Week 7 Writing Problem

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The Art of Problem Solving,
Intermediate Counting and Probability

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Problem Statement

From the set $\{1, 2, 3, \ldots, 20\}$, ten distinct numbers are chosen at random, forming a subset. Let M be the largest element among the ten numbers. Find the expected value of M, and simplify your answer as much as possible.

Solution

$$\Pr(M = m) = \frac{\binom{m-1}{9}}{\binom{20}{10}} \qquad (m = 10, ts, 20),$$

and hence

$$\mathbb{E}[M] = \sum_{m=10}^{20} m \Pr(M = m) = \sum_{j=1}^{20} \Pr(M \geq j) = \sum_{j=1}^{20} \left(1 - \frac{\binom{j-1}{10}}{\binom{20}{10}}\right).$$

Using the hockey-stick identity,

$$\sum_{j=1}^{20} \binom{j-1}{10} = \sum_{k=0}^{19} \binom{k}{10} = \binom{20}{11},$$

so

$$\mathbb{E}[M] = 20 - \frac{\binom{20}{11}}{\binom{20}{10}} = 20 - \frac{10}{11} = \frac{210}{11}.$$