$$a_N(x) = \sum_{n=1}^{N} \gamma_n b_n(x)$$

$$L(y, z)$$

$$s_i = -\frac{\partial L}{\partial z}\Big|_{z = a_{N-1}(x)}$$

$$b_N(x) = \underset{b \in \mathcal{A}}{\arg\min} \sum_{i=1}^{\ell} \left(b(x_i) - s_i\right)^2$$

$$\boxed{\sum_{i=1}^{\ell} L\left(y_i, a_{N-1}(x_i) + b(x_i)\right) \to \min_b}$$

$$\approx \sum_{i=1}^{\ell} \left(L\left(y_i, a_{N-1}(x_i)\right) + \frac{\partial L}{\partial z}\Big|_{a_{N-1}(x_i)} \cdot b(x_i) + \right.$$

$$+ \frac{1}{2} \underbrace{\frac{\partial^2}{\partial z^2} L(y_i, z)}_{h_i} \Bigg|_{z = a_{N-1}(x_i)} \cdot b^2(x_i)$$

$$= \sum_{i=1}^{\ell} \Big(\cancel{L(y_i, a_{N-1}(x_i))} - \underline{s_i b(x_i)} + \frac{1}{2} h_i \cdot b^2(x_i) \Big) \to \min_b$$

$$\boxed{\sum_{i=1}^{\ell} \Big(-s_i b(x_i) + \frac{1}{2} h_i b^2(x_i) \Big) \to \min_b} \quad (*)$$

$$\boxed{\sum_{i=1}^{\ell} \big(b(x_i) - s_i \big)^2 \to \min_b}$$

$$\Leftrightarrow \sum_{i=1}^{\ell} b^2(x_i) + s_i^2 - 2 s_i b(x_i) \to \min_b$$

$$\Leftrightarrow \sum_{i=1}^{\ell} b^2(x_i) - 2 s_i b(x_i) \to \min_b$$

$$\Leftrightarrow \boxed{2\sum_{i=1}^{\ell}\left(-s_i b(x_i) + \frac{1}{2} b^2(x_i)\right) \to \min_{b}} \quad (**)$$

$h_i = 1$

$$b(x) = \sum_{j=1}^{J} \underbrace{b_j}_{\text{ответ в листе } j} [x \in R_j]$$

попал ли объект в лист j

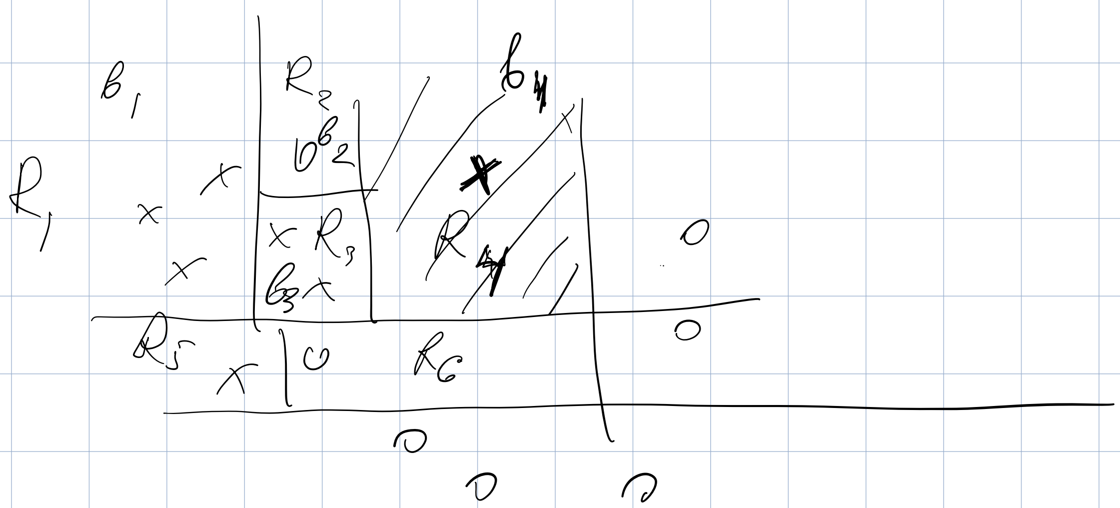
$R_1 \cup R_2 \ldots \cup R_J$

$\downarrow$

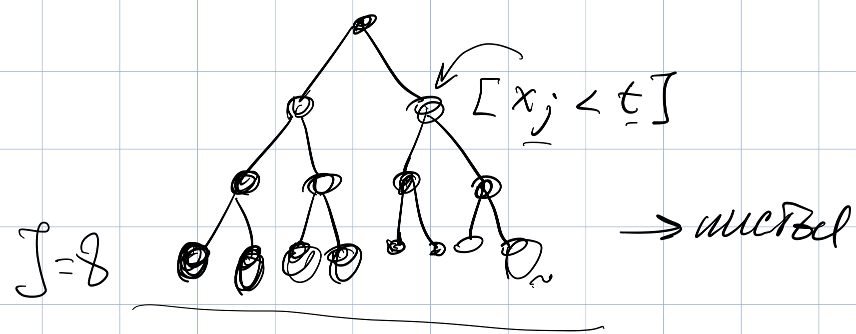
выборка

1) Число листьев J

2) $\|b\|_2^2 = \sum_{j=1}^{J} b_j^2$



$$\sum_{i=1}^{\ell}\left(-s_i b(x_i) + \frac{1}{2} h_i b^2(x_i)\right) + \gamma J + \frac{\lambda}{2}\sum_{j=1}^{J} b_j^2 \to \min_b \qquad (***)$$



$$\boxed{x_1 \quad x_2}$$

$$b(x_1) = b(x_2) = b$$

$$\sum_{j=1}^{J}\left[\left(-\sum_{i\in R_j} s_i\right) b_j + \frac{1}{2}\sum_{i\in R_j} h_i b_j^2 + \gamma + \frac{\lambda}{2} b_j^2\right] \to \min_b$$

$$\sum_{j=1}^{J}\Big[\Big(\underbrace{-\sum_{i\in R_j} s_i}_{=-S_j}\Big) b_j + \frac{1}{2}\Big(\lambda + \underbrace{\sum_{i\in R_j} h_i}_{H_j}\Big) b_j^2 + \gamma\Big] \to \min_{b}$$

$$b_j = \frac{S_j}{H_j+\lambda}$$

$$-\frac{1}{2}\sum_{j=1}^{J}\frac{S_j^2}{H_j+\lambda} + \gamma J = H(b)$$

$H(b)$ - ошибка дерева с оптимальными прогнозами в листьях

$$Q = H(R_m) - \frac{|R_l|}{|R_m|}H(R_l) - \frac{|R_r|}{|R_m|}H(R_r)$$

$\downarrow$
max

$[x_j < t]$

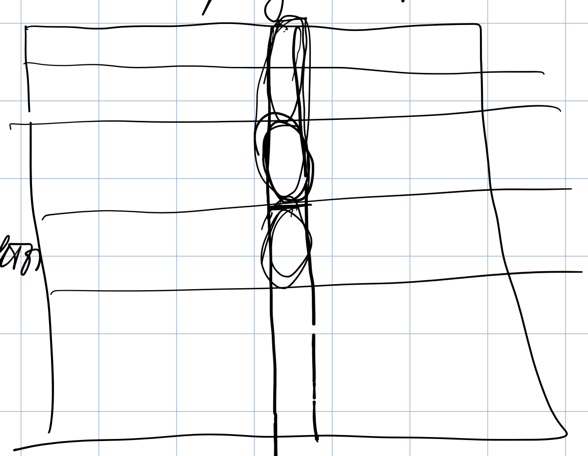
$$Q = H(R) - H(R_l) - H(R_r) \to \max$$

XGBoost.

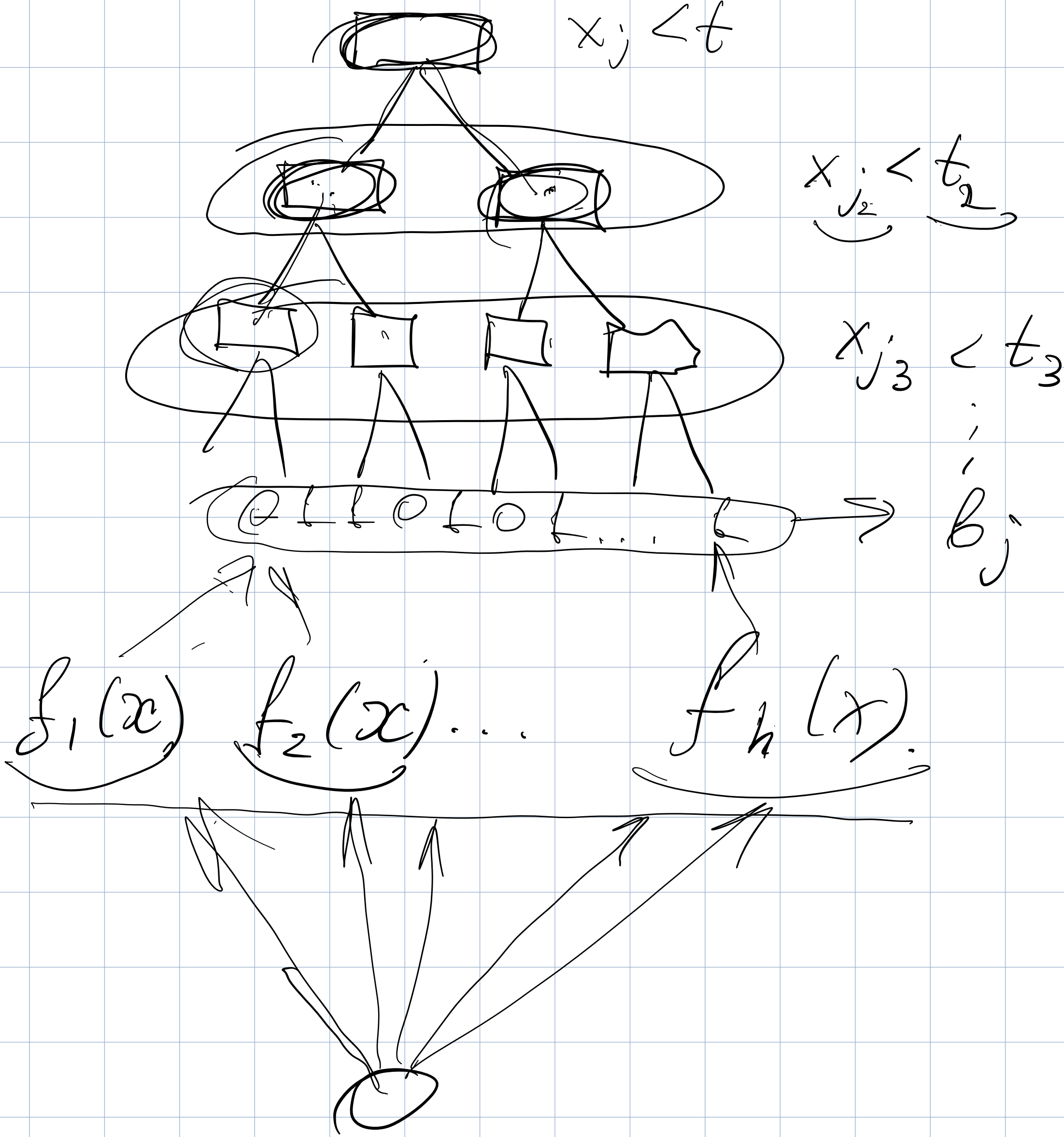
CatBoost.

признаки

объекты



Oblivious Trees


x_j < t
x_{j_2} < t_2
x_{j_3} < t_3
b_j
f_1(x)
f_2(x) . . .
f_h(x)