Case (III) Working Rule to finding P.I.

(When $f(x,y) = \sin(ax+by)$ OR $\cos(ax+by)$)

Then
$$\boxed{P.I. = \frac{\sin(ax+by)}{F(D^2, DD', D'^2)} = \frac{1}{F(-a^2, -ab, -b^2)} \sin(ax+by)}$$

Similarly for $\cos(ax+by)$, P.I. remains same.

Simply ↪ put $D^2 = -a^2$ ✓

$DD' = -ab$ ✓

and $D'^2 = -b^2$ ✓

Ex. (1) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$

$\longrightarrow (D^2 + 2DD' + D'^2)z = \sin(2x+3y)$

Here $a=2$, $b=3$

Now A.E. $\Rightarrow m^2 + 2m + 1 = 0$

$\Rightarrow (m+1)^2 = 0$

$\Rightarrow m = -1, -1$

$C.F. = f_1(y-x) + x f_2(y-x)$

And $P.I. = \frac{\sin(2x+3y)}{D^2 + 2DD' + D'^2}$

$$\Rightarrow \frac{1}{-4+2(-2\times 3)-9}\sin(2x+3y)$$

Acc to Rule:→ put $D^2 = -a^2$, $DD' = -ab$ and $D'^2 = -b^2$

$$\Rightarrow P.I. = \frac{\sin(2x+3y)}{-25}$$

Hence complete sol:→

$$Z = C.F. + P.I.$$

$$\text{OR } Z = f_1(y-x) + x f_2(y-x) - \frac{1}{25}\sin(2x+3y)$$

Ex. (2) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x+2y)$

$$\Longrightarrow D^3 - 4D^2D' + 4DD'^2 = 2\sin(3x+2y)$$

$$A.E. \Rightarrow m^3 - 4m^2 + 4m = 0$$

$$\Rightarrow m(m^2 - 4m + 4) = 0$$

$$\Rightarrow m = 0, 2, 2$$

$$C.F. = f_1(y) + f_2(y+2x) + x f_3(y+2x)$$

$$\text{Now } P.I. = \frac{2\sin(3x+2y)}{D^3 - 4D^2D' + 4DD'^2}$$

$$= \frac{2\sin(3x+2y)}{D(D^2 - 4DD' + 4D'^2)}$$

Here $a=3, b=2$

then $D^2 = -a^2$

put $DD' = -ab$

$D'^2 = -b^2$

which is

$D^2 = -9$

$DD' = -6$

$D'^2 = -4$

$$P.I = \frac{2\sin(3x+2y)}{D[-9-4(-6)+4(-4)]}$$

$$= \frac{2\sin(3x+2y)}{D[-9+24-16]}$$

$$= \frac{2\sin(3x+2y)}{D(-1)}$$

$$= -\frac{2}{D}\sin(3x+2y)$$

$$= -2\int \sin(3x+2y)\,dx \quad \left\{\because D = \int dx\right\}$$

$$= \frac{-2}{3}\left[-\cos(3x+2y)\right]$$

$$P.I. = \frac{2}{3}\cos(3x+2y)$$

Thus Complete solution

$$Z = C.F. + P.I.$$

$$= f_1(y) + f_2(y+2x) + x f_3(y+2x) + \frac{2}{3}\cos(3x+2y)$$

Ans.