University of the Basque Country

PhD Thesis

LOWER BOUNDS ON QUANTUM METROLOGICAL PRECISIONS

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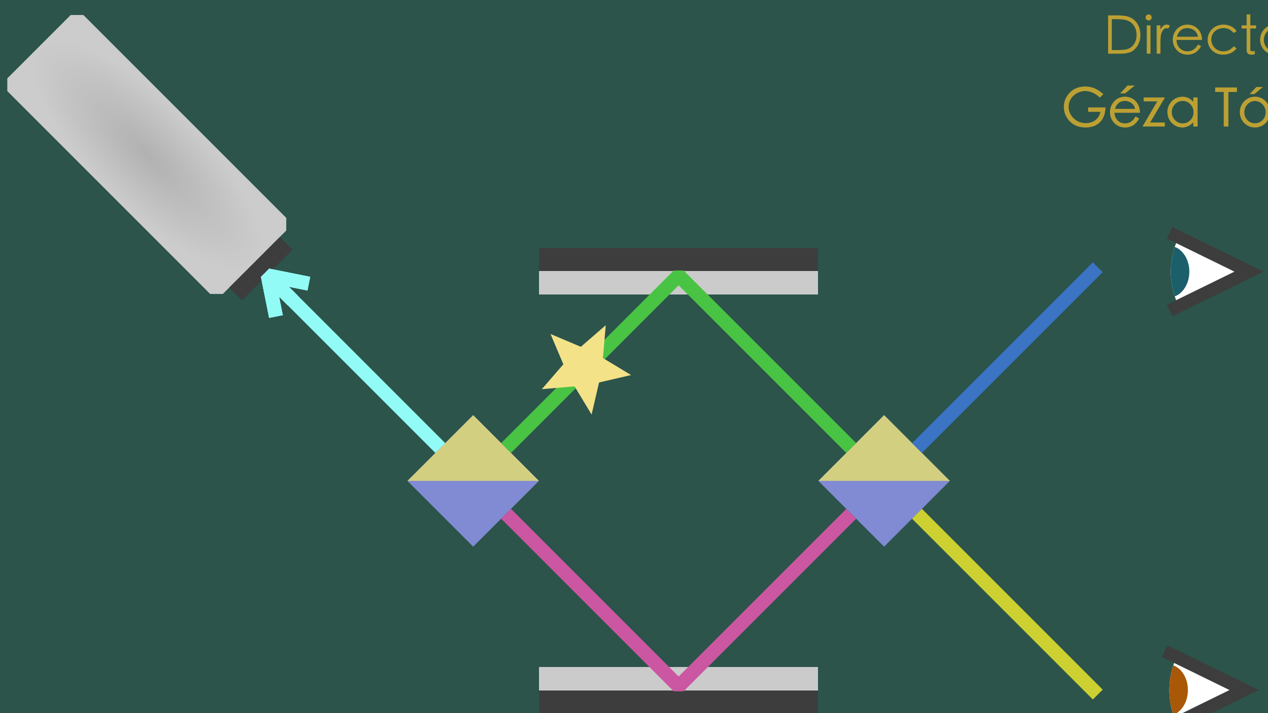

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Prologue

This work is part of the doctoral project of M. Sc. Iagoba Apellaniz in order to obtain the necesary qualification to promote himself to become a PhD. This work also tries to collect almost al the research discoveries done by the author on those previous years in a clear and concise way to make it understable for a general reader with a basic background in mathematics and physics.

The aim of this thesis is to present to the reader some important results of quantum metrology as well as guide possible interested ones into the fascinating field that is quantum metrology and its applications.

This is the prologe

Publications

Iagoba Apellaniz *et al*, *New J. Phys.* **17** 083027 (2015)
Detecting metrologically useful entanglement in the vicinity of Dicke states

Géza Tóth and Iagoba Apellaniz, *J. Phys. A: Math. Theor.* **47** 424006 (2014)
Quantum metrology from a quantum information science prespective

Preprints

Out of the scope of this thesis

Giuseppe Vitagliano *et al* 2014 *Phys. Rev. A* **89** 032307
Spin squeezing and entanglement for an arbitrary spin

Contents

Tables, figures and abbreviations used in this book

[Insert in a table]

SLD – Symmetric logarithmic derivative.

QFI – Quantum Fisher information

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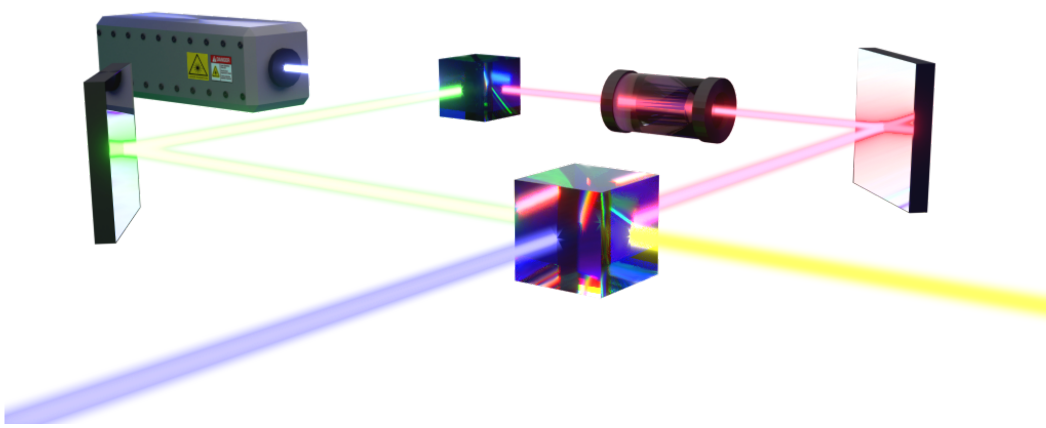

May 4, 2016

To my parents, my family
and to all the people
I have had around me those years.

Acknowledgments

I want to thank the people that support me in this endeavor especially my office and discussion mates and my director Géza Tóth for without whom my work would not be even started. I also want to thank more people especially those from the theoretical physics department of the University of the Basque Country and the unique very especial group for me from the Theoretical Quantum Optics group at the University of Siegen. I would really like to mention the names of all of them but I think it would be quite heavy for the average reader of this thesis. Thank you guys!

On the other hand I also felt very comfortable at my university, the University of the Basque Country, but I want to thank especially the people that make me grow in all ways as person.

1 Introduction

In the recent years...

The figure of merit for the precision is the inverse of the variance normalized with the number of particles, $(\Delta\Theta)^{-2}/N$. It has the following properties:

(i) The bigger it is the bigger is the precision

(ii) It is normalized so for the best separable state it is 1. For greater values than 1 it would be a non-classical sign.

SQL

$$(\Delta\Theta)^{-2} \leq N \tag{1}$$

HL

$$(\Delta\Theta)^{-2} \leq N^2 \tag{2}$$

This thesis consists of 4 well differentiated parts, apart from the current introduction, on which different topics are developed. In the first part, we will introduce the reader onto the research field of quantum metrology.

Brief comments on the notation: c_Θ and s_Θ stand for $\cos\Theta$ and $\sin\Theta$ respectively, probably some other trigonometry function is shortened.

2 Historical development of Quantum Metrology

METROLOGY, as the science of measuring, has played an essential role for the development of the technology as we know it today. It studies several aspects of the estimation process, such as which strategy to follow in order to improve the precision of an estimation. One of the most important figure of merit in this context is the achievable precision for a given system, independently of the strategy. We will show how to characterize it in the subsequents sections. And we will show as well different strategies to achieve the desired results. The metrology science also covers from the design aspects of a precise measuring device, until the most basic concepts of nature which lead in ultimate instance to the better understanding of the whole process.

In this sense, with the discovery of the Quantum Physics and the development of Quantum Mechanics, new doors for advances in metrology were open on the earlies decades of the 19th century. Later on, the Quantum Theory lead to the so-called field of Quantum Information which merges the notions of the theory of information and computer science, among others subfields, with the quantum mechanics. The role of the so-named entanglement, an exclusive feature of Quantum Mechanics, is essential in this context. Its complete understanding has integrated efforts of many researches world wide. Said this, the entanglement also is in the center of theoretical concepts included in Quantum Metrology.

On the other hand and with the aim of interpreting raw data, there are the statistics, without which many descriptions of the actual and past physical findings would lack of the rigorous interpretation needed for the complexity of data samples.

2.1 Background on statistics and theory of estimation

The main mathematical tools used on metrology belong to statistics with an emphasis on the estimation theory. The statistics makes the raw data of some subject under study comprehensible. It can be a set of different heights of a basketball team, or the age of graduated students, or a set of rapidly measured pressures of the air in the front part of a plane in order to estimate the velocity of the plane. It can be anything. The aim of this thesis is not to cover in all detail the subject of statistics, but to give the reader enough knowledge to understand the basis and properly follow this thesis.

2.2 Quantum Mechanics

The ubiquitous probabilistic nature of quantum mechanics is reflected on each corner of the field. This *ambiguity* of a system been quantum is exploited by many recent technologies.

The state

product states

Entanglement

Evolution

Unitary evolution

Markov

Limblad

Measurements

2.3 Quantum Metrology

Histograms of quantum states

Merging the probabilistic features of quantum mechanics with the estimation theory is not tibial.

The figure of merit for this purpose is the so-called Quantum Fisher Information.

3 Quantum metrology with Dicke like states

In this section we will show some results regarding the metrological usefulness of some unpolarized states. Recently it has been proven that among all states used for metrology the more polarized the state is the less margin to surpass the SQL the state has. All together, one can also show that the unpolarized states are able to perform better than the polarized ones. The more important figure of merit of such unpolarized but still useful states is the so-called unpolarized Dicke state, which consists of an equal number of particles pointing up and pointing down and it lives on the symmetric subspace.

One of the most particular features that this state has is that since it is a eigenstate of the collective operator J_z with eigenvalue zero, it mus have a very large uncertainty on the operators J_x and J_y. See Fig.

Since one cannot use the expectation value of the collective J_l operators to see how the state evolve under the magnetic field, one have to go at least a new level and consider the evolution of the variances.

3.1 Evolution of the expectation values

Here we show how the expectation values of collective operators needed in this section evolve under the unitary dynamics because of the influence of the homogeneous magnetic field.

$$J_x(\Theta) = J_x c_\Theta - J_y s_\Theta \tag{3}$$

4 Bounding quantum Fisher Information with observables

In the previous section we have shown how with few expectation values one would be able too bound from below the QFI. In this section we will show how the approach of bounding the precision with some expectation values can be further generalized.

Another approach must be taken to bound from below the QFI.

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