HOMEWORK 6

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SECTION 002 (MW 2:30 - 3:50 PM)

CSE 2320 - Homework 6 (You can solve all of P3 and parts of Knapsack. Monday Oct 22, we will cover Greedy and 0/1 for Knapsack.)

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Total points: 115/100 (points past 100 are bonus) Topics: Memoization, Greedy, Dynamic Programming (Knapsack: unbounded, 0/1, fractional)

P1 (4 pts) Given this solution information, for the **unbounded** Knapsack problem below, recover the choices that gave the optimal answer for knapsack capacity 19. Show your work (highlight or circle cells).

```
Item   |  A|  B|  C|  D|
Weight |  3|  4|  7|  8|
Value  | the item values are hidden as they should not be used in recovering the solution.
```

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
picked				A	B	B	A	C	D	D	A	B	B	A	C	C	D	A	B	B

Items picked for capacity 19: D, C, B

→ last item = B
weight of B is 4
19 − 4 = 15
→ pick C at 15
→ weight of C = 7
15 − 7 = 8
→ Pick D at 8, weight of D = 8
8 − 8 = 0

P2 (61 pts) Given the item types below, solve the following problems. Fill in the answer in the table and show your work below.

Item:	A	B	C	D
Weight:	3	4	6	7
Value:	4	7	10	12

	Unbounded Knapsack	0/1 Knapsack	Fractional Knapsack
Dynamic Programming	$$: 24 Items: C, B, B	$$: 23 Items: A, B, D	
Greedy	$$: 21 Items: B, B, B	$$: 23 Items: B, D, A	$$: 23.8 $ Items: B, D, C

a) (20 pts) Solve the **unbounded** Knapsack problem. Recover the items in the solution and show how you did that (e.g. highlight or circle cells). Show your work as done in class.

10 − 4 14 − 4

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
sol	0	0	0	4	7	7	10	12	14	14	17	19	21	22	24
picked	–	–	–	A	B	B	C	D	B	A	B	B	B	C	B
A, 3, 4	–	–	–	0, 4	1, 4	2, 4	3, 8	4, 11	5, 11	6, 14	7, 16	8, 18	9, 18	10, 21	11, 23
B, 4, 7	–	–	–	–	0, 7	1, 7	2, 7	3, 11	4, 14	5, 14	6, 17	7, 19	8, 21	9, 21	10, 24
C, 6, 10	–	–	–	–	–	–	0, 10	1, 10	2, 10	3, 14	4, 17	5, 17	6, 20	7, 22	8, 24
D, 7, 12	–	–	–	–	–	–	–	0, 12	1, 12	2, 12	3, 16	4, 19	5, 19	6, 22	7, 24

B = 14 − 4 ⇒ 10 ⇒ B ⇒ 10 − 4 = 10 − 4 = 6 ⇒ C ⇒ 6 − 6 = 0

b) (20 pts) Solve the **0/1** knapsack problem below (15pts). Use a star to show if the current item was used or not in the solution (8pts). Recover the items in the solution and show how you did that (e.g. highlight or circle cells) (7 pts). Show your work as done in class.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A, 3, 4	0	0	0	4*	4*	4*	4*	4*	4*	4*	4*	4*	4*	4*	4*
B, 4, 7	0	0	0	4	7*	7*	7*	11*	11*	11*	11*	11*	11*	11*	11*
C, 6, 10	0	0	0	4	7	7	10*	11	11	14*	17*	17*	17*	21*	21*
D, 7, 12	0	0	0	4	7	7	10	12*	12*	12*	17	19*	19*	22*	23*

D = 14 − 7 = 7 = C (we didn't pick) so go up = D

Actual value is 10. 10 < 11 so pick (11) → Actual value = 16. 16 < 17 so pick 17

c) (8 pts) What items will a Greedy algorithm based on the **ratio**, choose for an **unbounded** knapsack problem of size 14? Show your work.

Item : A B C D
Weight : 3 4 6 7
Value : 4 7 10 12

A → Value/Weight = 4/3 = 1.3
B → 7/4 = 1.75
C → 10/6 = 1.6
D → 12/7 = 1.71

→ Highest → B, weight of B = 4, Size = 14
14 − 4 = 10
→ Be greedy, pick B again 10 − 4 = 6
→ we can pick C, But Being greedy and pick → B
→ B, B, B 7 + 7 + 7 = 21$

P2) a) Recover the items in the Solution

→ unbounded knapsack

Max_weight → 14

B is at 14

B has a weight 4

14 − 4 = 10

B is at 10, weight of B is 4

10 − 4 = 6

C is at C, weight of C is 6

6 − 6 = 0

So picked items = C, B, B.

P2) b) 0/1 knapsack.

Max_value at D which is at weight 14

Weight of D 7

14 − 7 = 7 Remaining

↳ check the row above 17) Actual value is 11 but here we have not picked C. So go to a row above that is B.

weight of B = 4 7 − 4 = 3

↳ go to a Row above it at 3

which is A

Weight of A = 3

3 - 3 = 0

So the item picked → A, B, D

d) (8 pts) What items will a Greedy algorithm based on the **ratio**, choose for a **0/1** knapsack problem of size 14? Show your work.

A → value/weight = 4/3 = 1.3

B → 7/4 = 1.75

C → 10/6 = 1.6

D → 12/7 = 1.71

Highest → B, weight of B = 4 Size = 14

14 − 4 = 10

→ You cant choose B, Next Highest → D = 10 − 7 = 3

→ You cant choose A, Now choose A 3 → 3 = 0

→ B, D, A ⇒ 7 + 12 + 4 = 23$

e) (5 pts) What items will a Greedy algorithm based on the **ratio**, choose for a **fractional** knapsack problem of size 14? Assume you have only one of each item. Show your work.

A → 1.3

B → 1.75

C → 1.6

D → 1.71

Highest → B, weight of B = 4 → Size 14

14 − 4 = 10

→ Choose Next Highest → D → 10 − (weight of D) = 10 − 7 = 3

→ Choose C for a Fraction

→ B, D, C = 7 + 12 + 3 × 1.6 = 23.8

P3 (50 pts) Consider this recursive function:

```
int foo(int N){
    if (N <= 1) return 5;
    int res1 = 3*foo(N/2);
    int res2 =   foo(N-1);
    if (res1 >= res 2)
        return res1;
    else
        return res2;
}
```

a) (6 points) Write the recurrence formula for the TIME COMPLEXITY of this function, including the base cases for N>=0. You do NOT need to solve the recurrence. Remember not to confuse the time complexity of the function with what the function calculates.

b) (8 points) Draw the tree that shows the function calls performed in order to compute foo(5) (the root will be foo(5) and it will have a child for each recursive call.) Also show what each call returns by using an arrow pointing back from the child to the parent.

(P3)

a) what is the recurrence formula for the time complexity of the function, including the base case for $N>=0$.

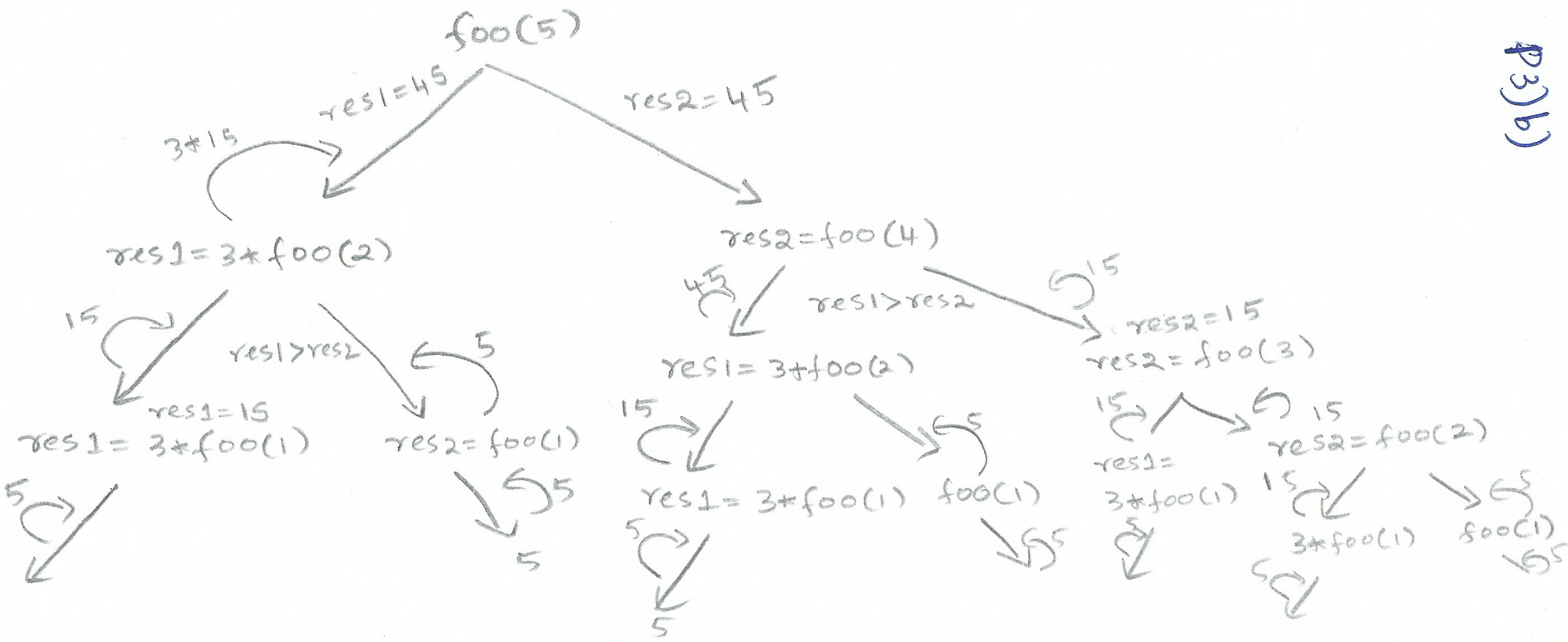
```
int foo (int N) {
if (N<=1) return 5;          -> Constant C
int res1 = 3*foo(N/2);       ->3T(N/2)
int res2= foo(N-1);          -> T(N-1)
if(res1 >= res2)
 return res1;
else
 return res2;
}
```

Recurrance formula :->

$$T(N) = 3T(N/2) + T(n-1) + C$$

$$T(N) = \begin{cases} C & \text{when } N<=1 \\ 3T(N/2) + T(N-1) & N>1 \end{cases}$$

P3)b)


foo(5)
res1=45
3*15
res2=45
res1=3*foo(2)
res2=foo(4)
15
res1>res2
5
res1=15
res1= 3*foo(1)
res2=foo(1)
5
5
5
45
res1>res2
15
res2=15
res1= 3*foo(2)
res2=foo(3)
15
5
res1= 3*foo(1)
foo(1)
5
5
5
15
15
res1=
3*foo(1)
res2=foo(2)
5
15
5
3*foo(1)
foo(1)
5
5