Chapter 4 : <u>Flow Of Incompressible Fluids In Conduit And Thin Layers</u>

<u>Important Terms ⇒</u>

- ★ **Friction factor :** **Ratio of wall shear stress to the product the density and velocity head.**

- ★ **Hydraulically Smooth :** **When further smoothening brings about no further reduction in the friction factor for a given Reynolds number.**

- ★ **Moody Diagram :** **1. Log-log plot of friction factor vs Reynolds number.**
  2. The frictional characteristics of round pipes(both smooth and rough) are summarized by a friction factor chart.

- ★ **Vena Contracta :** **The cross-section of minimum area at which the jet changes from a contraction to an expansion is called vena contracta.**

- ★ **Couette Flow :** **It is the flow of a viscous fluid in the space between two surfaces, one of which is moving tangentially relative to the other. The relative motion of the surfaces imposes a shear stress on the fluid and induces flow.**

Important Formulas ⇒

1. **SHEAR-STRESS DISTRIBUTION IN A CYLINDRICAL TUBE :**

$$\frac{dp}{dL} + \frac{2\tau}{r} = 0 \qquad\qquad \frac{\tau_w}{r_w} = \frac{\tau}{r}$$

2. **RELATION BETWEEN SKIN FRICTION AND WALL SHEAR :**

$$h_{fs} = \frac{2}{\rho}\frac{\tau_w}{r_w}\Delta L = \frac{4}{\rho}\frac{\tau_w}{D}\Delta L$$

3. **Friction Factor :**

$$f \equiv \frac{\tau_w}{\rho \bar{V}^2/2g_c} = \frac{2g_c\tau_w}{\rho\bar{V}^2}$$

4. **RELATIONS BETWEEN SKIN-FRICTION PARAMETERS :**

$$h_{fs} = \frac{2}{\rho}\frac{\tau_w}{r_w}\Delta L = \frac{\Delta p_s}{\rho} = 4f\frac{\Delta L}{D}\frac{\bar{V}^2}{2g_c}$$

$$f = \frac{\Delta p_s\, g_c D}{2\Delta\, L\rho\bar{V}^2}$$

5. **LAMINAR FLOW OF NEWTONIAN FLUIDS :**

$$u = \frac{\tau_w g_c}{2r_w\mu}(r_w^2 - r^2) \quad , \quad \frac{u}{u_{\max}} = 1 - \left(\frac{r}{r_w}\right)^2$$

6. **Average Velocity :**

$$\bar{V} = \frac{\tau_w g_c}{r_w^3 \mu} \int_0^{r_w} (r_w^2 - r^2) r \, dr = \frac{\tau_w g_c r_w}{4\mu} \quad , \quad \frac{\bar{V}}{u_{max}} = 0.5$$

7. **HAGEN-POISEUILLE EQUATION :** **Main use is in the experimental measurement of viscosity by measuring pressure drop and volumetric flow rate.**

$$\Delta p_s = \frac{32 \, \Delta L \, \bar{V} \mu}{g_c D^2} \quad , \quad f = \frac{16 \, \mu}{D \bar{V} \rho} = \frac{16}{N_{Re}}$$

8. **Velocity Distribution for Turbulent Flow :**

$$u^* \equiv \bar{V} \sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_w g_c}{\rho}}$$

$$u^+ \equiv \frac{u}{u^*}$$

$$y^+ \equiv \frac{y u^* \rho}{\mu} = \frac{y}{\mu} \sqrt{\tau_w g_c \rho}$$

where u^* = friction velocity
u^+ = velocity quotient, dimensionless
y^+ = distance, dimensionless
y = distance from wall of tube

9. **UNIVERSAL VELOCITY-DISTRIBUTION EQUATIONS :**

- **Viscous sublayer :** $u^+ = y^+$
- **Buffer layer :** $u^+ = 5.00 \ln y^+ - 3.05$
- **Turbulent layer :** $u^+ = 2.5 \ln y^+ + 5.5$

for the viscous sublayer: $y^+ < 5$
for the buffer zone: $5 < y^+ < 30$
for the turbulent core: $30 < y^+$

10. Average velocity :

$$\bar{V} = \frac{5\nu^2}{r_w^2 u^*} \int_0^{y_c^+} \left(0.4 u_c^+ + \ln \frac{y^+}{y_c^+}\right)(y_c^+ - y^+)\, dy^+ \quad , \quad \frac{\bar{V}}{u_{\max}} = \frac{1}{1 + (3.75\sqrt{f/2})}$$

11. Von-Karman equation :

$$\frac{1}{\sqrt{f/2}} = 2.5 \ln \left(N_{\mathrm{Re}} \sqrt{\frac{f}{8}} \right) + 1.75$$

12. Hydraulic Radius :

$$r_H \equiv \frac{S}{L_p}$$

where S = cross-sectional area of channel
L_p = perimeter of channel in contact with fluid

13. Darcy-Weisbach equation :

$$h_{fs} = \frac{2}{\rho} \frac{\tau_w}{r_w} \Delta L = \frac{\Delta p_s}{\rho} = 4f \frac{\Delta L}{D} \frac{\bar{V}^2}{2g_c}$$