differential of the $G + qI$ schedule and setting $\mathrm{d}(G + qI) = \mathrm{d}N = \mathrm{d}K = 0 \Rightarrow (Iq_{r-\pi} + qI'q_{r-\pi})\mathrm{d}r = (Iq_{r-\pi} + qI'q_{r-\pi})\mathrm{d}\pi$ or $\mathrm{d}r = \mathrm{d}\pi$, which establishes that the curve $(qI + G) \uparrow$ by the amount of $\pi \uparrow$.

Cont'd

- An equivalent interpretation: **"Loanable-funds" theory**. From the right part of eq.(32), we can see that the left-hand side of the equation is the actual time rate of growth of the economy's real stock paper assets (money, bonds, and equities,) and the right-hand side is the rate at which the public desires to add to its stocks of assets (desired saving).
- In equilibrium the actual rate of growth of the economy's paper assets must just equal the rate at which the public wishes to add to its assets. The actual real growth rate of government issued financial assets $\frac{\dot{M}+\dot{B}}{p}$ is equal to the government's deficit.

Cont'd

- Given that rate and given total taxes T, r adjusts to ensure that desired S exceeds the expected rate of increase in the real value of equities qI exactly by the real rate at which the government is expanding the public's claims upon it in the form of financial assets.
- Thus, e.g., given T, $G\uparrow$ raises $\frac{\dot{M}+\dot{B}}{p}$; in order for equilibrium to be restored, r must rise, thus diminishing I and therefore the rate of issuing equities, and stimulating S, to such a point that the new higher real rate of addition to government-issued finanial assets is consistent with the public's S and I plans.

Cont'd

- The above equation succinctly summarizes the traditional rationale for levying taxes as opposed to financing government expenditures by printing bonds or money: It is to protect the rate of growth of physical capital by limiting the extent to which private saving is diverted to accumulating claims on the government instead of (claims on) physical capital.
- The differentials of the r and Y having been determined in eqs (i)-(vi), the role of eq.(vii) is simply to determine the differential of p. See eq.(33).

Cont'd

- Thus, if the money supply is the only exogenous variable that changes, only p is affected, and it changes proportionately with the money supply. On the other hand, p does respond to the changes in Y and r that emerge from eqs (i)-(iv), $r \uparrow \rightarrow p \uparrow$, while $Y \uparrow \rightarrow p \downarrow$.
- However, as long as neither dp nor dM appears in eqs (i)-(vi), "money is a veil," having no effects on Y, N, $\frac{w}{p}$, C, I or r. Recall that in order to eliminate dp from the equation for dC we assumed that $M + B$ was zero initially.

Stability

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