HOMEWORK 2

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SECTION 002 (MW 2:30 - 3:50PM)

TASK 1

a) Give the answers for the two problems below and Justify your answer using the limit theorom and computing the actual limit.

a) is $2^{n+1} = O(2^n)$?

As per the notation, we know that

$f(n) = O(g(n)) \rightarrow$ Asymptotic upper Bound

Also According to the Limit theorom

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \text{ or } C \quad \text{—1}$$

where, $C \geq 0$ $C \neq \infty$

We have $f(n) = 2^{n+1}$ and $g(n) = 2^n$

Substitute these values in eq 1

$$\lim_{n\to\infty} \frac{2^{n+1}}{2^n} = \lim_{n\to\infty} \frac{\cancel{2^n} \cdot 2}{\cancel{2^n}}$$

$$= \lim_{n\to\infty} 2 = 2$$

2 is a constant and 2 is greater than 0.

Therefore, it is proved that

$2^{n+1} = O(2^n)$ is valid.

TASK 1

b) is $2^{2n} = O(2^n)$

As per the notation, we know that

$f(n) = O\,g(n) \rightarrow$ Asymptotic upper bound

Also according to the limit therorom

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \text{ or } c \quad \text{—— (1)}$$

where, $c \geq 0$ $\quad c \neq \infty$

we have $f(n) = 2^{2n}$ $\quad g(n) = 2^n$

Substitute these values in equation (1)

$$\lim_{n \to \infty} \frac{2^{2n}}{2^n} = \lim_{n \to \infty} \frac{2^n \cdot \cancel{2^n}}{\cancel{2^n}}$$

$$= \lim_{n \to \infty} 2^n = \underline{\underline{\infty}}.$$

Therefore, we proved that

$2^{2n} = O(2^n)$ is not valid.

TASK 2

a) Let $f(n) = \left(4/9\right)^0 + \left(4/9\right)^1 + \left(4/9\right)^2 + \cdots \left(4/9\right)^n$

Find Θ for $f(n)$

Here function $f(n)$ is in geometric series

with common ratio is $r = \dfrac{\left(4/9\right)^1}{\left(4/9\right)^0} = \left(4/9\right)$

and $0 < x < 1$ $\quad x = 4/9$

we have Finite summation

$$\sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1-x} \quad -(1)$$

Substitute $x = 4/9$ in eq (1)

$$\sum_{k=0}^{n} \left(4/9\right)^k = \frac{1 - \left(4/9\right)^{n+1}}{\left(1 - 4/9\right)}$$

$$= \left(4/9\right)^n \left(4/9\right)^1$$

Dominant term is 1

Hence $\displaystyle\sum_{k=0}^{n} \left(4/9\right)^k = \Theta(1)$

$$f(n) = \Theta(1)$$

b) Use the definition with constant to show

$f(n) = n\lg(n) - 15n + 14\sqrt{n}$ is $\Theta(n\lg(n))$

According to the definition,

There exist positive constants, c_0, c_1 and

n_0 such that,

$c_0 g(n) \leq f(n) \leq c_1 g(n)$ for all $n \geq n_0$

$c_0 n\lg(n) \leq n\lg(n) - 15n + 14\sqrt{n} \leq c_1 n\lg(n)$

$\div\ n\lg(n)$

$$c_0 \leq 1 - \frac{15n}{n\lg(n)} + \frac{14\sqrt{n}}{n\lg(n)} \leq c_1$$

Substitute $c_0 = 0.1$ $n = 1$ $c_1 = 2$

$$0.1 \leq 1 - \frac{15 \times 1}{1 \times \lg(1)} + \frac{14\sqrt{1}}{1 \times \lg(1)} \leq 2$$

$0.1 \leq 1 - 0 + 0 \leq 2$

$\boxed{0.1 \leq 1 \leq 2}$

for $c_0 = 0.1$ $c_1 = 2$ $n \geq 1$

$f(n) = \Theta(n\lg(n))$

TASK 3

COMPILATION INSTRUCTIONS

→ Connect to omega Server

→ Create a folder hw2 (mkdir) in the server
[mkdir hw2]

→ copy the code (search.c) into the folder
hw2

→ Make sure (Search.c) has read|write|Execute
If not do chmod 777 Search.c

→ COMPILE

```
gcc -o Search Search.c
```

→ EXECUTION INSTRUCTIONS

Run

```
./Search
```