Q1) Sol→ $\sum \frac{\sqrt{n}}{n^2+1}$ $\qquad U_n = \frac{\sqrt{n}}{n^2+1} \qquad U_{n+1} = \frac{\sqrt{n+1}}{(n+1)^2+1}$

by D-Alembert test ⇒ $\lim_{n\to\infty} \frac{U_{n+1}}{U_n}$

$$\lim_{n\to\infty} \frac{\sqrt{n+1}}{(n+1)^2+1} \times \frac{n^2+1}{\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{\cancel{\sqrt{n}}\left[\sqrt{1+\frac{1}{n}}\right]}{\cancel{n}\left[1+\frac{1}{n}\right]^2+1} \times \frac{n^{\cancel{2}}\left[1+\frac{1}{n}\right]}{\cancel{\sqrt{n}}}$$

$$\lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{n}}}{\left[1+\frac{1}{n}\right]^2+1} \times n\left[1+\frac{1}{n}\right]$$

$$\Rightarrow 0$$

Hence ~~the~~ It is divergent as $0 < 1$

$[k < 1]$

Q2) $\sum \sqrt[3]{n^3+1} - n$

Sol $\Rightarrow$ ~~$U_n = (n^3+1)^{1/3} + n$, $V_n = n + 1$~~

$U_n = (n^3+1)^{1/3} - n$

$U_{n+1} = [(n+1)^3+1]^{1/3} - (n+1)$

$U_{n+1} \Rightarrow [n^3+1+3n^2+3n+1]^{1/3} - (n+1)$

$\Rightarrow n\left[1+\frac{3}{n}+\frac{3}{n^2}+\frac{2}{n^3}\right]^{1/3} - (n+1)$

$\Rightarrow n\left[\left(1+\frac{3}{n}+\frac{3}{n^2}+\frac{2}{n^3}\right)^{1/3} - \left(1+\frac{1}{n}\right)\right]$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} \Rightarrow \frac{n\left[\left(1+\frac{3}{n}+\frac{3}{n^2}+\frac{2}{n^3}\right)^{1/3} - \left(1+\frac{1}{n}\right)\right]}{(n^3+1)^{1/3} - n}$$

$$\lim_{n\to\infty} \frac{\cancel{n}\left[\left(1+\frac{3}{n}+\frac{3}{n^2}+\frac{2}{n^3}\right)^{1/3} - \left(1+\frac{1}{n}\right)\right]}{\cancel{n}\left[\left[1+\frac{1}{n^3}\right] - 1\right]}$$

$= 0$

divergent as $0 < 1$

Q3) a) $\sum_{n=1}^{\infty} \frac{n^2}{3n}$

$U_n = \frac{n^2}{3n}$ $\qquad V_n = n$

$\lim_{n\to\infty} \frac{U_n}{V_n} = \frac{n^2}{3n} \times \frac{1}{n} \Rightarrow \frac{1}{3}$

$\frac{1}{3} < 1$ finite

$\sum U_n$ & $\sum V_n$ converse & diverge together

$\sum V_n = \sum \frac{1}{n^{1/3}}$ is of the form $\sum \frac{1}{n^p}$

Hence $p = \frac{1}{3} < 1$

So according to p series test $\sum V_n$ is divergent $\sum U_n$ is also divergent

Q(b) $\sum_{n=1} \frac{n!}{n^n}$

$U_n = \frac{n!}{n^n}$ $\qquad U_{n+1} = \frac{(n+1)!}{(n+1)^{(n+1)}}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} \Rightarrow \frac{(n+1)!}{(n+1)^{(n+1)}} \times \frac{n^n}{n!}$

$\lim_{n\to\infty} \Rightarrow \frac{\cancel{(n+1)!}}{(n+1)^n (n+1)} \times \frac{n^n}{n\cancel{(n+1)!}}$

$\lim_{n\to\infty} \frac{\cancel{n^{n-1}}}{\cancel{(n+1)}(n+1)^n}$

$\lim_{n\to\infty} \frac{n^n}{(n+1)^{n+1}\, n}$

$\Rightarrow 0$

~~diverge~~ The series is convergent as $0 < 1$

c) $\sum_{n=1}^{\infty} \frac{n!\,2^n}{n^n}$

$U_n = \frac{n!\,2^n}{n^n}$ $\qquad U_{n+1} = \frac{(n+1)!\,2^{(n+1)}}{(n+1)^{(n+1)}}$

by D'Alebert rule

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} \Rightarrow \frac{(n+1)!\,2^{(n+1)}}{(n+1)^{(n+1)}} \times \frac{n^n}{n!\,2^n}$

$$\lim_{n\to\infty} \frac{(n+1)!\, 2^{n}.2}{(n+1)^{(n+1)}} \times \frac{n^n}{n(n+1)!\, 2^n}$$

$$\lim_{n\to\infty} \frac{\cancel{2n^n}}{\cancel{(n+1)}} \quad 2\left[\frac{n^{n-1}}{(n+1)^{(n+1)}}\right] = 2 > 1$$

The series is divergent at $2 > 1$

d) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n \cdot 3^n}$

$$U_n = \frac{x^{n-1}}{n \cdot 3^n} \qquad U_{n+1} = \frac{x^{n+1-1}}{(n+1)\, 3^{(n+1)}}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} \Rightarrow \lim_{n\to\infty} \frac{x^n}{(n+1)3^{(n+1)}} \times \frac{n \cdot 3^n}{x^{n-1}}$$

$$\lim_{n\to\infty} \frac{\cancel{x^n}}{(n+1)\,\cancel{3^n}.3} \times \frac{n.\cancel{3^n}}{\cancel{x^n}.(-x)}$$

$$\lim_{n\to\infty} \frac{n}{3(-x)n\left[1+\frac{1}{n}\right]} \Rightarrow \lim_{n\to\infty} -\frac{1}{3}\left[\frac{1}{x}\right]$$

By D'Alembert test $\sum U_n$ Converges ~~if x>1~~ & diverges if x <1

e) $\frac{2}{1} + \frac{2\cdot5\cdot8}{1\cdot5\cdot9} + \frac{2\cdot5\cdot8\cdot11}{1\cdot5\cdot9\cdot13} + \cdots \infty$

$U_n = \frac{2\cdot5\cdot8\cdots(3n-1)}{1\cdot5\cdot9\cdots(4n-3)}$ $\quad U_{n+1} = \frac{2\cdot5\cdot8\cdots(3n+2)}{1\cdot5\cdot9\cdots(4n+1)}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} \Rightarrow \frac{2\cdot5\cdot8\cdots(3n+2)}{1\cdot5\cdot9\cdots(4n+1)} \times \frac{1\cdot5\cdot9\cdots(4n-3)}{2\cdot5\cdot8\cdots(3n-1)}$

$\cancel{\lim_{n\to\infty} 2\cdot5\cdot8\cdots(3n+1)}\ 3n$

$\lim_{n\to\infty} \frac{\cancel{2\cdot5\cdot8\cdots(3n-1)}(3n)(3n+1)(3n+2)}{\cancel{1\cdot5\cdot9\cdots(4n-3)}(4n-2)(4n-1)(4n)(4n+1)}$

$\frac{\cancel{2\cdot5\cdot8\cdots(3n-1)}}{\cancel{1\cdot5\cdot9\cdots(4n-3)}}$

$\lim_{n\to\infty} \frac{3\cancel{n}(3n+1)(3n+2)}{4\cancel{n}(4n-2)(4n-1)(4n+1)}$

$\lim_{n\to\infty} \frac{3n^2\left[3+\frac{1}{n}\right]\left[3+\frac{2}{n}\right]}{4n^3\left[4-\frac{2}{n}\right]\left[4-\frac{1}{n}\right]\left[4+\frac{1}{n}\right]} \Rightarrow 0$

~~Hence the~~ $0<1$ then the series is divergent

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1) $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \cdots \infty$

$\sum_{n=1}^{\infty} \frac{n}{(1+2^n)}$

$U_n = \frac{n}{1+2^n}$ $U_{n+1} = \frac{n+1}{1+2^{n+1}}$

D'Albert therm $\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n+1}{1+2^{n+1}} \times \frac{1+2^n}{n}$

$\lim_{n\to\infty} \Rightarrow \frac{\cancel{n}\left[1+\frac{1}{n}\right]}{1+2^n \cdot 2} \times \frac{2^n\left(1+\frac{1}{2^n}\right)}{\cancel{n}}$

$\lim_{n\to\infty} \Rightarrow \frac{\left[1+\frac{1}{n}\right]}{\cancel{2^n}\left[\frac{1}{2^n}+2\right]} \times \cancel{2^n}\left[1+\frac{1}{2^n}\right]$

$\Rightarrow \frac{1}{2} < 1$

then the series is divergent as $\frac{1}{2} < 1$