SimFL

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Syntax

$$T \in \textsc{TypeCon} \qquad C \in \textsc{DataCon}$$

$$\begin{aligned}
e \in \textsc{Expr} ::= {} & x \\
\mid {} & n \\
\mid {} & C \\
\mid {} & [\,] \\
\mid {} & [\,e\,\overline{\langle,\ e\rangle}\,] \\
\mid {} & e \bullet e \\
\mid {} & \texttt{(}\bullet\texttt{)} \\
\mid {} & \textsf{fun}\ x\ \texttt{->}\ e \\
\mid {} & e\ e \\
\mid {} & \textsf{let}\ x = e\ \textsf{in}\ e \\
\mid {} & \textsf{let rec}\ f\ x = e\ \overline{\langle \textsf{and rec}\ f\ x = e\rangle}\ \textsf{in}\ e \\
\mid {} & \textsf{case}\ e\ \textsf{of}\ \texttt{\{}\ p\ \texttt{->}\ e\ \overline{\langle \texttt{;}\ p\ \texttt{->}\ e\rangle}\ \texttt{\}} \\
\mid {} & \textsf{if}\ e\ \textsf{then}\ e\ \textsf{else}\ e
\end{aligned}$$

$$\begin{aligned}
\bullet \in \textsc{BinOp} ::= {} & s\overline{s} \\
v \in \textsc{Value} ::= {} & C\ \overline{v} \\
\mid {} & (\textbf{closure}\ x \to e, \rho)
\end{aligned}$$

Natural Semantics

$$\frac{}{\rho \vdash n \Rightarrow n}\ \textsc{Num} \qquad \frac{}{\rho \vdash C \Rightarrow C}\ \textsc{Con} \qquad \frac{(x,v) \in \rho}{\rho \vdash x \Rightarrow v}\ \textsc{Var}$$

$$\frac{}{\rho \vdash \textsf{fun } x \texttt{ -> } e \Rightarrow (\textbf{closure } x \to e, \rho)}\ \textsc{Fun}$$

$$\frac{\rho \vdash e_1 \Rightarrow (\textbf{closure } x \to e_3, \sigma) \qquad \rho \vdash e_2 \Rightarrow v_2 \qquad \sigma[x \mapsto v_2] \vdash e_3 \Rightarrow v_3}{\rho \vdash e_1\ e_2 \Rightarrow v_3}\ \textsc{App}$$

$$\frac{\rho \vdash e_1 \Rightarrow C\ \hat{v} \qquad \rho \vdash e_2 \Rightarrow v}{\rho \vdash e_1\, e_2 \Rightarrow C\ \hat{v}, v}\ \textsc{AppCons}$$

$$\frac{\rho \vdash e_1 \Rightarrow v_1 \qquad \rho[x \mapsto v_1] \vdash e_2 \Rightarrow v_2}{\rho \vdash \textsf{let } x = e_1 \textsf{ in } e_2 \Rightarrow v_2}\ \textsc{Let}$$

$$\frac{\rho[f \mapsto (\textbf{closure } x \to \textsf{let rec } f\ x = e_1 \textsf{ in } e_1, \rho)] \vdash e_2 \Rightarrow v}{\rho \vdash \textsf{let rec } f\ x = e_1 \textsf{ in } e_2 \Rightarrow v}\ \textsc{LetRec}$$

$$\frac{\sigma = \bigcup_{i=1}^{n}\{f_i \mapsto (\textbf{closure } x_i \to \textsf{let rec } f_1\ x_1 = e_1 \textsf{ and} \ldots \textsf{and } f_n\ x_n = e_n \textsf{ in } e_i, \rho)\} \qquad \rho[f \mapsto v \in \sigma] \vdash e_2 \Rightarrow v}{\rho \vdash \textsf{let rec } f_1\ x_1 = e_1 \textsf{ and} \ldots \textsf{and } f_n\ x_n = e_n \textsf{ in } e \Rightarrow v}\ \textsc{LetRec}*$$

$$\frac{\rho \vdash e \Rightarrow v \qquad v \triangleright p_i : \sigma_i \qquad \rho, \sigma_i \vdash e_i \Rightarrow v_i \qquad i \leq n}{\rho \vdash \textsf{case } e \textsf{ of } \{\ p_1 \texttt{ -> } e_1 \,;\ \ldots\,;\ p_n \texttt{ -> } e_n\} \Rightarrow v_i}\ \textsc{Case}$$

$$\frac{\rho \vdash e_1 \Rightarrow \textsf{True} \qquad \rho \vdash e_2 \Rightarrow v_2}{\rho \vdash \textsf{if } e_1 \textsf{ then } e_2 \textsf{ else } e_3 \Rightarrow v_2}\ \textsc{IfTrue} \qquad \frac{\rho \vdash e_1 \Rightarrow \textsf{False} \qquad \rho \vdash e_3 \Rightarrow v_3}{\rho \vdash \textsf{if } e_1 \textsf{ then } e_2 \textsf{ else } e_3 \Rightarrow v_3}\ \textsc{IfFalse}$$

$$\frac{\rho \vdash e_1 \Rightarrow v_1 \qquad \rho \vdash e_2 \Rightarrow v_2 \qquad \textsf{builtin } \bullet}{\rho \vdash e_1 \bullet e_2 \Rightarrow v_1 [\![\bullet]\!] v_2}\ \textsc{BuiltInOp}$$

$$\frac{\rho \vdash \textsf{fun a -> (fun b -> a } \bullet \textsf{ b)} \Rightarrow v \qquad \textsf{builtin } \bullet}{\rho \vdash (\,\bullet\,) \Rightarrow v}\ \textsc{BuiltInFun}$$

$$\frac{[\,] \hookrightarrow e \qquad \rho \vdash e \Rightarrow v}{\rho \vdash [\,] \Rightarrow v}\ \textsc{List}_1 \qquad \frac{1 \leq n \qquad [e_1,\ldots,e_n] \hookrightarrow e \qquad \rho \vdash e \Rightarrow v}{\rho \vdash [e_1,\ldots,e_n] \Rightarrow v}\ \textsc{List}_2$$

$$\frac{}{v \triangleright \_ : \varnothing}\ \textsc{MatchAny} \qquad \frac{}{v \triangleright x : \{x \mapsto v\}}\ \textsc{MatchVar}$$

$$\frac{\forall i \leq n,\ \ v_i \triangleright p_i : \sigma_i \qquad \bigcap_{j=1}^{n} \textsf{free}(p_j) = \varnothing \qquad \sigma = \bigcup_{j=1}^{n} \sigma_j}{C\ \hat{v} \triangleright C\ \hat{p} : \sigma}\ \textsc{MatchCons}$$

$$\frac{}{[\,] \hookrightarrow [\,]}\ \textsc{Nil} \qquad \frac{}{[e_1,\ldots,e_n] \hookrightarrow [\,]}\ \textsc{Nil}$$