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# CRITICAL BEHAVIOR OF THE ANTIFERROMAGNETIC ISING MODEL ON THE SQUARE LATTICE UNDER AN EXTERNAL MAGNETIC FIELD

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## INTRODUCTION

In the past century, the Ising model has been used in the study of magnetic properties of materials [1,2]. Although most of the problems treated using the Ising model do not have an exact solution, the results found using approximate techniques show very interesting behavior concerning phase transitions [3,4]. The study of the antiferromagnetic Ising model in the square lattice with interactions between first ($J_1$) and second neighbors ($J_2$) under the action of an external magnetic field is one such case. This case shows that the adoption of ferromagnetic interactions between second neighbors leads the system towards first-order phase transitions [4,5]. However, the influence of $J_2$ on the nature of phase transitions is still an open problem. Therefore, we will investigate the cases of intermediate $J_2$ under the action of an external magnetic field. The aim of this work is to understand the critical behavior of the antiferromagnetic Ising model with ferromagnetic second-neighbor interactions under an external magnetic field. To this end, we used the cluster mean-field (CMF) technique to investigate phase transitions of the model in various scenarios concerning the exchange couplings.

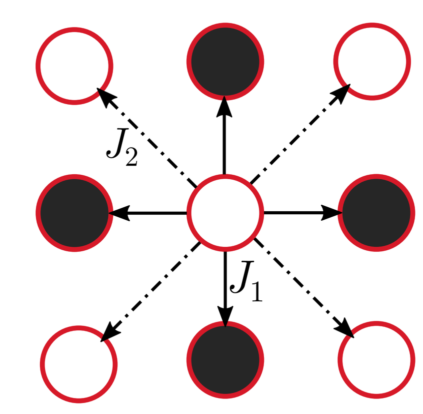


**Fig. 1** - First ($J_1$) and second ($J_2$) neighbor interactions in one site. Black and white circles represents spins up or down.

## MODEL AND METHOD

The Hamiltonian of the model is given by:

$$\mathcal{H} = \sum_{i,j} J_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i,$$

where $J_{ij}$ represents the coupling term between spins, $\sigma$ is the spin variable and $h$ is the external magnetic field. We use $J_1$ antiferromagnetic as energy scale and explore the cases of $J_2$ ferromagnetic, more in specific: $-1 \leq J_2/J_1 \leq 0$.

This problem does not has an exact solution, so we adopt an approximation technique known as cluster mean-field method. In the CMF technique, we divide the infinite lattice in finite clusters. As a result, the effective hamiltonian is also divided. Our CMF hamiltonian has an *intracluster* and an *intercluster* term:

$$\mathcal{H}_{CMF} = \mathcal{H}_{intra} + \mathcal{H}_{inter} - h\sum_i^{n_s}\sigma_i.$$

The *intracluster* term is obtained treating the interactions between spins inside the cluster exactly. The *intercluster* term is obtained by the mean-field aproximation:

$$\sigma_i\sigma_j \approx \sigma_i m_j + \sigma_j m_i - m_i m_j\,.$$

The local magnetization $m_i$ is calculated from:

$$m_i = \langle\sigma_i\rangle = \frac{\mathrm{Tr}\ \sigma_i\ e^{-\beta\mathcal{H}_{CMF}}}{\mathrm{Tr}e^{-\beta\mathcal{H}_{CMF}}},$$

where $\beta = 1/k_B T$, $T$ is temperature and $k_B$ is the Boltzmann constant ($k_B$=1).

To obtain the first order transitions we look to the Helmholtz free energy:

$$F = -T\ln(\mathrm{Tr}e^{-\beta\mathcal{H}}),$$

and the continuous transitions are detected by the order parameter of the Néel antiferromagnetic, obtained by the difference between the sublattices magnetizations.

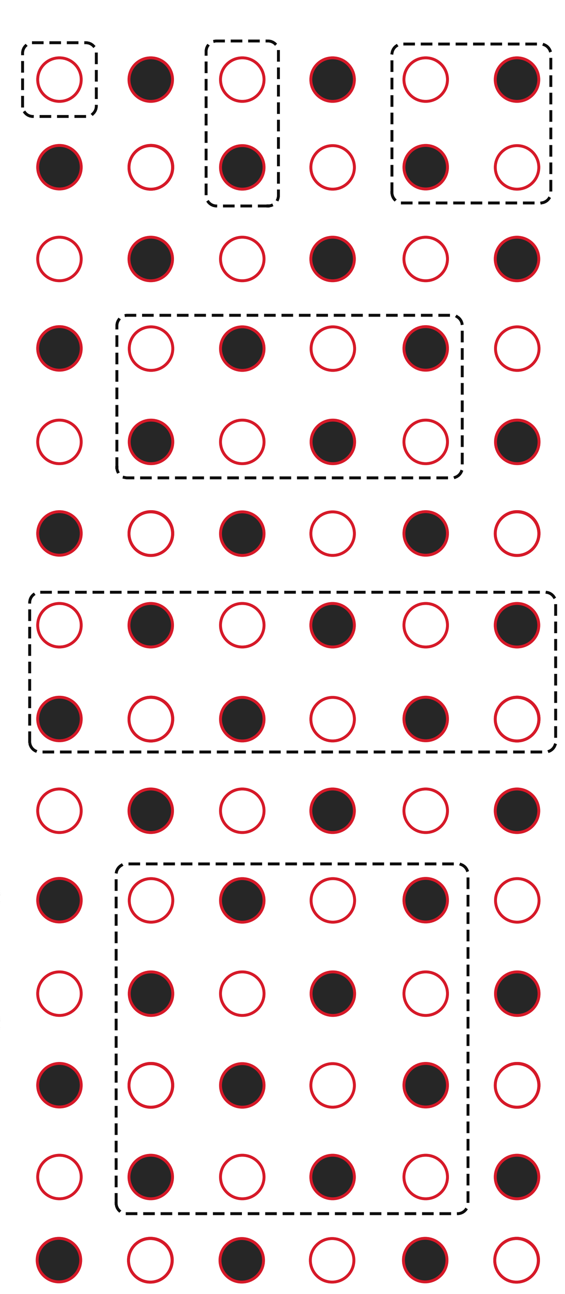

**Fig. 2** - Representation of the clusters adopted, with 1, 2, 4, 8, 12 and 16 sites.

## REFERENCES

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[3] M. Roos, et al, Phys. Rev. E 109 (2024) 014144
[3] Y. Kato, T. Misawa, Phys. Rev. B 93 (2015) 174419.
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## RESULTS

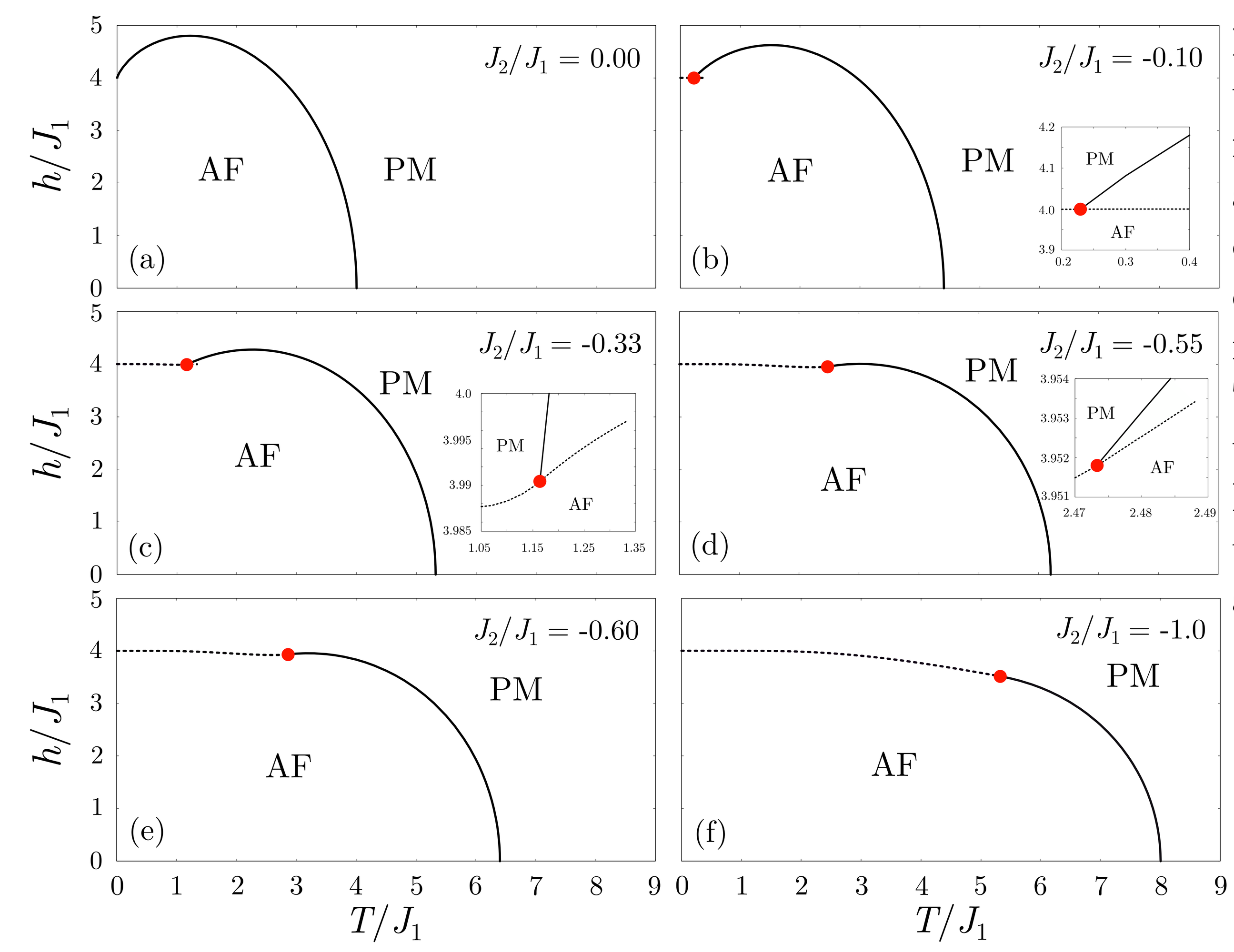


**Fig. 3** - Phase diagrams of temperature versus external magnetic field for $n_s$ = 1 and several $J_2$. Solid and dashed lines represent continuous and discontinuous phase transitions. There is a first-order phase transition curve inside the Néel AF phase close to the tricritical point, denoted by a red circle.

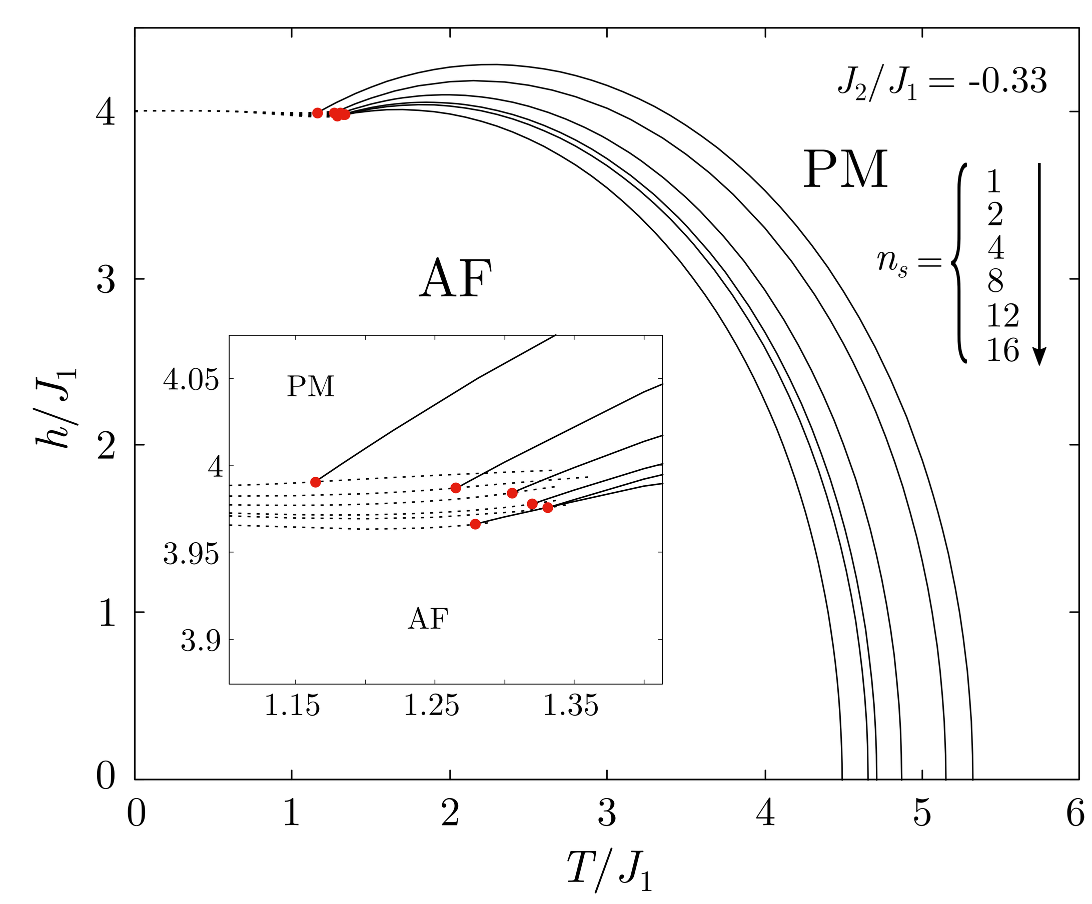


**Fig. 4** - Phase diagrams of temperature versus magnetic field for different cluster sizes and $J_2/J_1$ = -0.33. The continuous lines represent second-order phase transitions and dashed lines represent first-order phase transitions. The red dots are tricritical points. Close to the tricritical points we found a transition between two ordered phases with non-zero AF order parameter. This transition is discontinuous and extends over a small range of temperature, ending in a critical point.

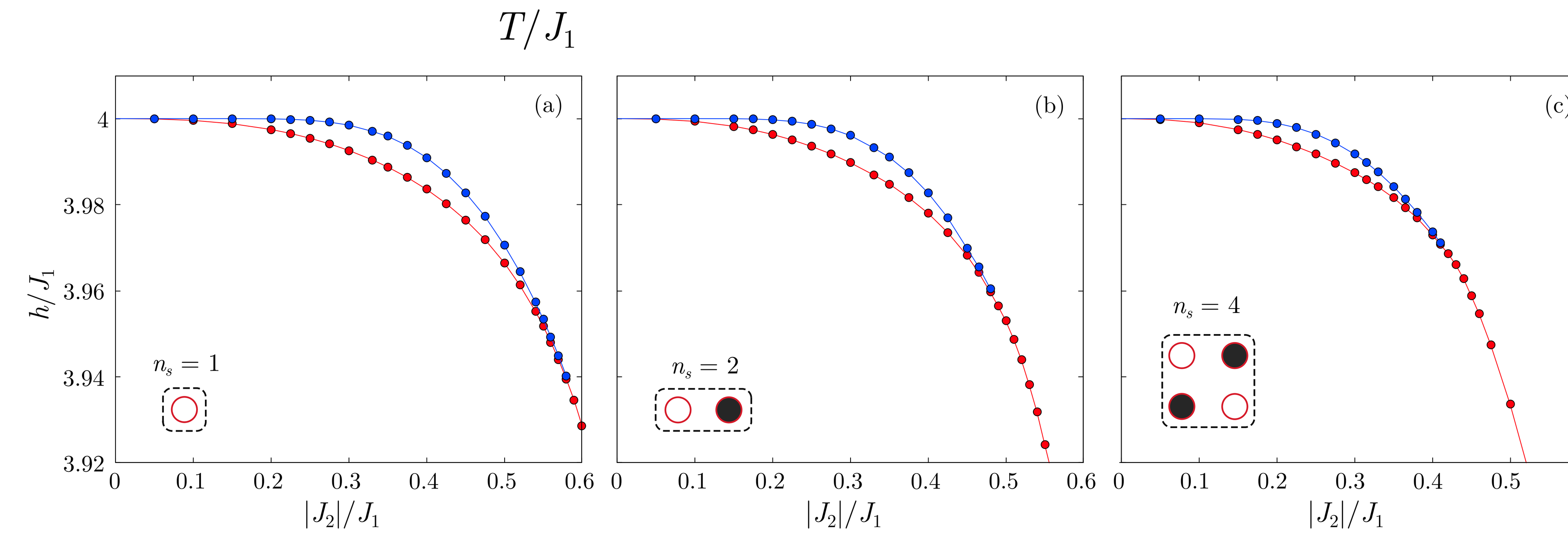


**Fig. 5** - External field coordinates of the tricritical points (TCP's) and critical points (CP's) for $n_s$ = 1, 2, and 4 (a, b, c) in finite temperatures. The TCP's are represented by red circles, while the CP's are represented in blue circles and are associated to the AF-AF transitions that appears in Fig. 3 and Fig. 4.

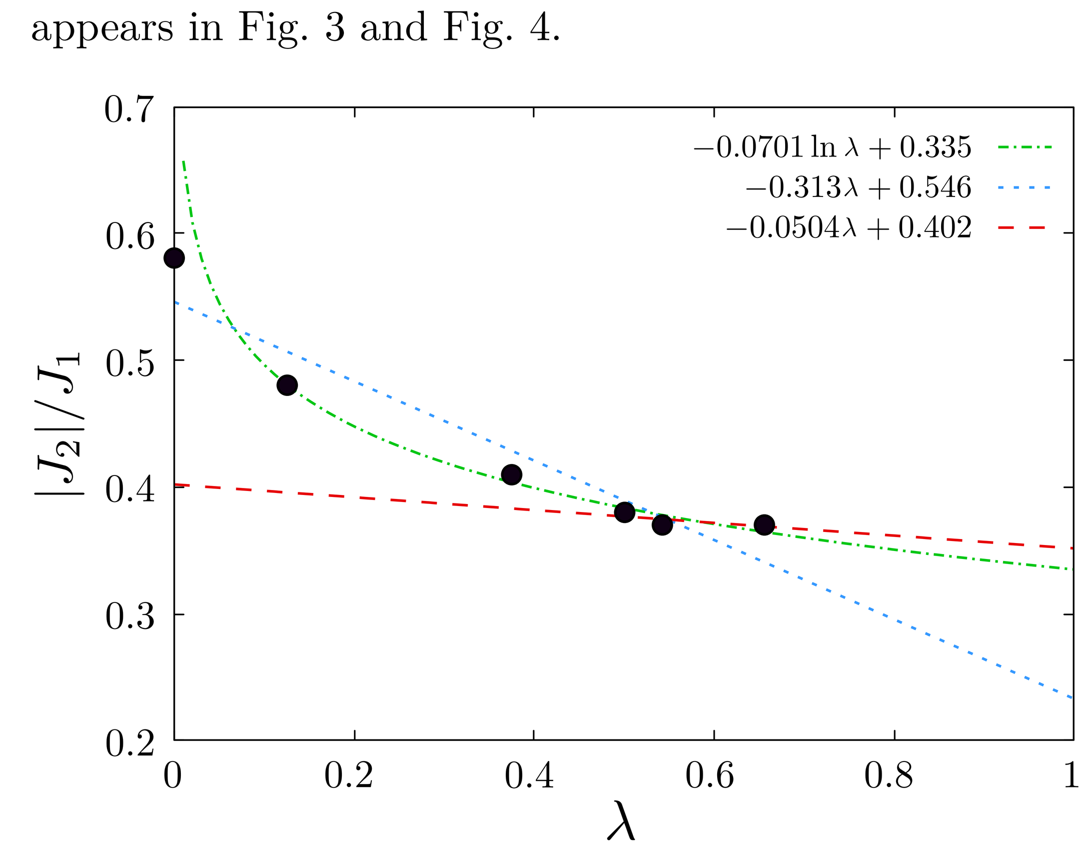


**Fig. 6** - Larger absolute value of second neighbor coupling in which a critical point is found for several cluster sizes. Where $\lambda$ is a parameter described as:

$$\lambda = \frac{N_b}{\frac{Z}{2}N_c}.$$

$N_b$ is the number of interactions adopted exactly, $N_c$ is the number of sites and $Z$ is the coordenation number of the lattice (Z=8) . The lines are tentatives of fitting the points.

## CONCLUSION

We find that a very small ferromagnetic $J_2$ change the field-induced low-temperature phase transition to first-order. Also, stronger second-neighbor interacions leads the TCP's to high temperatures and eliminates the CP's. In general, clusters with more sites decrease the critical temperature. The zero-temperature transition field remains the same in all cases. The AF-AF transition is mitigated by bigger clusters but still appears in all the cases investigated. Our findings suggest that both the onset of tricriticality and the transition between two AF phases are a feature of the model.