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TCS 301. T301

Paper ID and Roll No. to be filled in your Answer book

Roll. No.

B.tech

(SEM. III) – 2014

Discrete structure

[Time: 3 Hours] **Total Marks: 100**

Note: Attempt all question

5X4

Q:-1 Attempt any 4 questions

(a) Prove that by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer.

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(b) Prove that $(A \cap B) \times (B \cap C) = (A \times B) \cap (A \times C)$.

(c) Given $A = \{1, 2, 3, 4\}$. Consider the following relation on A:

$R = \{(1,1),(2,2),(2,3),(3,2),(4,2),(4,4)\}$

(i) Draw the direct graph.

(ii) Is R reflexive, symmetric, transitive or ant symmetric?

(iii) Find R^2

(d) Let R is an equivalence relation on the set $A = \{a,b,c,d\}$ defined by partitions $P= \{\{a,d\},\{b,c\}\}$. Determine the elements of equivalence relation and also find the equivalence classes of R.

(e) Prove that intersection of two equivalence relation is also an equivalence relation.

(f) Consider a function $f:A \rightarrow B$ and $g:B \rightarrow C$. Prove that if f , g and gof are one to one and onto then (ii) $(gof)^{-1} = f^{-1}og^{-1}$

Q:-2 **Attempt any 4 questions** **5X4**

(a) Show that a sub lattice of a distributive lattice is distributive lattice.

(b) Show that a sub lattice of a distributive lattice is distributive lattice.

(c) Draw the Hasse diagram of the following sets under the partial ordering relation "divides" and indicate those which are totally ordered. (i) {2,6,24} (ii) {3,5,15} (iii) {1,2,3,6,12} (iv) D_{42} (v) D_{32}

(d) What is complemented lattice and bounded lattice? Explain by taking a suitable example.

(e) Show that a sub lattice of a bounded lattice is also bounded lattice.

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(f) In any lattice the distributive inequalities

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

Q:-3 **Attempt any 2 questions** **10X2**

(a) Define Ring and Field with suitable example. Prove that if a,b ε R where (R, +, ·) is aring then

$$(a+b)^2 = a^2 + a.b + b.a + b^2 \text{ where } a^2 = a.a.$$

(b) Consider a group G={1,2,3,4,5,6}Under multiplication modulo 7.

(i) Find the multiplication table of G

(ii) Find 2^{-1}, 3^{-1} and 6^{-1}

(iii)Find the orders and subgroups generated by 2 and 3.

(iv) Is G cyclic?

(c) Prove that the order of each sub-group of finte group G is a divisor of the group G.

Q:-4 Attempt any 2 questions **10X2**

(a) Check the validity of following arguments

(i) $P, \neg q \square r, \neg p \rightarrow q \vdash r$

(ii) $P \rightarrow q, r \rightarrow \neg q \vdash p \rightarrow \neg r$

(b) Check the validity of the following arguments without using truth table:

"If the races are fixed or the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed."

(c) Give the symbolic form of the following statements:

(i) Some men are genius.

(ii) For every x, there is a greater positive integer.

9 (iii) Given any positive integer, there is a greater positive integer.

(iv) Everyone who likes fun will enjoy each of these plays

(v) All healthy people eat an apple a day.

Q:-5 Attempt any 2 questions **10X2**

(a) Solve the of the recurrence relation $a_r - 6a_{r-1} + 9a_{r-2} = (r+1)\,3^r$.

(b) Solve the of the recurrence relation $y_{n+2} - 2y_{n+1} + y_n = 0$ by using generating function method with the boundary condtions $y_0 = 2$, $y_1 = 4$.

(c)

(i) A class has 12 boys and four girls.Suppose three students are selected at random from class. Find the probability that they are all boys.

(iii) A pair of fair dice is thrown. If the two numbers appearing are different, find the probability that 1) sum is six (2) an a ace appears (c) the sum is 4 or less

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