

# Examination cover sheet

(to be completed by the examiner)

Course name: Hardware Verification	Course code: 2IMF20
Date: 27-10-2015	
Start time: 13:30	End time : 16:30
Number of pages: 5	
Number of questions: 6	
Maximum number of points/distribution of points over questions:100	
Method of determining final grade: divide total of points by 10	
Answering style: open questions	
Exam inspection: With your instructor	
Other remarks: It is not allowed to use study materials or a computer during the exam.	

## Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☐ Other:

### Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will **in any case** be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

# TECHNISCHE UNIVERSITEIT EINDHOVEN

Department of Mathematics and Computer Science

## Examination Hardware Verification (2IMF20)

Monday, October 27, 2015, 13h30 – 16h30.

Your answers should be formulated and written down clearly. First read **ALL** questions once!

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### Linear Time Logic (LTL)

1. **(15 points)** Suppose we have two users, *Peter* and *Betsy*, and a single printer device *Printer*. Both users perform several tasks, and every now and then they want to print their results on the *Printer*. Since there is only a single printer, only one user can print a job at a time. Suppose we have the following atomic propositions for *Peter* at our disposal:

- *Peter.request*: indicates that *Peter* has requested usage of the printer;
- *Peter.use*: indicates that *Peter* is using the printer;

For *Betsy*, similar predicates are defined. Specify in LTL the following properties:

- (a) Mutual exclusion, that is, only one user at a time can use the printer.
- (b) Finite time of usage, that is, a user can print only for a finite amount of time.
- (c) Absence of individual starvation, that is, if a user requests to print something, he/she eventually is able to do so.
- (d) Alternating access, that is, users must strictly alternate in printing.

### Computation Tree Logic (CTL)

2. **(20 points)** Consider an elevator system that services  $N > 0$  floors numbered 0 through  $N - 1$ . There is an elevator door at each floor with a call button and an indicator light that signals whether or not the elevator has been called. In the elevator cabin there are  $N$  send buttons (one per floor) and  $N$  indicator lights that inform to which floor(s) is going to be sent. For simplicity consider  $N = 4$ . Present a set of atomic propositions – try to minimise the number of propositions – that are needed to describe the following properties of the elevator system as CTL formulas and give the corresponding CTL formulas:

- (a) The doors are "safe", that is, a floor door is never open if the cabin is not present at the given floor.

- (b) The indicator lights correctly reflect the current requests. That is, each time a button is pressed, there is a corresponding request that needs to be memorised until completion (if ever).
- (c) The elevator only services the requested floors and does not move when there is no request.
- (d) All requests are eventually satisfied.

## Binary Decision Diagrams and Combinatorial Equivalence

3. (10 points) Using the ordering  $x_1 < x_2 < x_3 < x_4 < x_5$ , construct the ROBDD for the majority function:

$$\text{MAJ}(x_1, x_2, \dots, x_6) \equiv (x_1 + x_2 + x_3 + x_4 + x_5) \geq 3$$

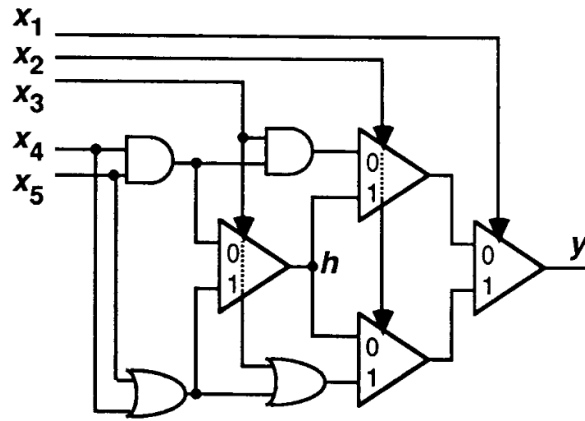


Figure 1: Realisation of a 5-bit majority function.

4. (15 points) Figure 1 shows the realisation of the majority function. Note that triangles are 2-input multiplexers. Prove using ROBDDs that this circuit is equivalent to the majority function defined in the previous question.

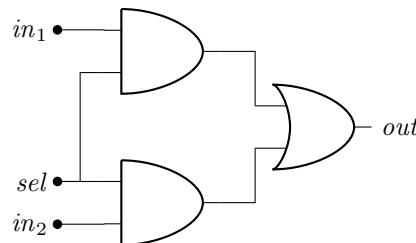


Figure 2: Realisation of a two input multiplexer

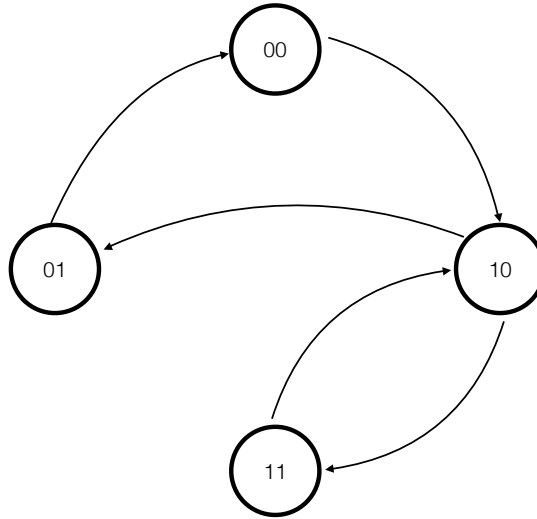


Figure 3: A transition system.

## And-Inverter Graphs

5. (10 points) Draw an AIG (And-Inverter-Graph) representing the 2-input multiplexer shown in Figure 2.

## Bounded Model Checking

6. (30 points) Assume a system composed of threads. A thread can be in four different states:
  - **idle**: the thread is inactive.
  - **running**: the thread is active.
  - **suspended**: the thread has been suspended.
  - **halted**: the thread has been stopped.

Figure 3 shows the transitions between these states assuming the following encoding. Each state is encoded as a sequence of two bits, noted  $a$  and  $b$ . State **idle** is encoded by the sequence 00, state **running** as the sequence 10, state **suspended** as 11, and state **halted** as 01. So, a thread starts in the **idle** state and makes a transition to the **running** state. Then it can either go to the state **suspended** or the state **halted**. Once **suspended** a thread goes back to the **running** state. Once **halted** a thread goes to the **idle** state. Consider the property that a thread is eventually halted, that is, it cannot run forever. Formally, this is expressed in LTL as the formula  $\mathbf{F}(\text{halted})$  or using the state

encoding  $\mathbf{F}(\neg a \wedge b)$ . In the following questions you will use Bounded Model Checking to prove or disprove this property.

- (a) Write down the transition relation as a Boolean function  $T(a, b, a', b')$ , where  $a$  and  $b$  denotes the values of the variables in the current state and  $a'$  and  $b'$  denote the values of the variables after a transition. This is the usual "prime" notation.
- (b) Define predicate  $I(a, b)$  that returns true if and only the values of  $a$  and  $b$  represent an initial state.
- (c) We will attempt BMC at a depth of 2, that is, the length  $k$  of paths is  $k = 2$ . We define three variables representing each state of such paths. That is, we define  $s_0 = a_0b_0$ ,  $s_1 = a_1b_1$ , and  $s_2 = a_2b_2$ . Write the Boolean formula encoding the validity of paths.
- (d) Some paths have a loop. Some paths do not have a loop. Let  $\mathbf{L}_2$  be the Boolean formula encoding the existence of a loop in paths of length 2. Write down this formula.
- (e) Now that you have defined the encoding for valid paths and the existence of a loop, what is the property that paths must satisfy? Write down this property in LTL.
- (f) Write down the encoding of this property for paths with a loop.
- (g) Write down the encoding of this property for paths without a loop.
- (h) Now you have all the pieces to build the global formula encoding the satisfiability of the LTL formula  $\mathbf{F}(\neg a \wedge b)$ . Write down the resulting property.
- (i) After simplifying as much as possible, write down the CNF formula that would be submitted to a SAT solver.
- (j) What can you conclude about the validity of the formula  $\mathbf{F}(\neg a \wedge b)$  ?