$$Q(a, X) = \frac{1}{\ell}\sum_{i=1}^{\ell} \mathcal{L}(a(x), y) \to \min_{a \in A}$$

$$\frac{1}{\ell}\sum_{i=1}^{\ell} \left(\langle w, x_i \rangle + \cancel{w_0} - y_i\right)^2 \to \min_{w}$$

$$\frac{1}{\ell}\|Xw - y\|^2 \to \min_{w}$$

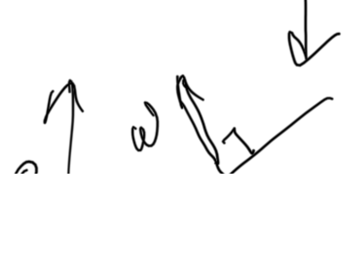
$X = \mathbb{R}^d$ $\quad Y = \{1, 2, 3 \dots n\}$

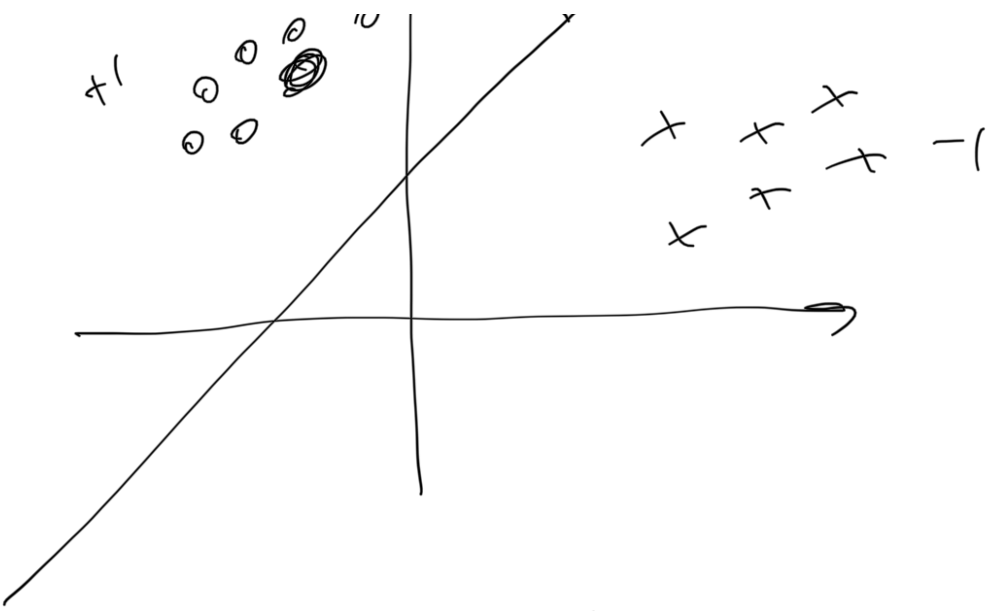
$Y = \{-1, +1\}$

$+1$

-1

$$\langle w, x \rangle + w_0 = 0$$





$$a(x) = \mathrm{sign}\left(\langle w, x\rangle + w_0\right)$$

$w \in \mathbb{R}^d \quad w_0 \in \mathbb{R}$ - сдвиг.

$$Q(a, X) = \boxed{\frac{1}{\ell} \sum_{i=1}^{\ell} [a(x_i) \neq y_i]} \to \min_a$$

Error rate

$$\frac{1}{\ell} \sum_{i=1}^{\ell} [a(x_i) = y_i]$$

"accuracy"

$$\frac{1}{\ell}\sum_{i=1}^{\ell}\left[\operatorname{sign}(\langle w, x_i\rangle) \neq y_i\right]$$

$$\Updownarrow$$

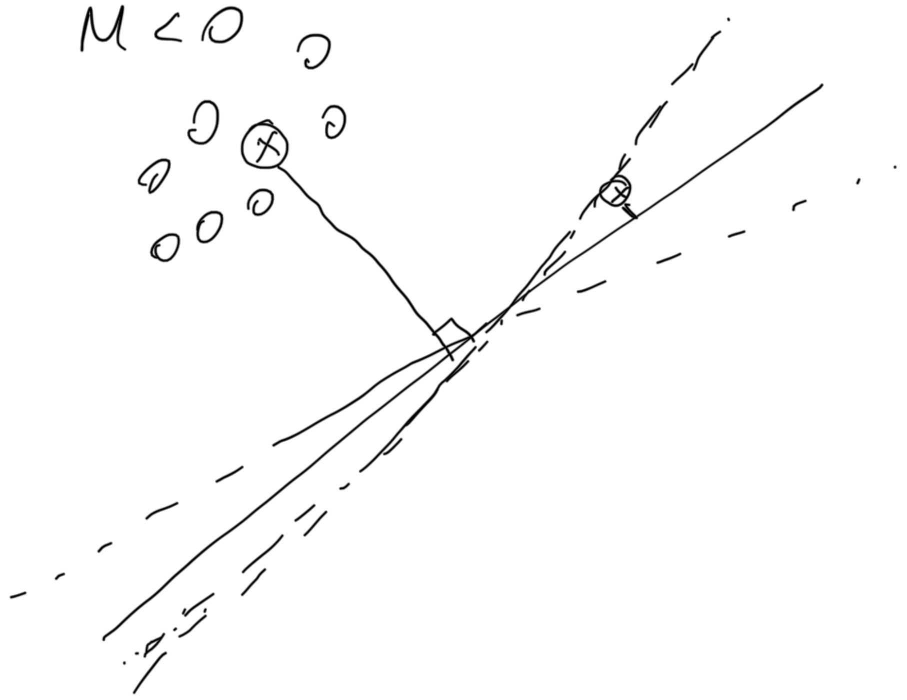
$$\frac{1}{\ell}\sum_{i=1}^{\ell}\left[\underbrace{y_i\langle w, x_i\rangle}_{M_i} < 0\right]$$

$M_i = y_i\langle w, x_i\rangle$ – margin

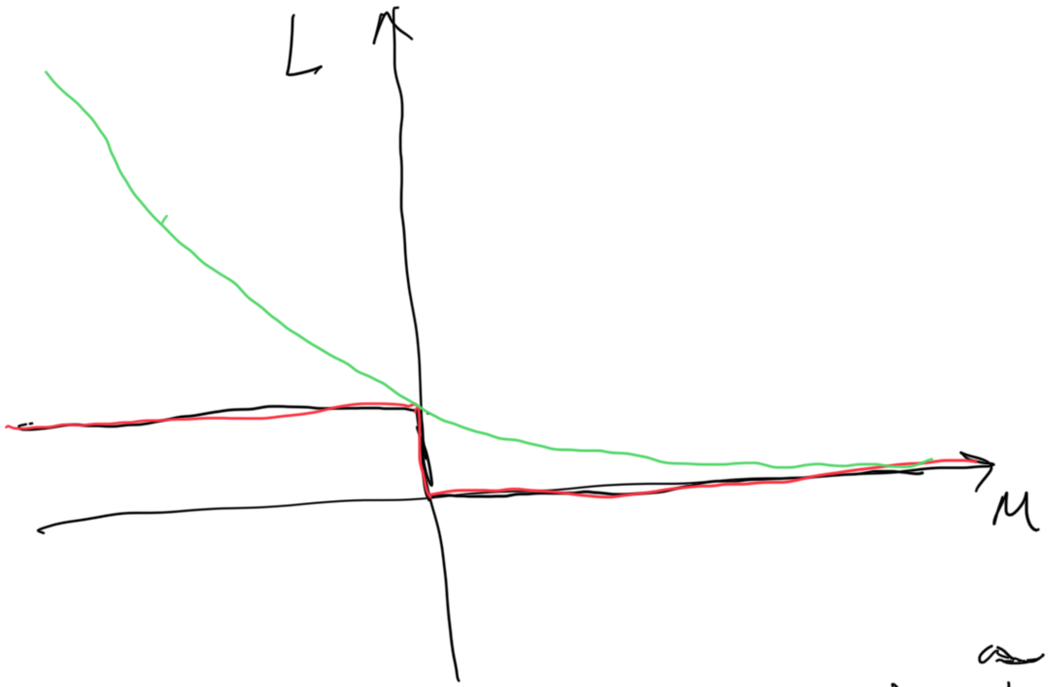
$M > 0$

$M < 0$

$\langle w, x_i\rangle \sim \rho(x_i, \text{гиперпл.})$



$L(M) = [M < 0]$



$$L(M) < \tilde{L}(M)$$

$$\nabla Q = \frac{1}{\ell}\sum_{i=1}^{\ell} \nabla \mathcal{L}.$$

$\tilde{L}(M) = \log(1+e^{-M})$ - лог. рег.

$\tilde{L}(M) = \max(0, 1-M)$ - SVM

$a(x) = [\quad]$ $\quad +1$, -1

$$a(x) = [b(x) > t].$$

$b(x) \in [0,1]$

$a(x) = \text{sign}(\langle w, x \rangle + w_0)$

$b(x) \in [0,1]$

(x_i, y_i)

$b(x_i)$ - вероятность принадлежности объекта x_i к классу +1

$b(x_i)$ $\quad x_i$ - +1

$1 - b(x_i)$ $\quad$ -1

$$b(x_i)^{[y_i = +1]} (1 - b(x_i))^{[y_i = -1]}$$

$\overset{\shortparallel}{p(x_i, y_i)}$

x_i, y_i

$p((x_1 y_1)(x_2 y_2) \dots (x_n y_n)) = p(x_1 y_1)\, p(x_2 y_2) \dots p(x_n y_n)$

$$= \prod_{i=1}^{\ell} b(x_i)^{[y_i = +1]} (1 - b(x_i))^{[y_i = -1]}$$

правдоподобие

max

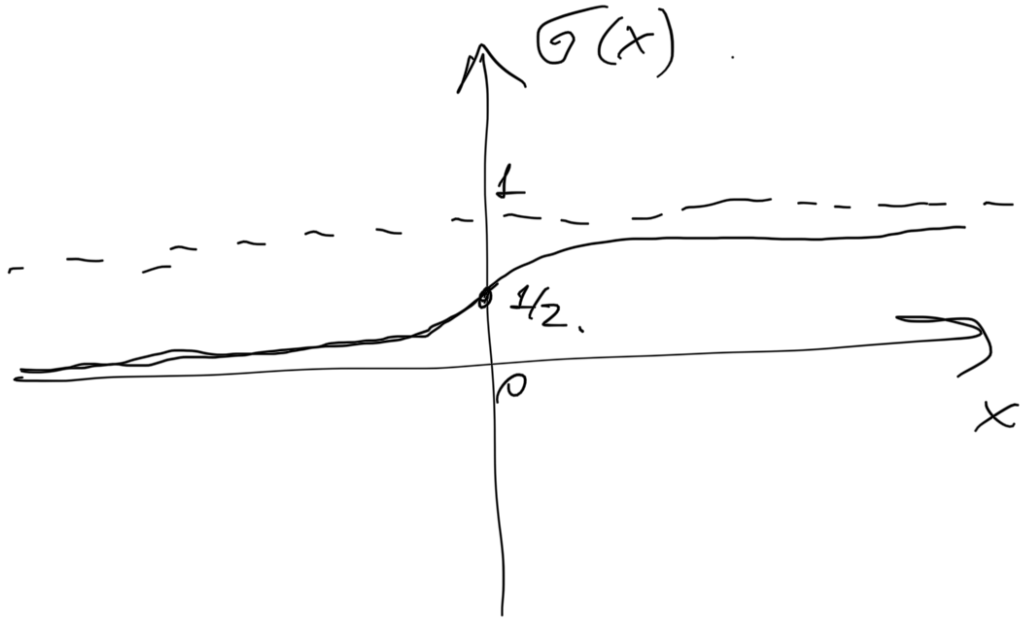
$$-\sum_{i=1}^{\ell} \left([y_i = +1] \log b(x_i) + [y_i = -1] \log(1 - b(x_i)) \right)$$

min

log loss.

$\langle w, x_i \rangle$

$$\underline{b(x)} = \sigma(\langle w, x_i \rangle) = \frac{1}{1+e^{-\langle w, x_i \rangle}}$$



$$-\sum_{i=1}^{\ell} \Big([y_i = +1] \log \frac{1}{1+e^{-\langle w, x_i \rangle}} +$$

$$+ [y_i = -1] \log \frac{\exp(-\langle w, x_i \rangle)}{1 + \exp(-\langle w, x_i \rangle)} \Big) =$$

$\exp(\langle w, x_i \rangle)$

$$\sum^{\ell} \qquad \frac{1}{\qquad} +$$

$$= -\sum_{i=1} \Big([y_i = +1] \log \frac{1}{1 + \exp(-\langle w, x_i \rangle)}$$

$$+ [y_i = -1] \log \frac{1}{1 + \exp(\langle w, x_i \rangle)} \Big) =$$

$$= -\sum_{i=1}^{\ell} \begin{cases} \log \dfrac{1}{1+\exp(-\langle w, x_i \rangle)}, & \text{если } y_i = +1 \\ \log \dfrac{1}{1+\exp(\langle w, x_i \rangle)}, & \text{если } y_i = -1 \end{cases}$$

$$-\sum_{i=1}^{\ell} \log \frac{1}{1 + \exp(\underbrace{-y_i \langle w, x_i \rangle}_{M})}$$

$$= \sum_{i=1}^{\ell} \log \left(1 + e^{-M}\right).$$

$$b(x) = \sigma(\langle w, x_i \rangle) = \frac{1}{1+e^{-\langle w, x_i \rangle}}$$

1 Accuracy

$$\frac{1}{\ell}\sum_{i=1}^{\ell}[y_i = a(x_i)]$$

	$y=+1$	$y=-1$
$a(x)=+1$	True Positive TP	FP
$a(x)=-1$	FN	TN

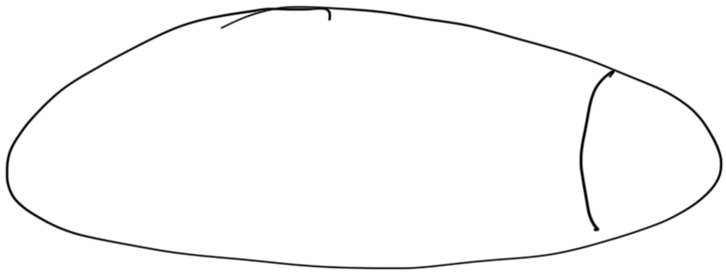
precision = TP

FP

precision = $TP+FP$ $\quad$ $FP+TP$

$$recall = \frac{TP}{TP+FN}$$

-1 -



$$F = \frac{2 \cdot precision \cdot recall}{precision + recall}$$