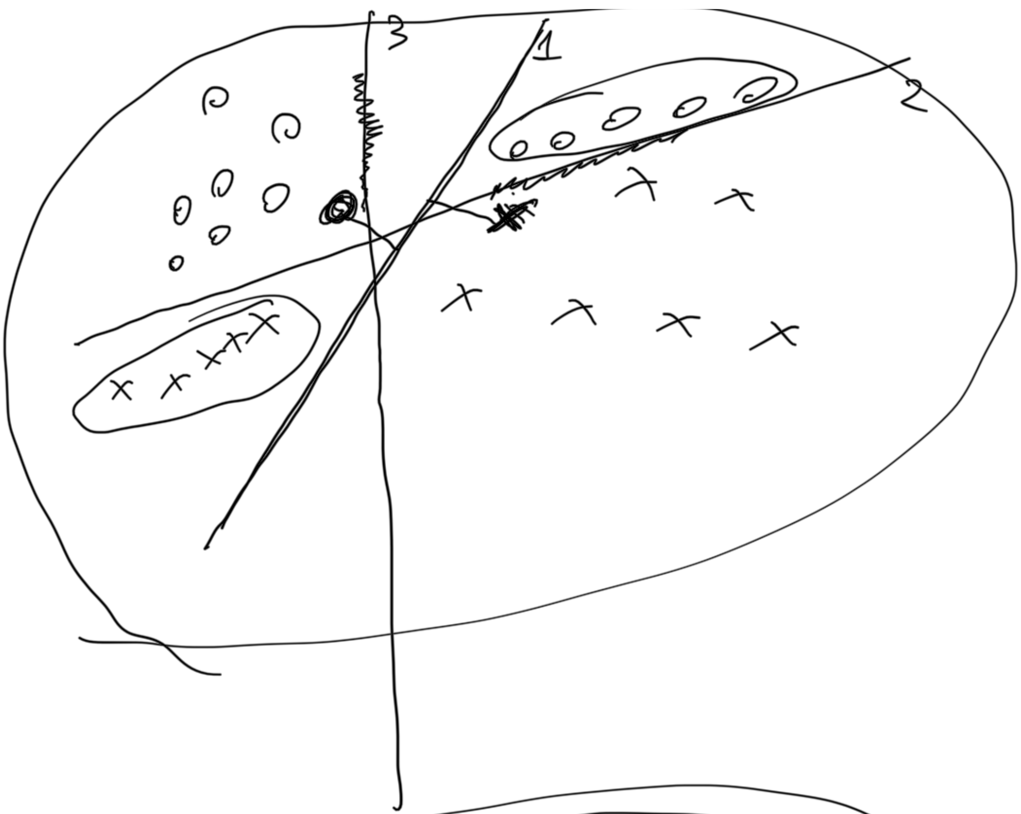

$$\alpha(x) = \operatorname{sign}(\langle w, x\rangle + b) \qquad w \in \mathbb{R}^d,\ b \in \mathbb{R}$$

$$\min_{x \in X} |\langle w, x\rangle + b| = 1 \quad (VV)$$

$x \in \mathbb{R}^d$

$\langle w, x\rangle + b = 0$

$$\rho(x_0, a) = \frac{|\langle w, x\rangle + b|}{\|w\|}$$

$$\frac{|\langle w, x\rangle + b|}{} \quad \min |\langle w, x\rangle + b|$$

$\min_{x \in X}$ $\|w\|$ $\|w\|$ $x \in X$

$= \frac{1}{\|w\|} \to \max$

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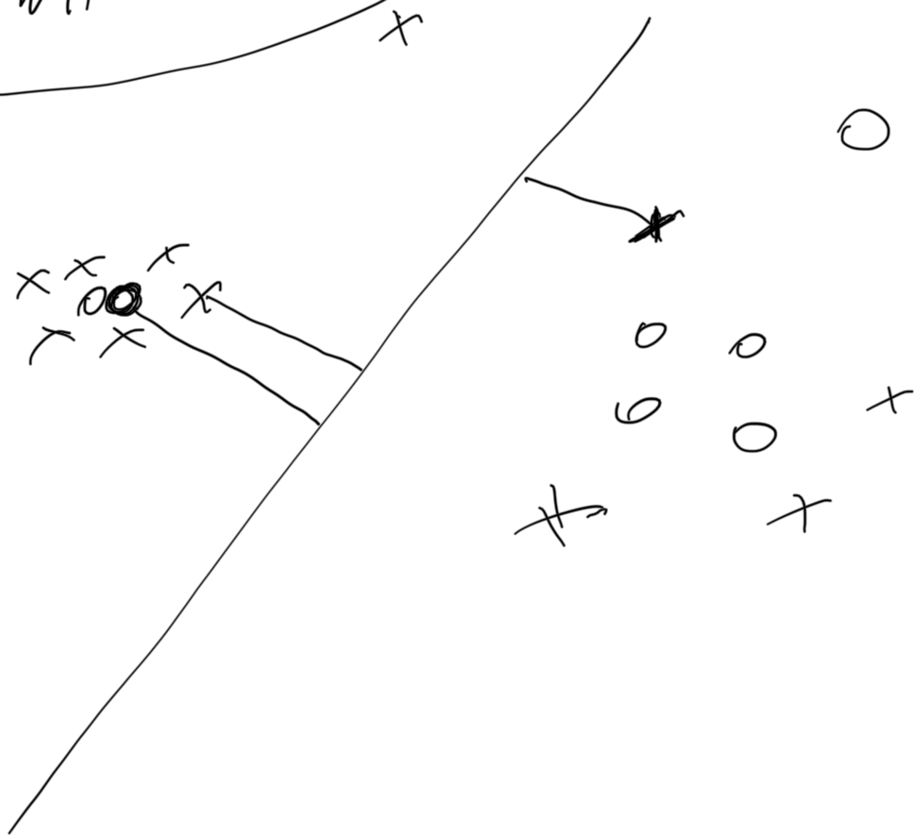


$\frac{1}{\|w\|} \to \max_{w}$

(3) $\min_{x \in X} |\langle w, x \rangle + b| = 1$ ✓

(1) $y_i(\langle w, x_i \rangle + b) \geq 0$ ✓

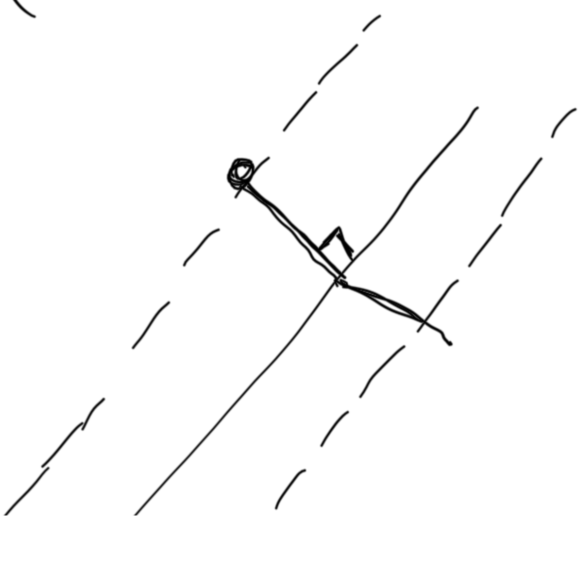
$M > 0$ $\Uparrow$

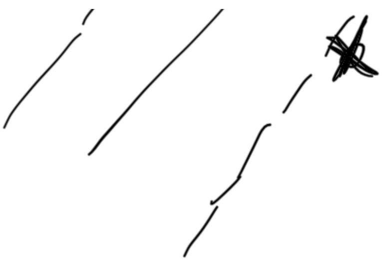
SVM ↙

(2) $\begin{cases} \frac{1}{2}\|w\|^2 \to \min\limits_{w} \\ y_i(\langle w, x_i \rangle + b) \geq 1 \quad i = 1, \dots, \ell \end{cases}$

$\forall i \Rightarrow y_i(\langle w, x_i \rangle + b) > 1$

$\min\limits_{x \in X} |y_i(\langle w^*, x_i \rangle + b^*)| = 1$



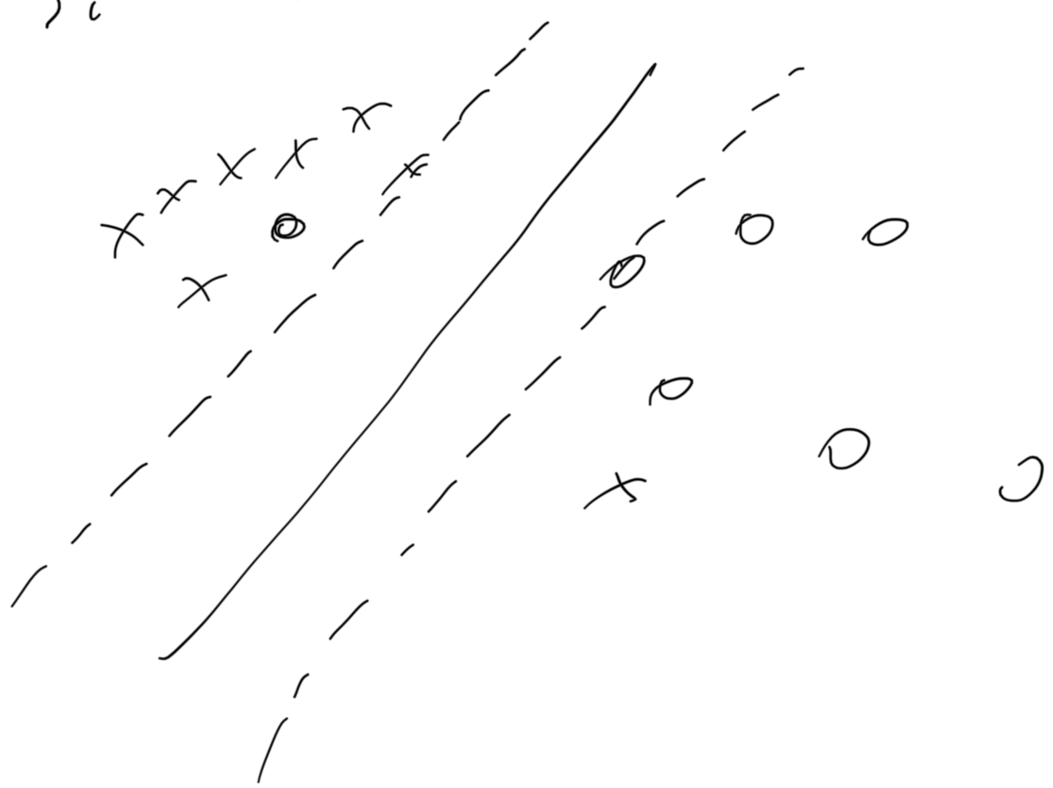


$$\exists\, x_i \in X : y_i\left(\langle w, x_i \rangle + b\right) < 0$$
$$\forall w, b$$

$$\begin{cases} \frac{1}{2}\|w\|^2 \to \min\limits_{w,b} \\ y_i\left(\langle w, x_i \rangle + b\right) \geqslant 1 - \xi_i & i = 1, \dots, l \end{cases}$$

$$(*) \quad \begin{cases} 3)\ \frac{1}{2}\|w\|^2 + C \sum\limits_{i=1}^{l} \xi_i \to \min\limits_{w,\xi,b} \\ 1)\ y_i\left(\langle w, x_i \rangle + b\right) \geqslant 1 - \xi_i, & i = 1, \dots, l \end{cases}$$

2) $\xi_i \geq 0$, $i = 1, \dots, \ell$



$$\begin{cases} \xi_i \geq 1 - y_i(\langle w, x_i \rangle + b) \\ \xi_i \geq 0 \end{cases} \qquad \sum_{i=1} \xi_i \to \min_{\xi}$$

$$\Downarrow$$

$$\xi_i = \max(0, 1 - y_i(\langle w, x_i \rangle + b))$$

$$\frac{1}{2}\|w\|^2 + C \sum_{i=1}^{\ell} \max(0, 1 - y_i(\langle w, x_i \rangle + b)) \to \min_{w, b}$$

$$\Updownarrow$$

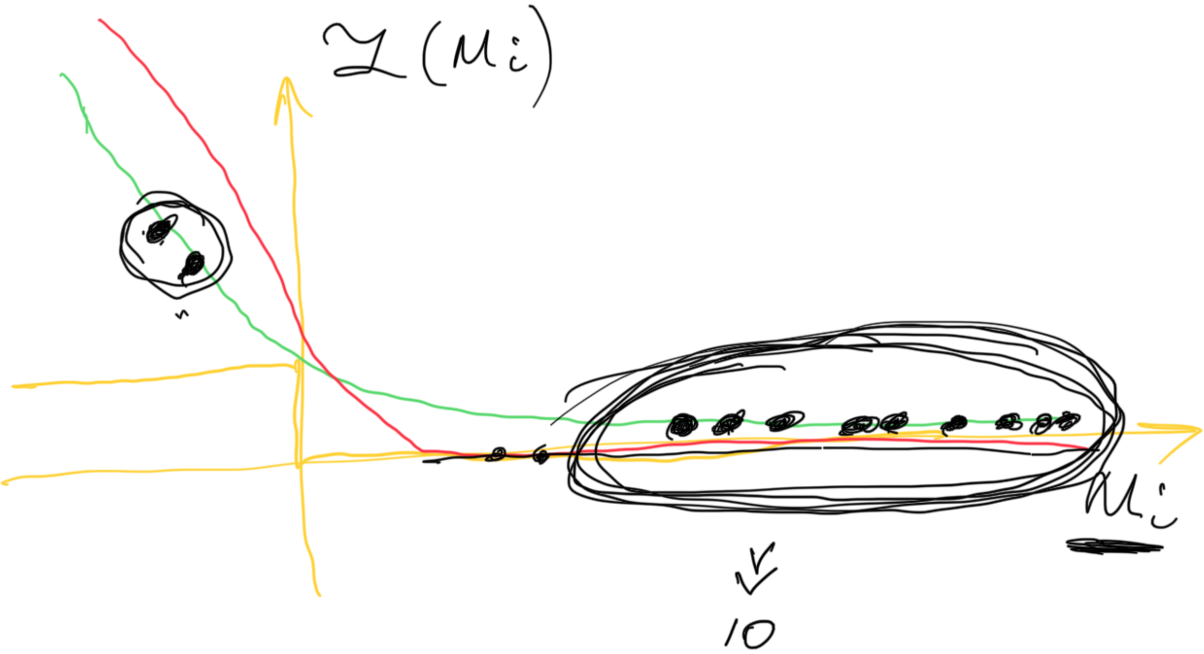
(*)

$$\frac{1}{\ell 2C}\|w\|^2 + \frac{1}{\ell}\sum_{i=1}^{\ell} \max\left(0, 1 - y_i\left(\langle w, x_i\rangle + b\right)\right) \to \min_{w,b}$$

$$\frac{1}{\ell 2C} = \alpha, \qquad y_i\left(\langle w, x_i\rangle + b\right) = M_i$$

$$\frac{1}{\ell}\sum_{i=1}^{\ell} \mathcal{L}(a(x_i), y_i) + \alpha\|w\|^2 \to \min_{w,b}$$

$$\mathcal{L}(a(x_i), y) = \max(0, 1 - M_i)$$



$Q \to \min$

$Q \to \min$

$$Q = \sum_{i=1}^{\ell} \tilde{\mathcal{L}}(M_i) \to \min$$

$$\operatorname{sign}(\langle w, x \rangle + b)$$