SimFL

February 2, 2024

Syntax

$$n \in \mathbb{Z} \qquad x \in \textsc{Var} \qquad C \in \textsc{DataCon}$$

$$\begin{aligned} s \in \textsc{Symbol} &::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{/} \mid \texttt{=} \mid \texttt{<} \mid \texttt{>} \mid \texttt{:} \mid \texttt{.} \\ \bullet \in \textsc{BinOp} &::= s\overline{s} \end{aligned}$$

$$\begin{aligned} e \in \textsc{Expr} ::= {} & x \\ & \mid n \\ & \mid C \\ & \mid \texttt{[]} \\ & \mid \texttt{[}e\,\overline{\texttt{,}\,e}\texttt{]} \\ & \mid e \bullet e \\ & \mid \texttt{(}\bullet\texttt{)} \\ & \mid \mathsf{fun}\ x\ \texttt{->}\ e \\ & \mid e\ e \\ & \mid \mathsf{let}\ x = e\ \mathsf{in}\ e \\ & \mid \mathsf{let\ rec}\ f\ x = e\ \overline{(\mathsf{and\ rec}\ f\ x = e)}\ \mathsf{in}\ e \\ & \mid \mathsf{case}\ e\ \mathsf{of}\ \texttt{\{}\ p\ \texttt{->}\ e\ \overline{(\texttt{;}\,p\ \texttt{->}\ e)}\ \texttt{\}} \\ & \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e \\ & \mid \texttt{(}\,e\,\texttt{)} \end{aligned}$$

$$p \in \textsc{Pattern} ::= \texttt{\_} \mid x \mid n \mid C\ \overline{p} \mid \texttt{(}\,p\,\texttt{)}$$

$$\begin{aligned} v \in \textsc{Value} ::= {} & C\ \overline{v} \\ & \mid \langle \rho, x \to e \rangle \end{aligned}$$

$$\begin{aligned} \rho, \sigma \in \textsc{Env} &::= \mathcal{P}(\textsc{Var} \times \textsc{Value}) \\ x \mapsto v &\stackrel{\mathsf{def}}{\equiv} (x, v) \\ \rho[x \mapsto v] &\stackrel{\mathsf{def}}{\equiv} \{x \mapsto v\} \cup \{x_i \mapsto v_i \in \rho \mid x_i \neq x\} \\ \rho, \sigma &\stackrel{\mathsf{def}}{\equiv} \sigma \cup \{x_i \mapsto v_i \in \rho \mid x_i \notin \mathsf{dom}(\sigma)\} \end{aligned}$$

Semantics

Pattern Matching

$$v \rhd p \rightsquigarrow \sigma \subseteq \textsc{Value} \times \textsc{Pattern} \times \textsc{Env}$$

$$\frac{}{v \rhd \_ \rightsquigarrow \varnothing}\ \textsc{MatchAny} \qquad \frac{}{v \rhd x \rightsquigarrow \{x \mapsto v\}}\ \textsc{MatchVar}$$

$$\frac{\forall i \leq n,\ \ v_i \rhd p_i \rightsquigarrow \sigma_i \qquad \bigcap_{j=1}^{n} \mathsf{free}(p_j) = \varnothing \qquad \sigma = \bigcup_{j=1}^{n} \sigma_j}{C\ \hat{v} \rhd C\ \hat{p} \rightsquigarrow \sigma}\ \textsc{MatchCons}$$

$$\mathsf{free}(p) \stackrel{\mathsf{def}}{\equiv} \begin{cases} \{x\} & \text{if } p = x \in \textsc{Var} \\ \bigcup_{i=1}^{n} \mathsf{free}(p_i) & \text{if } p = C\ p_1\ \ldots\ p_n \text{ and } C \in \textsc{Cons} \\ \varnothing & \text{otherwise} \end{cases}$$

Natural Semantics

$$\rho \vdash e \Downarrow v \subseteq \textsc{Env} \times \textsc{Expr} \times \textsc{Value}$$

$$\frac{}{\rho \vdash n \Downarrow n}\ \textsc{Num} \qquad \frac{}{\rho \vdash C \Downarrow C}\ \textsc{Cons} \qquad \frac{(x, v) \in \rho}{\rho \vdash x \Downarrow v}\ \textsc{Var}$$

$$\frac{}{\rho \vdash \mathsf{fun}\ x\ \texttt{->}\ e \Downarrow \langle \rho, x \to e \rangle}\ \textsc{Fun}$$

$$\frac{\rho \vdash e_1 \Downarrow \langle \sigma, x \to e_3 \rangle \qquad \rho \vdash e_2 \Downarrow v_2 \qquad \sigma[x \mapsto v_2] \vdash e_3 \Downarrow v_3}{\rho \vdash e_1\ e_2 \Downarrow v_3}\ \textsc{App}$$

$$\frac{\rho \vdash e_1 \Downarrow C\ \hat{v} \qquad \rho \vdash e_2 \Downarrow v}{\rho \vdash e_1\, e_2 \Downarrow C\ \hat{v}, v}\ \textsc{AppCons}$$

$$\frac{\rho \vdash e_1 \Downarrow v_1 \qquad \rho[x \mapsto v_1] \vdash e_2 \Downarrow v_2}{\rho \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Downarrow v_2}\ \textsc{Let}$$

$$\frac{\rho \begin{bmatrix} f_1 \mapsto \langle \rho, x_1 \to \mathsf{let\ rec}\ f_1\ x_1 = e_1\ \mathsf{and}\ \ldots\ \mathsf{and}\ f_n\ x_n = e_n\ \mathsf{in}\ e_1 \rangle \\ \vdots \\ f_n \mapsto \langle \rho, x_n \to \mathsf{let\ rec}\ f_1\ x_1 = e_1\ \mathsf{and}\ \ldots\ \mathsf{and}\ f_n\ x_n = e_n\ \mathsf{in}\ e_n \rangle \end{bmatrix} \vdash e \Downarrow v}{\rho \vdash \mathsf{let\ rec}\ f_1\ x_1 = e_1\ \mathsf{and}\ \ldots\ \mathsf{and}\ f_n\ x_n = e_n\ \mathsf{in}\ e \Downarrow v}\ \textsc{LetRec}$$

$$\frac{\rho \vdash e \Downarrow v \qquad v \rhd p_i \rightsquigarrow \sigma_i \qquad \rho, \sigma_i \vdash e_i \Downarrow v_i \qquad i \leq n}{\rho \vdash \mathsf{case}\ e\ \mathsf{of}\ \{\ p_1\ \texttt{->}\ e_1\,;\ \ldots\,;\ p_n\ \texttt{->}\ e_n \} \Downarrow v_i}\ \textsc{Case}$$

$$\frac{\rho \vdash e_1 \Downarrow \mathsf{True} \qquad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v_2}\ \textsc{IfTrue} \qquad \frac{\rho \vdash e_1 \Downarrow \mathsf{False} \qquad \rho \vdash e_3 \Downarrow v_3}{\rho \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v_3}\ \textsc{IfFalse}$$

$$\frac{\rho \vdash e_1 \Downarrow v_1 \qquad \rho \vdash e_2 \Downarrow v_2 \qquad \mathsf{builtin}\ \bullet}{\rho \vdash e_1 \bullet e_2 \Downarrow v_1 [\![\bullet]\!] v_2}\ \textsc{BuiltInOp}$$

$$\frac{\rho \vdash \mathsf{fun\ a\ \texttt{->}\ (fun\ b\ \texttt{->}\ a\ \bullet\ b)} \Downarrow v \qquad \mathsf{builtin}\ \bullet}{\rho \vdash (\,\bullet\,) \Downarrow v}\ \textsc{BuiltInFun}$$

$$\frac{[\,] \looparrowright e \qquad \rho \vdash e \Downarrow v}{\rho \vdash [\,] \Downarrow v}\ \textsc{List}_1 \qquad \frac{1 \leq n \qquad [e_1,\ldots,e_n] \looparrowright e \qquad \rho \vdash e \Downarrow v}{\rho \vdash [e_1,\ldots,e_n] \Downarrow v}\ \textsc{List}_2$$

$$\frac{}{[\,] \looparrowright [\,]}\ \textsc{Nil} \qquad \frac{}{[e_1,\ldots,e_n] \looparrowright [\,]}\ \textsc{Nil}$$