JuliaOpt

Optimization packages in Julia

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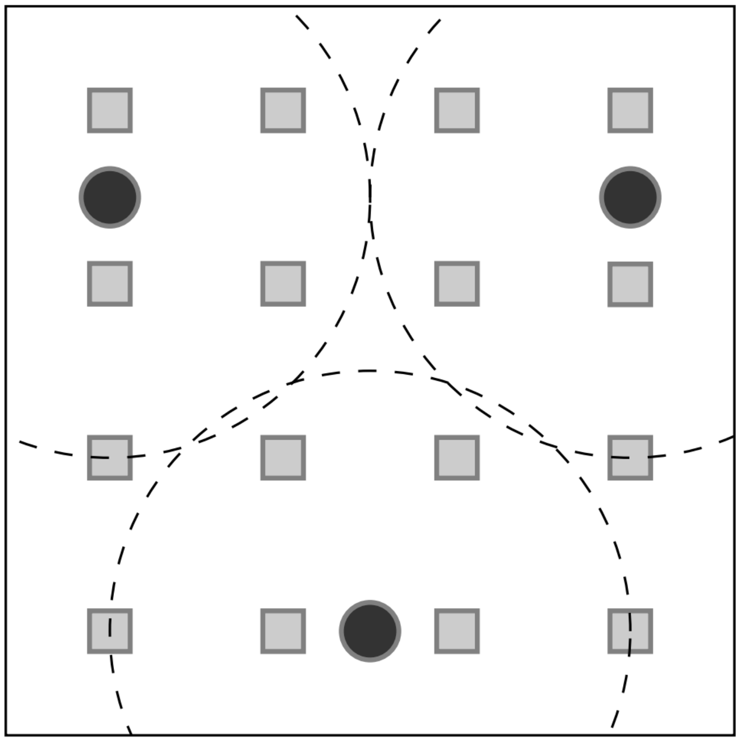
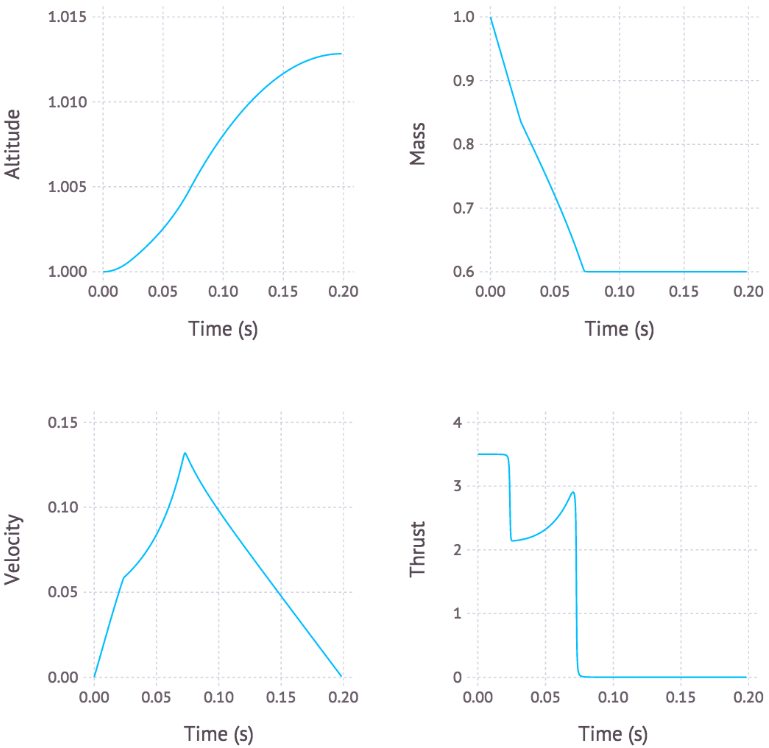
What is Optimization?

$$\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{subject to} & g_i(x) \leq 0 \quad \forall i \end{array}$$

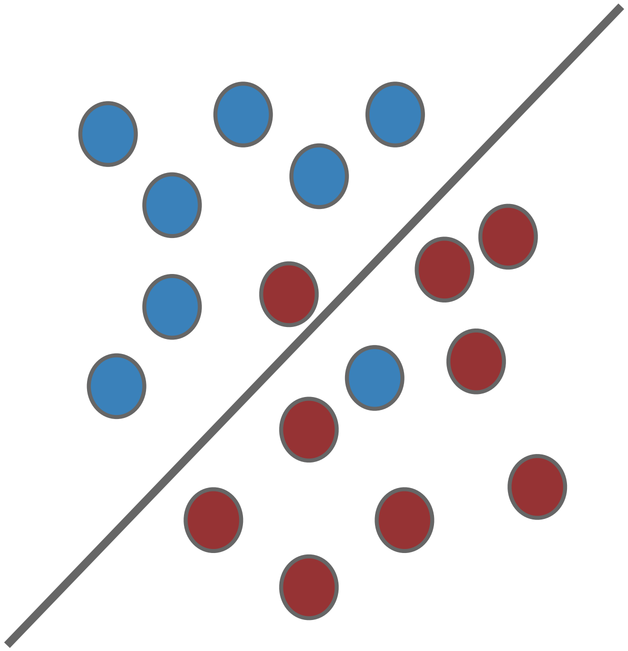
What is Optimization?

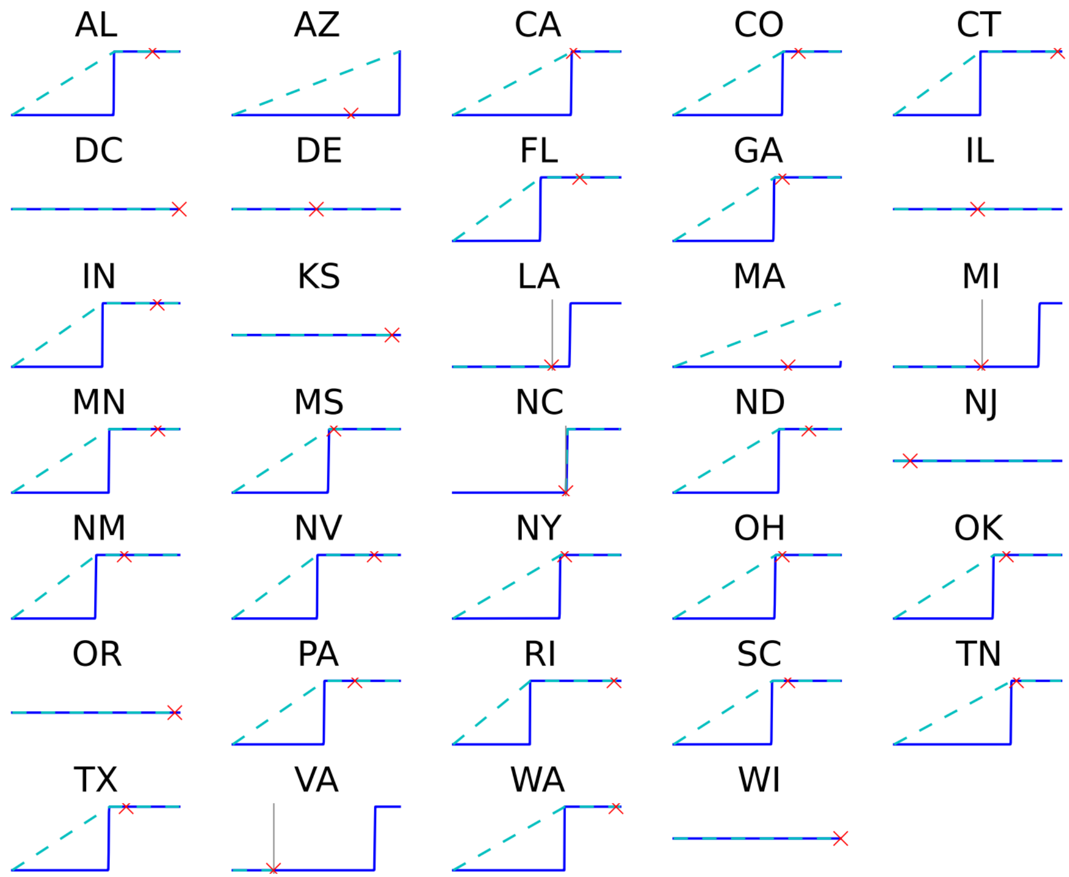




I. Dunning, J. Huchette, and M. Lubin. "JuMP: A modeling language for mathematical optimization."

What is Optimization?





M. Udell, and S. Boyd, “Maximizing a Sum of Sigmoids”

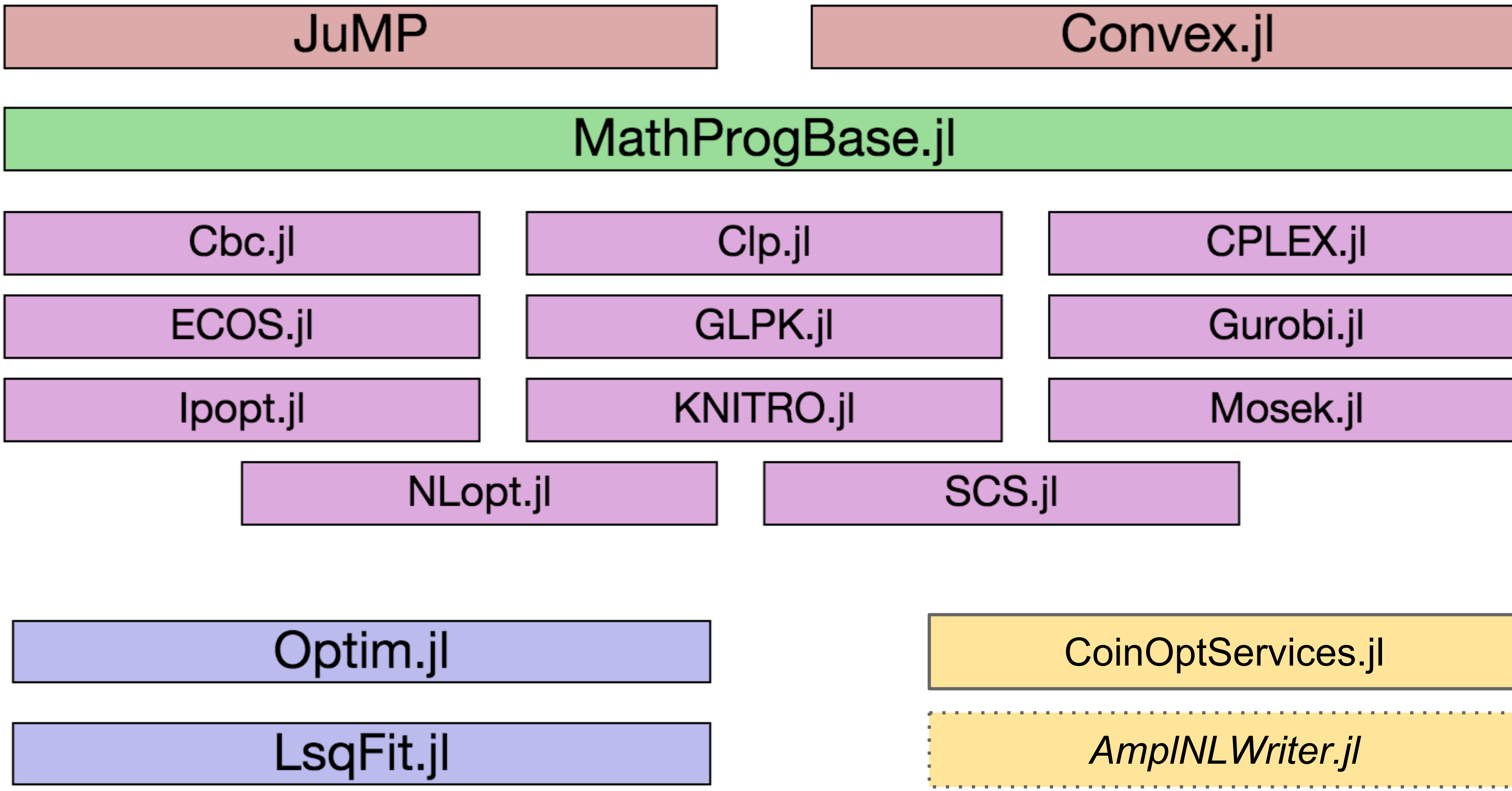
What is JuliaOpt? Packages for...

- **Modeling**: express optimization problems with in Julia code
- **Solving**: pure Julia routines, and wrappers for external solvers
- **Abstracting**: the “glue” between modeling, solving, and user code

JuliaOpt as an Organization

- Standards for packages: binaries, documentation, tests, integration
- A centralized guide to optimization in Julia http://www.juliaopt.org/
- Examples, documentation, guides
  - http://www.juliaopt.org/notebooks/index.html
  - https://github.com/JuliaOpt/juliaopt-notebooks

JuliaOpt Packages



Modeling with JuliaOpt - Two Paths

JuMP

- General linear, quadratic, nonlinear, integer optimization tool
- Callbacks for advanced integer programming
- Automatic generation of first- and second-order derivatives

Convex.jl

- **D**isciplined **C**onvex **P**rogramming
- Linear, second-order, semidefinite, exponential conic optimization
- Automatic validation of model convexity

JuMP - Julia for Math. Programming

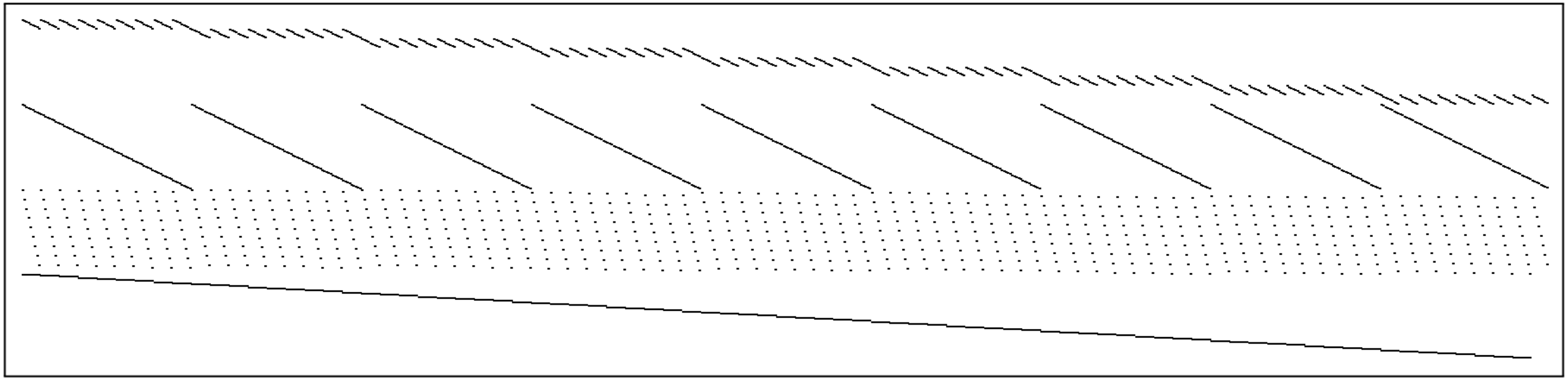
$$
\begin{aligned}
\min_{\mathbf{u},\mathbf{y}} \quad & \frac{1}{4}\Delta_x \left(\left(y_{m,0} - y_0^t\right)^2 + 2\sum_{j=1}^{n-1} \left(y_{m,j} - y_j^t\right)^2 + \left(y_{m,n} - y_n^t\right)^2 \right) + \\
& \qquad \frac{1}{4} a \Delta_t \left(2 \sum_{i=1}^{m-1} u_i^2 + u_m^2 \right) \\
\text{s.t.} \quad & {}^1\!/\!_{\Delta_t} \left(y_{i+1,j} - y_{i,j}\right) = \\
& \qquad \frac{1}{2h_2} \left(y_{i,j-1} - 2y_{i,j} + y_{i,j+1} + y_{i+1,j-1} - 2y_{i+1,j} + y_{i+1,j+1}\right) && \forall i \in I', j \in J' \\
& y_{0,j} = 0 && \forall j \in J \\
& y_{i,2} - 4y_{i,1} + 3y_{i,0} = 0 && \forall i \in I \\
& {}^1\!/\!_{2\Delta_x} \left(y_{i,n-2} - 4y_{i,n-1} + 3y_{i,n}\right) = u_i - y_{i,n} && \forall i \in I \\
& -1 \le u_i \le 1 && \forall i \in I \\
& 0 \le y_{i,j} \le 1 && \forall i \in I, j \in J
\end{aligned}
$$

$$
\begin{aligned}
\max_{x_{ijk} \in \{0,1\}} \quad & 0 \\
\text{subject to} \quad & \sum_{i} x_{ijk} = 1 \qquad \forall j, k \\
& \sum_{j} x_{ijk} = 1 \qquad \forall i, k \\
& \sum_{k} x_{ijk} = 1 \qquad \forall i, j \\
& \sum_{i=a}^{a+2} \sum_{j=b}^{b+2} x_{ijk} = 1 \qquad \forall a, b \in \{1, 4, 7\}, k
\end{aligned}
$$

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

$$\max_{x \geq 0} \quad c^T x$$
$$\text{subject to} \quad Ax = b$$

$$=$$



$$\sum_{i} x_{ijk} = 1 \qquad \forall j, k$$

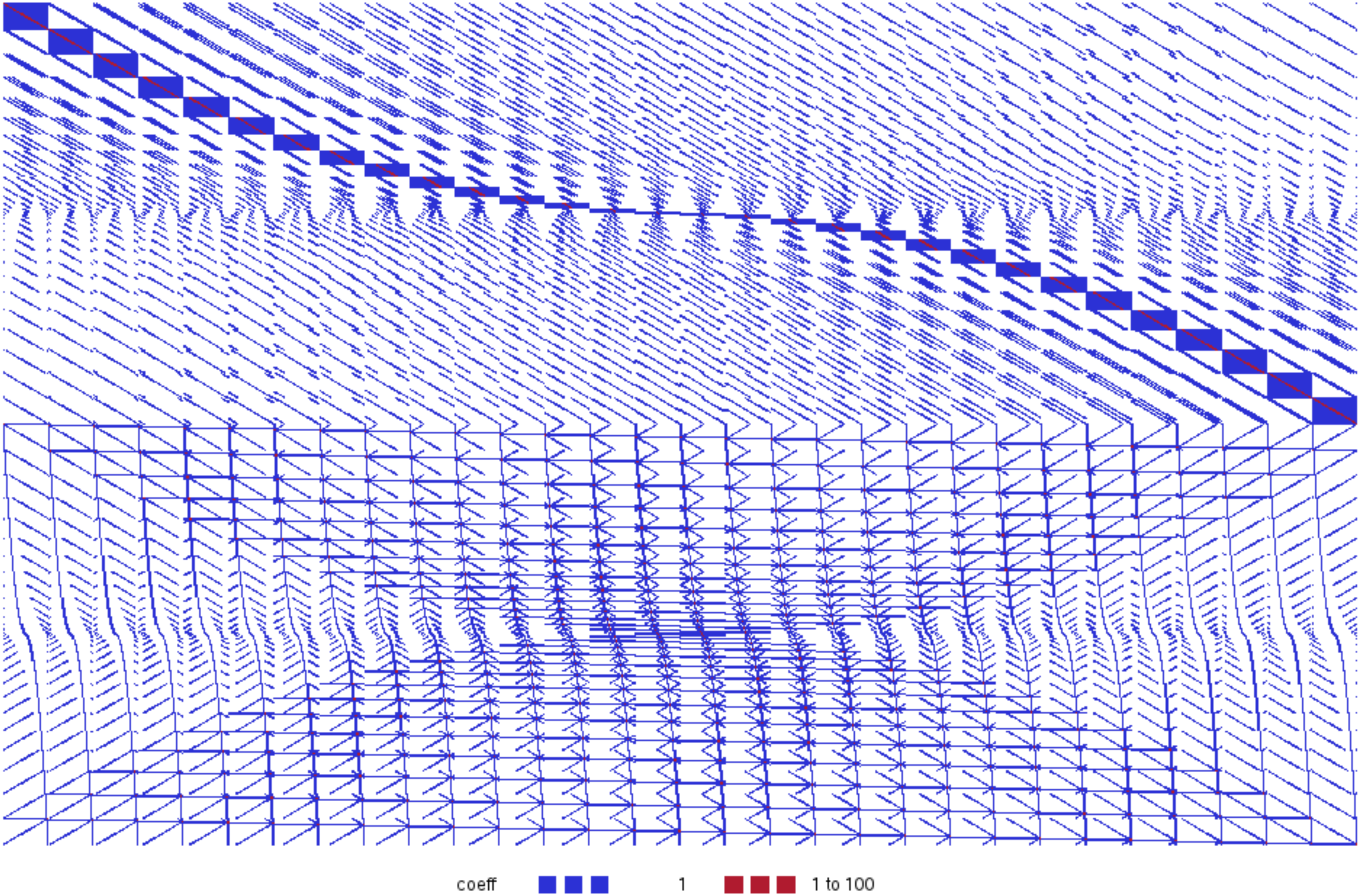
$$\sum_{j} x_{ijk} = 1 \qquad \forall i, k$$

$$\sum_{k} x_{ijk} = 1 \qquad \forall i, j$$

$$\sum_{i=a}^{a+2} \sum_{j=b}^{b+2} x_{ijk} = 1$$

$$\forall a, b \in \{1, 4, 7\}, k$$

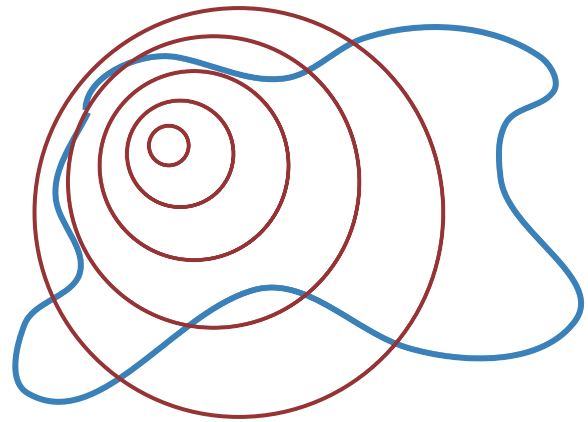
```
m = Model()

@defVar(m, x[i=1:9, j=1:9, v=1:9], Bin)

@addConstraint(m, cols[j=1:9,v=1:9],
    sum{x[i,j,v], i=1:9} == 1)

@addConstraint(m, rows[i=1:9,v=1:9],
    sum{x[i,j,v], j=1:9} == 1)

@addConstraint(m, digits[i=1:9,j=1:9],
    sum{x[i,j,v], v=1:9} == 1)

@addConstraint(m,
    cells[a=1:3:7,b=1:3:7,v=1:9],
    sum{x[i,j,v], i=a:a+2, j=b:b+2} == 1)

for i in 1:9, j in 1:9
    @addConstraint(m, x[i,j,S[i,j]] == 1)
end

solve(m)
```



http://blogs.sas.com/content/operations/2015/06/25/finding-the-beauty-in-optimization-models-visualizing-mps-data-sets/

JuMP for nonlinear optimization

$$\min_{x} \quad f(x)$$
$$\text{subject to} \quad g_i(x) \le 0 \quad \forall i$$



Solvers need *derivative-evaluating* functions

$$f(x),\ g_i(x)$$

$$\nabla f(x),\ \nabla g_i(x)$$

$$\nabla^2 f(x) + \lambda \Sigma_i \nabla^2 g_i(x)$$

JuMP + Automatic Differentiation

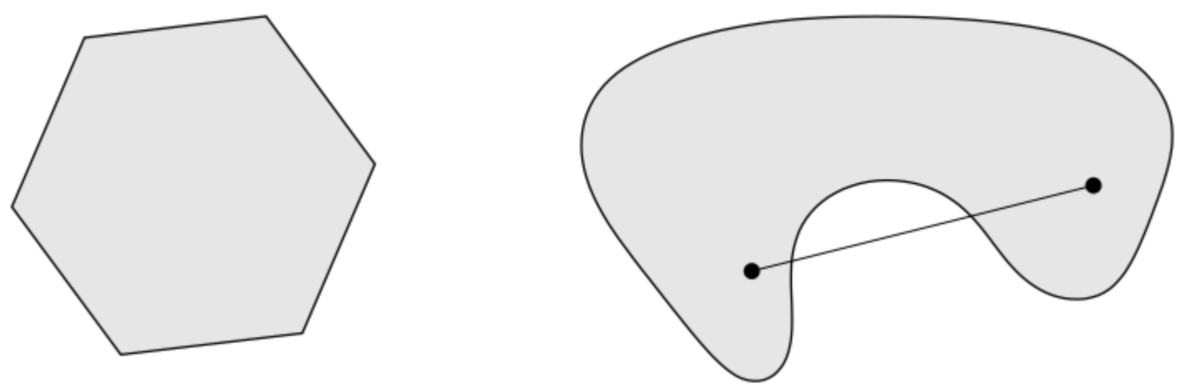
```
@setNLObjective(mod, Min, sum{ (bus[k].p_load +
  sum{ bus_voltage[k] * bus_voltage[branch[i].from] *
      (Gin[i] * cos(bus_angle[k] - bus_angle[branch[i].from]) +
       Bin[i] * sin(bus_angle[k] - bus_angle[branch[i].from])),
         i=in_lines[k] } +

  sum{ bus_voltage[k] * bus_voltage[branch[i].to] *
      (Gout[i] * cos(bus_angle[k] - bus_angle[branch[i].to]) +
       Bout[i] * sin(bus_angle[k] - bus_angle[branch[i].to])),
         i=out_lines[k] } +

  bus_voltage[k]^2*Gself[k] )^2,
   k in 1:nbus;
   bus[k].bustype == 2 || bus[k].bustype == 3})
```

$$\sum_k \left[g_k + \sum_m V_k V_m (G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)) \right]^2$$

Convex.jl - DCP in Julia

“DCP is a system for constructing mathematical expressions with known curvature from a given library of base functions.”

→ Specify your optimization problem & Convex.jl will analyze the convexity and reduce to a standard form.



http://stanford.edu/~boyd/cvxbook/bv_cvxslides.pdf

Max Volume Inscribed Ellipsoid

“Given **polyhedron** $C = \{x \mid a_i^T x \leq b_i,\ i = 1, \ldots, m\}$

find **ellipsoid** $\mathcal{E} = \{Bu + d \mid \|u\|_2 \leq 1\}$

that lies in the **interior of *C*** with maximum volume”

$$\begin{array}{ll} \text{maximize} & \log\det B \\ \text{subject to} & \sup_{\|u\|_2 \leq 1} I_C(Bu + d) \leq 0 \end{array}$$

Max Volume Inscribed Ellipsoid

$$\begin{array}{ll} \text{maximize} & \log\det B \\ \text{subject to} & \|Ba_i\|_2 + a_i^T d \le b_i, \quad i = 1, \ldots, m. \end{array}$$

Is that objective concave?

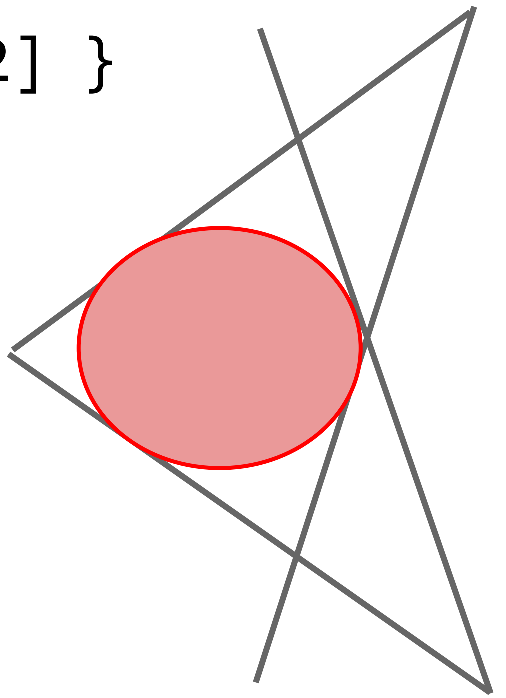
→ B is positive definite matrix...

→ det(B) = product of eigenvalues of B = +ve...

→ log of positive *x* = concave

Max Volume Inscribed Ellipsoid

```
using Convex

a = { [ 2, 1], [ 2,-1], [-1, 2], [-1,-2] }
B = Variable(2,2)
d = Variable(2)
p = maximize(logdet(B))
for i in 1:4
  p.constraints += norm(B*a[i]) +
                      dot(a[i],d) <= 1
end
solve!(p)
println(B.value)
println(d.value)
```



Which do I use?

- Convex but transformation not obvious or is painful? Nonlinear but structured (e.g. GP, exponential cones) → **Convex.jl**

- Complex indexing (3+ dimensions, not 1:n)? Nonlinear, nonconvex? Large scale linear/quadratic, solver callbacks? → **JuMP**

Solving your problems

Cbc.jl
Clp.jl
CPLEX.jl
ECOS.jl
GLPK.jl
Gurobi.jl
Ipopt.jl
KNITRO.jl
Mosek.jl
NLopt.jl
SCS.jl

Optim.jl
LsqFit.jl

CoinOptServices.jl
AmplNLWriter.jl

MathProgBase.jl

- Standard interface for optimization Julia
- Crucial to the success of JuliaOpt

Linear Programming	Mixed-integer Programming	Quadratic Programming
Conic Programming	MIP Callbacks	Nonlinear Programming
		Semidefinite Programming

MathProgBase.jl Design & Benefits

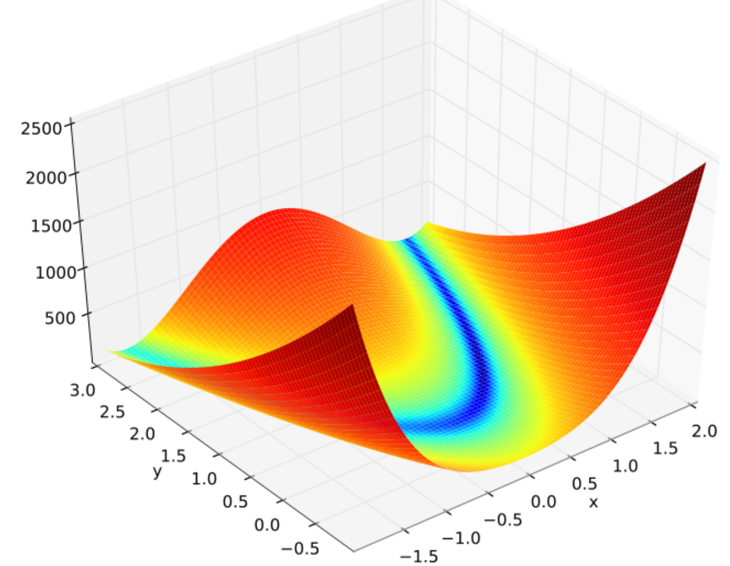
- “Don’t try to create interface/abstraction unless you have two or more cases”
  - Callbacks: “many states, one callback” vs “many states, many callbacks”
  - SDP interface
- Multiplier effect of participation
  - JuMP initial consumer, now also Convex.jl
  - Each added solver benefits all

JuliaOpt + MPB for new solvers

1. If making new pure Julia solver, can get problems to your solver easily
2. Compose solvers for new problem classes
    - e.g. “mixed-integer non-convex quadratically constrained optimization”
    - Solve as series of mixed-integer linear problems
    - JuMP → MathProgBase →

      MySolver → MathProgBase → AnyMILPSolver

Optim.jl

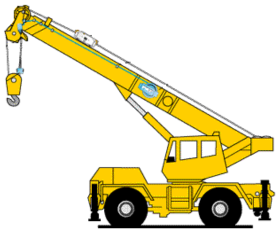


- Pure Julia routines for unconstrained and box-constrained optimization problems
- Line searches, BFGS, Levenberg-Marquadt, Nelder-Mead, etc.
- Can provide function and derivatives, or…
- Ask for derivatives to be provided by autodifferentiation (DualNumbers.jl)

https://en.wikipedia.org/wiki/Rosenbrock_function#/media/File:Rosenbrock_function.svg

```julia
function rosenbrock100(x::Vector)
  out = zero(eltype(x))
  for i in 1:div(length(x),2)
    out += 100*(x[2i-1]^2 - x[2i])^2 + (x[2i-1]-1)^2
  end
  out
end

@time optimize(rosenbrock100, zeros(100),
      method = :l_bfgs, iterations=21)
# elapsed time: 0.003834211 seconds
# Value of Function at Minimum: 3.419262

@time optimize(rosenbrock100, zeros(100),
      method = :l_bfgs, iterations=21, autodiff=true)
# elapsed time: 0.002318992 seconds
# Value of Function at Minimum: 0.000000
```

Building on JuliaOpt



- Packages using Optim.jl directly
  - KernelDensity.jl, KernelEstimator.jl, GaussianProcesses.jl, MachineLearning.jl, NLsolve.jl, QuantEcon.jl, RegERMs.jl, StochasticSearch.jl, TimeModels.jl, TrafficAssignment.jl
- Packages using JuMP (optionally)
  - Gadfly/Compose.jl, GraphLayout.jl

JuMP Extensions

- JuMPeR - Robust Optimization https://github.com/IainNZ/JuMPeR.jl
- JuMPChance - Chance Constraints https://github.com/mlubin/JuMPChance.jl
- StochJuMP - Stochastic Optimization https://github.com/joehuchette/StochJuMP.jl

Education, Academia & Industry

- JuliaOpt been used at 5+ universities around the world
- Many papers starting to appear using JuliaOpt
  - Vielma, J, et al. "Extended Formulations in Mixed Integer Conic Quadratic Programming"
  - Gorhan, Mackey. “Measuring Sample Quality with Stein’s Method”
  - Giordano, Broderick, Jordan. “Linear Response Methods for Accurate Covariance Estimates from Mean Field Variational Bayes”
- Several companies, incl. https://www.staffjoy.com/
- See http://juliaopt.org for latest info, email us!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Blake Johnson


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João Felipe Santos

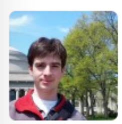

joehuchette
Joey Huchette

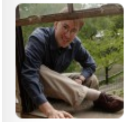

johnmyleswhite
John Myles White

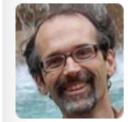

karanveerm
Karanveer Mohan


lindahua
Dahua Lin

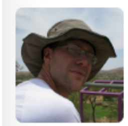

madeleineudell
Madeleine Udell

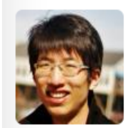

mlubin
Miles Lubin


stevengj
Steven G. Johnson


timholy
Tim Holy


tkelman
Tony Kelman


ulfworsoe
Ulf Worsøe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