# Unit 4

## Infinite Series

**Sequence:→** A sequence is a succession of number or terms formed according to some definite rule.

The nth term in sequence is denoted by $u_n$

**Example:→** if $u_n = 2n+1$

So by giving different value of n in $u_n$, we get different terms of sequence

like $u_1 = 3$, $u_2 = 2(2)+1 = 5$, $u_3 = 2(3)+1 = 7$...

- A sequence having unlimited number of terms is known as an Infinite sequence.

**Limit:→** If a sequence tends to a limit $l$, then we write

$$\boxed{\lim_{n\to\infty} u_n = l}$$

**Convergence of Sequence:→**

If the limit ($l$) of a sequence is finite, then sequence is called convergent.

If the limit ($l$) of a sequence does not tend to a finite number, then the sequence is said to be divergent.

**Ex:→** $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots \frac{1}{n^2} \ldots$ is a "convergent sequence".

$3, 5, 7, \ldots (2n+1)$ is a "divergent sequence".

**Bounded sequence:**→ $u_1, u_2, u_3 \ldots u_n$ is a bounded sequence if $u_n < K$ for every $n$.

**Monotonic Sequence:**→ The sequence is either increasing or decreasing, such sequence are called monotonic

**Ex.** (i) $1, 4, 7, 10 \ldots$ is an increasing (monotonic) sequence.

(ii) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ is a decreasing (monotonic) sequence.

(iii) $1, -1, 1, -1, 1 \ldots$ is not a monotonic sequence.

**Note:**→ "A sequence which is monotonic and bounded is a convergent Sequence."

**Questions:**→ **Determine the general term of each of the following Sequence. Prove that the following sequence are convergent.**

**Q.(1)**→ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots$

**Sol:**→ General term :- $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4} \ldots \frac{1}{2^n}$

$$u_n \Rightarrow \frac{1}{2^n}$$

Yes it is convergent, as $n \to \infty$ then sequence tends to zero (finite value)

**Q.2:→** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5} \dots$

**Sol:→** General term, $u_n = \frac{n}{n+1}$

Now checking the convergency:→

thus $\lim_{n\to\infty}\left(\frac{n}{n+1}\right) = \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}}\left(\frac{1}{1+1/n}\right) = \lim_{n\to\infty} \frac{1}{(1+1/n)}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{(1+1/n)} = 1$ (finite)/(No.).

Hence sequence is convergent.

**Q.3:→** Is the sequence $u_n = 3n$ is convergent?

**Sol:→** No, because $\lim_{n\to\infty} u_n = \lim_{n\to\infty} 3n \Rightarrow \infty$ (not tend to finite no.)

**Q.4:→** Is the sequence $u_n = 1/n$ is convergent?

**Sol:→** Yes, because $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n} = 0$ (finite number)

## Remember The following limits

(i) $\lim_{n\to\infty} x^n = 0$ if $x<1$ and $\lim_{n\to\infty} x^n = \infty$ if $x>1$

(ii) $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for all values of $x$

(iii) $\lim_{n\to\infty} \frac{\log n}{n} = 0$ (iv) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ (v) $\lim_{n\to\infty}(n)^{1/n} = 1$

(vi) $\lim_{n\to\infty} [n!]^{1/n} = \infty$ (vii) $\lim_{n\to\infty}\left[\frac{n!}{n}\right]^{1/n} = 1/e$ (viii) $\lim_{n\to\infty} nx^n = 0$ if $x<1$

(ix) $\lim\limits_{n\to\infty} n^{h} = \infty$ (x) $\lim\limits_{n\to\infty} \frac{1}{n^{h}} = 0$ (xi) $\lim\limits_{n\to\infty} \left[\frac{a^{x}-1}{x}\right] = \log a$

OR $\lim\limits_{n\to\infty} \frac{a^{1/n}-1}{1/n} = \log a$

(xii) $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

(xiii) $\lim\limits_{x\to 0} \frac{\tan x}{x} = 1$