Analyzing “Are You the One?”

Iain Dunning

February 28, 2022

1 Introduction

“Are You the One?” is an MTV dating game show[1]. There are twenty contestents, ten men and ten women, and they must each identify their “perfect match” (PM) before the show ends in order to collectively win a cash prize of one million dollars. The PM for each pair is decided by the producers of the show, and only limited information about who is and isn’t a match is provided to the contestents. To win the prize money they must use logical reasoning as well as follow their romantic intuition. This paper focusses on exploring the former aspect by showing how the information revealed at each stage in the game constrains the remaining possibilities, and what actions the contestents can take to maximize their chances of winning. We apply this to season four of the show, to see how this approach could work in practice.

The structure of this paper is as follows. First, in Section 2 we will describe the game more formally. In Section 3 we will provide some basic information about the “scale” of the challenge. In Section 4 we will show how one can determine the best couple to send to the “truth booth”. In Section 5 we similarly show how to pick the best couples to suggest in the “matching ceremony”. In Section 6 we will apply this all to season four of the show. Finally, we offer some concluding remarks and thoughts on future work in Section 7.

[1] `https://en.wikipedia.org/wiki/Are_You_the_One%3F`

2 Game Rules

There are ten men and ten women in the game. Each man has a woman who is his producer-decided "perfect match" (PM), and the PM for the woman is the same man – that is, it is symmetric[2]. At the start of the game no other information is provided, apart from anything that can be informally inferred by the participants own natural attraction to each other[3]. The game has ten rounds, and contestents can take two actions every round:

- **Challenge and "truth booth" (TB)** Each round the contestents partake in a physical challenge, normally with only men or only women actively competeting. The result of a typical challenge is that two or three couples get to go on a special date. If the contestents collude, they can increase the likelihood that certain couples will go on dates. The winners in some challenges can sometimes not only pick their own date, but also the other couples to go on the dates. The 14 to 16 contestents who do not go on the date get to pick one of the couples who did to go into the "truth booth" (TB). The TB is an oracle – it will reveal if the couple are a PM or not[4]. If they are, the couple is effectively removed from the game.

- **"Matching ceremony" (MC)** After the TB result is revealed, everyone remaining will attend a "matching ceremony" (MC). In alternating rounds, either the men will each pick a woman to match with, or vice versa. The selections are made one-by-one, which has no impact if the contestents pre-agree on who to pick but can cause issues otherwise. After everyone is matched, the number of PMs is revealed by the number of spotlights illuminated by the show producers. Critically, which contestents are PMs is not revealed. To win the game, contestents need all ten lights illuminated, and the prize pool is reduced if there are no

[2]There is a season of the show that allows for non-heterosexual PMs, but we will not cover this variant in this paper; generalizing to that case is left as an interesting exercise for the reader.

[3]Even a casual viewing of the show will indicate that the initial matches made on the show are very unlikely to PMs, and are mainly based on purely physical attraction even if many contestents will not acknowledge this until later weeks.

[4]Contestents typically consider this information much more helpful than the matching ceremony, but as we will see, depending on the choice it may not reduced the possibilities much.

lights corresponding to PMs other than those already revealed as PMs by the TB.

Collusion and strategic play is seemingly not actively discouraged by the show producers. Furthermore, the host of the show will often query the poor choices made by contestents, rather than adversarially try to encourage bad matches. As we will discuss in the following sections, these two actions per round are sufficient for the contestents to win.

3 Scale of the Challenge

It is obvious that there are a great many possible sets of matches, which we call *pairings*, of the contestents. Formally, a pairing is a set of ten matches such that every man and every woman are in exactly one match. The branch of mathematics that concerns itself with such topics is combinatorics[5] but we only need some of the basics of the field to reason about this game.

First, and most importantly, how many possible pairings are there? To calculate this, consider lining up all the men and all the women. For the first man in line, there are clearly ten possible choices. If we were to match the man to any of these choices, there would be nine choices left for the next man in line. If there are ten ways to make the first choice, then nine ways for the second, and eight ways for the third, we count the number of pairings using multiplication:

$$\begin{aligned}\text{Number of pairings} &= 10 \times 9 \times 8 \cdots \times 2 \times 1 \\ &= 10! \\ &= 3,628,800\end{aligned}$$

The notation 10! is read as "ten factorial", and frequently appears when counting permutations of sets. Another derivation for why 10! is the right number that uses permutations explicitly is as follows: consider lining up all the women first in some arbitrary order. We then line up all the men in some arbitrary order, and match the first person in each line together, the second person in each line together, and so on. The number of possible pairings then is the same number of ways to shuffle, or permute, the line of men, as each possible permutation creates a unique pairing.

[5] `https://en.wikipedia.org/wiki/Combinatorics`

This many pairings may seem daunting, but the contestents have two powerful tools (TB and MC) to reduce the number of possibilities each round. Here we introduce the idea of a "remaining" pairing: one of the 10! possible pairings that is consistent with all the evidence revealed so far. If we have only one one remaining pairing, we have won the game. Imagine if the contestents could halve the number of remaining pairings at each round. If they achieved that, it would take about 22 rounds – too long for the show. However if the contestents could reduce the number of pairings by a factor of ten each round, they'd finish around round seven.

To see how realistic that is, first assume we have no information available to us, and just consider TBs. If we put a couple into the TB, at random, what effect does that have in practice? There is a one-in-ten chance that the couple are a PM, and a nine-in-ten chance they are not[6]. If they are a PM, there is a ten-fold reduction in the number of remaining pairings to $9! = 362,880$ – it is like a new game with only nine men and nine woman. However, if they are not a PM, it is a bit more complicated. From the negative result from the TB, we know that any pairing that contains this couple can't happen. Returning to our original calculation, this is equivalent to saying the first man in line now has nine choices instead of the original ten. The second man in line, however, still has nine choices, as before, and so on down the line.

$$\begin{aligned}
\text{Number of pairings} &= 9 \times 9 \times 8 \cdots \times 2 \times 1 \\
&= 9 \times 9! \\
&= 3,265,920
\end{aligned}$$

This is a far less substantial reduction in the remaining possibilities than a PM would give us. We can calculate how much, on average, the first TB reduces the number of pairings:

$$\begin{aligned}
\text{Average reduction} &= \frac{1}{10}\frac{9!}{10!} + \frac{9}{10}\frac{9 \times 9!}{10!} \\
&= \frac{1}{10}\frac{9!}{10 \times 9!} + \frac{9}{10}\frac{9 \times 9!}{10 \times 9!} \\
&= \frac{1}{10}\frac{1}{10} + \frac{9}{10}\frac{9}{10} \\
&= \frac{82}{100}
\end{aligned}$$

[6]If it isn't clear why, consider the simpler game of one woman, ten men. There is clearly a one-in-ten chance for any given man to be her match.

At that rate, it would take far too long to win the game. Contestents in general seem to overestimate the power of the TB to add clarity because they fail to appreciate how little power a negative result can have if their is no prior information – but, as we will see later, they also overestimate how much power a positive result can have if we are already fairly sure a couple is a PM.

A key source of that prior information is the "matching ceremony" (MC). The ideal situation is that at the MC, the contestents will put forward a pairing that has ten PMs. However, this is of course very difficult to do early in the game. Even though the result of the MC doesn't reveal which matches are correct, it can reveal a signficant amount of information that when accumulated over multiple rounds radically reduces the number of possible pairings. Consider the most simple case, where we have no information and no TB results - we simply put forward a random pairing at the first MC. The reduction in the number of remaining pairings is determined by the number of lights.

First of all, if we get ten lights we are done. We cannot get nine lights, as that would be a contradiction – the remaining man and woman must be a PM[7]. If we get eight lights, we have two remaining men and two remaining woman, which would seem to mean $2! = 2$ possible pairings. However, observe that one of those two possible pairings is the one we just tried, and we know it wasn't right - therefore, there is actually only one possible pairing of the remaining men and woman. But, how many ways could we get eight lights from ten couples in the first place? Another equivalent way to phrase that is, how many *combinations* are there of eight of the ten couples? There is a standard formula for this [8] of the form

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

We can think of $n!$ as "number of ways to pick the n couples", the $k!$ as the "number of ways pick the k revealed by the MC", and the $(n-k)!$ as the "number of ways to pick the $n-k$ not revealed". Applying that to the eight light case, we get $\binom{10}{8} = \frac{10!}{8! \times 2!} = \frac{10 \times 9}{2} = 45$ ways to pick the eight couples. There is only one way to arrange the remainder, so 45 possible pairings left.

[7] On the show, they move slowly to illuminate the lights to build suspense, but after eight lights they quickly move from nine to ten

[8] https://en.wikipedia.org/wiki/Combination

The problem becomes more difficult for seven lights and lower. We know there a $\binom{10}{7} = 120$ ways to pick seven of the ten couples. But how many ways to arrange the remaining contestents such that none of the pairings has a couple that corresponds to the three lights that didn't come on? One way to think about this is to pick an arbitrary seven of the MC couples, and leave aside the other three MC couples. Line up the three men in one line, and the three women in another line, so that the couples are side be side. Our question is equivalant to: "how many ways can we reorder the men in the line such that no man is in their starting position?" This is because we know that all the starting positions corresponds to non-PMs, but any other arrangement is a possibility. These arrangements are known as *derangements* and the number of them is known as the *subfactorial*, sometimes denoted as $!n$[9]. The formula is a bit more complicated than the formula for combinations, so we won't reproduce it here. It turns out that !3 is 2, so given an arbitrary selection of seven of the ten couples are the PMs, we have just two ways to arrange the remainder. We can check this by hand: assume M_1 was matched to W_1, M_2 to W_2, M_3 to W_3. We know any remaining pairing must have M_1 matched to W_2 or W_3. If M_1 is matched to W_2, that means M_2 must be matched to W_3 because if M_2 matched to W_1, M_3 would be forced to match with W_3 which we know is not a PM. That is one of the two pairings, and the other pairing can be derived by matching M_1 to W_3. Given we have 120 way to pick the seven, we have a 240 possible pairings after getting seven lights.

We can now calculate how many pairings are remaining after each number

[9] `https://en.wikipedia.org/wiki/Derangement`

of lights we might receive in the first MC:

$$
\begin{aligned}
0 \text{ lights} &= !10 = 1,334,961 \\
1 \text{ lights} &= \binom{10}{1} \times !9 = 10 \times 133,496 = 1,334,960 \\
2 \text{ lights} &= \binom{10}{2} \times !8 = 45 \times 14,833 = 667,485 \\
3 \text{ lights} &= \binom{10}{3} \times !7 = 120 \times 1,854 = 222,480 \\
4 \text{ lights} &= \binom{10}{4} \times !6 = 210 \times 265 = 55,650 \\
5 \text{ lights} &= \binom{10}{5} \times !5 = 252 \times 44 = 11,088 \\
6 \text{ lights} &= \binom{10}{6} \times !4 = 210 \times 9 = 1,890 \\
7 \text{ lights} &= \binom{10}{7} \times !3 = 120 \times 2 = 240 \\
8 \text{ lights} &= \binom{10}{8} \times !2 = 45 \times 1 = 45 \\
9 \text{ lights} &= 1 \text{ remaining pairing} \\
10 \text{ lights} &= 1 \text{ remaining pairing}
\end{aligned}
$$

The above gives some intuition for how much the number of possible pairings can be decreased in just a single round. Even getting no lights in round one reduces the number of possibilites by a factor of three. In round one of season four, the first TB had a negative result – reducing the number of possible pairings slightly from 3,628,800 to 3,265,920. More positively, they received three lights at the first MC. Ignoring the TB result, we know that three lights reduced the possible pairings by more than a factor of ten, from 3,628,800 to 222,480, and at that pace they would finish by round seven. By combining these results, and over multiple rounds, we can make even stronger inferences, although doing so precisely by hand gets more and more complicated. We will instead use an enumerative approach – that is, we will use a computer to generate all 3.6 million possible pairings, and then eliminate all pairings that are inconsistent with the information revealed so far. Using a computer, we can determine that the combined effect of the first TB and the first MC reduced the possibilities from 3,628,800 to 205,176 – a good start on the road to getting down to one remaining pairing.

4 Playing the Truth Booth

There are two decisions to be made each round. One is which couples to pick for the MC, and one is which couple to test in the TB. In practice, the contestents might not have full discretion. For the MC, it is not uncommon to have one or more contestents unwilling to collude and instead acting solely on their own belief in who their PM is. For the TB, only couples that go on dates can be tested, and the couples that go on dates can only be somewhat controlled. In season four, the contestents were somewhat effective at controlling who to go into the TB, but did not necessarily make optimal choices. In this section we will discuss what an optimal choice is, at least in a by-the-numbers approach. We will discuss other considerations at the end of the section.

One key assumption we will make is that we do not assign any probability of a given couple as being more likely to be a PM than any other unless the results so far suggest it. Sometimes we might be able to make better choices if we can increase or decrease these probabilities based on other information, but in practice contestents often don't demonstrate any particular ability to pick their own PM let alone the PMs for others, especially in early rounds.

One natural strategy that contestents often try is to test the couple that is "most likely" to be a PM. For our purposes we can define "most likely" as the couple that appears the most in all remaining possible pairings. For example, in season four round one we learn that Prosper and Tori are not a PM from the TB, and then learn there are three PMs among the couples at the first MC. All the couples in that first MC are instantly more likely than any arbitrary other couple, and we can also say that the two couples that involved Prosper and Tori are the "most likely" – simply because for those two contestents there are slightly fewer possibilities remaining. In this case, sending Prosper-Camille or Tyler-Tori to the TB would follow the "most likely" strategy.

The problem with this strategy, especially later in the game, is related to the point explored in the previous section: if we know nothing else and the TB reveals a PM then we reduce the number of remaining pairings a lot, but if it doesn't reveal a PM we don't learn much. More generally, if a couple is very likely to a PM (appears in a large fraction of the remaining possible pairings), and they are then revealed to a PM, we learn little. It is also true the other way round: if a couple are very unlikely to be a PM, and are revealed to not be, we also learn little.

An alternative strategy then is to test the couple that will *reveal the most information*, regardless of the outcome. In other words, we want to test the couple that will reduce the number of possible pairings as much as possible in all scenarios. We can define a quantity for each possible couple as follows:

> Let C be a couple we are considering testing. Let N be the number of remaining pairings. Let N_C be the number of those pairings where C is a PM. The information gain for testing a couple C is $I_C = \min(N_C, N - N_C)$.

To see how this works in practice, consider a scenario where in round five there are 1,000 remaining pairings ($N = 1000$). Suppose Couple A appears in 900 of those pairings ($N_A = 900$), and Couple B appear in 500 ($N_B = 500$). Then the information gain for testing Couple A is $I_A = \min(900, 1000 - 900) = 100$, and the information gain for testing Couple B is $I_B = \min(500, 1000 - 500) = 500$. We say that the information gain for Couple B is more because either way we halve the remaining possibilities, whereas Couple A may only reduce the possibilities by a tenth.[10]

The contestents on season four had a tendency to not follow this strategy, nor even necessarily picking the most likely couple. Instead, they sometimes employed an alternative strategy that emphasized couples who were considered by at least one member of the proposed couple to be a guaranteed PM, but that the other contestents considered very unlikely to be one. By proving beyond doubt that these "sure-thing" couples are not in fact PMs, the contestents in the couple are forced to move on and try to find their true PM rather than stay in an incorrect couple. It is possible that this is a better strategy in the long run, as it also allows for more diverse pairings to be tested in MCs.

[10] Note we could also make an argument in expectation rather than a worst-case analysis. As an exercise for the reader: does it make a difference?

5 Playing the Matching Ceremony

Playing the MC is similar to playing the TB, except with a more complicated decision to make. The ideal play is the one that reduces the possibilities remaining the most – in fact, ideally to just one by selecting the ten-PM pairing. Calculating what such a possibility-reducing pairing might be is tricky because unlike the binary outcome of the TB, we can get any number of lights between zero and ten for a given MC lineup. This quickly gets quite complicated and extremely unrealistic for the players to do, and is more interesting in the context of a specific game.

We will instead analyze whether we need to worry about strategy much at all. We do this by analyzing a thousand games where we assume the contestents play a purely random strategy. That is, at every MC, they suggest ten couples completely independent of any other information – there is no opinion, there are not even any TB results, nor use of previous MC results. We select one pairing as the true pairing (M_1 with W_1, M_2 with W_2, and so on) and "report" the number of lights each randomly selected pairing obtains. After each round, we record the number of possible pairings left, determined by eliminating any pairing that is inconsistent with that all MCs so far. For example, from above we know that the worst-case scenario is no lights in round one, which would leave us with 1,334,961 pairings. The best case scenario is we get the correct answer in round one, ten lights, but also that there is one-in-three-million chance of this happening – unlikely to happen in only a thousand games, although of course possible. Given there is a range of pairings left after each round, we report six different numbers:

- The number of games that are effectively "won" because there is only one pairing left.
- The minimum, across all simulated games, of the number of pairings left after each round.
- The maximum, like the minimum.
- The tenth percentile across all simulated games. This means we take the number of pairings left in all thousand games at a given round, sort the list, and take the hundredth entry.
- The ninetieth percentile across all simulated games. This means taking the nine-hundredth entry in that sorted list.

- The median, or fiftieth percentile. This means taking the five-hundredth entry.

Below is a table of these results:

Round	Won?	Min	10^{th}	Median	90^{th}	Max
Round 1	0	11088	667485	1334960	1334961	1334961
Round 2	0	1180	81493	439792	493992	671904
Round 3	0	128	13843	92271	182242	328100
Round 4	0	10	3077	32416	67744	125534
Round 5	0	5	723	7396	24798	47276
Round 6	2	1	166	2126	8538	20339
Round 7	7	1	43	636	2829	7966
Round 8	12	1	15	184	890	3478
Round 9	35	1	4	54	292	2543
Round 10	103	1	1	16	91	633

The median outcome after a full ten rounds is to have only 16 pairings left, or 54 pairings at the end of round nine, just from random MC pairings. If we also have nine rounds of TB results at that point, picked with any care at all, it seems that we would be quite likely to be able to narrow it down to one. It is worth considering the extremes though: in at least one of the thousand games, we still had 633 pairings left at the end of round ten. On the other hand, after round nine we have 35 of the 1000 games with only one pairing left, and ten percent of games have four pairings or less left. Note that having only one pairing left doesn't mean we guessed the true pairing – instead, we just inferred enough information to not have to guess it.

One takeaway from this is that the games organizers can say that the contestents are quite unlikely to win by playing truly randomly. On the other hand, with just a bit of smartness we can say that they very likely can win if they do anything better. For example, what if we do something slightly smarter than above by picking a random MC pairing each time from the set of remaining pairings? For the first round this is identical to above (as all pairings are possible) but begins to diverge meaningfully as the rounds go on. Consider a simulated game above where we have only eight possible pairings left at round seven. With the first approach, we would be quite likely to try MC pairing that is impossible but now we will have a one-in-eight chance of getting ten lights. We would thus expect to do much better, and in fact we do:

Round	Won?	Min	10^{th}	Median	90^{th}	Max
Round 1	0	11088	667485	1334960	1334961	1334961
Round 2	0	3080	81992	439792	493902	493992
Round 3	0	47	13950	88875	178876	184541
Round 4	0	30	2456	22284	64567	67574
Round 5	0	3	338	4992	20457	24432
Round 6	5	1	57	1026	4890	8884
Round 7	24	1	9	215	1436	3443
Round 8	103	1	1	41	346	1164
Round 9	279	1	1	8	81	424
Round 10	527	1	1	1	18	168

In contrast to only 35 of the 1000 games having only one remaining pairing at the end of round nine, we now have that result in one-quarter of games, meaning guaranteed wins in or before round 10 MC. After round ten, the majority of games have only one remaining pairing. The worst-cases are no longer so bad – for example, our maximum remaining pairings at round ten before was 633, but is now only 168. This demonstrates that if we are able to apply just a bit of logic, the MC is a very powerful tool.

6 Season Four

Season four starred:

- **Women**: Camille, Emma, Kaylen, Francesca, Victoria, Alyssa, Mikala, Julia, Nicole, Tori
- **Men**: Prosper, John, Giovanni, Asaf, Cam, Sam, Cameron, Morgan, Stephen, Tyler

We start the game with 3,628,800 possible pairings. We will use $\binom{\text{Man}}{\text{Woman}}$✓ to denote a couple that is also a PM (even if the contestents don't yet know it), and $\binom{\text{Man}}{\text{Woman}}$✗ for a couple that is not.

Round One

In this first round there is little information to go on, and the couples who went on the first date and thus were eligible for the TB were based mainly on first phyiscal impressions. On this basis, $\binom{\text{Prosper}}{\text{Tori}}$✗ went into the TB –

they were not a PM, and so the number of possible pairings was reduced only slightly from 3,628,800 to 3,265,920.

At the MC there were three lights for the following pairing: $\binom{\text{Prosper}}{\text{Camille}}$✗, $\binom{\text{John}}{\text{Emma}}$✗, $\binom{\text{Giovanni}}{\text{Kaylen}}$✗, $\binom{\text{Asaf}}{\text{Francesca}}$✗, $\binom{\text{Cam}}{\text{Victoria}}$✗, $\binom{\text{Sam}}{\text{Alyssa}}$✓, $\binom{\text{Cameron}}{\text{Mikala}}$✓, $\binom{\text{Morgan}}{\text{Julia}}$✗, $\binom{\text{Stephen}}{\text{Nicole}}$✓, $\binom{\text{Tyler}}{\text{Tori}}$✗. That was a substantial number of matches given this point in the game, and reduced the pairings remaining by more than a factor of ten, from 3,265,920 to 205,176. Of these correct couples, $\binom{\text{Sam}}{\text{Alyssa}}$✓ and $\binom{\text{Cameron}}{\text{Mikala}}$✓ appeared to have relatively strong beliefs they were PMs. Unforunately, similarly strong beliefs were held by $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ and $\binom{\text{Asaf}}{\text{Francesca}}$✗.

Round Two

John was convinced that Julia was his perfect match, at least partly because they grew up in the same state. The others (and Julia) were not convinced, so to try to dissuade John of the two of them were sent to the TB. As they were not in the first MC, they had a lower chance of being a PM than some other choices (8% vs 33% for $\binom{\text{Prosper}}{\text{Camille}}$✗ and $\binom{\text{Tyler}}{\text{Tori}}$✗). The best choice for information gain would have been in this case the most likely couples – anyone in the first MC, but especially the two couples just mentioned who would reduce the count to 66,744. These two are more likely because they involve a TB participant from round one. Despite this, convincing a couple or even individual that they are a PM may overall be a better strategy if it helps a contestent "move on" and try different couplings. In this case $\binom{\text{John}}{\text{Julia}}$✗ were not a PM, so the number of remaining pairings decreased slightly from 205,176 to 189,152.

Only two couples from the first MC were coupled again in the second MC - $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ and $\binom{\text{Cameron}}{\text{Mikala}}$✓ (who all believed themselves to be PMs). $\binom{\text{Sam}}{\text{Alyssa}}$✓ would reunite for subsequent MCs, but $\binom{\text{Stephen}}{\text{Nicole}}$✓ (especially Stephen) had no idea they were a PM until near the end. Despite all this, three lights were achieved from the pairing: $\binom{\text{Prosper}}{\text{Emma}}$✓, $\binom{\text{John}}{\text{Nicole}}$✗, $\binom{\text{Giovanni}}{\text{Kaylen}}$✗, $\binom{\text{Asaf}}{\text{Camille}}$✗, $\binom{\text{Cam}}{\text{Julia}}$✓, $\binom{\text{Sam}}{\text{Francesca}}$✗, $\binom{\text{Cameron}}{\text{Mikala}}$✓, $\binom{\text{Morgan}}{\text{Alyssa}}$✗, $\binom{\text{Stephen}}{\text{Tori}}$✗, $\binom{\text{Tyler}}{\text{Victoria}}$✗. This strong result from a substantially different set of pairings reduced the pairings remaining by more than a factor of ten again, from 189,152 to 16,570. Of the correct couples, it seemed that $\binom{\text{Prosper}}{\text{Emma}}$✓ and $\binom{\text{Cam}}{\text{Julia}}$✓ didn't show much indication that they believed they were PMs, and neither would be recognized as such until near the end of the game (round ten in the latter case).

With an overall reduction already of 3,628,800 to 16,570 pairings, we can look at the "most likely" couples by counting the percentage of the 16,570 pairings remaining in which a couple is a PM:

1. $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ (60.7%) (rounds one and two)
2. $\binom{\text{Cameron}}{\text{Mikala}}$✓ (60.7%) (rounds one and two)
3. $\binom{\text{John}}{\text{Nicole}}$✗ (23.8%) (round two)
4. $\binom{\text{Prosper}}{\text{Camille}}$✗ (23.7%) (round one)
5. $\binom{\text{Stephen}}{\text{Tori}}$✗ (23.7%) (round two)
6. $\binom{\text{Cam}}{\text{Julia}}$✓ (23.6%) (round two)
7. $\binom{\text{Morgan}}{\text{Julia}}$✗ (23.6%) (round one)
8. $\binom{\text{Tyler}}{\text{Tori}}$✗ (23.6%) (round one)
9. $\binom{\text{Prosper}}{\text{Emma}}$✓ (23.0%) (round two)
10. $\binom{\text{John}}{\text{Emma}}$✗ (23.0%) (round one)

Fascinatingly, most are wrong! All couples have appeared in one of the MCs – this will change in later rounds – and the top two have appeared in both MCs. Some contestents don't appear at all (Asaf, Sam, Francesca, Victoria, Alyssa), and some appear multiple times (Julia, Emma, Tori, John, Prosper). $\binom{\text{Sam}}{\text{Alyssa}}$✓ missing is particularly notable as there was a reasonably high confidence in them by the contestents – they were not tested in round two, and are in eleventh place. At this stage, it is difficult to guess winners, but we can start making smarter choices about what to test.

Round Three

In round three the opportunity to test a couple that was strongly suspected to be a PM, $\binom{\text{Cameron}}{\text{Mikala}}$✓, was tested in the TB and was found to be true. While testing likely couples is wasteful later in the game, at this stage it was one of the best choices they could have made – only $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ would have reduced the remaining pairings as much. This is because while they seemed the most likely to the contestents, in practice the probability was fairly close to the information-revealing-ideal of 50%: 60.7%. With this uncertainty resolved, the first successful TB effectively reduces the game to a nine-couple sub-game. As discussed in Section 3, a nine-couple game already reduces the number of pairings by a factor of ten ($10! \to 9!$) but when combined with information already revealed, the effect is to reduce the number of pairings by almost a factor of two: 16,570 to 10,053. In relative terms, this was the most second most impactful TB in the game as measured by ratio of remaining pairings before the TB to remaining pairings after.

There were six repeat matches at the MC, including the revealed TB result: $\binom{\text{Prosper}}{\text{Emma}}$✓, $\binom{\text{Giovanni}}{\text{Kaylen}}$✗, $\binom{\text{Asaf}}{\text{Camille}}$✗, $\binom{\text{Sam}}{\text{Alyssa}}$✓, $\binom{\text{Cameron}}{\text{Mikala}}$✓, $\binom{\text{Stephen}}{\text{Tori}}$✗. The remaining matches were all new: $\binom{\text{John}}{\text{Victoria}}$✓, $\binom{\text{Cam}}{\text{Nicole}}$✗, $\binom{\text{Morgan}}{\text{Francesca}}$✗, $\binom{\text{Tyler}}{\text{Julia}}$✗. This gave four lights, including the one guaranteed light. The high level of overlap with previous rounds was not ideal and led to a smaller relative reduction than the first two rounds but still substantial: 10,053 to 1,269. The result had less-than-ideal psychological effects as well: there was no indication that $\binom{\text{John}}{\text{Victoria}}$✓ was considered as one of the lights, and recurring couples that were incorrect like $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ and $\binom{\text{Asaf}}{\text{Camille}}$✗ were reinforced.

The number of pairings remaining is already less than 0.04% of the original number of possibilities, after just three rounds. The top ten couples are:

1. $\binom{\text{Cameron}}{\text{Mikala}}$✓ (100%) (TB round 3)
2. $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ (61.8%) (rounds one, two, three)
3. $\binom{\text{Sam}}{\text{Alyssa}}$✓ (38.4%) (rounds one and three)
4. $\binom{\text{Stephen}}{\text{Tori}}$✗ (35.9%) (rounds two and three)
5. $\binom{\text{Prosper}}{\text{Emma}}$✓ (35.2%) (rounds two and three)
6. $\binom{\text{Asaf}}{\text{Camille}}$✗ (30.3%) (rounds two and three)
7. $\binom{\text{John}}{\text{Victoria}}$✓ (26.6%) (round three)
8. $\binom{\text{Tyler}}{\text{Julia}}$✗ (24.1%) (round three)
9. $\binom{\text{Morgan}}{\text{Francesca}}$✗ (23.9%) (round three)
10. $\binom{\text{Cam}}{\text{Nicole}}$✗ (23.8%) (round three)

All couples are still those that have appeared in MC, although some only in this most recent MC, and all men and all woman appear now in the top ten. $\binom{\text{Sam}}{\text{Alyssa}}$✓ were missing at the end of round two but now appear near the top. $\binom{\text{Prosper}}{\text{Emma}}$✓ and especially $\binom{\text{John}}{\text{Victoria}}$✓ are not properly recognized by the contestents as possible PMs.

With two contestents removed from the game, and two couples confirmed as not being PMs, it is interesting to consider if any other couples can be "proved" to be impossible. It turns out that, despite there being only 1,269 possibilities left, all couples not obviously excluded are still possible. Some are definitely less likely than others: the most unlikely is $\binom{\text{Giovanni}}{\text{Alyssa}}$✗ who appear in only 44 of 1,269 pairings (3.5%). More interestingly, $\binom{\text{Asaf}}{\text{Kaylen}}$✓ is the seventh-most unlikely (54 pairings, 4.3%) but is a PM.

Round Four

Coming into round four, by far the best choices of couple to test in the TB was either $\binom{\text{Sam}}{\text{Alyssa}}$✓ or $\binom{\text{Giovanni}}{\text{Kaylen}}$✗, with the former being very slightly better. Instead, due to set relatively poor set of candidates couples available to pick from, $\binom{\text{Asaf}}{\text{Tori}}$✗ (9.1%), was picked – mainly because of how flirtatious they had been the night before. They were not a PM, and little information was revealed overall - reducing the possibilities from 1,269 to 1,153, far less effective than the previous round. Moreover, there was not much of a prior that they were a match, so there was little psychological benefit. This was the second TB for Tori, reducing her pool of possible men to seven (as Cameron was matched to Mikala), the smallest number of possibilities for any contestent. If $\binom{\text{Sam}}{\text{Alyssa}}$✓ was chosen instead there would have been only 487 pairings remaining before the MC.

There were only five repeat matches at the MC, down from six the previous round: $\binom{\text{John}}{\text{Victoria}}$✓, $\binom{\text{Giovanni}}{\text{Kaylen}}$✗, $\binom{\text{Asaf}}{\text{Camille}}$✗, $\binom{\text{Sam}}{\text{Alyssa}}$✓, $\binom{\text{Cameron}}{\text{Mikala}}$✓. The new matches were $\binom{\text{Prosper}}{\text{Nicole}}$✗, $\binom{\text{Cam}}{\text{Emma}}$✗, $\binom{\text{Morgan}}{\text{Tori}}$✓, $\binom{\text{Stephen}}{\text{Julia}}$✗, $\binom{\text{Tyler}}{\text{Francesca}}$✗. This combination achieved four lights, although no good conclusions were drawn from this result. In particular little note was taken of $\binom{\text{John}}{\text{Victoria}}$✓ being in two successful MCs, as $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ and $\binom{\text{Asaf}}{\text{Camille}}$✗ (in Camille's eyes) were dominating the focus. One of the new couples this week, $\binom{\text{Stephen}}{\text{Julia}}$✗, would prove a serious obstacle – they had an extremely high mutual belief they were a PM, which would not be resolved until the round eight MC. This four light result was the least effective so far but still reduced the remaining pairings by a factor of five from 1,153 to 219.

We are now only have 0.006% of the initial set of pairings, and the top ten has five PMs:

1. $\binom{\text{Cameron}}{\text{Mikala}}$✓ (100%) (TB round 3)
2. $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ (66.7%) (rounds one, two, three, four)
3. $\binom{\text{Asaf}}{\text{Camille}}$✗ (53.9%) (rounds two, three, four)
4. $\binom{\text{Sam}}{\text{Alyssa}}$✓ (49.8%) (rounds one, three, four)
5. $\binom{\text{John}}{\text{Victoria}}$✓ (40.6%) (rounds three and four)
6. $\binom{\text{Prosper}}{\text{Emma}}$✓ (24.2%) (rounds two and three)
7. $\binom{\text{Stephen}}{\text{Tori}}$✗ (23.3%) (rounds two and three)
8. $\binom{\text{Morgan}}{\text{Tori}}$✓ (19.6%) (round four)
9. $\binom{\text{Stephen}}{\text{Julia}}$✗ (19.2%) (round four)
10. $\binom{\text{Prosper}}{\text{Nicole}}$✗ (18.7%) (round four)

Tori, having been in the TB twice, is in general going to appear more "densely" with the remaining men, and indeed appears in the top ten with her PM Morgan after just one round in the MC. Unforunately the same two couples who are not PMs appear high on the list, representing excellent TB choices. Because $\binom{\text{Asaf}}{\text{Kaylen}}$✓ each appear in such high-probability couples, the chances they are a PM are tiny – third-least-likely, appearing in only four of the remaining 219 pairings. For similar reasons, $\binom{\text{Giovanni}}{\text{Francesa}}$✓ appears in only seven. There are still no non-obvious zero-possibility couples at the end of round four although it is getting close: $\binom{\text{Giovanni}}{\text{Alyssa}}$✗ and $\binom{\text{Sam}}{\text{Kaylen}}$✗ appear in only two pairings.

Round Five

Coming into round five, $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ were dominating a lot of the conversation. They were the only couple other than $\binom{\text{Cameron}}{\text{Mikala}}$✓ to have been together at all MCs, and Giovanni had previously expressed extremely strong sentiments about Kaylen – including marriage and children. Despite that, by round five he had somewhat walked that back although Kaylen was still very committed. They were one of the contenders for the TB in round five and were selected, where it was revealed to all but maybe most importantly them both that they were not a PM. This was a suboptimal choice by just the numbers ($\binom{\text{Sam}}{\text{Alyssa}}$✓, $\binom{\text{Asaf}}{\text{Camille}}$✗ and even $\binom{\text{John}}{\text{Victoria}}$✓ were better), but probably the best choice for the game overall. As a couple who were in all MCs so far, they also provided some of the easiest to interpret information – one of the lights in all previous MCs was definitely not theirs. Resolving $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ reduced the possibility signficantly and was the best TB since $\binom{\text{Cameron}}{\text{Mikala}}$✓ was revealed in round three: from 219 to 73 pairings.

Even though it was suboptimal, this TB result tipped the game into a interesting new state. This is the first moment in which some couples are provably not PMs, even without confirmation by the TB. One of the easiest reasons to see why is simply that $\binom{\text{Sam}}{\text{Alyssa}}$✓ is now very likely (70 of 73 pairings). Not counting possible couples with Sam and Alyssa, the following additional couples are no longer possible: $\binom{\text{John}}{\text{Camille}}$✗, $\binom{\text{Asaf}}{\text{Victoria}}$✗, $\binom{\text{Asaf}}{\text{Julia}}$✗, $\binom{\text{Cam}}{\text{Camille}}$✗. The logic behind these pairs is simply because both members of each couple appear in highly-likely other couples – $\binom{\text{Asaf}}{\text{Camille}}$✗ is still a highly likely outcome at this point. It would be absolutely suboptimal to send any of these couples to the MC, and fortunately none were in round five or any subsequent round.

There were six repeat couples in the round five MC. $\binom{\text{Cameron}}{\text{Mikala}}$✓ were of course a match, $\binom{\text{Asaf}}{\text{Camille}}$✗ matched for the fourth round in a row, and $\binom{\text{Sam}}{\text{Alyssa}}$✓ matched for the fourth time. $\binom{\text{Cam}}{\text{Emma}}$✗ matched for the second round in a row, and two couples from round one returned: $\binom{\text{Morgan}}{\text{Julia}}$✗ and $\binom{\text{Stephen}}{\text{Nicole}}$✓. The new couples were $\binom{\text{Prosper}}{\text{Victoria}}$✗, $\binom{\text{John}}{\text{Tori}}$✗, $\binom{\text{Giovanni}}{\text{Francesca}}$✓, $\binom{\text{Tyler}}{\text{Kaylen}}$✗. The later two were forced due to the TB result, and while none of these couples were particularly likely to be PMs, none of them were particularly unlikely either. This MC resulted in four lights again, and $\binom{\text{Giovanni}}{\text{Francesca}}$✓ being correct was not seemingly suspected by anyone. The number of pairings decreased from 71 to 21, not as large a relative change as previous rounds but at this point it is harder to make such large jumps.

The contestents certainly did not understand at this point how few possibilities remained, and had some convictions about the PMs that were off-base. The top ten remaining couples were

1. $\binom{\text{Cameron}}{\text{Mikala}}$✓ (21/21) (TB round 3)
2. $\binom{\text{Asaf}}{\text{Camille}}$✗ (20/21) (rounds two, three, four, five)
3. $\binom{\text{Sam}}{\text{Alyssa}}$✓ (18/21) (rounds one, three, four, five)
4. $\binom{\text{Prosper}}{\text{Emma}}$✓ (8/21) (rounds two and three)
5. $\binom{\text{Tyler}}{\text{Francesca}}$✗ (7/21) (round four)
6. $\binom{\text{Stephen}}{\text{Nicole}}$✓ (6/21) (rounds one and five)
7. $\binom{\text{Morgan}}{\text{Julia}}$✗ (6/21) (rounds one and five)
8. $\binom{\text{John}}{\text{Victoria}}$✓ (6/21) (rounds three and four)
9. $\binom{\text{Giovanni}}{\text{Julia}}$✗ (6/21) (never matched)
10. $\binom{\text{Stephen}}{\text{Tori}}$✗ (5/21) (rounds two and three)

We still only have five PMs in the top ten, the same as last round even though the number of pairings has shrunk to just 21. Asaf and Camille appear to be a near-sure-thing but are not – if they were to go to the TB in round six, the game would be over (in theory) as there would only be the true pairing left. $\binom{\text{Prosper}}{\text{Emma}}$✓ appear high but were not a strong consideration at this point in the game, perhaps because it had been multiple rounds since they last matched. $\binom{\text{Tyler}}{\text{Francesca}}$✗ are surprisingly likely given they appeared only in one MC, but were not being considered either (Francesca was very much focussed on Asaf, who was also not her PM). $\binom{\text{Stephen}}{\text{Nicole}}$✓ correctly appear highly likely, but this opportunity has been completely missed, as $\binom{\text{Stephen}}{\text{Julia}}$✗ have a high degree of conviction they are a PM (only two in 21 chance). $\binom{\text{Giovanni}}{\text{Julia}}$✗ also appears highly, and is the first time a couple who have never matched appeared in the top ten. This is perhaps one of the harder matches to informally determine the likelihood of as it can only be determined by excluding other pairings. The contestents almost unanimously believe at this point, correctly, that Giovanni and Julia are not a PM. Unforunately for them, Giovanni is convinced that she is his PM, which slows down their information discovery and creates a massive distraction, as it somewhat reinforces Stephen's incorrect belief in $\binom{\text{Stephen}}{\text{Julia}}$✗.

The number of couples that are impossible is now substantial and almost all involve involve one of Cameron, Asaf, Sam, Mikala, Camille, and Alyssa. There are two exceptions: $\binom{\text{Stephen}}{\text{Francesca}}$✗, and $\binom{\text{Tyler}}{\text{Nicole}}$✗. Stephen is very invested in Julia, but Tyler floats for much of the game – and $\binom{\text{Tyler}}{\text{Nicole}}$✗ are matched in the round six MC, even though they are 100% guaranteed not a match.

Round Six

Going into round six, there is one best choice for the TB ($\binom{\text{Propser}}{\text{Emma}}$✓, who will leave us with eight or thirteen possibilites) and many reasonable choices. There also many choices that could reveal very little, because many possible couplings appear in only a single pairing. The contestents, eager to make progress and having the chance, selected $\binom{\text{Sam}}{\text{Alyssa}}$✓ to go into the TB. This had a positive psychological effect, and also made some manual elimination of possibilities much easier. This is because $\binom{\text{Sam}}{\text{Alyssa}}$✓ and $\binom{\text{Cameron}}{\text{Mikala}}$✓ were both round one MC matches, as were $\binom{\text{Giovanni}}{\text{Kaylen}}$✗, meaning one of the remaining round one MC matches must be a PM. Furthermore, $\binom{\text{Giovanni}}{\text{Kaylen}}$✗ and $\binom{\text{Sam}}{\text{Alyssa}}$✓ were in round three, placing restrictions on who the four couples could be. While this may have helped the contestents, it does not reduce our possibilites much – from 21 to 18.

The couples in round six MC were: $\binom{\text{Prosper}}{\text{Kaylen}}$✗, $\binom{\text{John}}{\text{Emma}}$✗ (second time), $\binom{\text{Giovanni}}{\text{Francesca}}$✓ (second time), $\binom{\text{Asaf}}{\text{Camille}}$✗ (fifth time), $\binom{\text{Cam}}{\text{Victoria}}$✗ (second time), $\binom{\text{Sam}}{\text{Alyssa}}$✓ (TB), $\binom{\text{Cameron}}{\text{Mikala}}$✓ (TB), $\binom{\text{Morgan}}{\text{Tori}}$✓ (second time), $\binom{\text{Stephen}}{\text{Julia}}$✗ (second time), $\binom{\text{Tyler}}{\text{Nicole}}$✓. This produced four lights for the third week in a row, and again it was not clear if the contestents were able to infer much correclty from this result. $\binom{\text{Stephen}}{\text{Julia}}$✗ was a particularly bad distraction, as there was strong belief it might be a PM. As mentioned earlier, $\binom{\text{Tyler}}{\text{Nicole}}$✗ was not in any pairings, making this a particularly poor MC. Despite its failings, the MC result further restricted the number of possible pairings to just 8, down from 18.

Despite the number of pairings left being so small, it is not possible to prove any couples are PMs other than those already identified. The couples that appear in at least three pairings are:

1. $\binom{\text{Cameron}}{\text{Mikala}}$✓ (8/8) (TB round 3)
2. $\binom{\text{Sam}}{\text{Alyssa}}$✓ (8/8) (TB round 6)

3. $\binom{\text{Asaf}}{\text{Camille}}$✗ (7/8) (rounds two, three, four, five, six)

4. $\binom{\text{Giovanni}}{\text{Julia}}$✗ (5/8) (never matched)

5. $\binom{\text{Tyler}}{\text{Kaylen}}$✗ (3/8) (round five)

6. $\binom{\text{Stephen}}{\text{Tori}}$✗ (3/8) (rounds two and three)

7. $\binom{\text{Prosper}}{\text{Nicole}}$✗ (3/8) (round four)

8. $\binom{\text{Morgan}}{\text{Francesca}}$✗ (3/8) (round three)

9. $\binom{\text{John}}{\text{Emma}}$✗ (3/8) (rounds one and six)

With the only two correct couples being the two TBs, this is arguably the strangest "top" list yet. $\binom{\text{Giovanni}}{\text{Julia}}$✗ is the most interesting – they've never been matched, most of the contestents have no belief in them being a PM, and Giovanni is convinced they are. Perhaps more stark is the number of couples that can't be a PM. The most potential matches a contestent can have at this point is eight, but in practice none have more than five, many having four. Asaf has only two, Camille or Kaylen, but has demonstrated no interest in Kaylen and is increasingly convinced that Francesca is his PM - already provably not possible. If the contestents made one correct assumption (that $\binom{\text{Asaf}}{\text{Camille}}$✗ are not a PM) then they could determine the solution. If they made any other assumption about a couple, however, they would be unlikely to be able to solve it at this point. Furthermore, it is likely that the assumption they made would wrong, which would not help matters. For example, they might assume $\binom{\text{Stephen}}{\text{Julia}}$✗ were a PM – a couple that appears only in one of the remaining eight pairings.

Round Seven

With so few possibilities remaining, there are many reasonable choices for couples to send to the TB. Of the options available, $\binom{\text{Cam}}{\text{Victoria}}$✗ were sent, at least partially due to a reasonably strong interest from Victoria in Cam. They were also one of the round one and round six couples, meaning they had been part of relatively successful MCs. They were the fourth couple from the first MC to be tested, and although they were not a PM, this was still helpful to manually determining the solution. In particular, there were six couples from the round one MC, of which one had to be a PM, and similar situation for round six (two from seven). This includes some couples that were suspected to be PMs but weren't (like $\binom{\text{Stephen}}{\text{Julia}}$✗) but also some there were ($\binom{\text{Morgan}}{\text{Tori}}$✓).

In terms of possible pairings, this meant that there were now only six remaining going into the MC. If they were to construct a slate of only the most likely couples, it would have performed poorly! If were to follow a "greedy" strategy, taking at each point the most likely possible remaining couple, you would suggest the following two-light pairing: $\binom{\text{Sam}}{\text{Alyssa}}$✓, $\binom{\text{Cameron}}{\text{Mikala}}$✓, $\binom{\text{Asaf}}{\text{Camille}}$✗, $\binom{\text{Morgan}}{\text{Francesca}}$✗, $\binom{\text{John}}{\text{Emma}}$✗, $\binom{\text{Giovanni}}{\text{Julia}}$✗, $\binom{\text{Tyler}}{\text{Tori}}$✗, $\binom{\text{Stephen}}{\text{Victoria}}$✗, $\binom{\text{Propser}}{\text{Nicole}}$✗, $\binom{\text{Cam}}{\text{Kaylen}}$✗. This would have only given two lights, which would cause a "blackout".

The contestents instead followed their usual loose strategy – but, by picking just enough new couples, reduced the possible pairings to just one! The pairing they suggested obtained four lights: $\binom{\text{Prosper}}{\text{Victoria}}$✗, $\binom{\text{John}}{\text{Kaylen}}$✗, $\binom{\text{Giovanni}}{\text{Emma}}$✗, $\binom{\text{Asaf}}{\text{Francesca}}$✗, $\binom{\text{Cam}}{\text{Nicole}}$✗, $\binom{\text{Sam}}{\text{Alyssa}}$✓, $\binom{\text{Cameron}}{\text{Mikala}}$✓, $\binom{\text{Morgan}}{\text{Tori}}$✓, $\binom{\text{Stephen}}{\text{Julia}}$✗, $\binom{\text{Tyler}}{\text{Camille}}$✓. This result was met with dismay instead of joy, despite being the effective end of the game. They were disappointed that they had yet another week with just four lights, and moreover drew an incorrect implication: that the two non-guaranteed lights are quite likely to be $\binom{\text{Stephen}}{\text{Julia}}$✗ and $\binom{\text{Morgan}}{\text{Tori}}$✓. The former would prove particularly dangerous.

Round Eight

Although the game was effectively over, for the contestents it very much was not. Both Giovanni and Stephen were convinced that Julia was their PM, when she was actually Cam's – someone she had barely spoken to. Giovanni ended up winning a date with Julia, much to the consternation of Stephen. However, this was a blessing to most of the contestents, as they could now send them to the TB and resolve a situation they perceived as hopeless. This represented the second time in the TB for Giovanni and Julia, and this alone reduces the possibilities quite a bit. While we know the number of pairings remaining is one, this is unclear to the contestents. One of the simplest pieces of information to use though is TB results, and from TB alone we are down to 17,688 pairings remaining. Too much to nail down, perhaps, but useful when combined with other information and intuition.

For the matching ceremony, the contestents had begun to feel the lack of progress. The most lights achieved was still four, and another unsuccessful (even if expected) TB added to the pressure. The contestents decided they needed to shake up the couples at the MC, encouraging randomized matches. The men were responsible for picking, but not all of them followed the randomization strategy. In particular Stephen picked Julia, and Asaf picked Camille. This unfortunately led to just two lights from the already-confirmed matches, and thus caused a "blackout" - reducing the prize pool from $1,000,000 to $750,000. There was one benefit though, in that some of the contestents were now fairly sure of the true pairing. As $\binom{\text{Stephen}}{\text{Julia}}$✗ and $\binom{\text{Asaf}}{\text{Camille}}$✗ had finally been disproved, the possibilities collapsed. A simple analysis based on this round's MC and past TBs alone shows Giovanni has only five possibilities left, as does Julia. Making assumptions like $\binom{\text{Morgan}}{\text{Tori}}$✓, and then flowing through the early weeks strongly suggests $\binom{\text{Cam}}{\text{Julia}}$✓, and $\binom{\text{Giovanni}}{\text{Francesca}}$✓, and ultimately gives a pairing that can be found to be consistent with all known data. Played optimally, this means a round nine victory was definitely possible.

Round Nine

After the shock of the prize pool reduction in round eight, the contestents were eager to not make any more mistakes. Having the somewhat-surprising result that $\binom{\text{Stephen}}{\text{Julia}}$✗ and $\binom{\text{Asaf}}{\text{Camille}}$✗ were not matches also convinced most of contestents to listen to the small set of contestents, particularly Tori, who were trying to systematically identify the PMs. It is difficult to be sure, but at least based on what dialogue was shown in the show, it was at the start of round nine that Tori potentially correctly identified all ten PMs, or at least a signficant fraction.

Based on this, as well as some chemistry, the contestents sent $\binom{\text{Prosper}}{\text{Emma}}$✓ to the TB, and thus guaranteed they would not have another prize pool reduction this week. Additionally, with now only seven contestents left unmatched, we have only 5040 possible pairings left. Using all other TB results, we have only 2286 pairings, making it even more likely that the contestents could identify the correct one.

At this point, Tori had prepped everyone with their PM, but couldn't control who actually picked who in the MC. In round nine, the seven remaining woman were picking from the men. Camille correctly picked Tyler based on the strategy, declaring they were not a romantic match. Victoria correctly picked John based on the strategy – when asked if they were a PM by the host of the show, John declared he no longer cares about the game anyway and they have that in common. Kaylen incorrectly picks Stephen, and it is unclear if this was expected by Tori or not. Similarly Nicole incorrectly picks Cam in a similar fashion. Julia then picks Morgan, for no apparent reason apart from a rejection of the strategy. Tori is upset as this severely constrains the remaining picks. Francesca picks Giovanni because of the strategy, to the surprise of Asaf who has not yet bought into the strategy. Finally, Tori has no choice but to pick Asaf – already confirmed to not be a PM by the TB. Despite this, they achieve six lights, and thus have everything they need to win in the final round, assuming no one picks incorrectly intentionally.

Round Ten

In the final round $\binom{\text{Cam}}{\text{Julia}}$✓ are sent to the TB to get a fourth confirmed match. Based on TB results alone at this point there are only 504 possible pairings left. Julia is the person who visited the TB the most - in round two (John), round eight (Giovanni) and in round ten (Cam).

The only uncertainty comes from the MC, where Asaf knows his PM is Kaylen (unless a mathematical error has been made, as he repeatedly points out) but his "heart" is with Francesca. Francesca, as proven in the previous MC, is fully bought into the strategy (and winning) at this point, as are seemingly all other contestents. Ultimately Asaf does pick Kaylen, and thus the game is won.

Progress by Round

While the game was ultimately won, we know it could have been won sooner and with all prize money intact. It is interesting to consider where the information to win ultimately came from. The two sources, TB and MC, are both important at least psychologicallly and its hard to fully disentangle them. For example, a confirmed PM from a TB means that all subsequent MCs have the PM in them, so we can't measure the effect of the MC alone. If we put that aside, we can measure the number of remaining pairings at the end of each round using four different factors:

- Remaining unmatched contestents.
- Using just TB results.
- Using just MC results.

- Using both.

The counts are summarized below:

Round	Remaining	TB	MC	Both
Round 1	10! = 3,628,800	3,265,920	222,480	205,176
Round 2	10! = 3,628,800	2,943,360	19,520	16,570
Round 3	9! = 362,880	287,280	2,176	1,269
Round 4	9! = 362,880	252,000	409	219
Round 5	9! = 362,880	225,360	54	21
Round 6	8! = 40,320	23,520	11	8
Round 7	8! = 40,320	20,808	1	1
Round 8	8! = 40,320	17,688	1	1
Round 9	7! = 5,040	2,286	1	1
Round 10	1	504	1	1

One can observe the MC is by far the most powerful source of information, rapidly cutting down the possibilites. With just MC information alone we are still left with only one pairing at the end of round seven, although the particular MC pairings trialled are of course influenced by the TB-confirmed PMs.

7 Conclusion

In this paper we have shown how even a seemingly small amount of revealed information can rapidly cut down a search space from millions to just tens. We have used permutations, combinations, and derangements to calculate exactly how many possibilities remain under various circumstances and to inform strategy. We have also employed simulation to understand expected information gain from different sources. Finally, we analyzed a season of the game and showed how the players actions were sometimes suboptimal but ultimately good enough to win (most of) the prize.

There are many follow ups one could persue, including generalizing the problem to include an 11th man or woman who will be unmatched in the end, as well as a version of the game where any contestent can match with any other contestent. For the basic game, strategies could be developed that could be easily applied by hand with minimal assistance or even without paper and pen. Hopefully, future contestents will benefit from this and find their perfect matches promptly and efficiently.