AN INVESTIGATION INTO THE PRACTICAL USES OF SPHERICAL NON-EUCLIDE-AN GEOMETRY IN GAMES

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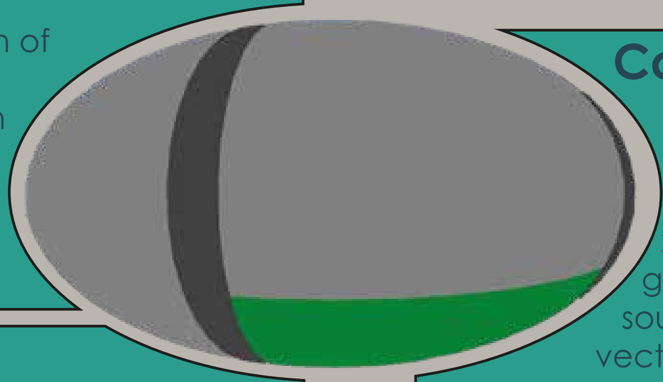
Introduction

The aim of this project is to investigate the uses, and potential uses, of non-Euclidean geometry, and try to implement a viable method for the implementation of a Spherical geometry in the Unity game engine.

Euclidean Geometry

Euclidean geometry is defined by a collection of rules as set out in "The Elements" (Euclid, 300 BCE) and has formed the basis of geometry in all modern society. As a result of Euclidean geometry being a key aspect of society, it has an influence on video games (Guimar-aes, Mello & Velho, 2015).



Non-Euclidean Geometry

Non-Euclidean geometry, however, exists in two main forms Hyperbolic and Spherical (Manning, 1963). The primary difference between these geometries is curvature. Euclidean geometry has the zero curvature nature people are accustomed to. However, Hyperbolic geometry possesses negative curvature, whilst Spherical possesses positive curvature.

Manual Curvature

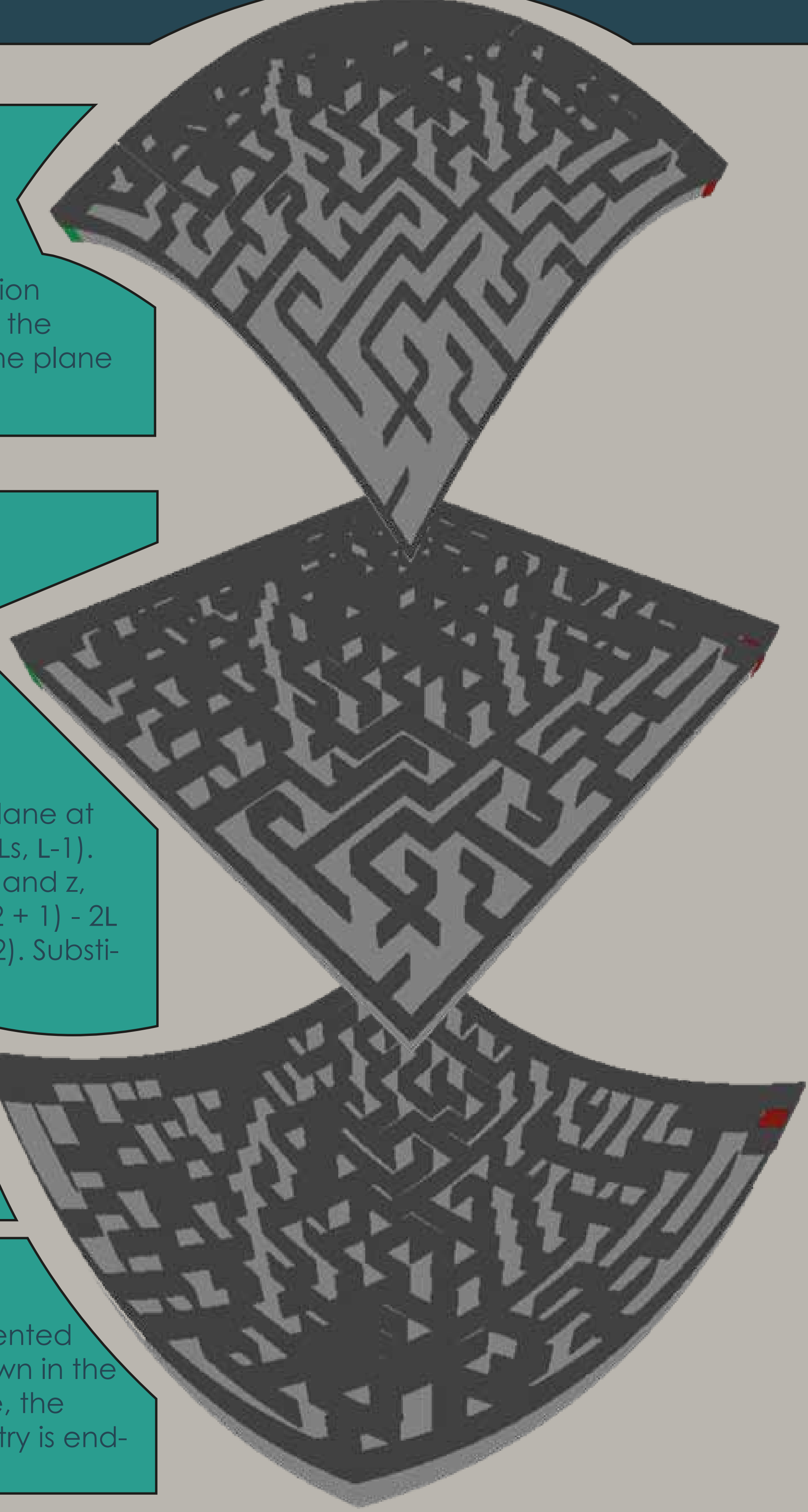
Another system that has been used to create a non-Euclidean geometry is manually curving around the player. With this, a system can be created where the terrain can be both negatively and positively curved. This has been done with the simple equation $Vf = Vo + (Vo - P)^2 * C$, where Vf is final vertex position, Vo is original vertex position, P is the camera position and C is a curvature variable. This allows for the terrain to be warped around the player.

Stereographic Projection

In stereographic projection the is the point furthest away from the plane. Allowing for the mapping of all points onto a flat plane from the Spherical geometry (Coxeter, 1989). Utilising this form of projection can cause depth issues. This will only begin to become apparent in the hemisphere closest to the origin, as the line plotting the sphere to the plane tends towards the tangent of the sphere.

Calculating Stereographic Projection

The equation for the sphere in standard form is $x^2 + y^2 + z^2 = r^2$. However, the sphere can be given a radius (r) of 1, leaving $x^2 + y2 + z2 = 1$. If the south pole is then used it can be ascertained that the vector from the viewing point (0, 0, -1) to the point of the sphere can be (0, 0, -1) + L(r, s, 1), where r and s are points on the plane at (r, s, 0), and where L is a multiple. This can then be expressed as (Lr, Ls, L-1). This can then be substituted into the equation for the sphere for x, y and z, giving $(Lr)^2 + (Ls)^2 + (L-1)^2 = 1$. Simplifying this gives $L^2(r^2 + s^2 + 1) - 2L = 0$. Solving this for L gives us the result of L = 0 or $L = (2 / 1 + r^2 + s^2)$. Substituting the non 0 value into the vector (Lr, Ls, L-1) (the vector for the point from the point of viewing) gives the resultant vector $(2r / (r^2 + s^2 + 1), 2s / (r^2 + s^2 + 1), (1 - r^2 - s^2) / (r^2 + s^2 + 1)) = R$ (where R is a point on the sphere). If the values for s and r are then substituted the new points of the stereographic projected plane can be ascertained.



Conclusion

Non-Euclidean geometry is not only something that can be implemented in games but something that can add another layer to them as shown in the very successful game Animal Crossing (Nintendo, 2001). Furthermore, the potential mechanics that could branch from non-Euclidean geometry is endless, whether this is simply graphics or physical too.

References

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