DISTRIBUTED OPTIMIZATION IN ENERGY COMMUNITIES: A FOCUS ON FLEXIBILITY PROVISION

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Abstract

The emergence of energy communities (ECs) as new stakeholders in modern electrical grids presents an opportunity to address the challenges associated with integrating distributed energy resources, such as increased uncertainty and variability in generation, transmission grid saturation, or diminished power quality, among others. The ability of an EC to provide flexibility is therefore paramount. However, data privacy and investment costs are the main drawbacks to achieving optimal operation. This paper proposes a distributed optimization framework designed for the economic dispatch (ED) of ECs, employing the auxiliary problem principle (APP) decomposition technique. The primary focus is to study the effects of flexibility provision in real-time control while addressing data privacy concerns and minimizing communication infrastructure investments. The proposed algorithm is tested in a case study, providing insights into the selection of the control parameters of the APP framework to improve the convergence properties.

1 Introduction

The modern electrical grid is undergoing a transformative shift characterized by increased integration of distributed generation, distributed storage, demand response, and power electronics. As the grid evolves, system operators face the challenge of enhancing hosting capacity to accommodate these diverse energy resources. The emergence of energy communities (ECs) as new stakeholders in the energy market holds promise for addressing this challenge through the provision of flexibility services. To achieve the optimal operation of these community networks, the use of the traditional centralized energy management systems (EMS) is hindered by physical space requirements, extensive communication infrastructure, and high computational cost.

To address these limitations, we propose a distributed approach to an economic dispatch (ED) problem using the auxiliary problem principle (APP) decomposition technique [1]. This assumes that the network size is small enough to accept the ED results as a feasible solution (neglecting network constraints).

Previous research has utilized the APP technique for solving ED problems. For instance, [2] explored the correlation between APP control parameters and convergence speed in ED problems, whereas [3] proposed a self-adaptative strategy to adjust these parameters based on iterative information. However, these studies primarily focus on large-scale ED without considering grid constraints necessary for flexibility provision.

A comparable approach to ours has been proposed by [4], employing the alternating direction method of multipliers (ADMM) technique. However, this method necessitates agents to share information with a central coordinator, a requirement we aim to circumvent in our approach.

In the framework we propose, agents within the EC possess local sensing, communication, and computation capabilities. The EC owns the microgrid, agents share information only with their neighbors, and there is no need for a central coordinator. To illustrate the effectiveness of this approach, we present a case study that considers flexible assets within an EC. Through this study, we investigate the framework's responsiveness to system demands in a potential real-time control scenario. Our findings demonstrate the feasibility and advantages of this distributed optimization approach, highlighting its capability to optimally manage connected assets without relying on a centralized controller. This not only enhances the scalability of the solution but also ensures the protection of agents' sensitive data, effectively addressing concerns regarding privacy and security.

2 Problem Formulation

We consider an EC network as an undirected interconnected graph, comprising a set of n agents or nodes symbolized by $\{a_1, a_2, ..., a_n\}$, and a series of communication links between agents a_i and a_j denoted by pairs (i, j). Agents are all physically connected behind a meter; however, the communication links may or may not be identical to physical connections and serve the purpose of sharing information (Fig. 1). For the rest of the paper, every two agents sharing a link (i, j) are said to be neighbors.

Each agent a_i is equipped with some units capable of providing flexibility, like dispatchable generators, batteries, or flexible loads. In addition, each agent possesses a critical load that must be supplied.

The EC is connected to the utility grid and holds an elec-

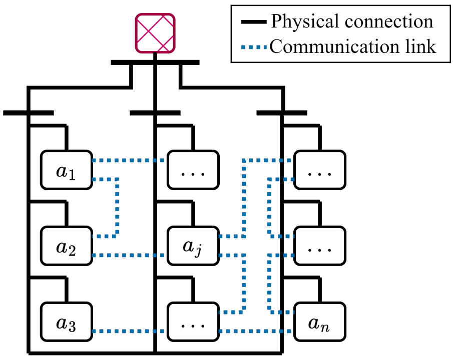

Figure 1: Energy Community scheme

tricity supply contract with a retailer or other stakeholder. The prices for purchasing and selling electricity on the utility grid may fluctuate over time.

We are interested in determining the optimal dispatch of active power from flexible units over a predetermined planning horizon that spans T periods, to minimize the operational costs of the EC, subject to delivering flexibility services that impose constraints on the amount of power purchased from the utility grid.

Let $\mathbf{p}_i \in \mathbb{R}^T$ represent the aggregated active power setpoint of the flexible units of agent a_i, and let $C_i(\mathbf{p}_i)$ denote its cost function. Similarly, let $\mathbf{p}^p$, $\mathbf{p}^s \in \mathbb{R}^T$ be the active power purchased and sold in the utility grid, respectively, by the EC, whereas $\mathbf{c}^p$, $\mathbf{c}^s \in \mathbb{R}^T$ denote each purchase and sale price over the planning horizon, which have known values. Finally, let $\mathbf{p}_i^D \in \mathbb{R}^T$ represent the active power demand of the critical load of agent a_i.

The ED of the network can be obtained by solving the following optimization problem:

$$\begin{aligned}
\min_{\substack{(\mathbf{p}_i,\mathbf{p}^p,\mathbf{p}^s,\\ i=1\ldots n)}} \quad & \sum_{i=1}^{n} C_i(\mathbf{p}_i) + \mathbf{c}^{p\intercal}\mathbf{p}^p - \mathbf{c}^{s\intercal}\mathbf{p}^s && \text{(1a)}\\
\text{subject to} \quad & \sum_{i=1}^{n} \mathbf{p}_i + \mathbf{p}^p = \sum_{i=1}^{n} \mathbf{p}_i^D + \mathbf{p}^s && \text{(1b)}\\
& \underline{\mathbf{p}}^p \le \mathbf{p}^p \le \overline{\mathbf{p}}^p && \text{(1c)}\\
& \mathbf{p}_i \in \mathcal{F}_i, \quad i = 1\ldots n, && \text{(1d)}
\end{aligned}$$

where $\underline{\mathbf{p}}^p$ and $\overline{\mathbf{p}}^p$ are the lower and upper bounds of the active power that the EC can purchase from the grid, imposed for instance by flexibility requirements, and $\mathcal{F}_i$ is the feasible set encompassing technical constraints for each unit of agent a_i (such as lower and upper bounds, state of charge, ramps, etc.).

In the next section, we develop the distributed formulation for problem (1).

3 Distributed Algorithm

To overcome the limitations of the centralized approach, by using the APP decomposition technique we eliminate the need for a central controller and restrict the amount of information shared. Within this framework, the original problem (1) is decomposed into n smaller subproblems, with each agent tasked with one of these subproblems.

Each agent is equipped with a local controller that operates autonomously. The optimal operation can be attainable after a succession of iterations where these controllers update the parameters of their assigned subproblems, solve them independently, and exclusively share nonsensitive data with neighboring agents.

To facilitate the decomposition of (1) into smaller subproblems, it is essential to recognize that $\mathbf{p}^p$ and $\mathbf{p}^s$ are global variables requiring consensus among all agents, but the specific allocation of this power to individual agents does not affect the total cost for the EC. Consequently, we can assume an identical quantity assigned to each agent, leading to the definitions $\mathbf{p}_i^p = \frac{\mathbf{p}^p}{n}$ and $\mathbf{p}_i^s = \frac{\mathbf{p}^s}{n}$ which represent the average power exchanged by the agents with the utility grid. This allows us to rewrite (1a) as

$$\sum_{i=1}^{n} f_i(\mathbf{p}_i, \mathbf{p}_i^p, \mathbf{p}_i^s) = \sum_{i=1}^{n} \left(C_i(\mathbf{p}_i) + \mathbf{c}^{p\intercal}\mathbf{p}_i^p - \mathbf{c}^{s\intercal}\mathbf{p}_i^s\right), \qquad (2)$$

which is now decomposable, and where f_i is the cost function for agent a_i.

Constraint (1c) can also be rewritten as a set of constraints, each pertaining to an individual agent a_i:

$$\underline{\mathbf{p}}^p \le \mathbf{p}_i^p n \le \overline{\mathbf{p}}^p, \qquad (3)$$

and

$$\mathbf{p}_i^p = \mathbf{p}_j^p, \qquad j \in \Omega_i, \qquad (4)$$

where Ω_i is the set of indices corresponding to the neighbors of agent a_i. Constraints (4) are for consistency and dictate that the purchased power assigned to agent a_i must match that assigned to each of its neighboring agents.

To advance towards the decomposition of (1b), we introduce an auxiliary variable $\mathbf{p}_{ij} \in \mathbb{R}^T$ representing the amount of active power an agent a_i virtually exchanges with its neighbor a_j. Together with consistency constraints to ensure that the power sent by one agent equals the power received by the other, we can rewrite (1b) for each agent a_i as:

$$\mathbf{p}_i + \mathbf{p}_i^p - \sum_{j\in\Omega_i} \mathbf{p}_{ij} = \mathbf{p}_i^D + \mathbf{p}_i^s \qquad (5)$$

and

$$\mathbf{p}_{ij} + \mathbf{p}_{ji} = 0, \qquad \forall j \in \Omega_i. \qquad (6)$$

Problem (1) can then be restated equivalently as:

$$\begin{aligned}
\min_{\substack{(\mathbf{p}_i,\mathbf{p}_i^p,\mathbf{p}_i^s,\mathbf{p}_{ij},\\ i=1\ldots n)}} \quad & (2) && \text{(7a)}\\
\text{subject to} \quad & (3), (4), (5), (6), (1d), \quad i = 1\ldots n && \text{(7b)}
\end{aligned}$$

Now only the consistency constraints (4) and (6) prevent the problem from decomposing. To address this, the APP employs an Augmented Lagrangian relaxation to relocate these constraints to the objective function as penalty terms, resulting in the following Lagrangian expression:

$$
\begin{aligned}
\mathcal{L} = &\sum_{i=1}^{n} f_i \\
&+ \sum_{i=1}^{n} \sum_{j \in \Omega_i} \left(\boldsymbol{\nu}_{ij}^{\intercal} (\mathbf{p}_i^p - \mathbf{p}_j^p) + \frac{\gamma}{2} \left\| \mathbf{p}_i^p - \mathbf{p}_j^p \right\|_2^2 \right) \\
&+ \sum_{i=1}^{n} \sum_{j \in \Omega_i} \left(\boldsymbol{\lambda}_{ij}^{\intercal} (\mathbf{p}_{ij} + \mathbf{p}_{ji}) + \frac{\gamma}{2} \left\| \mathbf{p}_{ij} + \mathbf{p}_{ji} \right\|_2^2 \right),
\end{aligned} \tag{8}
$$

where $\boldsymbol{\nu}_{ij}$ and $\boldsymbol{\lambda}_{ij} \in \mathbb{R}^T$ correspond to the dual variables of constraints (4) and (6) respectively, and γ is a chosen penalty factor.

The inclusion of quadratic cross terms between $\mathbf{p}_i^p$ and $\mathbf{p}_j^p$ and between $\mathbf{p}_{ij}$ and $\mathbf{p}_{ji}$ within the two-norm expressions of (8) still hinders decomposition feasibility. Consequently, the APP linearizes these quadratic norms to facilitate decoupling, thereby enabling the decomposition.

For simplicity of notation, denote $\mathbf{x}_i$ as the vector of decision variables $[\mathbf{p}_i^{\intercal}, \mathbf{p}_i^{p\intercal}, \mathbf{p}_i^{s\intercal}, \mathbf{p}_{ij}^{\intercal}]^{\intercal}$, and A_i as the feasible space encompassed by constraints (3), (5), (1d). If the linearization is made around the current point of operation at iteration k, then (7) can be decomposed into the following two-stage subproblem for each agent a_i:

Stage 1: Solve for iteration k+1

$$
\begin{aligned}
\mathbf{x}_i^{k+1} := \underset{\mathbf{x}_i^{k+1} \in A_i}{\operatorname{argmin}} \quad & f_i^{k+1} \\
&+ \frac{\beta}{2} \left\| \mathbf{p}_i^{p\,k+1} - \mathbf{p}_i^{pk} \right\|_2^2 + \frac{\beta}{2} \sum_{j \in \Omega_i} \left\| \mathbf{p}_{ij}^{k+1} - \mathbf{p}_{ij}^{k} \right\|_2^2 \\
&+ \sum_{j \in \Omega_i} \mathbf{p}_i^{p\,k+1\,\intercal} \left(\gamma (\mathbf{p}_i^{pk} - \mathbf{p}_j^{pk}) + \boldsymbol{\nu}_{ij}^{k} \right) \\
&+ \sum_{j \in \Omega_i} \mathbf{p}_{ij}^{k+1\,\intercal} \left(\gamma (\mathbf{p}_{ij}^{k} + \mathbf{p}_{ji}^{k}) + \boldsymbol{\lambda}_{ij}^{k} \right),
\end{aligned} \tag{9}
$$

Stage 2: Update the dual variables

$$
\boldsymbol{\nu}_{ij}^{k+1} := \boldsymbol{\nu}_{ij}^{k} + \alpha (\mathbf{p}_i^{p\,k+1} - \mathbf{p}_j^{p\,k+1}) \qquad \forall j \in \Omega_i, \tag{10}
$$

$$
\boldsymbol{\lambda}_{ij}^{k+1} := \boldsymbol{\lambda}_{ij}^{k} + \alpha (\mathbf{p}_{ij}^{k+1} + \mathbf{p}_{ji}^{k+1}) \qquad \forall j \in \Omega_i \tag{11}
$$

Stage 2 is executed by incorporating both shared information from a_i neighbors and the solution of problem (9). $\mathbf{p}_{ji}^{k+1}$ refers to the information shared by a_j about the virtually exchanged power with a_i, whereas $\mathbf{p}_j^{p\,k+1}$ refers to the power purchased from the utility grid assigned to a_j. This information is again used to solve (9) in the next iteration. β and α are predefined positive factors.

Convergence is achieved in iteration k once $\mathbf{p}_{ij}^k + \mathbf{p}_{ji}^k$ and $\mathbf{p}_i^{pk} - \mathbf{p}_j^{pk}$ are less than or equal to a certain error tolerance ϵ for all agents a_i and for each of its neighbors.

In summary, Algorithm 1 describes the necessary steps to achieve the optimal value of problem (1) through this decomposition scheme.

Convergence of the algorithm is guaranteed when the objective cost function f_i is convex [1]. However, since the focus of this scheme is on real-time optimization to bolster the responsiveness of ECs to system demands, it operates under the key assumption that the environmental conditions, including power demand, renewables availability, and grid prices, are subject to continuous fluctuations. Consequently, the optimal operating conditions also evolve dynamically. In this context, the algorithm is designed to operate continuously, actively guiding the operating points of controllable devices towards the optimal configuration.

Algorithm 1 Economic Dispatch decomposition using APP

```
Set k=0, Set α, β, γ to some positive value
for Each agent a_i do
    Initialize p_i^{p0} p_j^{p0}, p_ij^0, p_ji^0, ν_ij^0 and λ_ij^0 to some value for each neighbour j
end for
repeat
    for Each agent a_i do in parallel
        Solve (9) for x_i^{k+1}
        Share p_i^{p k+1} and p_ij^{k+1} with each neighbour j
        Update ν_ij^{k+1} and λ_ij^{k+1} using (10) and (11) for each neighbor j, with incoming information p_j^{p k+1} and p_ji^{k+1}
        Set k = k + 1
    end for
until (p_i^{pk} − p_j^{pk} and p_ij^k + p_ji^k) ≤ ϵ   ∀j ∈ Ω_i, ∀i
```

4 Flexibility Services

In contemporary energy systems, flexibility services play a pivotal role in optimizing both operational efficiency and economic viability, offering various benefits to the ECs themselves and other stakeholders such as the distribution system operators (DSOs).

For ECs, flexibility services manifest in strategies like Time-of-Use (ToU) optimization and Control of Maximum Load (kWMax Control), which enable ECs to save significantly on tariff costs. ToU optimization involves intelligently shifting energy consumption from high-price intervals to low-price intervals or even implementing complete load shedding during peak-price periods. This service is embedded in (9) by considering the grid projected prices $\mathbf{c}^p$ and $\mathbf{c}^s$ in f_i in the objective function. Similarly, kWMax Control focuses on reducing the maximum load, often referred to as peak shaving. This reduction can be achieved through load shifting or shedding and can be provided by setting $\overline{\mathbf{p}}^p$ to the desired profile.

For DSOs, for instance, flexibility services such as grid capacity management and congestion management enable them to reduce costs, while providing ECs with additional revenues. Grid capacity management utilizes consumer flexibility to optimize operational performance and asset dispatch by mitigating peak loads, prolonging component lifetimes, and ensuring a more even distribution of loads across the grid. Congestion management involves preventing thermal overload of system components by curtailing peak loads. Consumer flexibility offers an alternative avenue by potentially deferring or even obviating the need for costly grid investments in reinforcements. Consequently, both these services for DSOs can be provided by controlling the limits for the power purchased from the grid, $\underline{\mathbf{p}}^p$ and $\overline{\mathbf{p}}^p$.

5 Case Study and Results

To test the proposed scheme, we use an 11-bus network with two zones, each based on the IEEE 5-bus system, with a central bus that connects both zones with the utility grid. Each zone contains 3 loads, 4 dispatchable generators, a photovoltaic plant (PV), and a storage system. The planning horizon will encompass 24 periods, each lasting 1 hour.

We aim to investigate the algorithm's capability to determine the optimal operating level under two different environmental conditions. In test (a), the system is operating at the optimal value when a change is introduced to the environment in the 10th iteration. This alteration entails a smooth update of loads, solar irradiance, and grid price forecasts, denoting conditions in which operating parameters remain relatively stable over time without significant fluctuations. For these updates, we use real data series to reflect real variations after 1 hour of normal operating conditions. In test (b), a flexibility request is also scheduled simultaneously, resulting in sudden alterations in purchasable power (i.e. a reduction in $\overline{\mathbf{p}}^p$). In contrast to test (a), this aims to replicate abrupt external variations. It is important to note that the impact of these variations is relative to the speed of the algorithm, as we will see now.

To assess the convergence of Algorithm 1, we use a tolerance of $\epsilon = 10^{-5}$ whereas the parameters α, β and γ are defined according to [5] and [2] for best performance, as relative to a parameter c as $\alpha = c$, $\beta = 2c$, $\gamma = c$. The results are shown in Fig. 2.

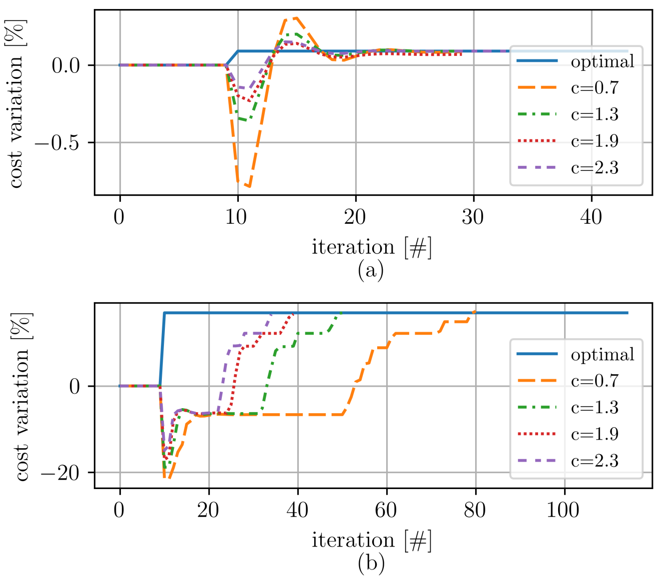

Figure 2: (a) Convergence sensitivity under normal conditions (b) Convergence sensitivity on flexibility request scheduling

In test (a), our smooth changes in input entail a step increase of approximately 0.1% in the optimal operating cost, as shown in Fig. 2 (a). Here, we have found that convergence is achieved in a relatively stable number of iterations for a wide range of values of c, reducing the fluctuation of intermediate iterations as it is increased.

In test (b), the same changes of test (a) together with the scheduled flexibility request produces an increase of 16.9% in the optimal operating cost as shown in Fig. 2 (b). In this case, small values of c produce slower convergence, while the number of iterations stabilizes when c is increased.

In both tests, high values of c increase slowly both the number of iterations and the error concerning the optimal value of the cost function. In our scenario, a good choice would be $c = 2.3$, resulting in a convergence rate of approximately 25 iterations for both smooth and abrupt variations.

6 Conclusion

Our investigation focused on the convergence rate of the suggested algorithm, rather than CPU time. This deliberate choice paves the way for further research, enabling the exploitation of parallelized computations and examination of convergence efficiency while taking into account communication delays. The resulting error has not been assessed either because, in a real-time application, the algorithm should continue to iterate rather than stop when the convergence criteria are met, meaning that it may be further decreased. In any case, the selection of the tuning parameters plays a vital role in achieving an acceptable convergence rate and error. However, the convergence properties remain stable under different sizes of perturbations.

Another noteworthy aspect is the assumption that the EC owns the microgrid. This introduces the additional challenge of allocating costs and benefits among community members. Adjustments to the problem formulation may be necessary when considering a virtual community connected through the public grid and when their supply contracts differ. This brings new research possibilities.

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References

[1] G. Cohen, "Auxiliary problem principle and decomposition of optimization problems," *Journal of optimization Theory and Applications*, vol. 32, pp. 277–305, 1980.

[2] Y. Ren, Y. Tian, and H. Wei, "Parameter evaluation of auxiliary problem principle for large-scale multi-area economic dispatch," *International Transactions on Electrical Energy Systems*, vol. 24, no. 12, pp. 1782–1790, 2014.

[3] Y. Ren and S. Fei, "The Auxiliary Problem Principle with Self-Adaptive Penalty Parameter for Multi-Area Economic Dispatch Problem," *Algorithms*, vol. 8, no. 2, pp. 144–156, Jun. 2015, number: 2 Publisher: Multidisciplinary Digital Publishing Institute.

[4] P. Olivella-Rosell, F. Rullan, P. Lloret-Gallego *et al.*, "Centralised and Distributed Optimization for Aggregated Flexibility Services Provision," *IEEE Transactions on Smart Grid*, vol. 11, no. 4, pp. 3257–3269, 2020, arXiv: 1907.08125.

[5] B. Kim and R. Baldick, "Coarse-grained distributed optimal power flow," *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 932–939, May 1997.