		$a(x)$: 1	−1
y_i	1	TP	FN
	−1	FP	TN

$$\text{Accuracy} = \sum I\{y_i = a(x_i)\}$$

precision, Recall → F-score

~~ROC-AUC~~

PR-AUC

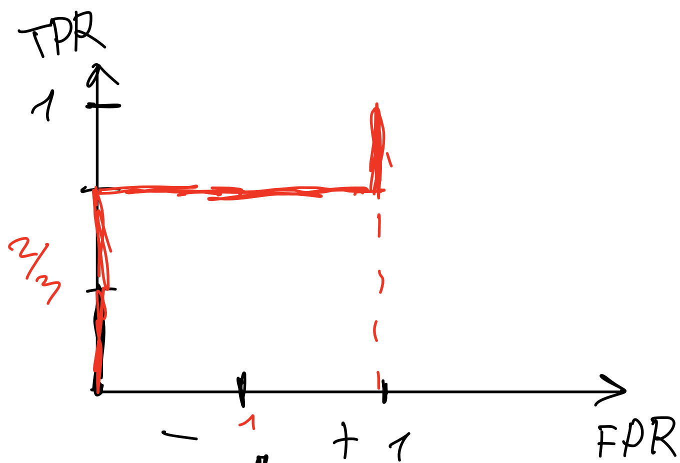
$$FPR = \frac{FP}{FP+TN}$$

$$TPR = \frac{TP}{TP+FN}$$

Задача 1.2.

$b(x)$	y_i
0,2	−1
0,4	+1
0,1	−1
0,7	+1
0,05	+1

ROC-AUC



0,05	0,1	0,2	0,4	0,7
+1	−1	−1	+1	+1

$t = 0{,}8$

$$\text{ROC-AUC} = \frac{2}{3}$$

0,05	0,1	0,2	0,4	0,4	0,7
+1	-1	-1	+1	-1	+1

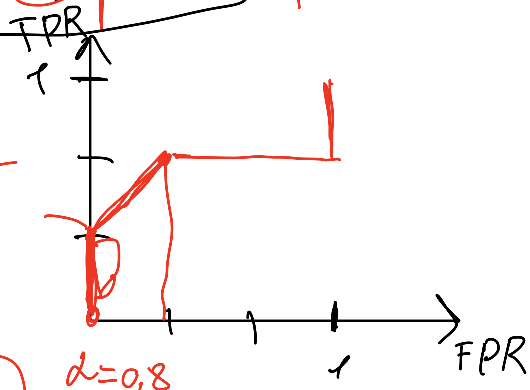
$\alpha = 0{,}8$

3

+1

$FPR = \frac{1}{3}$

$TPR = \frac{2}{3}$



$$\boxed{\begin{gathered} \frac{4}{9} + \frac{1}{9} + \frac{2}{36} \\ \frac{20}{36} + \frac{2}{36} = \frac{22}{36} \end{gathered}}$$

Задача 1.3.

$b(x)$ выдаёт случайную перестановку объектов X в ранжированном списке.

$E(AUC)$

$$I = \begin{cases} 1 & p \\ 0 & (1-p) \end{cases}$$

$E(I) = p$

$$AUC = \frac{1}{l_+ l_-} \sum_{i<j} I\{y_{(i)} = -1\} I\{y_j = +1\}$$

l_+ — кол-во положит. объектов в X

$E(AUC)$

$$E(AUC) = \frac{1}{l_+ l_-} \sum_{i<j} E\, I\{y_{(i)} = -1\} I\{y_j = +1\}$$

$$P\{y_i = -1,\ y_j = +1\} = \frac{\ell_-\,\ell_+\,(\ell-2)!}{\ell!} =$$

$$\begin{array}{c} \quad - \qquad\qquad + \\ [\ \_\ \_\ \_\ \mid \_\ \_\ \_\ \_\ ] \\ \ \ell_- \qquad\qquad \ell_+ \end{array} = \frac{\ell_-\,\ell_+}{\ell(\ell-1)}$$

$$E(AUC) = \frac{1}{\cancel{\ell_+\ell_-}} \sum_{i<j} \frac{\cancel{\ell_-\ell_+}}{\ell(\ell-1)} =$$

$$= \frac{1}{\ell(\ell-1)} \sum_{i<j} 1 = \frac{1}{\cancel{\ell(\ell-1)}} \cdot \frac{\cancel{\ell(\ell-1)}}{2} = \frac{1}{2}$$

Многокл. классификация.

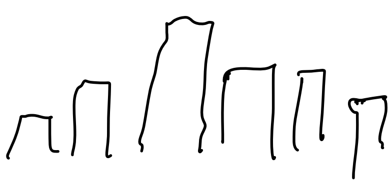
one-vs-All

K классикат.

all-vs-vs

C_K^2

$$\begin{array}{cccccc} 1 & \dots & & & & K \\ \end{array}$$
$$[\,0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\,]\ ?$$



Эмпирические лоссы

$$\frac{1}{l}\sum_{i=1}^{l} L(y_i, a(x_i)) \to \min_{w}$$

$$\Downarrow \; l \to \infty$$

$$\underbrace{E(L(y, a(x))|x)}$$

$$L(y, a(x)) = (y - a(x))^2$$

$$E(L) = E((\overset{iid}{y} - a(x))^2 | x) =$$

$$= \int_{-\infty}^{+\infty} (y - a(x))^2 p(y|x)\,dy \to \min_{a}$$

$$\frac{\partial}{\partial a} E(L) = \int_{-\infty}^{\infty} \cancel{-2}(y - a(x)) p(y|x)\,dy =$$

$$\underbrace{\int_{-\infty}^{\infty} y\, p(y|x)\,dy} - \int_{-\infty}^{+\infty} a(x) p(y|x)\,dy =$$

$$= E(y|x) - a(x) = 0$$

$$a(x) = E(y|x)$$

$$Q(a, X) = \sum_{i=1}^{l} L(y_i, a(x_i))$$

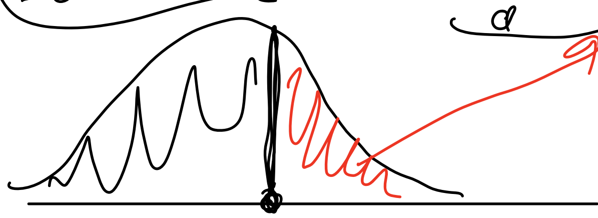
$$L(y_i, a(x_i)) = \begin{cases} (1-\alpha)(a(x_i)-y_i), & a(x_i) > y_i \\ \alpha(y_i - a(x_i)), & a(x_i) < y_i \end{cases}$$

$$E(L \mid x) = \int_{-\infty}^{a} (1-\alpha)(a-y)\, p(y \mid x)\, dy \; +$$

$$\int_{a}^{+\infty} \alpha (y-a)\, p(y \mid x)\, dy \; =$$

$$\frac{\partial E(L \mid x)}{\partial a} = \int_{-\infty}^{a} (1-\alpha)\, p(y \mid x)\, dy - \int_{a}^{+\infty} \alpha\, p(y \mid x)\, dy$$

$$(1-\alpha) \int_{-\infty}^{a} p(y \mid x)\, dy - \alpha \int_{a}^{+\infty} p(y \mid x)\, dy = 0$$



a

$1 -$

$$(1-\alpha)\, p(y \le a \mid x) = \alpha\,(1 - p(y \le a \mid x))$$

$$\underline{p(y \le a \mid x) = \alpha}$$