SAMPLING AND ANALOG-TO-DIGITAL CONVERSION

- Advancement in digital technology has given rise to inexpensive, light weight, flexible, programmable and easy to reproduce (manufacture) discrete time systems.
- This makes it preferable to process discrete signals over continuous signals.
- To accurately convert and represent a continuous time signal in a discrete form, sampling theorem is used.
- Sampling also guarantees recovery of the continuous time signal from its discrete representation using the knowledge of samples taken uniformly.

Sampling

- Analogue signals are converted to discreted time signal (digitized) through sampling and quantization.
- The sampling rate (thus number of samples/second) must be large enough to permit the analog signal to be reconstructed from the samples with sufficient accuracy.
- **Sampling theorem** determines the proper (lossless) sampling rate for a given signal.
- Application of sampling include: signal processing, communication theory and A/D circuit design.

Sampling theorem or Nyquist Theorem

- A signal g(t) whose spectrum is band limited to B Hz thus G(f)=0 for |f|> B can be reconstructed exactly (without error) from its discrete time samples taken uniformly at a rate of R samples per second provided R>2B Hz.
- **Sampling theorem** states that, "a signal can be completely represented and reproduced if it is sampled at the rate f_s, which is greater than or equal to twice the maximum frequency of the given signal f_m."

$$f_s \geq 2f_m \text{ Hz}$$

where f_m = maximum frequency present in the signal, f_m = B

- Thus, the minimum *uniform* sampling frequency for a perfect signal recovery is f_s=2B Hz, called the **Nyquist rate**.

Nyquist rate or Nyquist interval

- When the sampling rate is exactly equal to $2f_m$ Hertz, then is referred to as **Nyquist rate**.
- And it is the minimum sampling rate for proper reconstruction of a sampled signal..
- The corresponding interval, $T_s = \frac{1}{2fm}$ seconds and is referred to as the **Nyquist interval**.

Example 1

- Determine the Nyquist rate and Nyquist interval for an analogue signal expressed by: $x(t)= 3 \cos 50\pi t + 10 \sin 300\pi t – \cos 100\pi t$
- Solution:
- First we denote frequencies present in the signal as: ω_1, ω_2 and ω_3

  So that $x(t) = 3 \cos \omega_1 t + 10 \sin \omega_2 t – \cos \omega_3 t$

  Therefore: $\omega_1 t = 2\pi f_1 t = 50\pi t$ which gives $f_1 = 25$ Hz

  $\omega_2 t = 2\pi f_2 t = 300\pi t$ which gives $f_2 = 150$ Hz

  $\omega_3 t = 2\pi f_3 t = 100\pi t$ which gives $f_3 = 50$ Hz
- We determine the highest frequency as 150 Hz

  Nyquist rate, $f_s = 2fm = 2x150 = 300Hz$,

  Nyquist interval = 1/300 = 0.0033 seconds

Example 2:

- Find the Nyquist rate and the Nyquist interval for the signal

$$x(t) = \frac{1}{2\pi}\cos(2000\pi t)\cos(600\pi t).$$

- Solution:

From $2cos\omega_1 t\cos\omega_2 t = \cos(\omega_1 t + \omega_2 t) + \cos(\omega_1 t - \omega_2 t)$ we get

$cos\omega_1 t\cos\omega_2 t = \frac{1}{2}\cos(2000\pi t)\cos(600\pi t)$; $\omega_1 = 2000\pi$, $\omega_2 = 600\pi$

$$x(t) = \frac{1}{2\pi}\cos(2000\pi t)\cos(600\pi t)$$

$$= \frac{1}{2\pi} \text{ x } \frac{1}{2}[\cos(2000\pi t + 600\pi t) + \cos(2000\pi t - 600\pi t)]$$

Example 2: continued

$$= \frac{1}{4\pi}\left[\cos(2600\pi t) + \cos(1400\pi t)\right]$$

$\omega_1 t = 2\pi f_1 t = 2600\pi t$ which gives $f_1 = 1300$ Hz

$\omega_2 t = 2\pi f_2 t = 1400\pi t$ which gives $f_2 = 700$ Hz

The highest frequency fm = 1300Hz

Therefore: Nyquist rate, fs = 2fm = 2x1300 = 2600Hz

Nyquist interval = 1/2600 = 384.6 milliseconds

Effect of under-sampling

- Effect of undersampling is called aliasing
- When a continuous-time bandlimited signal is sampled at a rate lower that Nyquist frequency, $f_s < 2f_m$ then critical information about the sampled signal is lost.
- Aliasing is a phenomenon in which the frequency component in the frequency spectrum takes the identity of a lower frequency component in the spectrum of the sampled signal.
- Aliasing makes it impossible to recover the original signal from the sampled signal.

Effect of under-sampling continued

- Since any information contains a large number of frequencies, it becomes difficult to select the sampling frequency.
- So a signal has to be first passed through a low-pass filter (pre-aliasing filter) which blocks all the frequencies above f_m.
- The process is known as band limiting of the original signal.
- After band limiting, maximum frequency can be fixed at f_m.
- **TO AVOID ALIASING THEREFORE:**

i. Prealias filter must be used to limit the band of frequencies of the signal to f_m.

ii. Sampling frequency fs must be selected such that $f_s > 2f_m$.

Sampling of bandpass signals

- If a signal is bandlimited then a different signal must be used to sample the signal. (unlike the previous section which dealt with low-pass signals)
- The bandpass signal x(t) whose maximum bandwidth is $2f_m$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth..
- Here f_m is the maximum frequency component in the signal.
- Since the bandpass signal has a bandwidth of $2f_m$, then the sampling rate has to be $2 \times 2f_m = 4f_m$ samples per second

Example 3

- Consider a spectral range of the basspass signal is 20kHz to 82kHz. The bandwidth, $2f_m$ = 82kHz – 20kHz = 62kHz
- The minimum sampling rate = 2 x 2fm = 2 x 62 kHz = 124kHz
- Generally, the range of minimum sampling frequencies is specified for bandpass signals, it lies between 4fm and 8fm samples per second.
- Therefore = (2 x bandwidth) to (4 x bandwidth)

  = 2 x 62kHz to 4 x 62 kHz

  = 124 kHz to 248 kHz

SAMPLING TECHNIQUES

- There are three types of sampling techniques:

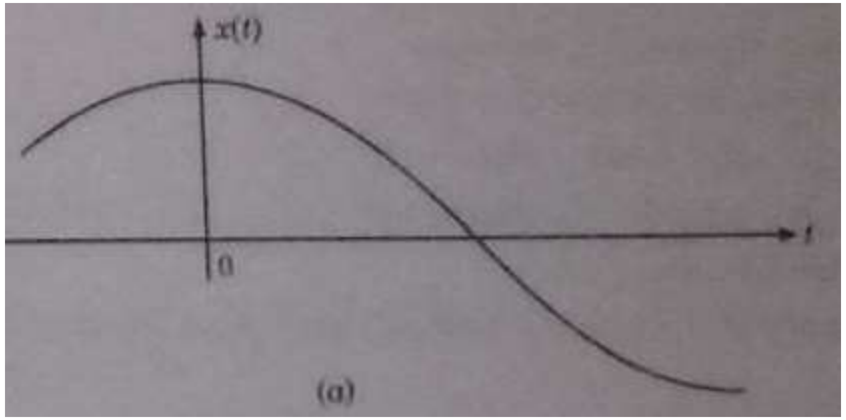
i. Instantaneous (or ideal) sampling

ii. Natural sampling

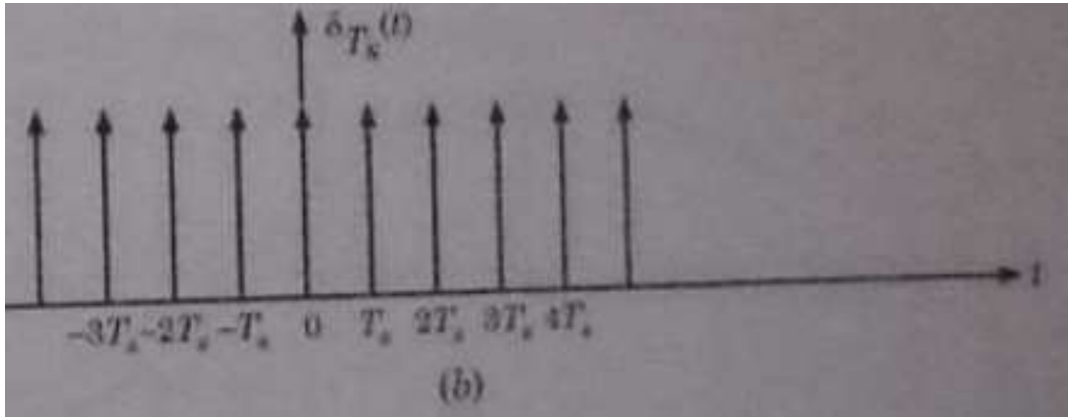
iii. Flat-top sampling

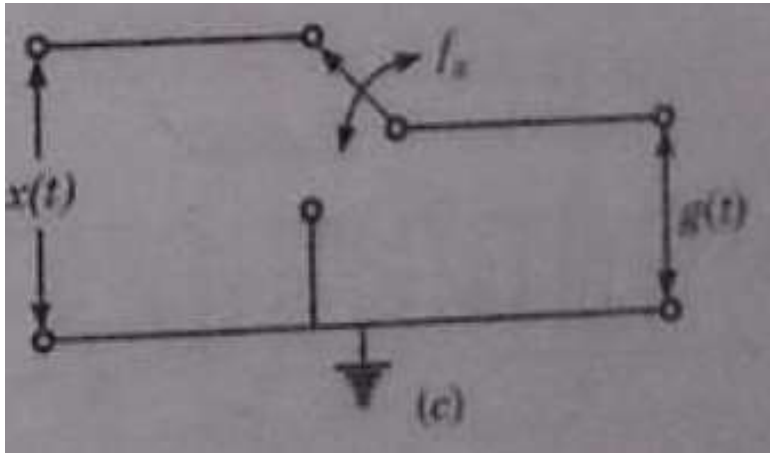
- Instantaneous sampling is also called ideal sampling while natural sampling and flat-top sampling are called practical sampling methods.

Ideal sampling or instantaneous sampling or impulse sampling



- The sampling function is a train of impulses.
- Fig a: shows the analogue signal
- Fig b shows the sampling function
- Fig c: shows the switching sampler (a circuit to produce instantaneous or ideal sampling).





The working principle of the switching sampler

- The circuit consists of a switch.
- Assuming that the closing time ‘t’ of the switch approaches zero, then the output g(t) of these circuit will contain only instantaneous values of the input signal x(t).
- Since the width of the pulse approaches zero, the instantaneous sampling gives a train of impulses of height equal to the instantaneous value of the input signal x(t) at the sampling instant.

- The train may be represented as:

$$\delta Ts(t) = \sum_{n=-\infty}^{\infty} \delta(t - nTs)$$

Where n= the nth sample, T_s = sampling interval

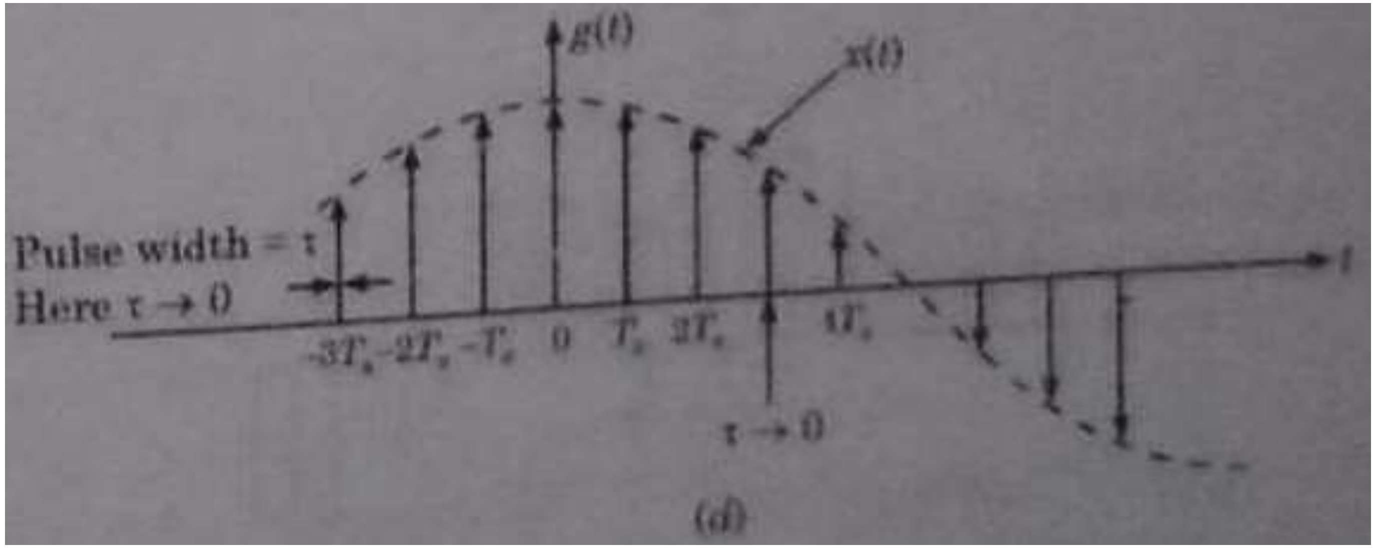
This is known as the sampling function and the waveform is shown in fig b

- The sampled signal g(t) is expressed as the multiplication of x(t) and impulse $\delta T_s(t)$
- Thus: g(t) = x(t) – $\delta Ts(t)$

$$= x(t) \sum_{n=-\infty}^{\infty} \delta(t - nTs) \qquad \text{or}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t - nTs)$$

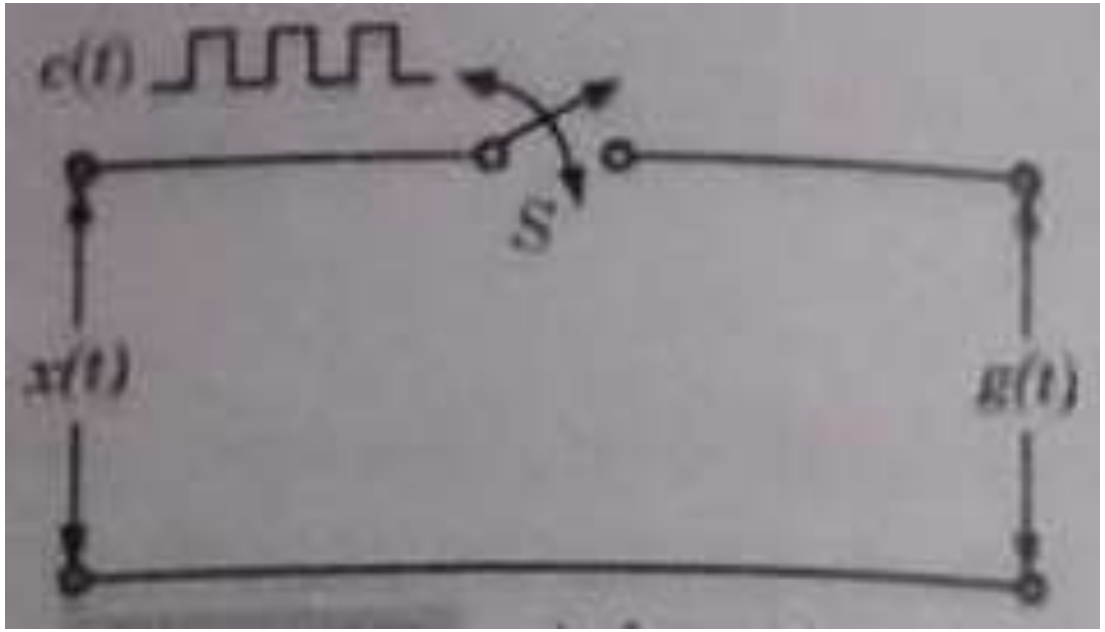
- The expression gives the spectrum of ideally sampled signal. Fig d



- However, it may be noted that ideal or instantaneous sampling is possible only in theory since it is impossible to have a pulse whose width approaches zero.

Natural sampling

- Here the pulses have finite width equal to τ .
- Consider an analog continuous-time signal x(t) to be sampled at the rate of $f_s > 2f_m$
- Again consider a sampling function c(t) which is a train of periodic pulses of width τ and frequency equal to f_s Hz
- The functional diagram of a natural sampler is shown.



- A sampled signal g(t) is obtained by multiplication of sampling function c(t)and input signal x(t).
- We see from the fig that:

  therefore g(t) = x(t) when c(t) = A

  and g(t) = 0 when c(t) = 0

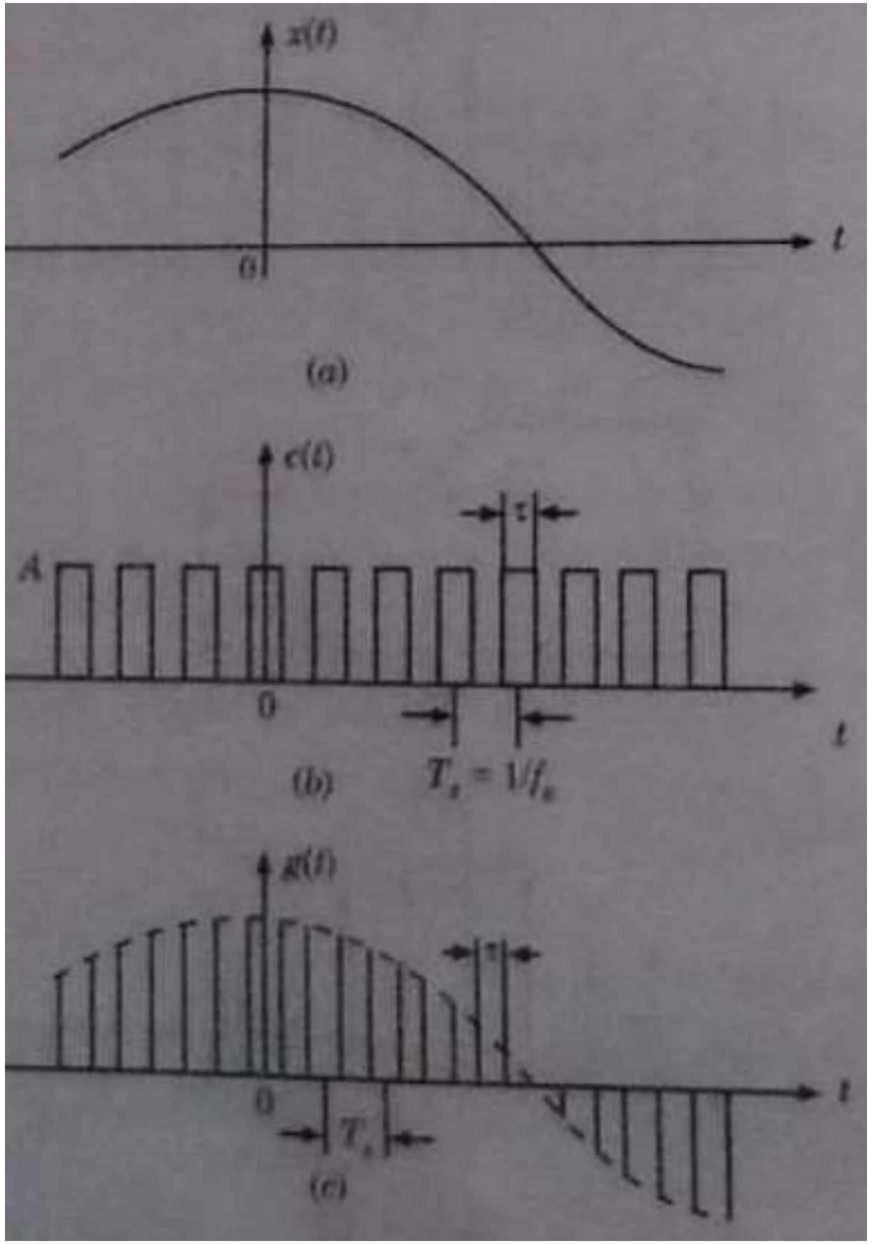
  where A is the amplitude of c(t).
- See the waveforms of signals x(t), c(t) and g(t) in the next slide.

- waveforms of signals x(t), c(t) and g(t):

Fig. a: is x(t) = is the continuous time signal

Fig b: is c(t) = Sampling function waveform i.e. periodic pulse train

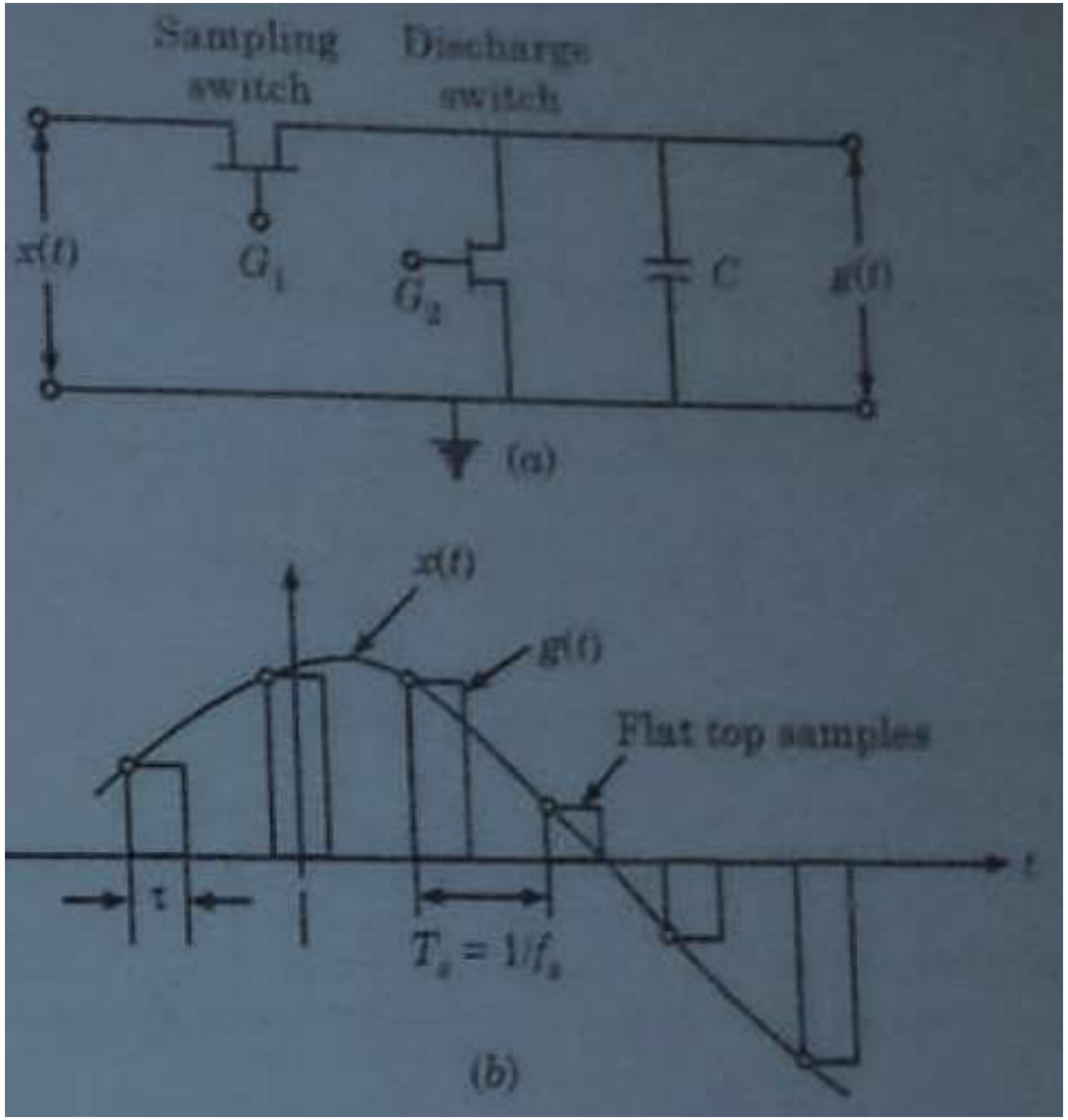
Fig c: is g(t) = naturally sampled signal waveform i.e. periodic pulse train



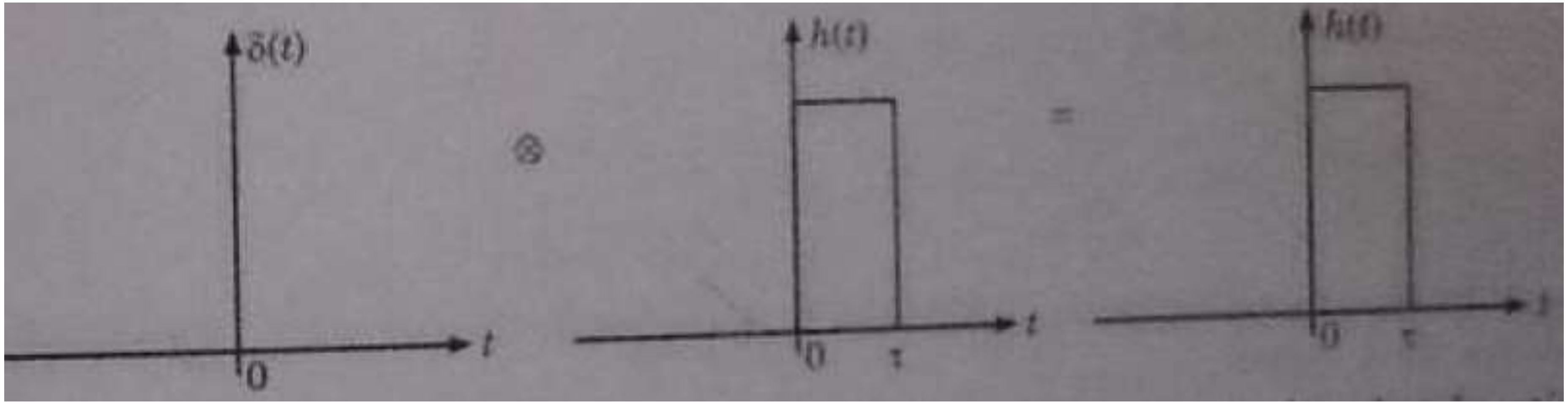
Flat Top Sampling or Rectangular Pulse Sampling

- The top of the samples remains constant and is equal to the instantaneous value of the baseband signal x(t) at the start of sampling.
- The width (duration) of each sample is τ and sampling rate is equal to $f_s = \frac{1}{T_S}$

- Fig a): A functional diagram of a sample and hold circuit which is used to generate that flat top samples.
- Fig b): a general waveform of flat top sampling.
- Note: only starting edge of the pulse represents instantaneous value of the baseband signal x(t).



- Also the flat top pulse of g(t) is mathematically equivalent to the convolution of instantaneous sample and a pulse h(t) as depicted as seen in figure below
- Note: width of pulse g(t) is determined by width h(t)
- and the sampling instant is determined by delta function.



- From the fig. the starting edge of the pulse represents the point where baseband signal sampled and width is determined by function h(t).
- Therefore $g(t) = s(t) \oplus h(t)$
- In this modified equation (modified from $f(t) \oplus \delta(t) = f(t)$) we are taking s(t) in place of $\delta(t)$. Note that $\delta(t)$ is a constant amplitude delta function whereas s(t) is a varying amplitude train of impulse.
- Meaning we are taking s(t) which is an instantaneously sampled signal and this is convolved with function h(t) as in equation.

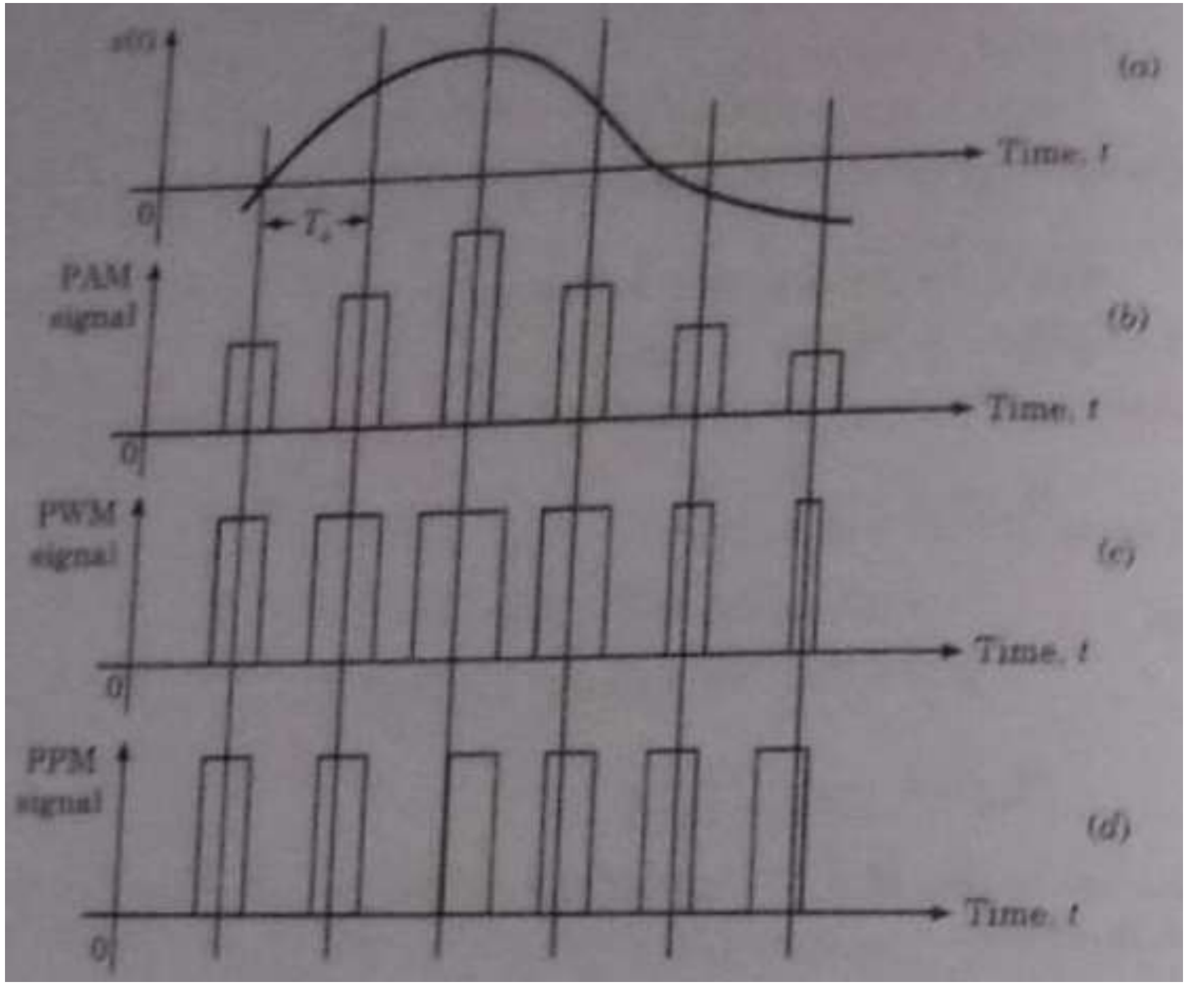
Analogue Pulse Modulation Methods

- In pulse modulation methods, the carrier is no longer a continuous signal but consists of a pulse train.
- Some parameter of which is varied according to the instantaneous value of the modulating signal.
- There are two major types of Analogue modulation systems:
   i. Pulse Amplitude Modulation (PAM)
   ii. Pulse Time Modulation (PTM)
      a. Pulse Width Modulation (PWM)
      b. Pulse Position Modulation (PPM)
- Since the modulating signal is an analogue signal, this technique are called Analogue pulse modulating techniques.

Analogue Pulse Modulation Methods continued

- Pulse Amplitude Modulation
  - The amplitude of the pulse of the carrier pulse train is varied in accordance with the modulating signal
- Pulse Time Modulation
  - The timing of the pulses of the carrier pulse train is varied in accordance with the modulating signal.
- Pulse Width Modulation (PWM) also called Pulse Duration Modulation
  - The width of the pulses of the carrier pulse train is varied in accordance with the modulating signal.
- Pulse Position Modulation
  - The position of pulses of the carrier pulse train is varied in accordance with the modulating signal.
- Remember the sampling frequency of the carrier pulse train must be satisfy the Nyquist theorem.

Analogue Pulse Modulation Methods continued



Drawbacks of PAM