Nengo Summerschool 2019

Biologically Detailed Networks and Neuron Models

Peter Duggins, Andreas Stöckel







Motivation & Background
Why care about biological detail?



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- II Conductance-based n-LIF neurons \leftarrow Andreas How to computationally exploit nonlinear dendritic interaction



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- III Detailed NEURON Models and the NEF ← Peter
 How to integrate detailed neuron models into the NEF
- IV nengo-bio hands-on ← Andreas
 Build networks adhering to Dale's principle and exploit dendritic computation

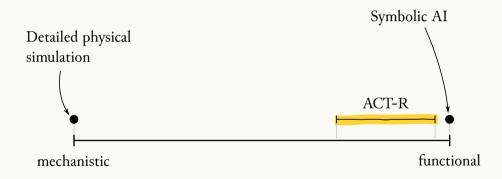
PART I

Motivation & Background

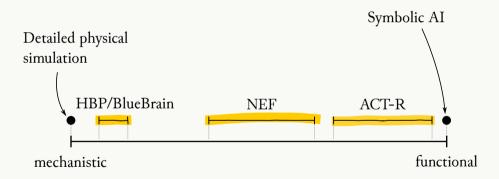


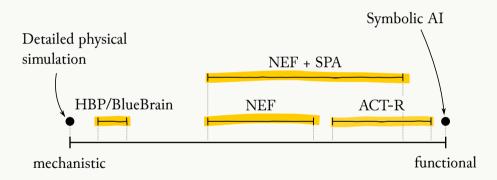


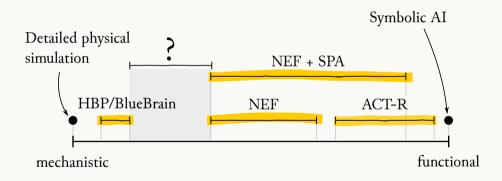


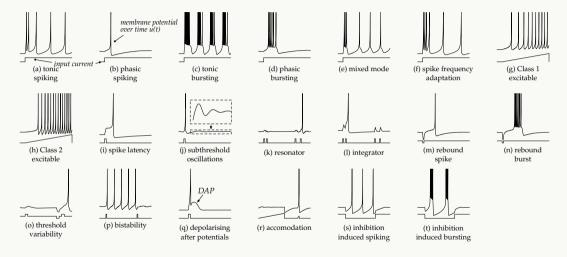




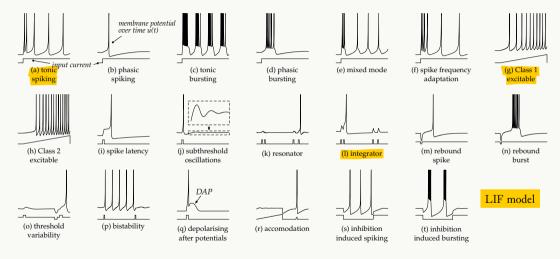








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- Can we *harness* these details to perform computation?

• Example:

- Excitatory and inhibitory channels interact nonlinearly.
- → Can we exploit this nonlinearity systematically?

Motivation — Recreating Idiosyncrasies of the Brain

1 Study Perturbations

- ▶ Drugs
- Neurological/mental disorders



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 Model additional pathways for information processing/learning



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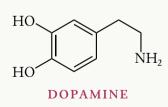
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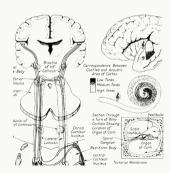
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3 Constrain Models

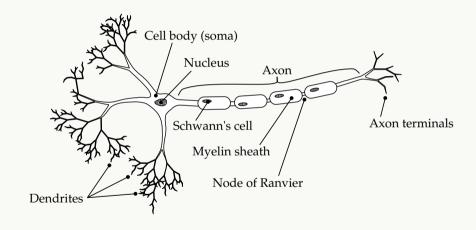
 Constrain models to available biophysical data



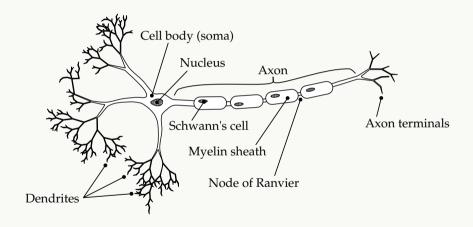




Neurobiology — Idealized "Textbook" Neuron

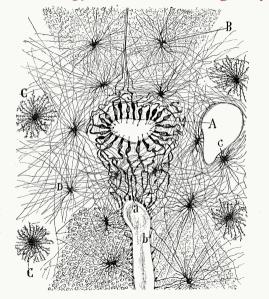


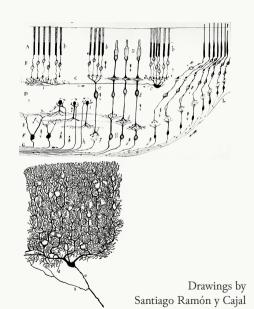
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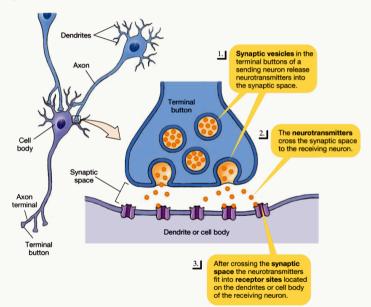
 $\textbf{ 1} \ \, \text{Dendrites collect input} \longrightarrow \textbf{ 2} \ \, \text{Integrated in soma} \longrightarrow \textbf{ 3} \ \, \text{Output spikes travel along axon}$

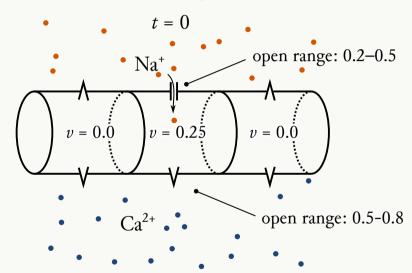
Neurobiology — Neural Heterogeneity

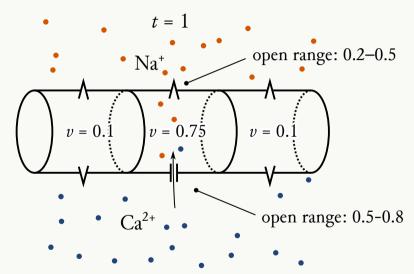


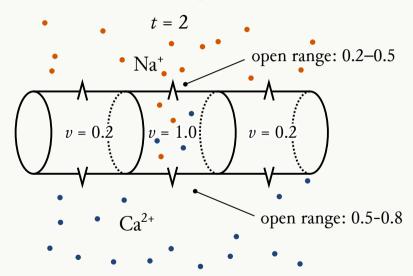


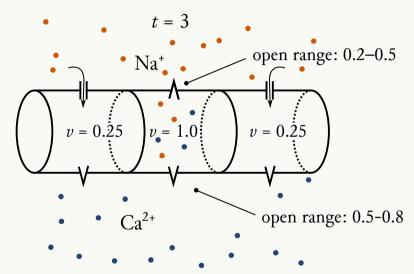
Synapses — Biophysics











Comparing LIF and Biology

Complexity			NEF compatibility		
	LIF	BIO		LIF	BIO
Geometry	point	compartmental	Tuning curve	$A=f(\mathbf{x})$	$A = f(\mathbf{x}, t)$
Synapse	current	conductance	Inputs	linear filter	synaptic nonlinearity
Dynamics	integrate, leak	ion channels	Dynamics	synapse dominates	neuron dominates
	voltage reset	cable equation	Decoders	static	time-dependent
Simulation	fast	slow	Estimates	$\sum_j a_i(t) * \mathbf{d}_i^f$?

PART II

Conductance-based *n*-LIF neurons

Reversal potentials

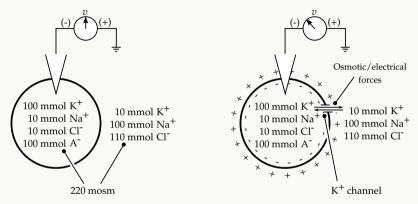
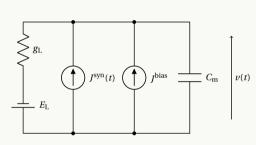


Illustration adapted from Reichert, 2000, Neurobiology.

Ion channels possess a specific *reversal potential* corresponding to the combination of *ion species* they are permeable for.

Current vs. Conductance-Based LIF

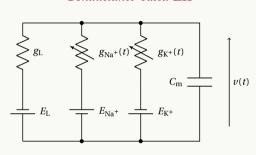
Current-based LIF



$$C_{\rm m}\dot{u}(t) = J^{\rm bias}$$

 $+ \alpha J^{\rm syn}(t)$
 $+ g_{\rm L}(E_{\rm L} - u(t))$

Conductance-based LIF

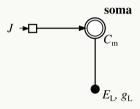


$$C_{\rm m}\dot{u}(t) = g_{\rm E}(t)(E_{\rm E} - u(t))$$

+ $g_{\rm I}(t)(E_{\rm I} - u(t))$
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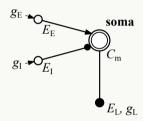
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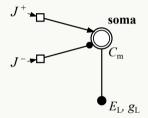


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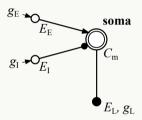


$$C_{\rm m}\dot{u}(t) = J^{\rm bias}$$

$$+ \alpha \left(J^{+}(t) - J^{-}(t)\right)$$

$$+ g_{\rm L}(E_{\rm L} - u(t))$$

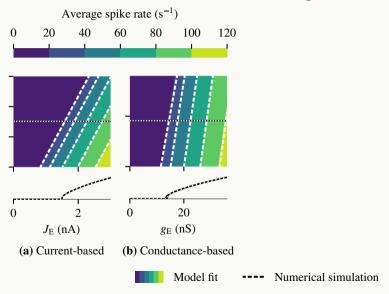
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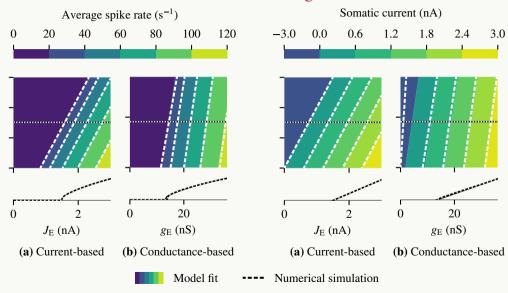
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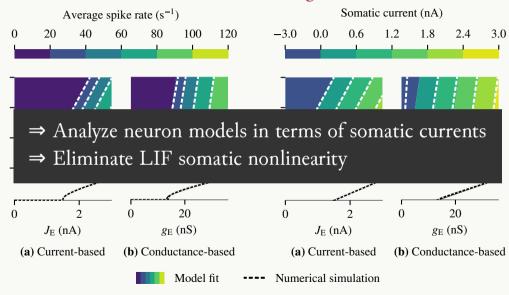
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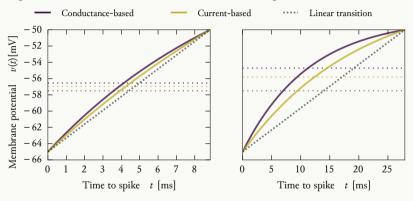


Current vs. Conductance-Based LIF Tuning Curves



Single-compartment conductance-based LIF

• For firing rates $\gg 0$, conductance-based LIF is boring

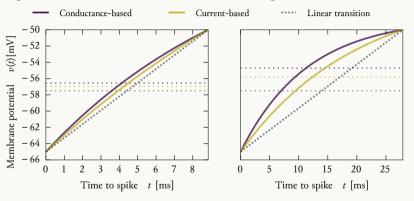


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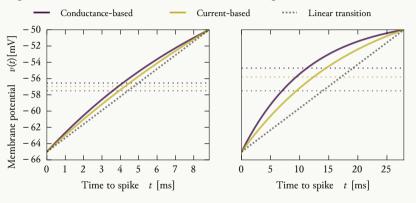


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Single-compartment conductance-based LIF

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• Can assume average membrane potential $\bar{u} \Rightarrow \text{just a skewed current-based LIF!}$

$$C_{\rm m}\dot{u}(t) = g_{\rm E}(t)\alpha_{\rm E} + g_{\rm I}(t)\alpha_{\rm I} + g_{\rm L}(E_{\rm L} - u(t))$$

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- Extensions

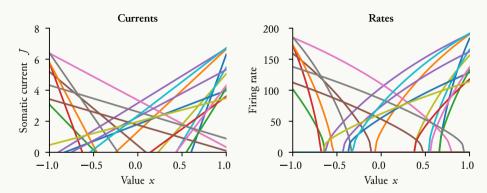
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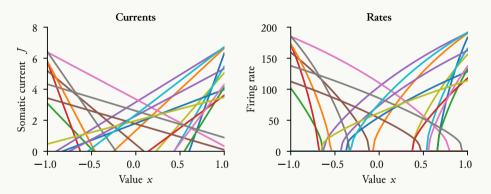
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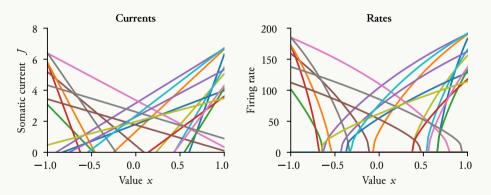
Extensions

- 3 Account for *sub-threshold currents* in the optimization process
- **4** Take *dendritic non-linearity* into account

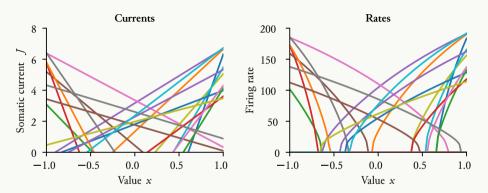




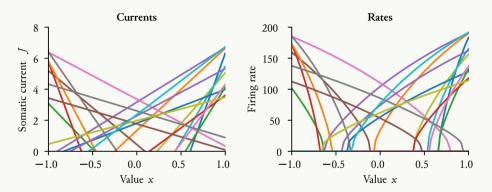
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- Firing rates correspond to a current J: "We want a current J if the neuron represents x"
- ⇒ Find a current-decoder for each individual post-neuron (instead of population-wise)

2 Nonnegative Weight Optimization

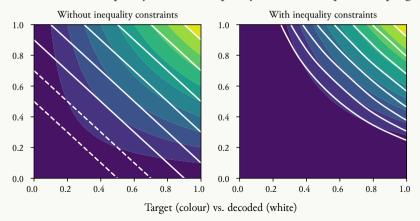
• Assume a post-neuron receives both excitatory and inhibitory input from each pre-neuron $i \Rightarrow$ weight vectors \vec{w}_i^+ , \vec{w}_i^- .

$$\begin{split} \min_{\vec{w}_i^+, \vec{w}_i^-} & \ \frac{1}{2} \sum_{k=1}^N \left\| \vec{w}_i^+ \vec{a}_k^+ - \vec{w}_i^- \vec{a}_k^- - J \left(\langle \vec{e}_i, f(\vec{x}_k) \rangle \right) \right\|_2^2 \\ & = \frac{1}{2} \left\| \vec{w}_i' A' - \vec{\jmath} \right\|_2^2 \quad \text{where } \vec{w}_i' = (\vec{w}_i^+, \vec{w}_i^-), \ A' = (A^+, -A^-)^T, \\ & \quad \text{and } (\vec{\jmath})_k = J \left(\langle \vec{e}_i, f(\vec{x}_k) \rangle \right), \end{split}$$
 subject to $\vec{w}_i^+ \geq 0, \vec{w}_i^- \geq 0$

• Can remove rows/columns from A' and $\vec{w_i}$ to account for Dale's principle (e.g. only 20% of pre-neurons are inhibitory, 80% are excitatory)

3 Account for sub-threshold currents

- Tuning curves are not injective (one-to-one): multiple x map onto a zero output rate
- ⇒ If the target rate is zero, we do not care about the current, as long as the rate is zero
- ⇒ Turn zero-rates into inequality instead of equality constraints (quadratic programming)



4 Take *dendritic nonlinearity* into account

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- Dendritic nonlinearity ${\cal H}$ converts synaptic state to somatic current

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- Example:

Current-based model:
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$$\begin{split} \min_{\vec{w}_i^+, \vec{w}_i^-} \quad & \frac{1}{2} \sum_{k=1}^N \left\| H(\vec{w}_i^+ \vec{a}_k^+, \vec{w}_i^- \vec{a}_k^-) - J \big(\langle \vec{e}_i, f(\vec{x}_k) \rangle \big) \right\|_2^2 \\ \text{subject to } \vec{w}_i^+ \geq 0, \vec{w}_i^- \geq 0 \end{split}$$

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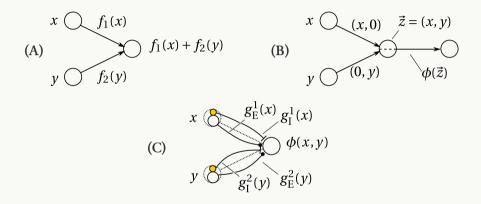
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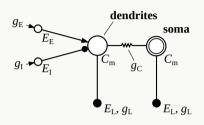
- Questions:
 - Can we find more interesting *H*?
 - Can we exploit H as a computational resource?
 - Under which conditions can we efficiently solve the optimization problem?

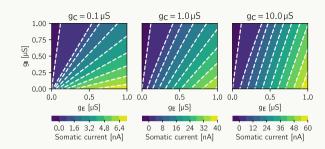
Computing Multivariate Nonlinear Functions



Example: Multiplication, compute $\phi(\vec{z}) = \phi(x, y) = x \cdot y$

Two-compartment LIF neuron

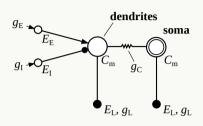


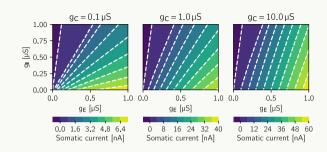


• Dendritic nonlinearity:

$$H(g_{\rm E}, g_{\rm I}) = \frac{b_1 + b_2 g_{\rm E} + b_3 g_{\rm I}}{a_1 + a_2 g_{\rm E} + a_3 g_{\rm I}}$$

Two-compartment LIF neuron



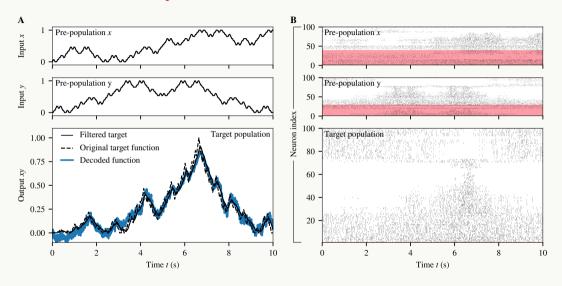


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 Can still formalize weight-optimization as convex quadratic programming problem, guaranteed to find global optimum

Results – Dendritic computation (I)

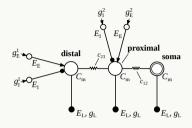


Results – Dendritic computation (II)

	Experiment setup						
	Standard LIF			Two comp. LIF $g_{\rm C}=50{\rm nS}$		Two comp. LIF $g_{\rm C}=100{\rm nS}$	
\mathbf{Target}	no relaxation	A standard	B two-layer	© standard	noise model	standard	noise model
x+y	$5.1 \pm 0.6\%$	$5.5 \pm 1.1\%$	$11.0 \pm 1.3\%$	$\textbf{3.2}\pm\textbf{1.1}\%$	$9.1 \pm 1.2\%$	$5.1 \pm 1.2\%$	$11.5 \pm 1.3\%$
$x \times y$	$26.2 \pm 0.4\%$	$21.5 \pm 6.6\%$	$15.4 \pm 4.0\%$	$13.9 \pm 2.9\%$	$11.9\pm1.8\%$	$18.2 \pm 4.0\%$	$14.3 \pm 2.1\%$
$\sqrt{x \times y}$	$14.1 \pm 0.4\%$	$19.7 \pm 6.1\%$	$16.3 \pm 3.0\%$	$9.7 \pm 2.6\%$	$\textbf{7.1}\pm\textbf{1.0}\%$	$13.3 \pm 4.2\%$	$8.9 \pm 1.7\%$
$(x \times y)^2$	$44.5 \pm 0.6\%$	$33.0 \pm 6.6\%$	$18.7\pm6.7\%$	$27.7 \pm 4.1\%$	$27.4 \pm 4.1\%$	$34.3 \pm 5.3\%$	$30.3 \pm 4.3\%$
x/(1+y)	$6.0 \pm 0.4\%$	$5.2 \pm 0.7\%$	$9.5 \pm 0.8\%$	$\textbf{3.4}\pm\textbf{1.0}\%$	$10.0 \pm 1.6\%$	$5.3 \pm 1.3\%$	$14.0 \pm 1.9\%$
$\ (x,y)\ $	$8.0 \pm 0.4\%$	$5.7 \pm 1.1\%$	$10.5 \pm 1.0\%$	$\textbf{3.1}\pm\textbf{1.3}\%$	$8.9 \pm 1.2\%$	$4.3 \pm 1.8\%$	$12.3 \pm 1.8\%$
atan(x, y)	$10.3 \pm 0.3\%$	$8.6 \pm 1.0\%$	$13.4 \pm 1.1\%$	$\textbf{5.8}\pm\textbf{1.3}\%$	$8.4 \pm 1.0\%$	$7.0 \pm 1.2\%$	$12.7 \pm 1.6\%$
$\max(x, y)$	$14.9 \pm 0.3\%$	$10.0 \pm 0.9\%$	$11.3 \pm 1.4\%$	$\textbf{5.5}\pm\textbf{0.9}\%$	$7.7\pm0.9\%$	$7.3\pm0.9\%$	$9.7\pm1.0\%$

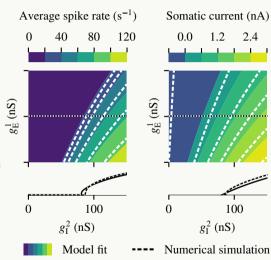
http://arxiv.org/abs/1904.11713

Three-compartment LIF neuron

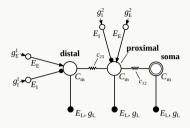


• Dendritic nonlinearity:

$$\left(\frac{b_1^1+b_2^1g_{\rm E}+b_3^1g_{\rm I}}{a_1^1+a_2^1g_{\rm E}+a_3^1g_{\rm I}}\right)\cdot \left(\frac{b_1^2+b_2^2g_{\rm E}+b_3^2g_{\rm I}}{a_1^2+a_2^2g_{\rm E}+a_3^2g_{\rm I}}\right)$$



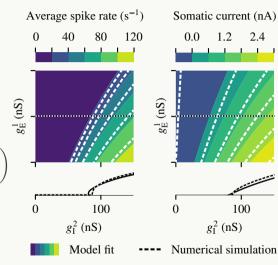
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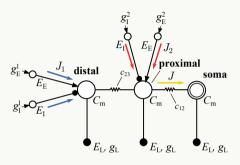


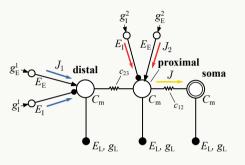
• Dendritic nonlinearity:

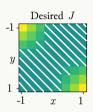
$$\left(\frac{b_1^1+b_2^1g_{\rm E}+b_3^1g_{\rm I}}{a_1^1+a_2^1g_{\rm E}+a_3^1g_{\rm I}}\right)\cdot \left(\frac{b_1^2+b_2^2g_{\rm E}+b_3^2g_{\rm I}}{a_1^2+a_2^2g_{\rm E}+a_3^2g_{\rm I}}\right)$$

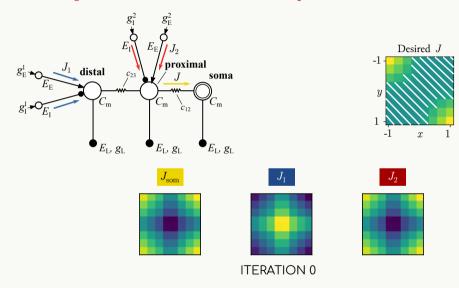
 For n-LIF, n ≥ 3: can formalize weight-optimization as iterative trust-region-based optimization problem; not guaranteed to find global optimum

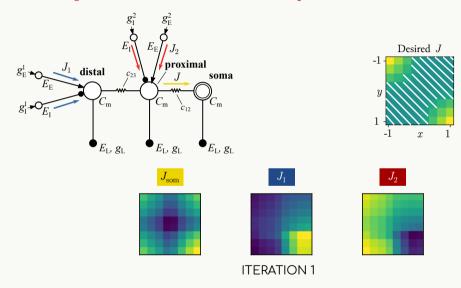


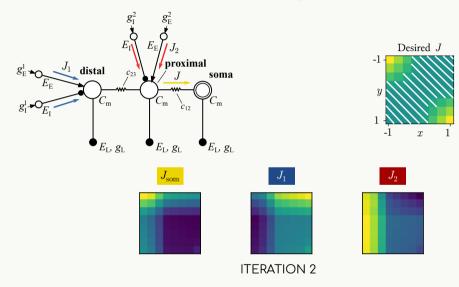


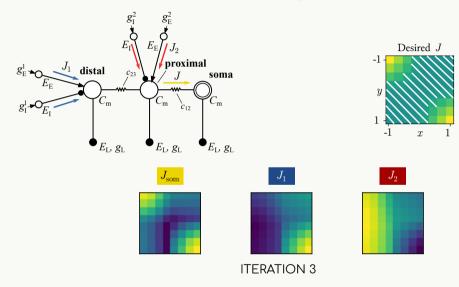


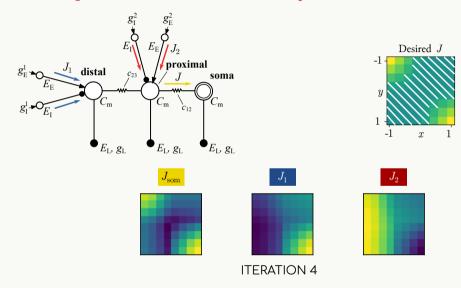


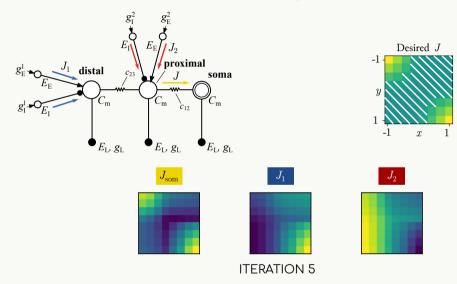


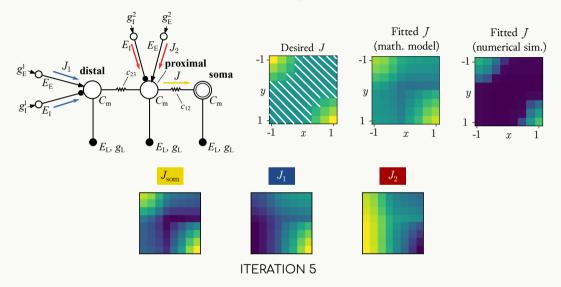










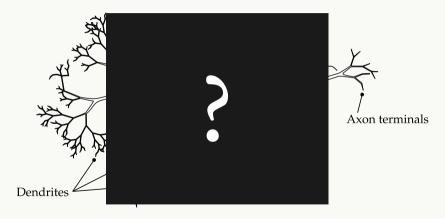


PART III

Detailed Neuron Models and the NEF

"Black Box" Modelling

Goal: methods to plug arbitrarily-complex neuron models into Nengo



Neuron Model

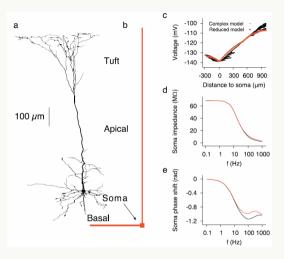


Figure: Morphology and electrophysiology for (a) a detailed reconstruction of a layer 5 pyramidal neuron and (b) a reduced model with 7 anatomical sections, 20 compartments, and 9 ion channels [?].

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Want: accurate state-space representation, transformation, dynamics

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Setup:

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- emulate bias with randomly-weighted connection

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Training: calculate readout decoders dout:

- Simulate bioneuron network
- Collect activities a(t) and targets $\mathbf{x}(t)$
- $\mathbf{d}^{out} = \operatorname{solver}(a(t), \mathbf{x}(t))$

NEF Principles 0-2

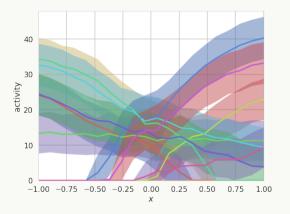


Figure: Heterogeneous bioneuron activities with randomly distributed e_j , α_j , β_j .

NEF Principles 0-2

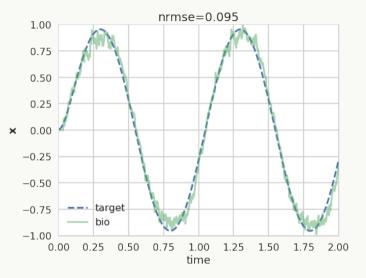


Figure: Principle 1: Representation

NEF Principles 0-2

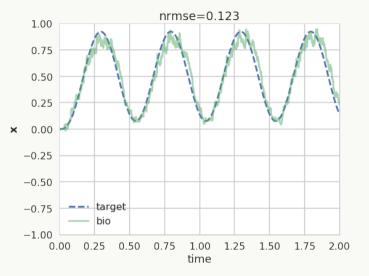


Figure: Principle 2: Transformation

Principle 3: Dynamics

Theory:

- $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$
- natural neural dynamics $\Rightarrow A'\mathbf{x} + B'\mathbf{u} \Rightarrow \mathbf{d}^{ff}, \mathbf{d}^{fb}$
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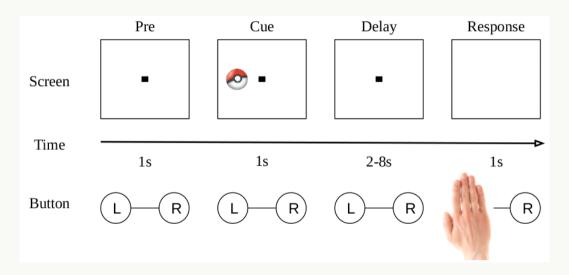
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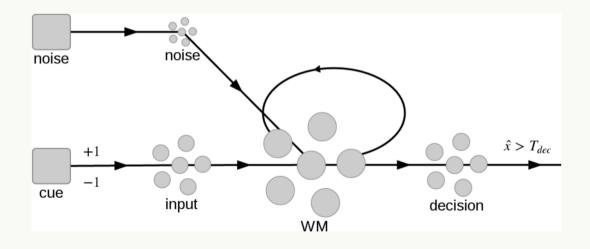
Possible approaches:

- supervise with training spikes "unrolling" the recurrence
- open up the "black box" to characterize internal dynamics
- project to/from higher dimensions with random weights

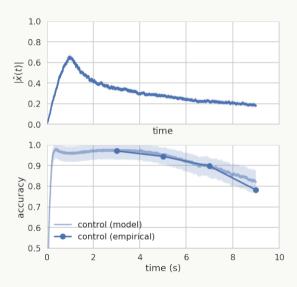
Application: Working Memory



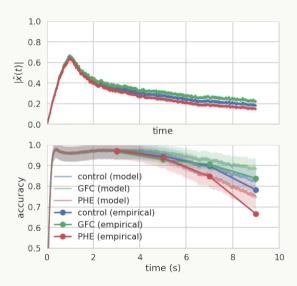
Working Memory Model



Drugs Affect Working Memory



Drugs Affect Working Memory



nengo-bio hands-on

PART IV

PART V

Summary & Conclusion

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Thank you for your attention!