Case (v) P.I. of any function :-

$$P.I. = \frac{\phi(x,y)}{F(D,D')}$$

factorized to get

$$= \frac{\phi(x,y)}{(D-m_1D')(D-m_2D')\ldots(D-m_nD')}$$

then

$$\boxed{P.I. = \frac{1}{D-mD'}F(x,y) = \int \phi(x, c-mx)\,dx}$$

Example (1) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

$\sim \quad (D^2 + DD' - 6D'^2) = y\cos x$

A.E. :- $m^2 + m - 6 = 0$

$\Rightarrow m = 2, -3$

C.F. $= f_1(y+2x) + f_2(y-3x)$

$$P.I. = \frac{y\cos x}{D^2+DD'-6D'^2}$$

$$= \frac{y\cos x}{(D-2D')(D+3D')} \quad \left\{ \begin{array}{c} \text{factorizing} \\ D^2+DD'-6D'^2 \end{array} \right\}$$

$$= \frac{1}{D-2D'} \int (c+3x)\cos x \quad [\text{put } y = c+3x]$$

$$= \frac{1}{D-2D'}\left[(c+3x)\sin x + 3\cos x\right]$$

$$= \frac{1}{D-2D'}\left[y\sin x + 3\cos x\right] \quad \left[\begin{array}{c}\text{Again put back} \\ c+3x = y\end{array}\right]$$

$$= \int\left[(c-2x)\sin x + 3\cos x\right]dx$$

put $y = c-2x$

$$\Rightarrow (c-2x)(-\cos x) - 2\sin x + 3\sin x$$

$$= -y\cos x + \sin x$$

they complete sol:- $z = f_1(y+2x) + f_2(y-3x) + \sin x - y\cos x$