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CRITICAL BEHAVIOR OF THE ANTIFERROMAGNETIC ISING MODEL ON THE SQUARE LATTICE UNDER AN EXTERNAL MAGNETIC FIELD

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INTRODUCTION

Trying to understand the magnetic properties of materials using the ising model has proved to be a very efficient alternative in recent decades [1,2]. Although most of the problems treated using the ising model do not have an exact solution, the results found using approximate techniques show great qualitative results [3,4]. The study of the antiferromagnetic ising model in the square lattice with interactions between first (J_1) and second neighbors (J_2) under the action of an external magnetic field is one such case. This case shows that the adoption of ferromagnetic interactions between second neighbors leads the system towards first-order transitions [4,5], but little is described about the influence of J_2 on changing the nature of the phase transition in non-boundary cases. We will therefore investigate the cases of intermediate J_2 under the action of an external magnetic field.

OBJETIVE

The aim of this work is to understand the critical behavior of the antiferromagnetic Ising model with ferromagnetic second-neighbor interactions under an external magnetic field. To this end, we used the approximate cluster mean-field technique (CMF) and analyzed various case scenarios.

METODOLOGY

The hamiltonian of the model is given by:

$$\mathcal{H} = \sum_{i,j}^{n_s} J_{ij}\sigma_i\sigma_j - h\sum_{i}^{n_s}\sigma_i,$$

where J_{ij} represent the coupling term between spins, σ is the spin variable, h is the external magnetic field and n_s is the number of sites. We define J_1=1 and antiferromagnetic. In the other hand, we explore the cases of J_2 ferromagnetic, in specific: $-1 \leq J_2/J_1 \leq 0$.

This problem does not has a exact solution, so we adopt a aproximation technique known as cluster mean-field. The CMF technique divide the infinite lattice in finite clusters, as a result, the effective hamiltonian is also divided. Our CMF hamiltonian has an *intracluster* and an *intercluster* term:

$$\mathcal{H}_{CMF} = \mathcal{H}_{intra} + \mathcal{H}_{inter} - h\sum_{i}^{n_s}\sigma_i.$$

The *intracluster* term is obtained treating the interactions of spins inside the cluster exactly. The *intercluster* term is obtained by the mean-field aproximation:

$$\sigma_i\sigma_j \approx \sigma_i m_j + \sigma_j m_i - m_i m_j.$$

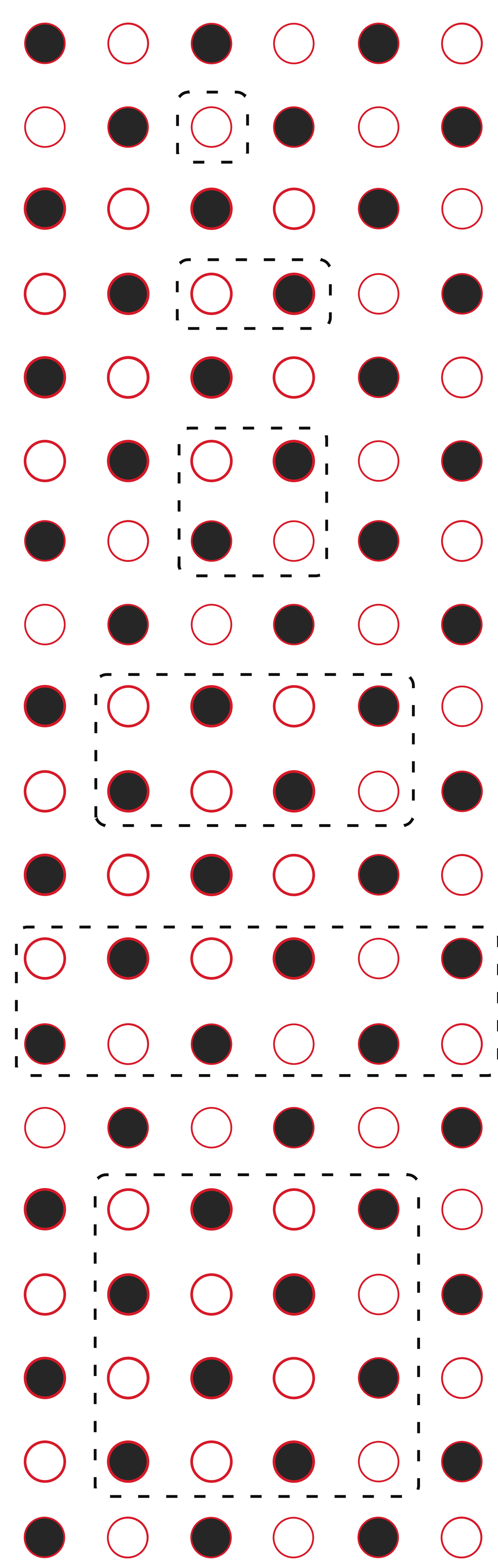

Fig. 1 - Representation of the clusters adopted, with 1, 2, 4, 8, 12 and 16 sites. Black and white circles represents spins up or down.

The local magnetization m_i is calculated with:

$$m_i = \langle\sigma_i\rangle = \frac{\mathrm{Tr}\ \sigma_i\ e^{-\beta\mathcal{H}_{CMF}}}{\mathrm{Tr} e^{-\beta\mathcal{H}_{CMF}}},$$

where Beta = $1/k_B T$, T is temperature and k_B is the Boltzmann constant (k_B=1). To obtain the first order transitions we look to the Helmholtz free energy:

$$F = -T\ln(\mathrm{Tr}e^{-\beta\mathcal{H}}),$$

and the continuous transitions are detected by the order parameter of the Néel antiferromagnetic phase, such as:

$$\psi_{AF} = \frac{1}{n_s}\left|\sum_{i}^{n_s} m_i(-1)^i\right|.$$

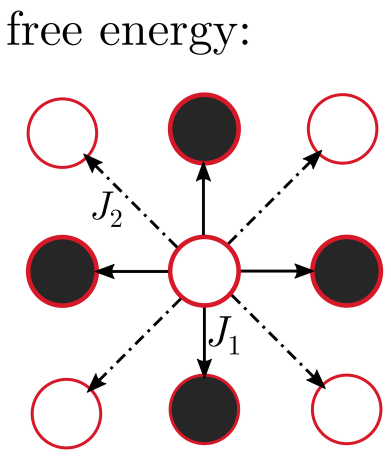

Fig. 2 - First (J_1) and second (J_2) neighbor interactions in one site.

REFERENCES

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RESULTS

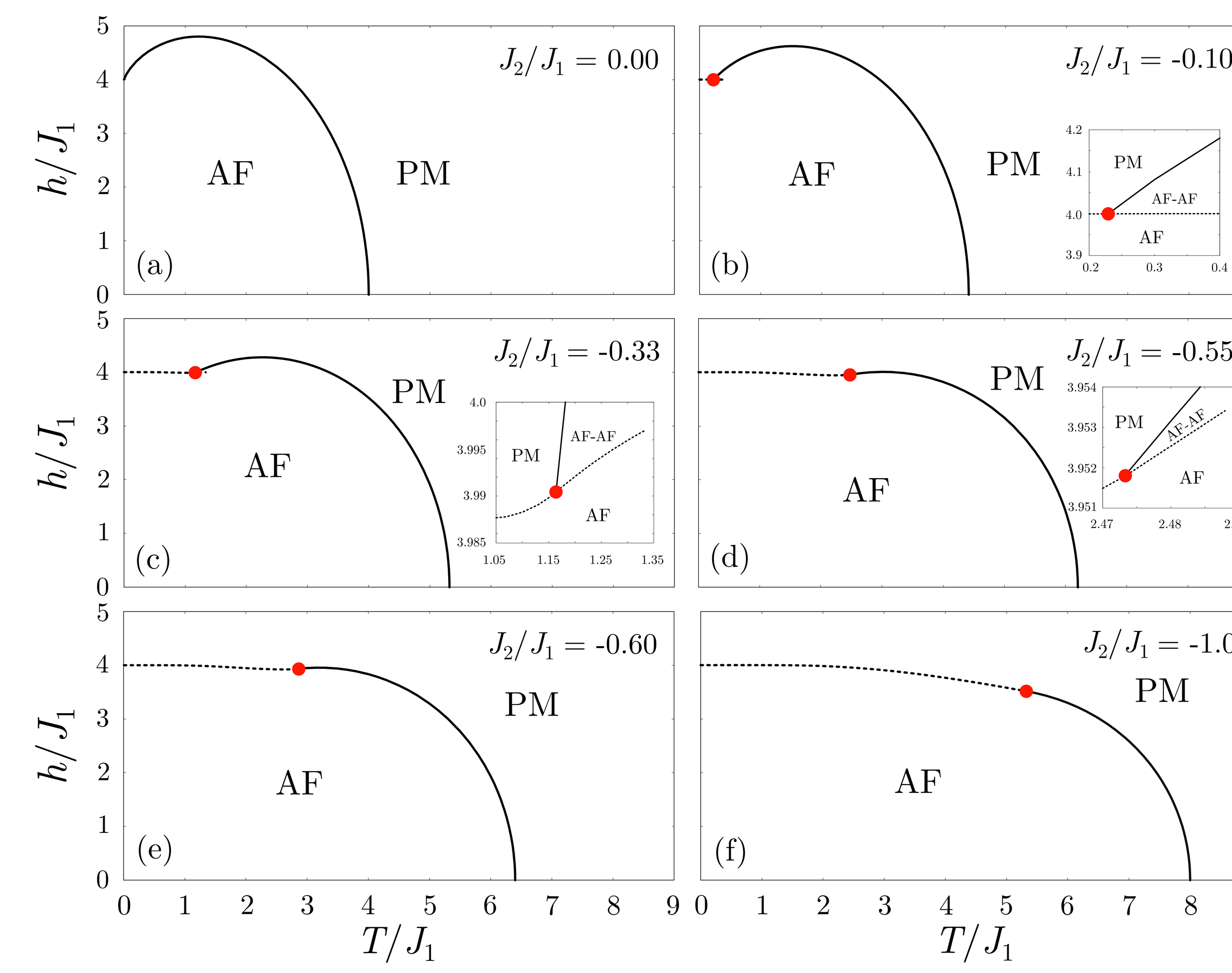

Fig. 3 - Phase diagrams of temperature versus external magnetic field for n_s = 1 and differents J_2. The lines represents continuous and discontinuous transitions, and are represented by such. The region AF-AF shows the appereance of a second Néel AF phase closely to the tricritical point, denoted by a red circle.

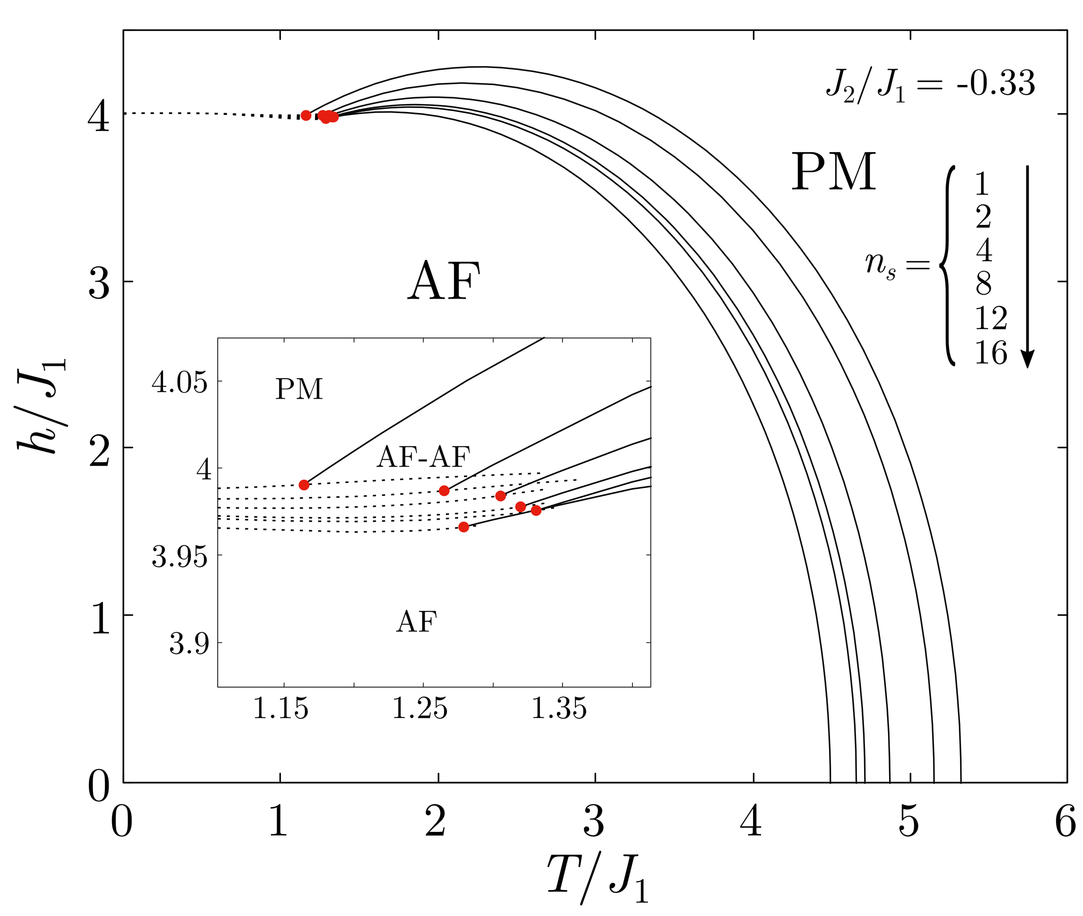

Fig. 4 - Phase diagrams of temperature versus magnetic field for differents n_s and J_2 = -0.33. The continuous lines represents second order transitions and dashed lines represents first order transitions. The red dots are tricritical points.

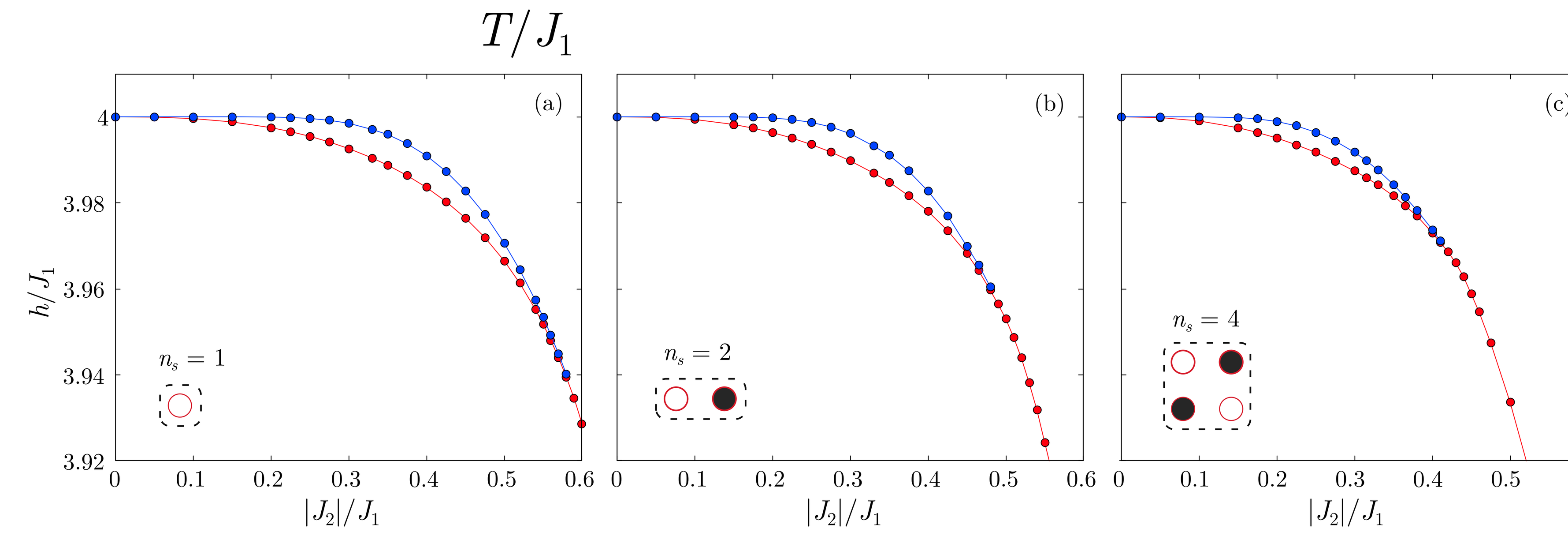

Fig. 5 - Dispositions of tricritical points (TCP's) and critical points (CP's) for n_s = 1,2, and 4 (a,b,c) in finite temperatures. The TCP's are represented by red circles, while the CP's are represented in blue circles and are associated to the AF-AF transitions that appears in Fig. 3 and Fig.4.

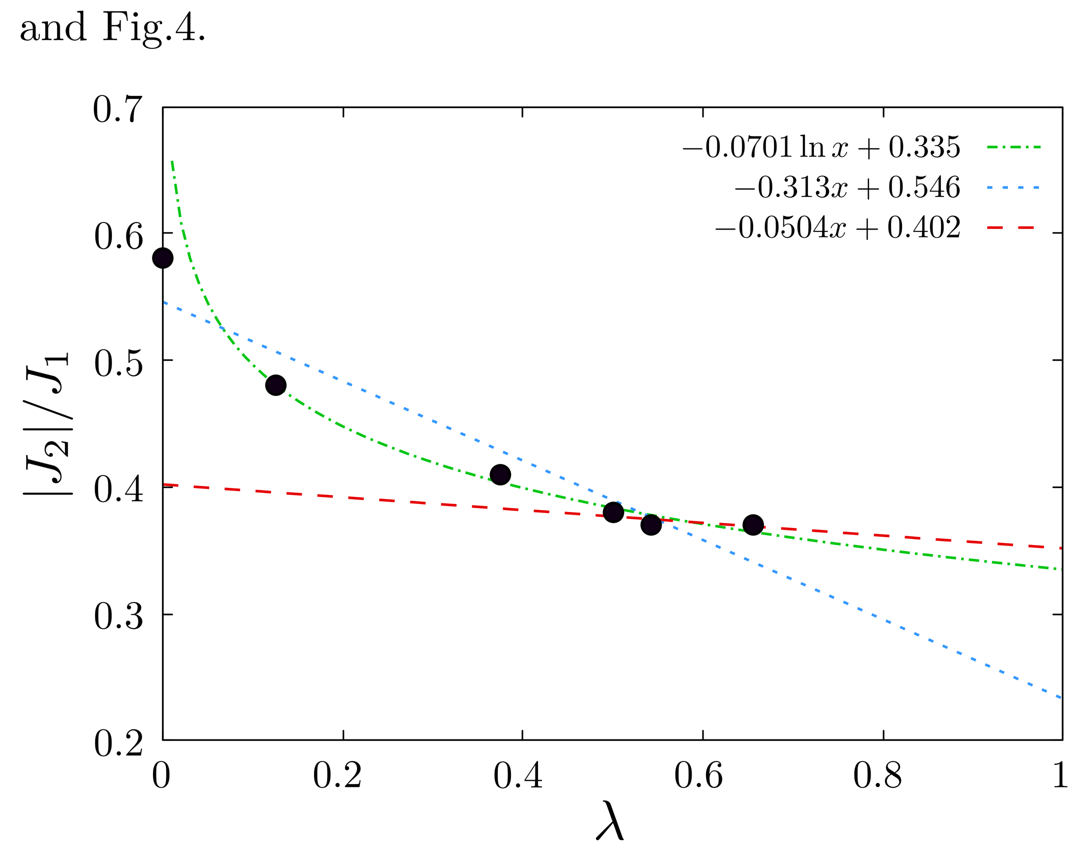

Fig. 6 - Disposition of the last CP (black circles) in wich cluster size in relation to J_2 and λ. Where λ is a parameter described as:

$$\lambda = \frac{N_b}{\frac{Z}{2}N_c}.$$

N_b is the number of interactions adopted exactly, N_c is the number of sites and Z is the coordenation number of the lattice (Z=8) . The lines are tentatives of fitting the points.

CONCLUSION

We find that the minor introduction of J_2 ferromagnetic change the low temperatute transitions to first-order. Also, stronger second neighbors interacions heads the TCP's to high temperatures and vanishes the CP's. In general, clusters with more sites decrease the critical temperature and favors first order transitions. The critical magnetic field remains the same in all cases. The AF-AF phase is mitigated by bigger clusters but still appearing in all the cases investigated. Our findings suggest that both the onset of tricriticality and the transition between two AF phases are a feature of the model.