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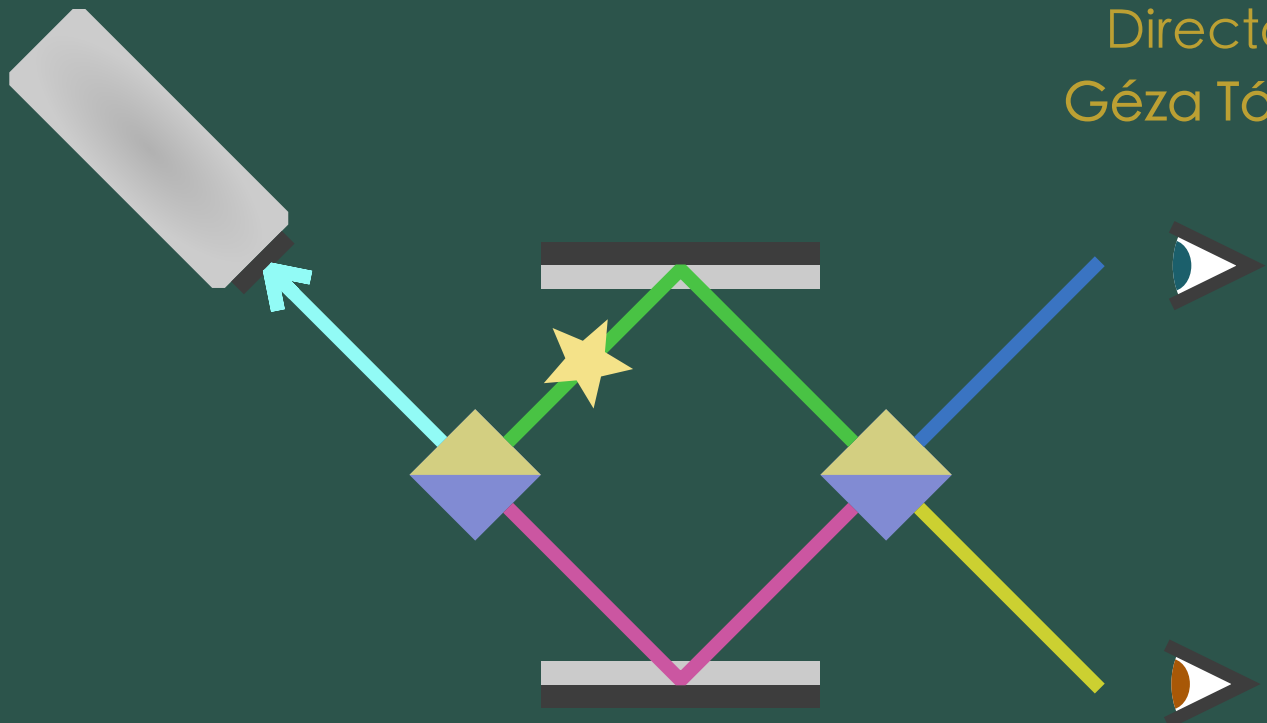
PhD Thesis



# LOWER BOUNDS ON QUANTUM METROLOGICAL PRECISIONS

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# Prologue

This work is part of the doctoral project of M. Sc. Iagoba Apellaniz in order to obtain the necessary qualification to promote himself to become a PhD. This work also tries to collect almost all the research discoveries done by the author on those previous years in a clear and concise way to make it understandable for a general reader with a basic background in mathematics and physics.

The aim of this thesis is to present to the reader some important results of quantum metrology as well as guide possible interested ones into the fascinating field that is quantum metrology and its applications.

This is the prologue



# Publications

Iagoba Apellaniz *et al* 2015 *New J. Phys.* **17** 083027  
Detecting metrologically useful entanglement in the vicinity of Dicke states

## Preprints

### Out of the scope of this thesis

Géza Tóth and Iagoba Apellaniz 2014 *J. Phys. A: Math. Theor.* **47** 424006  
Quantum metrology from a quantum information science perspective

Giuseppe Vitagliano *et al* 2014 *Phys. Rev. A* **89** 032307  
Spin squeezing and entanglement for an arbitrary spin





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# Tables, figures and abbreviations used in this book

[Insert in a table]

SLD – Symmetric logarithmic derivative.

qFI – Quantum Fisher information





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# Lower bounds on quantum metrological precisions

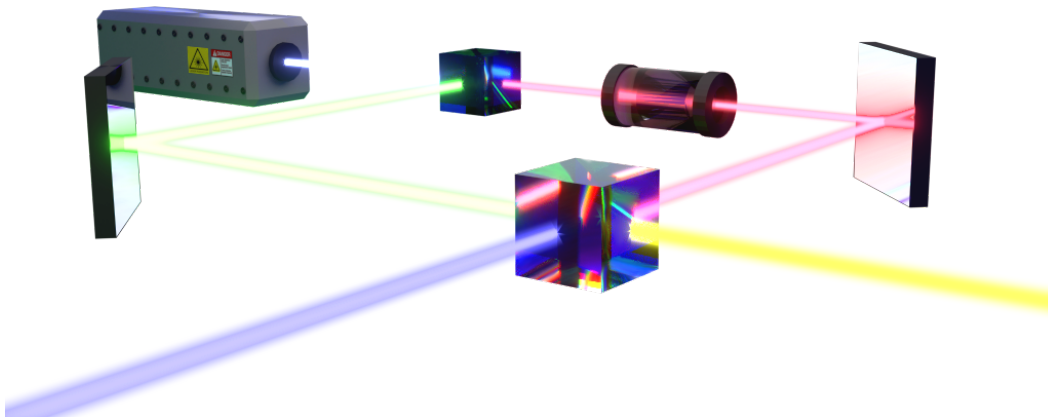
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Prof. Géza TÓTH



April 29, 2016



*To my parents, my family and all the people I have had around those years*





# Acknowledgments

I want to thank the people that support me in this endeavour especially my office and discussion mates and my director Géza Tóth for without whom my work would not be even started. I also want to thank more people especially those from the theoretical physics department of the University of the Basque Country and the unique very especial group for me from the Theoretical Quantum Optics group at the University of Siegen. I would really like to mention the names of all of them but I think it would be quite heavy for the average reader of this thesis. Thank you guys!

On the other hand I also felt very comfortable at my university, the University of the Basque Country, but I want to thank especially the people that make me grow in all ways as person.



## 1

# Introduction

THIS thesis is the final work of the PhD project spinning around the subject of Quantum Metrology.



## 2

## Historical development of Quantum Metrology

**M**ETROLOGY as the science of measuring has played an essential role for the development of the technology as it is nowadays. It studies several aspects of the actions involved on a measurement. Metrology covers from the physical design of a precise measuring device until the most basic concept of nature which lead in ultimate instance to the better understanding of the process.

In this sense, with the discovery of the Quantum Physics and the development of Quantum Mechanics, new doors for advances in metrology were open on the earlies decades of the 19th century. Later on, the Quantum Theory lead to the so called field of Quantum Information which merges the notions of the computer science, among others, with the quantum mechanics. The role of the so-called entanglement, an exclusive feature of Quantum Mechanics, is essential in this context. Its complete understanding has integrated efforts of many researches world wide. Said this, the entanglement also is in the center of theoretical concepts included in Quantum Metrology.

On the other hand and with the aim of interpreting raw data, there are the statistics, without which many descriptions of the actual and past physical findings would lack of the rigorous interpretation needed for the complexity of data samples.

### 2.1 Background on probability theory

One of the main tools required for the theoretical advance on metrology is the so-called probability theory. And for that reason, we will go trough the points of probability theory in this section.

The probability theory analyses a set of random data, in most cases outcomes of some process,

and with such analysis essential features or information in principle not so visible from the raw data is obtained. For example, we can have samples of the outcomes of a coin, 1 for heads and 0 for

## 2.2 Classical estimation theory

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## 2.3 Step in quantum estimation theory

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## 2.4 Quantum Metrology

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## Quantum metrology with Dicke like states

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## Bounding quantum Fisher Information with observables

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5

Accuracy bound for gradient field estimation with atomic ensembles

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## References

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