# Bat algorithm: Recent advances

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***Abstract*—The bat algorithm (BA) is a nature-inspired algorithm, which has recently been applied in many applications. BA can deal with both continuous optimization and discrete optimization problems. The literature has expanded significantly in the past few years, this paper provides a timely review of the latest developments. We also highlight some topics for further research.**

## I. Introduction

Nature-inspired algorithms have become a very promising alternative for solving very hard optimization problems in sciences and engineering. In the last two decades, many nature-inspired algorithms have been developed. The inspirations for developing such nature-inspired algorithms often come from biological, chemical and physical processes in nature. In addition, some algorithms were developed by drawing characteristics that based on sociology, history or even sports [24]. A brief review and taxonomy were proposed in the paper by Fister et al. [14]. According to the current literature, some popular nature-inspired algorithms are as follows:

- Ant colony optimization (ACO) [6], based on ant foraging behaviour.
- Artificial bee colony (ABC) [16], based on the behaviour of honey bees.
- Cuckoo search (CS) [12], based on the brooding behaviour of cuckoo species.
- Firefly algorithm (FA) [8], inspired by the flashing behaviour of tropical fireflies.
- Particle swarm optimization (PSO) [19], based on the flocking behavior of birds.
- and many evolutionary algorithms [3].

However, this short list of algorithms is just a tip of the algorithm iceberg, because there are more than 100 different algorithms in the literature. Therefore, it is not possible to cover even a fraction of these algorithms in one paper. Therefore, this paper is devoted to the bat algorithm (BA) which belongs to swarm intelligence [14]).

BA was developed in 2010 and significant progress has been made in the last 4 years. The aim of this paper is to review the bat algorithm and its recent developments, with an emphasis on the recent publications in 2013 and 2014 [7], [33]. We also discuss the latest improvements and applications concerning the bat algorithm.

The structure of this paper is as follows. Section 2 outlines the fundamentals of the bat algorithm, while Sections 3 and 4 review the recent variants of BA. Section 5 summarizes the recent applications, and Section 6 describes some important topics for the future work. Finally, Section 7 concludes briefly.

## II. Basics of bat algorithm

The standard bat algorithm was developed by Xin-She Yang in 2010 [30], [29]. The main characteristics in the BA are based on the echolocation behavior of microbats. As BA uses frequency tuning, it is in fact the first algorithm of its kind in the context of optimization and computational intelligence. Each bat is encoded with a velocity $v_i^t$ and a location $x_i^t$, at iteration $t$, in a $d$-dimensional search or solution space. The location can be considered as a solution vector to a problem of interest. Among the $n$ bats in the population, the current best solution $x_*$ found so far can be archived during the iterative search process.

Based on the original paper by Yang [30], the mathematical equations for updating the locations $x_i^t$ and velocities $v_i^t$ can be written as

$$f_i = f_{\min} + (f_{\max} - f_{\min})\beta, \tag{1}$$

$$v_i^t = v_i^{t-1} + (x_i^{t-1} - x_*)f_i, \tag{2}$$

$$x_i^t = x_i^{t-1} + v_i^t, \tag{3}$$

where $\beta \in [0, 1]$ is a random vector drawn from a uniform distribution.

In addition, the loudness and pulse emission rates can be varied during the iterations. For simplicity, we can use the following equations for varying the loudness and pulse emission rates:

$$A_i^{t+1} = \alpha A_i^t, \tag{4}$$

and

$$r_i^{t+1} = r_i^0[1 - \exp(-\gamma t)], \tag{5}$$

where $0 < \alpha < 1$ and $\gamma > 0$ are constants.

The pseudocode of the basic bat algorithm is presented in Algorithm 1. The main parts of the bat algorithm can be summarized as follows:

- First step is *initialization* (lines 1-3). In this step, we initialize the parameters of algorithm, generate and also evaluate the initial population, and then determine the best solution $\mathbf{x}_{best}$ in the population.

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**Algorithm 1** Original Bat algorithm

**Input:** Bat population $\mathbf{x_i} = (x_{i1}, \ldots, x_{iD})^T$ for $i = 1 \ldots Np$, $MAX\_FE$.
**Output:** The best solution $\mathbf{x}_{best}$ and its corresponding value $f_{min} = \min(f(\mathbf{x}))$.

```
1:  init_bat();
2:  eval = evaluate_the_new_population;
3:  f_min = find_the_best_solution(x_best); {initialization}
4:  while termination_condition_not_meet do
5:     for i = 1 to Np do
6:        y = generate_new_solution(x_i);
7:        if rand(0,1) > r_i then
8:           y = improve_the_best_solution(x_best)
9:        end if{ local search step }
10:       f_new = evaluate_the_new_solution(y);
11:       eval = eval + 1;
12:       if f_new ≤ f_i and N(0,1) < A_i then
13:          x_i = y; f_i = f_new;
14:       end if{ save the best solution conditionally }
15:       f_min=find_the_best_solution(x_best);
16:    end for
17: end while
```

- The second step is: *generate_the_new_solution* (line 6). Here, virtual bats are moved in the search space according to updating rules of the bat algorithm.
- Third step is a *local_search_step* (lines 7-9). The best solution is being improved using random walks.
- In forth step *evaluate_the_new_solution* (line 10), the evaluation of the new solution is carried out.
- In fifth step *save_the_best_solution_conditionaly* (lines 12-14), conditional archiving of the best solution takes place.
- In the last step: *find_the_best_solution* (line 15), the current best solution is updated.

In the general context of exploration and exploitation as well as genetic operators, we can analyze the roles of the main components of the standard BA. In essence, frequency tuning essentially acts as mutation because it varies the solutions mainly locally. However, if mutation is large enough, it can also leads to global search. Certain selection is carried out by applying a selection pressure that is relatively constant due to the use of the current best solution $x_*$ found so far. Compared with genetic algorithms, there is no explicit crossover; however, mutation varies due to the variations of loudness and pulse emission. On the other hand, the variations of loudness and pulse emission rates can also provide an autozooming ability in the sense that exploitation becomes intensive as the search is approaching the global optimality. This essentially switches an explorative phase to an exploitative phase automatically.

## III. The Brief History of the Bat Algorithm

Since the appearance of the original paper on the bat algorithm [30], the literature started to expand with a wide range of applications. The original paper outlined the main formulation of the algorithm and applied the bat algorithm to study function optimization with promising results. In fact, studies indicated that BA can perform better than genetic algorithms and particle swarm optimization [7], [33].

Then, Yang extended the BA to solve multi-objective optimization [31]. In addition, Yang and Gandomi applied the BA to engineering optimization with extensive results [32]. Probably the first hybrid variant of the bat algorithm was introduced by Wang et al. [28] and Fister proposed a hybrid bat algorithm [13]. Furthermore, discrete bat algorithms and parallelization versions also appeared.

## IV. Recent Variants of the Bat Algorithm

There are many new variants of the bat algorithm in the recent literature. In this brief paper, we summarize some the latest variants in Table I. For example, Mallikarjuna et al. proposed [23] a binary bat algorithm for solving the well-known economic load dispatch problem with the valve-point effect, and they concluded that their binary bat algorithm has many advantages. One of the biggest advantages is that BA can provide very quick convergence at the initial stage and can automatically switch from exploration to exploitation when the optimality is approaching. In addition, Sabba and Chikhi [25] proposed the so-called discrete binary bat algorithm by using the sigmoid function. In their paper [1], a new variant of BA called CBA (chaotic bat algorithm) by using chaotic maps to replace the uniform distribution used in the standard bat algorithm.

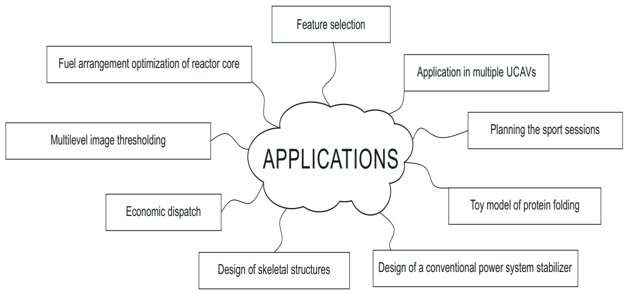


Fig. 1. Recent variants of bat algorithms

Meanwhile, Gandomi and Yang [15] also introduced chaos into the BA to increase its global search mobility. Furthermore, Yilmaz et al. [34] improved the explorative mechanism of BA by modifying the equation of the pulse emission rate and loudness of the bats. In optimizing benchmark functions, their modified variant showed superior results compared with the original bat algorithm. On the other hand, Zhou et al. [35] incorporated the cloud model to the bat algorithm and developed the cloud model based algorithm (CMBA). Another very interesting variant was developed by Li and Zhou [21], and they used complex-valued encoding. Their approach can increase the diversity of the population and expand the search dimensions.

More recently, Fister et al. applied the self-adaptation of novel parameter control mechanisms into the BA [10], [9],

[13]. To work on limited hardware, a compact bat algorithm (cBA) was proposed in [5]. In addition, Cai et al [4] applied Gaussian walks instead of uniform random walks to improve the local search capability.

TABLE I
RECENT APPLICATIONS USING THE BAT ALGORITHM.

| BA Variant | Author | Reference |
|---|---|---|
| Binary | Mallikarjuna et al. | [23] |
| Discrete binary bat | Sabba and Chikhi | [25] |
| Chaotic bat algorithm | Afrabandpey et al. | [1] |
| Chaotic bat algorithm | Gandomi and Yang | [15] |
| Modified BA | Yilmaz et al. | [34] |
| Cloud model bat | Zhou et al. | [35] |
| Self-adaptive | Fister et al. | [10] |
| Hybrid self-adaptive | Fister et al. | [9] |
| Complex valued | Li and Zhou | [21] |
| Compact bat algorithm | Dao et al. | [5] |
| BA with Gaussian walk | Cai et al. | [4] |

## V. RECENT APPLICATIONS USING THE BAT ALGORITHM

Accompanying the developments of the new variants and enhancements of the bat algorithm, many new applications of the bat algorithm have also appeared since 2013. In fact, some progress has been made in a diverse range of applications, including engineering design, image processing, feature selection, planning sport training sessions and others. Table II lists some of the recent applications using BA.

For example, in the context of engineering optimization, new applications can be summarized as follows:

- Kashi et al. [17] used the BA algorithm and implemented it into loading pattern optimization of nuclear core. Their results showed that BA is a very promising for such problems.
- Latif and Palensky [20] applied the bat algorithm to study a well-known nonlinear economic dispatch optimization problem. They proposed some modifications to the original bat algorithm and the proposed algorithm is very robust for optimizing economic dispatch problems.
- Sambariya and Prasad [26] applied the BA for the design of a power system stabilizer.
- Kaveh and Zakian [18] proposed the enhanced BA for the size optimization of skeletal structures consisting of truss and frame structures.

In addition, Alihodzic and Tuba [2] applied the BA to the multilevel image thresholding. Multilevel image thresholding belongs to image processing techniques. They modified the original bat algorithm with elements from DE and ABC. Experiments showed that their BA variant improved the quality of the results significantly and also improved the convergence speed. Furthermore, Li and Peng [22] used the BA in designing multiple UCAVs. Taha et al. [27] proposed a hybridized BA with a Naive Bayes classifier and used it on a feature selection problems. On the other hand, Fister et al. [11] used the BA to solve challenging problems in the context of planning sport sessions for athletes. Planning sport training is still a very challenging task for sport trainers. However, their paper showed that metaheuristic algorithms such as the BA can be very promising tools for planning proper training.

## VI. KEY ISSUES AND OPEN QUESTIONS

Despite the success of the bat algorithm and its diverse applications, there are still a few key issues that need more research. Here we highlight three areas: convergence rates, parameter tuning and large-scale problems.

The standard bat algorithm works very well, but there is no rigorous mathematical analysis to link the parameters with convergence rates. In principle, the convergence behaviour should be controlled by the BA parameters. In practice, it has been observed that the bat algorithm converges very quickly at the early stage and then the convergence rate slows down, and the bat algorithm is thus very useful to find good solutions to some tough problems in a quick time. However, the accuracy may be limited if the number of function evaluations is not high. Therefore, one of the key questions is how to improve the convergence rates at the later stage during the iterations. Various methods including hybrid approaches try to improve the bat algorithm, and it would be useful to use mathematical theory to guide such research.

In the bat algorithm, the algorithm-dependent parameters such as $\alpha$ and $\gamma$ require tuning, which is also true for all metaheuristic algorithms. In most applications, we use $\alpha = \gamma = 0.9$ or any fixed values. It is still not clear what the best values are for most applications, and some systematic parameter tuning should be carried out. Though such parameter settings may be problem-dependent and thus it is not an easy task. In addition, it can be expected that the use of varied parameter values during the iterations may be advantageous. The proper control of such parameters can be important. For example, at the moment, the loudness $A$ and pulse emission rate $r$ are varied in a monotonic manner; however, it may have some advantages to enhance the performance of the bat algorithm by using non-monotonic variations of $A$ and $r$. But how to vary them to achieve the best performance is still an open problem.

Most applications concerning the bat algorithm in the literature are small-scale or moderate-scale problems with a few or at most a few dozens design variables. It is highly needed that large-scale applications should be tested. In addition, most applications are continuous and partly discrete (with one or a few integer variables). It would be useful to apply it to truly NP-hard problems such as the travelling salesman problem and other real-world applications.

## VII. CONCLUSIONS

As the literature of the bat algorithm and its variants has expanded in the last few years, we have provided a short but timely review of the latest developments. Some new variants have been summarized and some key issues have been highlighted. Obviously, more studies are highly

TABLE II
RECENT APPLICATIONS USING THE BAT ALGORITHM.

| Application | Author | Reference |
|---|---|---|
| Fuel arrangement optimization of reactor core | Kashi et al. | [17] |
| Multilevel image thresholding | Alihodzic and Tuba | [2] |
| Economic dispatch | Latif and Palensky | [20] |
| Feature selection | Taha et al. | [27] |
| Planning the sport sessions | Fister et al. | [11] |
| Application in Multiple UCAVs | Li and Peng | [22] |
| Design of a conventional power system stabilizer | Sambariya and Prasad | [26] |
| Toy model of protein folding | Cai et al. | [4] |
| Design of skeletal structures | Kaveh and Zakian | [18] |

needed. In addition to the three main directions outlined in the previous section, other directions of research can also be very useful. For example, it will be useful to combine the bat algorithm with traditional algorithms such as gradient-based methods to see if it can improve the performance even further. Applications to data mining and telecommunications as well as transport engineering can also be very fruitful. These challenges and application areas mean more opportunities for further research.

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