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UNIT-1

Definition of set → A set is any well defined class or collection of objects.
By a well-defined collection we mean that there exists a rule with help of which it is possible to tell wheather a given object belongs or does not belong to the given collection.
The objects in sets may be anything, Numbers, people, mountains, rivers etc.

Example →

(i) The Numbers 2, 4, 6 and 1
(ii) The Countries India, Burma and Afghanistan
(iii) The rivers in India
(iv) The set of all triangles in a plane.
(v) The Numbers 1, 3, 5, 7.

A set may be described by actually listing the objects belonging to it.
For example, let the elements of the set A be a, e, i, o u then we write
A = {a, e, i, o, u}. This is called tab form of the set.

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Finite and Infinite Sets
(Countable set and Uncountable set),

* finite Set → A set is said to be finite if it consists of a specific Number of different elements i.e if in counting the different members of the set the counting process can come to an end.

Example Let A be the set of the days of the week. Then A is finite.

* Infinite set → A set is said to be infinite set if it Consists Number of Infinite set.

Example → The set of Real Numbers

* Countable set → An infinite set A said to be Countable set if it is equivalent to be the Set of Natural Numbers.
A set which is either empty, finite or Countable is called countable set

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otherwise uncountable.

Example → Countable set →

set of Natural Numbers
$N = \{1, 2, 3, 4 \ldots\ldots\}$

Uncountable set → set of Real Numbers

$R = \{-\infty \ldots -4, -3, -2, -1, 0, 1, 2, 3 \ldots \infty\}$

* Singleton set :- If a set contains only one elements, then it is called a singleton set.

Example $A = \{x : x$ is an even prime Number$\}$

* The power set of a set A is the set which consists of all the subsets of the set A. it is denoted by P(A). and Number of element of set is 2^N. where N is Number of element.

* Sub set :- If a set A contains element which are all the elements of set B as well, then A is known as the subset of B.

$P(A) = \{\phi, \{-9\}, \{1,3\}, \{6\}, \{-9, 13\}, \{13, 6\}, \{6, -9\}, \{-9, 13, 6\}\}$.

Venn-Euler Diagrams

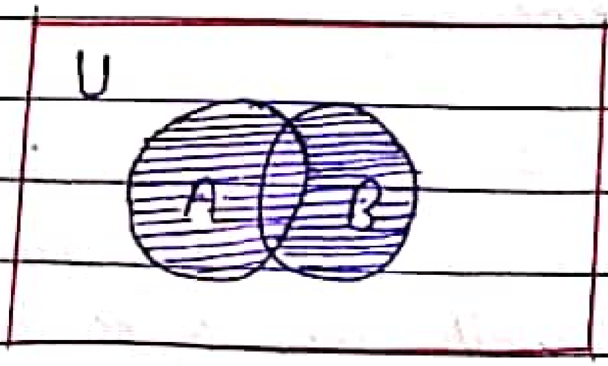
We consider a pictorial representation of sets. These pictures consist of rectangular and ~~circles~~ closed curves usually circles. These combinations of rectangles and circles are called Venn-Euler diagrams. or simply Venn-diagrams.

Union of Sets

Let A and B be two sets. The union of A and B is the set of all elements which are in set A or in B. We denote the union of A and B by $A \cup B$.

Symbolically $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Example If $A = \{1, 3, 5, \ldots\}$ and $B = \{2, 4, 6, \ldots\}$ then $A \cup B = \{1, 2, 3, 4, 5, 6, \ldots\}$



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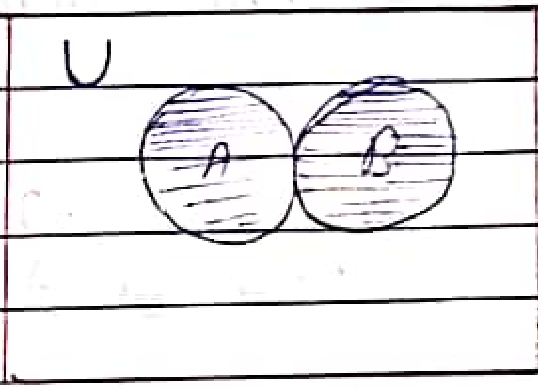
* Intersection of Sets :

Let A and B be two sets. The intersection of A and B is the set of all elements which are both in A and B.

Example → Let $A = \{1, 3, 7, 8\}$ and $B = \{2, 4, 9, 11\}$

$A \cap B = \phi$



(operation on set)

Laws of Algebra of Sets → If A, B and C are any sets then.

(i) $A \cup B = B \cup A$
and $A \cap B = B \cap A$ } → Commutative laws.

(ii) $A \cup (B \cup C) = (A \cup B) \cup C$
and $A \cap (B \cap C) = (A \cap B) \cap C$ } → Associative laws.

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(iii) $A \cup A = A$ and $A \cap A = A$ } → Idempotent laws.

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ } → Distributive laws

(v) $A - (B \cup C) = (A - B) \cap (A - C)$ and $A - (B \cap C) = (A - B) \cup (A - C)$ } → De Morgan's laws

(vi) $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$ } De-Morgan's laws.

Q 1. To prove that $A \cup B = B \cup A$.

proof We have $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

$\Leftrightarrow x \in B$ or $x \in A$

$\Leftrightarrow x \in B \cup A$

Consequently, $A \cup B \subset B \cup A$ and $B \cup A \subset A \cup B$

Hence $\boxed{A \cup B = B \cup A}$

Q2. To prove that $A \cap (B \cap C) = (A \cap B) \cap C$.

proof → We have $x \in A \cap (B \cap C)$

$\Rightarrow x \in A$ and $x \in (B \cap C)$

$\Leftrightarrow x \in A$ and $(x \in B$ and $x \in C)$

$\Leftrightarrow (x \in A$ and $x \in B)$ and $x \in C$

(By law of tautology)

$\Leftrightarrow (x \in A \cap B) \cap C$

Consequently, $A \cap (B \cap C] \subseteq (A \cap B) \cap C$

and $(A \cap B) \cap C \subseteq A \cap (B \cap C)$

Hence $A \cap (B \cap C) = (A \cap B) \cap C$.

Q3. To prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

proof :- We have $x \in A \cap (B \cup C)$

$\Leftrightarrow x \in A$ and $x \in (B \cup C)$

$\Leftrightarrow x \in A$ and $(x \in B)$ or $x \in C)$

$\Leftrightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$ (By tautology)

$\Leftrightarrow (x \in A \cap B)$ or $(x \in A$ and $x \in C)$

$\Leftrightarrow x \in A \cap B$ or $x \in A \cap C$

$\Leftrightarrow x \in (A \cap B) \cup x \in (A \cap C)$

Consequently $A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$.

and $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$.

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

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Homework

if A and B are any sets then prove that

(i) $A \subseteq A \cup B$ and $B \subseteq A \cup B$.

(ii) $A \subseteq B \Rightarrow A \cup B = B$.

(iii) $A - B \subseteq A$.

(iv) $(A - B) \cup A = A$.

(v) $(A - B) \cap B = \phi$.

Soln (i) Let $x \in A \Rightarrow x \in A$ or $x \in B$.

$\Rightarrow x \in A \cup B$.

Therefore $A \subseteq A \cup B$. —— ①

Similarly $B \subseteq A \cup B$ —— ②

from ① and ②

(ii) Let $A \subseteq B$ then to prove that

$A \cup B = B$

Let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$.

$\Rightarrow x \in B$ or $x \in B$.

$\Rightarrow x \in B$.

Consequently

$A \cup B \subseteq B$.

But $B \subseteq A \cup B$.

Hence $A \subseteq B \Rightarrow A \cup B = B$.

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(iii) let x be an arbitrary element of the set $A-B$. Then

$x \in A-B \Rightarrow x \in A$ and $x \notin B$.

$\Rightarrow x \in A$.

Consequently $A-B \subseteq A$.

(iv) Suppose $A \subseteq B$

$x \in A \cup B \Rightarrow x \in A$ or $x \in B$.

$\Rightarrow x \in B$ or $x \in B$.

$\Rightarrow x \in B$

Consequently $A \cup B \subseteq B$.

let $x \in A-B \Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A$.

Consequently

$A-B \subseteq A$.

(v) $x \in (A-B) \cap B$.

$\Rightarrow x \in A-B$ and $x \in B$.

$\Rightarrow x \in A$ and $x \notin B$ and $x \in B$.

$\Rightarrow x \in A$ and $(x \notin B$ and $x \in B)$

$\Rightarrow x \notin B$ and $x \in B$.

But there is no element x which satisfies both $x \notin B$ and $x \in B$.

Therefore there is no element in $(A-B) \cap B$.

i.e $(A-B) \cap B = \phi$.

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* The Product of Set + Cartesian Product

Let $A = \{a, b\}$ and $B = \{b, c, d\}$. The set of distinct ordered pairs

$C = \{(a,b), (a,c), (a,d), (b,c), (b,d)\}$

in which the first component of each pair is an element of A while the second is an element of B, is called the product of set $= A \times B$. Thus if A and B are arbitary sets define

$$A \times B = \{(x,y) : x \in A, y \in B\}$$

Example 1:- $X = \{1, 2, 3\}$ and $Y = \{1, 2, 3, 4\}$

Then Cartesian product

$$X \times Y = \{1,2,3\} \times \{1,2,3,4\}$$
$$= \{(1,1), (1,2), (1,3), (1,4)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\}$$

Example 2:- Let $P = \{a, b, c\}$ and $Q = \{k, l, m, n\}$. Determine the Cartesian product of P and Q.

Soln The Cartesian product of P and Q is

$$P \times Q = \{a, b, c\} \times \{k, l, m, n\}$$
$$= \{(a,k)\ (a,l)\ (a,m)\ (a,n)$$
$$(b,k)\ (b,l)\ (b,m)\ (b,n)$$
$$(c,k)\ (c,l)\ (c,m)\ (c,n)\}$$

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Example 3: Let $R = \{1,2,3\}$ and $S = \{4,5,6\}$. Determine the Cartesian product.

$R \times S = \{1,2,3\} \times \{4,5,6\}$

$= \{(1,4)\ (1,5)\ (1,6),\ (2,4)\ (2,5)\ (2,6)$
$(3,4)\ (3,5)\ (3,6)\}$

Power set : The power set is a set which includes all the subsets including the empty set and the original set itself. It is also a type of sets.

If set $A = \{x,y,z\}$ is a set, then all its subsets $\{x\}, \{y\}, \{z\}$ $\{x,y\}, \{y,z\}, \{x,z\}, \{x,y,z\}$ $\{\}$ & are the elements of power set.
Power set of A, $P(A) = \{x\}, \{y\}, \{z\}$ $\{x,y\}, \{y,z\}, \{x,z\}$ $\{x,y,z\}$ and $\{\}$

Example find the power set of $z = \{2,7,9\}$ and total Number of elements.

Soln power set $P(z) = \{\{\}\ \{2\}\ \{7\}\ \{9\}$
$\{2,7\}\ \{2,9\}\ \{7,9\}$
$\{2,7,9\}\}$

total Number of element
$= 8 = 2^3$

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Relation → Let A and B be two sets. A relation from A to B is a subset of $A \times B$. Symbolically, R is a relation from A to B is a subset of $A \times B$. Symbolically R is a relation from A to B iff $R \subseteq A \times B$.

* **Domain of a relation** → Let R be a relation from A to B, i.e let R be a subset of $A \times B$. The domain D of the relation R is the set of all first elements of the ordered pairs which belongs to R.

Symbolically,

$$D = \{x : x \in A \text{ and } (x, y) \in R \text{ for some } y \in B\}.$$

The range of the relation R is the set of all second elements of the ordered pair which belongs to R.

symbolically,

$$E = \{y : y \in B \text{ and } (x, y) \in R \text{ for some } x \in A\}.$$

Example · let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$

Relation $R = A \times B$

$$= \{(1,a)\ (1,b)\ (1,c)\ (2,a)\ (2,b)\ (2,c)\ (3,a)\ (3,b)\ (3,c)\ (4,a)\ (4,b)\ (4,c)\}$$

Domain = $\{1, 2, 3, 4\}$

Range = $\{a, b, c\}$

Type of Relation ⤵

(1) Identity relation in a set ⤵

Let A be a set. The relation I_A defined by $I_A = \{(x, y) : x \in A, y \in A, x = y\}$

is called the Identity relation in A.

If $A = \{1, 3, 3, 4, 5\}$

$I_A = A \times A$

$= \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$

$I_A = \{(1,1) (2,2) (3,3) (4,4) (5,5)\}$

(2) Universal relation in a set ⤵

Let A be any set and R be the set $A \times A$. Then R is called the universal relation in A.

Equivalence Relation ↓

Let R be a relation in a set A. Then R is an equivalence relation in A iff.

i. Reflexive relation → Let R be a relation in a set A i.e let R be a subset of $A \times A$. Then R is called a reflexive relation if $(a,a) \in R$ $\forall\ a \in R$

Thus R is reflexive if we have aRa $\forall\ a \in A$.

(ii) Symmetric Relation → Let R be a relation in a set A, i.e let R be a subset of $A \times A$. Then R is said to be a symmetric relation if $(a,b) \in R \Rightarrow (b,a) \in R$.

Thus R is symmetric if we have bRa whenever we have aRb.

(iii) Anti-Symmetric relation → Let R be a relation in a set A i.e let R be a subset of $A \times A$. Then R is said to be an anti-symmetric relation if $(a,b) \in R$ and $(b,a) \in R$ $\Rightarrow a = b$.

(iv) Transitive Relation → Let R be a relation in a set A, i.e, let R be a subset of A×A. Then R is said to be a transitive relation if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$.

Example Let $A = \{1,2,3,4\}$ and $R = \{(1,1), (1,3), (2,2)$

soln $(2,4), (3,1), (3,3), (4,2), (4,4)\}$.

$(1,1)(2,2), (3,3)(4,4) \in R \Rightarrow$ reflexive.

$(2,4) \in R \Rightarrow (4,2) \in R \Rightarrow$ symmetric and $(3,1) \in R$, $(1,3) \in R$
$\Rightarrow (3,3) \in R \Rightarrow$ Transitive

Inverse Relation ⇒

Definition → Let R be a relation from A to B. Then inverse relation of R, denoted by R^{-1} is a relation from B to A defined by

$$R^{-1} = \{(y,x) : y \in B, x \in A, (x,y) \in R\}.$$

Example → Let $A = \{a,b,c\}$, $B = \{1,2,3\}$
and $R = \{(a,1), (a,3), (b,3), (c,3)\}$.
Then $R^{-1} = \{(1,a), (3,a), (3,b), (3,c)\}$.

Example → Consider a Relation $R(\leq)$ on the set $A = \{2,3,4,5\}$. Determine its inverse

$R = \{(2,2)(2,3)\}$

<u>Relation</u> = $A \times A$

$= \{2,3,4,5\} \times \{2,3,4,5\}$

$= \{(2,2),(2,3),(2,4),(2,5),(3,2),(3,3)$
$(3,4),(3,5),(4,2)(4,3)(4,4)$
$(4,5),(5,2)(5,3)(5,4)(5,5)\}$

$R = \{(2,2),(2,3),(2,4),(2,5),(3,3),$
$(3,4),(3,5),(4,4),(4,5)(5,5)\}$

$R^{-1} = \{(2,2)(3,2)(4,2),(5,2),(3,3)$
$(4,3)(5,3)(4,4),(5,4)(5,5)\}$

<u>H.Q</u>

Q1. Consider the following relation R on the set of +ve integers. find its inverse
$R = \{(1,1),(1,2),(1,3),(2,1),(3,1)(3,2),$
$(2,3)\}$.

Q2. Let $P = \{1,2,3,4,5\}$ find R^{-1}

Q3. Let $Q = \{1,2,3,6\}$ find R^{-1}

Composition of Relation :-

Composition OF Relation → Consider a relation R_1 from A to B and R_2 be a relation from B to C. Then Composition R_1 and R_2 denoted by $R_1 \circ R_2$ is the relation from A to C and is defined by.

* Example → Let P and Q be the relation on set $A = \{1, 2, 3, 4\}$ defined by

$P = \{(1,2)\ (2,2), (2,3), (2,4), (3,2), (4,2), (4,3)\}$

$Q = \{(2,2), (2,3), (3,2), (3,3), (2,4), (4,1), (4,2)\}$

find (i) $P \circ P$ (ii) $P \circ Q$ (iii) $P \circ P \circ Q$

Soln (i) $P \circ P = \{(1,2)\ (1,3)\ (1,4)\ (2,2)\ (2,3)\ (2,4)\ (3,2)\ (3,3)\ (3,4)\ (4,2)\ (4,3)\ (4,4)\}$

P	P
* 1 → 2	1 2
# 2 → 2	2 → 2 * # x o
• 2 → 3	• 2 → 3 * # x o
+ 2 → 4	2 → 4 * # x o
x 3 → 2	3 → 2 • □
o 4 → 2	4 → 2 +
□ 4 → 3	4 → 3 +

(ii) P

	P	Q	
*	1→2	2→2	* □ • +
□	2→2	2→3	* □ • +
o	2→3	3→2	o
×	2→4	3→3	o −
•	3→2	3→4	o −
+	4→2	4→1	×
−	4→3	4→2	×

PoQ = {(1,2) (1,3) (2,2) (2,3) (2,4) (2,1) (3,2) (3,3) (4,2) (4,3) (4,4)}.

(iii) PoP Q

	PoP	Q	
*	1→2	2→2	* + B C
o	1→3	2→3	* + B C
×	1→4	3→2	o − E
+	2→2	3→3	o − A E
−	2→3	3→4	o − A
•	2→4	4→1	× • D F
A	3→3	4→2	• F
B	4→2		
C	3→2		
D	3→4		
E	4→3		
F	4→4		

PoPoQ = {(1,2), (1,3), (1,4), (1,1), (2,2), (2,3), (2,4), (2,1), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3)}

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Q.4 ~~Let~~ Let $P = \{2, 3, 4, 5\}$. Consider the relation R and S on P defined by
$R = \{(2,2), (2,3), (2,4), (2,5)\ (3,4)\ (3,5)$
$(4,5), (5,3)\}$
$S = \{(2,3), (2,5), (3,4), (3,5), (4,2),$
$(4,3), (4,5), (5,2), (5,5)\}$

Find the following composition of the relation R and S.

(i) RoS (ii) RoR (iii) SoR

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Function ⤵

Definition → A function f from a set P into a set Q is a relation from P to Q such that each element of P is related to exactly one element of the set Q. It is denoted as $f : P \to Q$ and read as "f is a function from P to Q"

* Domain of a Function ⤵

Let f be a function from P to Q. The set P is called domain of the function f.

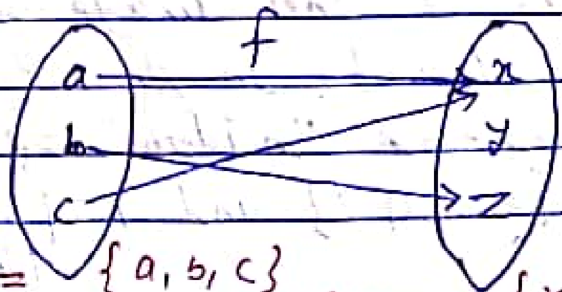
* Co-Domain of a function ⤵

Let f be a function from P to Q. The set Q is called co-domain of the function f.

* Range of a function ⤵

The Range of a function is the set of Image of its domain.

Example



Domain = {a, b, c}
Co-domain = {x, y, z} Range = {x, z}.

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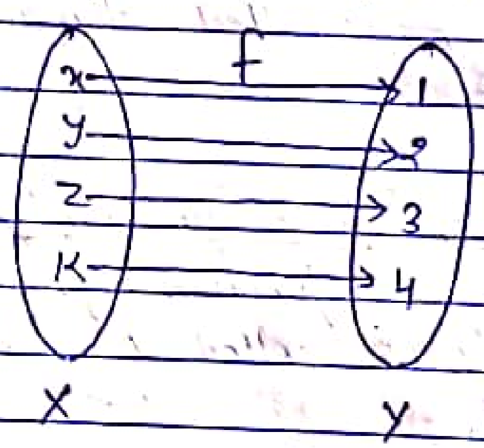
TYPES OF FUNCTION :-

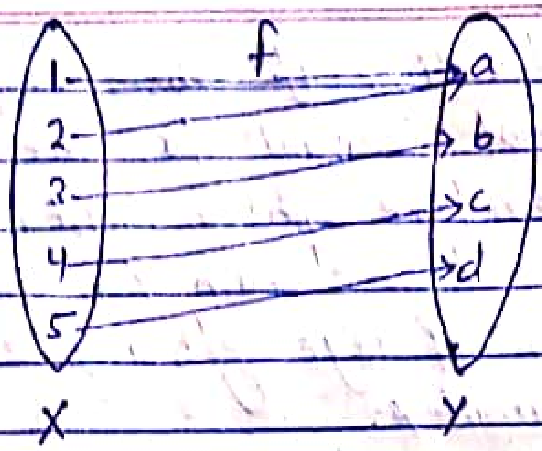
① Injective (one to one) Function :-
Let $f: X \to Y$. The function f is called one-one or injective if different element in X have different image in Y.

Example → Consider $X = \{x, y, z, k\}$ and $Y = \{1, 2, 3, 4\}$ and f is function from X to Y such that
$f = \{(x,1), (y,2), (z,3), (k,4)\}$



② Surjective (onto) Function :-
Let $f: X \to Y$. The function f is called Surjective function if each element in Y, is the image of at least one element in X.

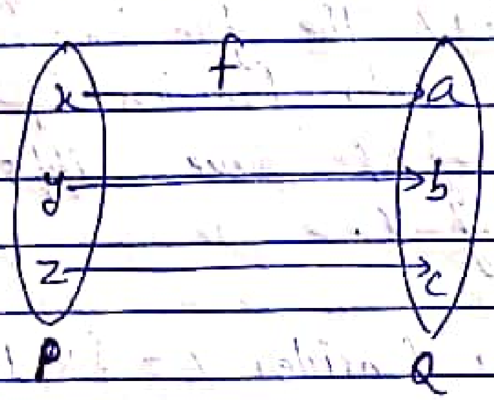
Example → Consider $X = \{1, 2, 3, 4, 5\}$
$Y = \{a, b, c, d\}$ and $f = \{(1,a) (2,a) (3,b) (4,c), (5,d)\}$



(3) Bijection (one to one-onto) Functions :-

A function which is both Injective (one to one) and surjective (onto) is called a bijection (one to one-onto) function.

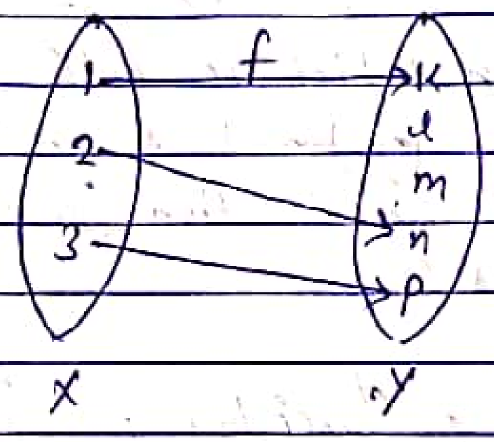
Example → Consider $P = \{x, y, z\}$
$Q = \{a, b, c\}$ and $f: P \to Q$ such that
$f = \{(x,a), (y,b), (z,c)\}$



(4) Into Functions :-
Let $f: X \to Y$. The function f is called an

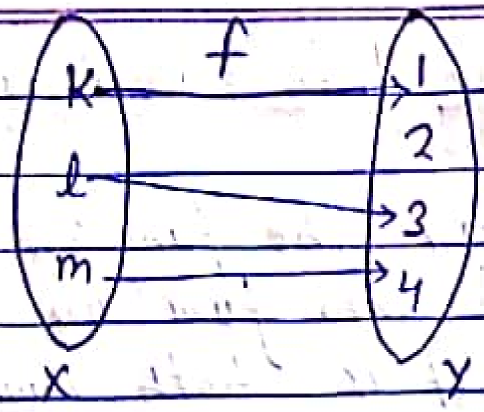
into function if the range of f is not equal to the co-domain Y. Therefore, there must be an element of co-domain Y which is not the Image of any element of domain X.

Example → Consider $X = \{1, 2, 3\}$, $Y = \{k, l, m, n, p\}$ and $f: X \to Y$ such that $f = \{(1,k), (2,n), (3,p)\}$



⑤ One - One Into Function]
Let $f: X \to Y$, The function f is called one-one into function if different elements of X have different unique image of Y.

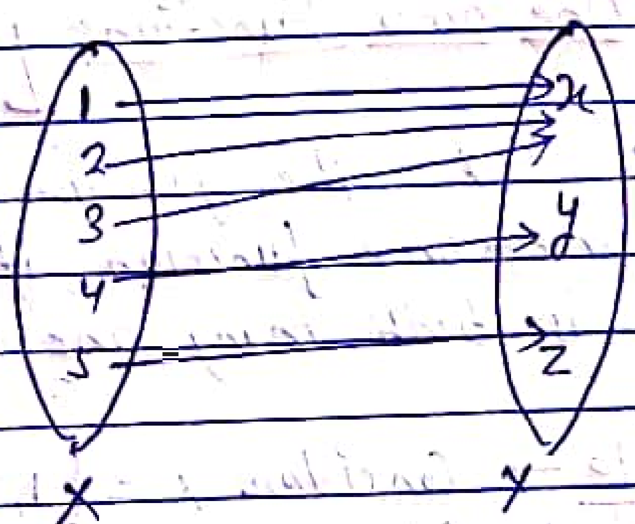
Example → Consider $X = \{k, l, m\}$
$Y = \{1, 2, 3, 4\}$ and $f: X \to Y$ such that
$f = \{(k,1), (l,3), (m,4)\}$.



(6) Many one functions] let $f : X \to Y$.

The function f is said to be many one function there exist two or more than two different in X having the same image in Y.

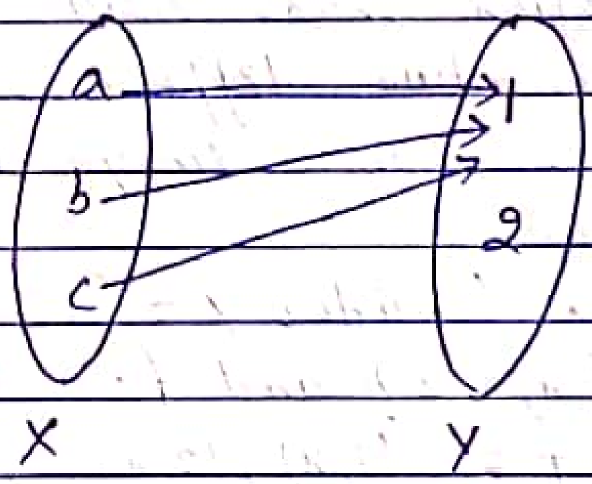
Example → Consider $X = \{1, 2, 3, 4, 5\}$, $Y = \{x, y, z\}$ and $f : X \to Y$ such that $f = \{(1, x), (2, x), (3, x), (4, y), (5, z)\}$.



(7) Many One Into Functions :-

Let $f: X \to Y$. The function f is called many one into function if and only if it is both many one and into function.

Example → Consider $X = \{a, b, c\}$, $Y = \{1, 2\}$ and $f: X \to Y$ such that
$f = \{(a,1), (b,1), (c,1)\}$



(8) Many One onto Functions :-

Let $f: X \to Y$. The function f is called many one-one function if and only if it is both many one and onto.

Example → Consider $X = \{1, 2, 3, 4\}$
$Y = \{k, l\}$ and $f: X \to Y$
such that $f = \{(1,k), (2,k), (3,l), (4,l$

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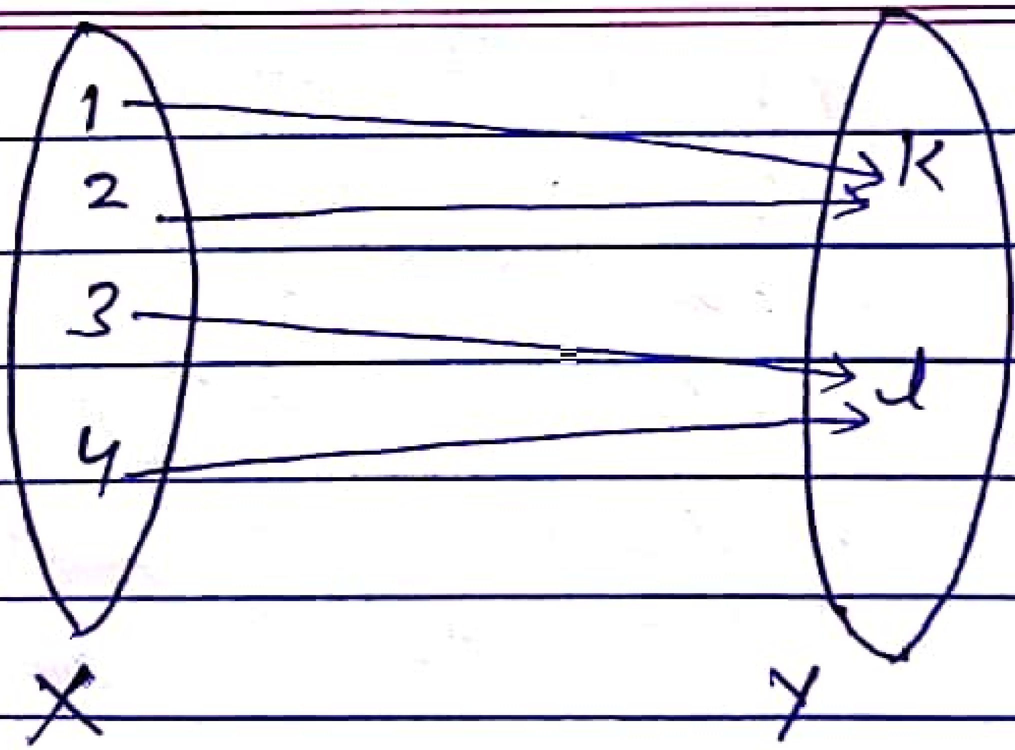

Compositions of Function]

Consider function $f: A \to B$ and $g: B \to C$. The composition of f with g is a function from A to C defined by $(g \circ f)(x) = g[f(x)]$ and is denoted by $g \circ f$ and is denoted by $g \circ f$.
To find the composition of f and g first find the image of x under f and then find the image of $f(x)$ under g.

Example – Let $X = \{1, 2, 3\}$, $Y = \{a, b\}$ and $Z = \{5, 6, 7\}$. Consider the function $f = \{(1, a); (2, a), (3, b)\}$ and $g = \{(a, 5), (b, 7)\}$

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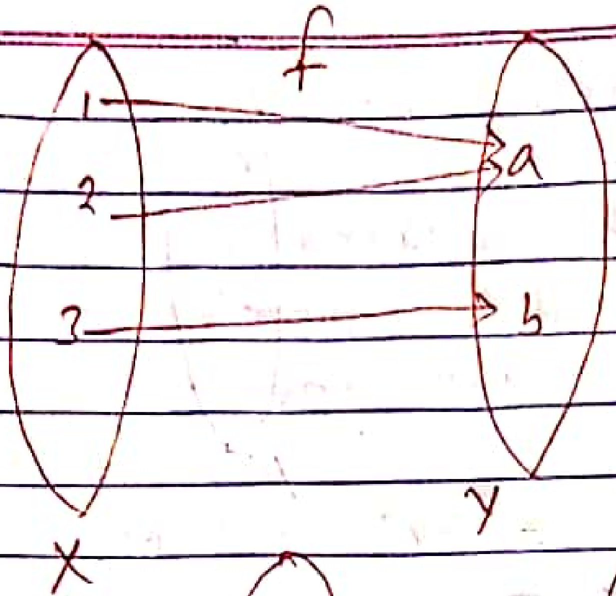
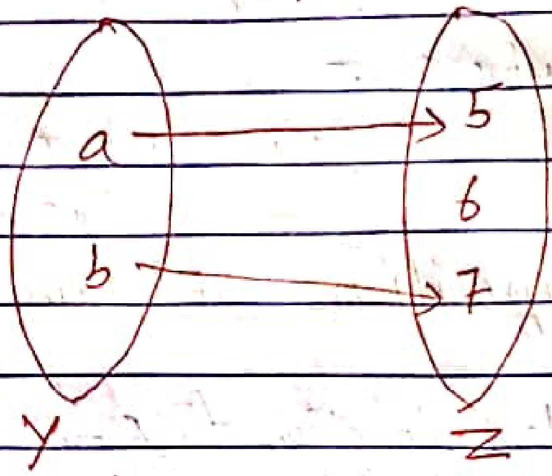
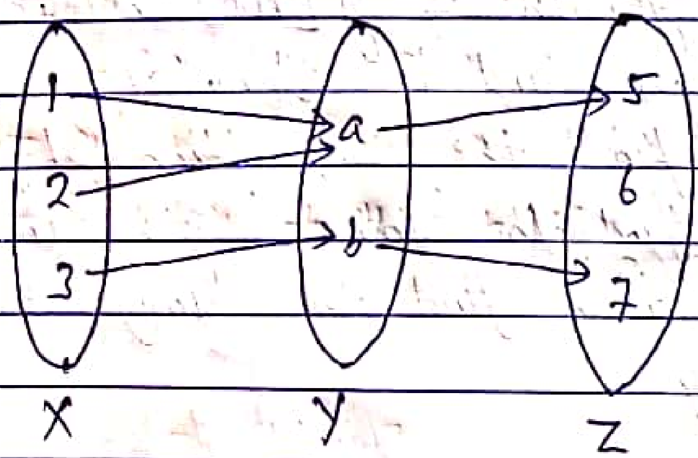
(i)



(ii)



Soln The composition function gof



$(g\circ f)(1) = g[f(1)] = g(a) = 5$

$(g\circ f)(2) = g[f(2)] = g(a) = 5$

$(g\circ f)(3) = g[f(3)] = g(b) = 7$

Example :- Consider f, g and h, all functions on the integers by $f(n) = n^2$, $g(n) = n+1$ and $h(n) = n-1$

Determine

(i) $h \circ f \circ g$ (ii) $g \circ f \circ h$ (iii) $f \circ g \circ h$.

Soln (i) $h \circ f \circ g(n) = n+1$

$h \circ (f(n+1)) = (n+1)^2$

$h((n+1)^2) = (n+1)^2 - 1$

$= n^2 + \cancel{1} + 2n - \cancel{1}$

$$\boxed{h \circ f \circ g(n) = n^2 + 2n}$$

(ii) $g \circ f \circ h(n) = n-1$

$g \circ f(n-1) = (n-1)^2$

$g((n-1)^2) = (n-1)^2 + 1$

$= n^2 - 2n + 1 + 1$

$$\boxed{g \circ f \circ h(n) = n^2 - 2n + 2}$$

(iii) $f \circ g \circ h = f \circ g \circ h(n)$

$= f \circ g \circ h(n) = n-1$

$= f \circ g(n-1) = (n-1) + 1$

$= n - \cancel{1} + \cancel{1}$

$= f \circ g(n-1) = n$

$$\boxed{f \circ g \circ h = f(n) = n^2}$$

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Example ⇒ consider the functions $f, g : R \to R$ defined by

$f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$

find the composition functions

(i) $f \circ f$ (ii) $f \circ g$ (iii) $g \circ f$

Sol → (iii) $g \circ f$

$g \circ f(x) = g[f(x)] = g(x^2 + 3x + 1)$

$= 2(x^2 + 3x + 1) - 3$

$= 2x^2 + 6x + 2 - 3$

$\boxed{g \circ f(x) = 2x^2 + 6x - 1}$

(i) $f \circ f = f[f(x)] = f(x^2 + 3x + 1)$

$= (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$

$= (x^2)^2 + (3x)^2 + (1)^2 + 2(x^2)(3x) + 2(3x)(1) + 2(x^2)(1)$
$+ 3x^2 + 9x + 3 + 1$

$= x^4 + 9x^2 + 1 + 6x^3 + 6x + 2x^2 + 3x^2 + 9x + \ldots$

$\boxed{f \circ f(x) = x^4 + 6x^3 + 14x^2 + 15x + 5}$

(ii) $f \circ g$

$= f(2x - 3) = (2x - 3)^2 + 3(2x - 3) + 1$

$= (2x)^2 + (-3)^2 + 2(2x)(-3) + 6x - 9 + \ldots$

$= 4x^2 + \cancel{9} - 12x + 6x - \cancel{9} + 1$

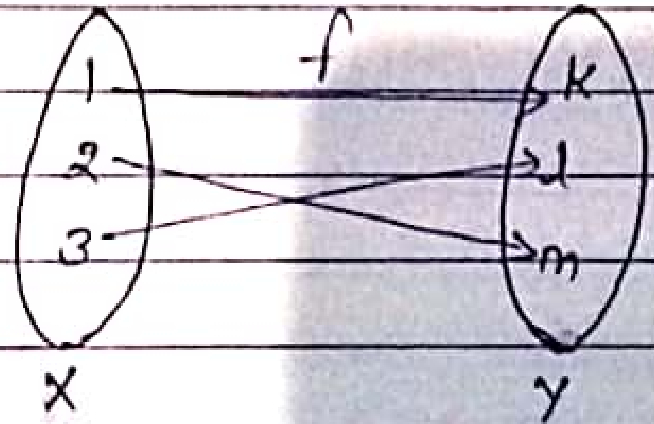
$\boxed{f \circ g = 4x^2 - 6x + 1}$

INVERTIBLE (INVERSE) FUNCTIONS]

A function $f: x \rightarrow y$ is Invertible if and only if it is a bijection function

Example → Consider $X = \{1,2,3\}$, $Y = \{k,l,m\}$ and $f: X \rightarrow Y$, such that

$$f = \{(1,k),(2,m),(3,l)\}.$$



The inverse function of f