General Rules for finding the P.I. of Linear PDEs (with const. coeffici of nth order

let diff. equation is

$$F(D, D')z = f(x,y)$$

then $P.I. = \dfrac{f(x,y)}{F(D,D')}$

Case ① If $f(x,y) = e^{ax+by}$

then

$$\boxed{P.I. = \frac{e^{ax+by}}{F(D,D')} = \frac{e^{ax+by}}{F(a,b)}}$$

Just put $D = a$ and $D' = b$

(but condition $F(D,D') \neq 0$)

If $F(D,D') = 0$, then

$$\boxed{P.I. = \frac{x\, e^{ax+by}}{\frac{\partial}{\partial D} F(D,D')}}$$

Again $\partial/_{\partial D} F(D,D') = 0$, then

$$\boxed{P.I. = \frac{x^2 e^{ax+by}}{\frac{\partial^2}{\partial D^2} F(D,D')}}$$

Example (1)

Solve $\dfrac{\partial^3 z}{\partial x^3} - 3\dfrac{\partial^3 z}{\partial x^2 \partial y} + 4\dfrac{\partial^3 z}{\partial y^3} = e^{x+2y}$

$\Rightarrow (D^3 - 3D^2D' + 4D'^3) = e^{x+2y}$ —— (1)

(Remember $D = \partial/\partial x$ and $D' = \partial/\partial y$)

Then A.E. of (1)

$\Rightarrow m^3 - 3m^2 + 4 = 0$ (for A.E. put $D = m$ and $D' = 1$)

$m = -1$ is a root of A.E., other roots are

-1	1	-3	0	4
		-1	4	-4
	1	-4	4	0
	m^2	m	const	

$(m+1)(m^2 - 4m + 4) = 0$

or $(m+1)(m-2)^2 = 0$

$\Rightarrow m = -1, 2, 2$

Thus C.F. $= f_1(y-x) + f_2(y+2x) + x\, f_3(y+2x)$

$$P.I. = \frac{e^{x+2y}}{D^3 - 3D^2D' + 4D'^3} \quad \text{—— (2)}$$

Here e^{ax+by} comparing, so $a = 1, b = 2$

put $D = 1$ and $D' = 2$ in Eq. (2)

then $P.I. = \dfrac{e^{x+2y}}{1 - 3(1)(2) + 4(2)^3} = \dfrac{e^{x+2y}}{1-6+32} = \dfrac{e^{x+2y}}{27}$

Thus complete sol.:- $z = C.F. + P.I.$

or $z = f_1(y-x) + f_2(y+2x) + x\, f_3(y+2x) + \dfrac{e^{x+2y}}{27}$

Ans.

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Ex. (2) $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$

$\sim (4D^2 - 4DD' + D'^2) z = e^{x+2y}$

Now A.E.

$\Rightarrow 4m^2 - 4m + 1 = 0$ (∵ for A.E. put $D = m, D' = 1$)

$\Rightarrow m = \frac{4 \pm \sqrt{16 - 4(4)}}{2 \times 4}$

or $m = \frac{4 \pm 0}{8} = 1/2$ (two times)

So $\boxed{m = 1/2, 1/2}$

OR $(2m-1)^2 = 0 \Rightarrow m = 1/2, 1/2$

then C.F. $= f_1\left(y + \frac{x}{2}\right) + x f_2\left(y + \frac{x}{2}\right)$

$P.I. = \frac{e^{x+2y}}{4D^2 - 4DD' + D'^2}$ {Here comparing it e^{ax+by}, we have $a = 1$ and $b = 2$}

put $D = 1$ and $D' = 2$ we get

$P.I. = \frac{e^{x+2y}}{4(1) - 4(1)(2) + (2)^2} = \boxed{\frac{e^{x+2y}}{0}}$? Not possible

thus

$P.I. = \frac{x e^{x+2y}}{8D - 4D'}$

put again $D = 1$ and $D' = 2$, we get $P.I. = \frac{x e^{x+2y}}{8(1) - 4(2)}$

Which also becomes zero (denominator)
thus again differentiate the denominator
w.r.t. D and multiplying by
x in Numerator, we get

$$P.I. = \frac{x^2 e^{x+2y}}{8}$$

Thus Complete sol:-

$$z = C.F. + P.I.$$

$$= f_1(2y+x) + x f_2(2y+x) + \frac{x^2 e^{x+2y}}{8}$$

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