Cheat Sheet for 2320

(Last updated: 5/8/2018)

a) Logarithm and exponential formulas:

1	2	3	4
$\log_a x = \frac{\log_b x}{\log_b a}$	$a^{\log_b x} = x^{\log_b a}$	$(x^a)^b = (x^b)^a$	$x^a * x^b = x^{a+b}$

b) Summation of consecutive values: $1+2+3+ \ldots +n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

c) Summation of squares: $1+2^2+3^2+ \ldots +n^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

d) Summation of Arithmetic series (where $a_i = a_1 + (i-1)d$): $\sum_{i=1}^{n} a_i = n\frac{(a_1 + a_n)}{2} = \frac{n}{2}[2a_1 + (n-1)d]$

e) Summation of Geometric Series: $1+x+x^2+ \ldots x^n$

0<x<1	***x>1***	***x=1***
$\sum_{k=0}^{n} x^k \le \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$	$\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$	$\sum_{k=0}^{n} 1^k = n+1$

f) Harmonic series: $\ln(n+1) \le \sum_{k=1}^{n} \frac{1}{k} \le \ln n + 1$

g) $\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}$, *for* $|x|<1$ (CLRS pg.1148)

h) $\sum_{k=0}^{\infty} \frac{1}{k!} = e = 2.718281... \cong 2.72$

i) Approximation by integrals (CLRS, 1154):

f(x) monotonically **increasing** $x \le y \Rightarrow f(x) \le f(y)$	***f(x)*** is monotonically **decreasing** $x \le y \Rightarrow f(x) \ge f(y)$
$\int_{m-1}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m}^{n+1} f(x)dx$	$\int_{m}^{n+1} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m-1}^{n} f(x)dx$

j) Radix sort (optimal r): r = min{b, floor(lg N)}

k) Master Theorem: Let $a \geq 1$ and $b > 1$, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence: $\boldsymbol{T(n) = aT(n/b) + f(n)}$, where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ has the following asymptotic bounds:

1. If $f(n) = O(n^{\log_b a-\varepsilon})$ for some constant $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \lg n)$.
3. If $f(n) = \Omega(n^{\log_b a+\varepsilon})$, for some constant $\varepsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta(f(n))$.

l) L'Hospital rule: If $\lim\limits_{n\to\infty} f(n)$ and $\lim\limits_{n\to\infty} g(n)$ are both 0 or $\pm\infty$ and if $\lim\limits_{n\to\infty} \frac{f'(n)}{g'(n)}$ is a constant or $\pm\infty$, then:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)} .$$

m) $f(n) = \boldsymbol{O}(g(n))$ if there exist positive constants c_0 and n_0 such that:

$$\boldsymbol{f(n)} \leq c_0\, g(n) \quad \text{for all } n \geq n_0.$$

Theorem: if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, then $f(n) = \boldsymbol{O}(g(n))$.

n) $f(N) = \boldsymbol{\Omega}(g(n))$ if there exist positive constants c_0 and n_0 such that:

$$c_0\, g(n) \leq \boldsymbol{f(n)} \quad \text{for all } n \geq n_0.$$

Theorem: if $\lim_{n\to\infty} \frac{g(n)}{f(n)} = c$, then $f(n) = \boldsymbol{\Omega}(g(n))$.

o) $f(n) = \boldsymbol{\Theta}(g(n))$ if there exist positive constants c_0 , c_1 and n_0 such that:

$$c_0\, g(n) \leq \boldsymbol{f(n)} \leq c_1\, g(n) \quad \text{for all } n \geq n_0.$$

Theorem: if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \mathbf{c \neq 0}$, then $f(n) = \boldsymbol{\Theta}(g(n))$.