

INDEX NUMBERS

An index number which is always expressed in terms of a base of 100, is statistical device used to measure changes in price, quantity or value of a group of related items over a period of time. In order to bring in the idea of time, the following standard convention is used

P_o, Q_o, V_o for price, quantity, or value at base time point

P_n, Q_n, V_n for price, quantity or value at some other time point

In selecting the base period for a particular index, two rules should be observed:

- The period selected should as much as possible be one of economic normalcy or stability-relative stability.
- The base period should be recent so that comparisons will not be unduly affected by changing technology, product, quantity, e.t.c. – recent period.

CONSTRUCTION/TYPES OF PRICE INDICES

INDEX RELATIVES OR SIMPLE INDEX NUMBER

An index relative is the name given to an index number which measures the changes in a distinct commodity i.e.

$$I_p = P_n/P_o \times 100, I_q = q_n/q_o \times 100$$

EXAMPLE

The prices and quantities of some household items sold by a supermarket over two years are as follows;

	2007		2008	
Item	Price	quantity	Price	quantity
Milk	250	500	300	450
Soap	1000	1000	1500	800
Matches	400	600	500	630

Determine the price and quantity relatives for these items for the period of 2008 (2007=100)

$$\text{Milk} \Rightarrow I_p = P_n/P_o = 300/250 \times 100 = 120$$

$$\text{Soap} \Rightarrow I_p = 1500/1000 \times 100 = 150$$

$$\text{Matches} \Rightarrow I_p = 500/400 \times 100 = 125$$

$$\text{Milk} \Rightarrow I_q = 450/500 \times 100 = 90$$

$$\text{Matches} = 360/600 \times 100 = 105$$

$$\text{Soap} \Rightarrow I_q = 800/1000 \times 100 = 80$$

TIME SERIES OF RELATIVES

It is usually necessary to see how the values of an index relative change over time. Given the values of some commodity over time, there are two distinct ways in which relatives can be calculated:

- a. **Fixed - base relatives** these are found by calculating relatives for each value of time series based on the same fixed time point
- b. **Chain – base relatives** these are found by calculating relatives for each value of a time series based on the immediately preceding time point.

EXAMPLE

Given the prices of a commodity for six years, calculate fixed based and chain based indices.
(2002=100)

Year	Prices
2002	4563
2003	4245
2004	4841
2005	4644
2006	4290
2007	5166

- a. Fixed - base (2002 = 100)
 $2003 \Rightarrow 4245/4563 \times 100 = 93.0$
 $2004 \Rightarrow 4841/4563 \times 100 = 106.1$
 $2005 \Rightarrow 4644/4563 \times 100 = 101.8$
 $2006 \Rightarrow 4290/4563 \times 100 = 109.3$
 $2007 \Rightarrow 5166/4563 \times 100 = 106.7$
- b. Chain-base
 $2003 \Rightarrow 4245/4563 \times 100 = 93.0$
 $2004 \Rightarrow 4841/4245 \times 100 = 114.0$
 $2005 \Rightarrow 4644/4841 \times 100 = 95.9$
 $2006 \Rightarrow 4290/4644 \times 100 = 113.9$
 $2007 \Rightarrow 5166/5290 \times 100 = 97.7$

COMPOSITE INDICES

A composite index number is an index number which is obtained by combining the information from a set of economic commodities of like kind. Such a composite index number normally requires that each component be weighted. Composite indices are of two types namely weighted and unweighted indices.

- a. **Un weighted index numbers**
 - (i) Simple aggregate index
 - (ii) Mean of price relatives

Simple Aggregate Index

$SAI = \frac{\sum P_n}{\sum P_o} \times 100$ for prices

Year	A	B	C	TOTAL
2000	11.4	18.5	11.7	41.6
2002	14.8	19.7	15.2	49.7
2004	16.8	18.7	14.4	50.4

If 2000 = 100

$SAI_{2002} = 49.7/41.6 \times 100 = 119.5$

$SAI_{2004} = 50.4/41.6 \times 100 = 121.2$

Mean of Price Relatives

$\Pi = (\sum (P_n/P_o) \times 100)/k$

Where k = number of items

	2007		2008		
Item	Price	Qty	Price	Qty	Ip
Milk	250	500	300	450	120
Soap	1000	1000	1500	800	150
Matches	400	600	500	630	125

$\Pi = 395/3 = 131.66$

b. weigh

Weighted index numbers

- (i) weighted average of relatives
- (ii) weighted aggregate index

Variants of weighted aggregate indices are:

1. Laspeyres index
2. Paasche index
3. Fishers ideal index
4. Marshal - Edgeworth index

Weighted Average of Relatives

This method involves calculating index relatives for each of the given components, then using the given weights to obtain a weighted average of the relatives. It is computed as;

$$I = \frac{\sum WI}{\sum W}$$

Example			Standard		
Components	Prices		quantity		
Of mix	2007	2008	(W)	(I)	WI
A	1.50	3.00	8	200	1600
B	3.40	4.25	3	125	375
C	10.40	8.84	<u>1</u>	85	<u>85</u>
-			12		2060

$$I_p, A = 3.00 \times 100 = 200$$

$$B = 4.25 \times 100 = 125$$

$$C = 8.84 \times 100 = 85$$

$$I_{AG} = 2060/12 = 171.7$$

Weighted Aggregate Index

This method involves multiplying each component value by its corresponding weight and adding these products to form an aggregate. It is computed as:

$$I_{AG} = \frac{\sum WV_n}{\sum WV_o} \times 100$$

Example

Items	Prices		weight		
	2007	2008	W	WVo	WVn
A	1.50	3.00	8	12.00	24.00
B	3.40	4.25	3	10.20	12.75
C	10.40	8.84	1	<u>10.40</u>	<u>8.84</u>
				32.60	45.59

$$I_{AG} = 45.59/32.60 \times 100 = 139.8$$

Laspeyres indices

A Laspeyres index is a special case of a weighted aggregate index which always uses base time period weights. It is most commonly associated with price and quantity indices. It is computed as follows:

$$L_p = \frac{\sum q_0 p_n}{\sum q_0 p_0} \times 100$$

$$L_q = \frac{\sum p_0 q_n}{\sum p_0 q_0} \times 100$$

Paasche indices

A paasche index is a weighted aggregate index which uses current time period as weights. It is computed as;

$$P_p = \frac{\sum q_n p_n}{\sum q_n p_0} \times 100$$

$$P_q = \frac{\sum p_n q_n}{\sum p_n q_0} \times 100$$

Example

The following data relate to a set of commodities used in a particular process.

Item	2007		2008	
	price	Qty	price	Qty
A	36	100	40	95
B	80	12	90	10
C	45	16	41	18
D	5	1100	6	1200

Calculate Laspeyres and Paasche price indices for 2008.

Item	Laspeyres		Paasche	
	qopn	poqo	qn timer="0.01"pn	qn timer="0.01"po
A	4000	3600	3800	3420
B	1080	960	900	800
C	656	720	738	810
D	<u>6600</u>	<u>5500</u>	<u>7200</u>	<u>6000</u>
	12336	10780	12638	11030

$$L_p = 12336/10780 \times 100 = 114.4$$

$$P_p = 12638/11030 \times 100 = 114.6$$

Fisher's ideal index

This is simply the geometric mean of the Laspeyre's and Paasche's indices. It is defined as follows;

$$F_p = \sqrt{L_p \times P_p}$$

$$F_q = \sqrt{L_q \times P_q}$$

Fisher's ideal index represents a compromise between the Laspeyre's and Paasche indices.

Note that whereas a Laspeyre's index tends to overestimate changes during period of inflation, a Paasche's index tends to under estimate changes. Therefore, as an average between the two, a Fisher's index is expected to more precisely reflect the current position than either of the two.

Example from our last exercise,

$$F_p = \sqrt{114.4 \times 114.6} = 114.5$$

Marshall Edgeworth's index

This is an alternative to the Laspeyre's and Paasche's indices, it uses the arithmetic mean of the quantities or price of the current and base time points as weighting factors. It is computed as follows:

$$M_p = \frac{\sum P_n (q_o + q_n)}{\sum P_o (q_o + q_n)} \times 100$$

$$M_q = \frac{\sum Q_n (p_o + p_n)}{\sum Q_o (p_o + p_n)} \times 100$$

Example

The prices and quantity demanded of commodities A, B, and C in the current and base year are given below (1990 = 100).

Commodity	1990		2000	
	price	qty	price	qty
A	4	50	10	40
B	3	10	9	2
C	2	5	4	2

Construct Marshal Edgeworth price and quantity indices.

Item	pn	qo + qn	pn (qo + qn)	po	po (qo+qn)
A	10	90	900	4	360
B	9	12	108	3	36
C	4	7	<u>28</u>	2	<u>14</u>
			1036		410

$$M_p = 1036/410 \times 100 = 252.$$

Item	qn	po + pn	qn (po + pn)	qo	qo (po + pn)
A	40	14	560	50	700
B	2	12	24	10	120
C	2	6	<u>12</u>	5	<u>30</u>
			596		850

$$M_q = 596/850 \times 100 = 70.1$$

Uses of index numbers

1. To show relative changes in an economic variable e.g. prices, costs, income, wages, e.t.c. over time.
2. To reflect general economic condition e.g. cost of living, standard of living ,e.t.c.
3. It provides useful inputs for planning, budgeting and making economic forecasts.
4. Trade unions often use price indices to support negotiation for better wage.

5. It is useful for comparing performance, economic conditions, productivity e.t.c. overtime and among towns, industries, cities, countries, e.t.c.
6. Governments use various types of indices as input when formulating economic policies.

Limitations to the use of indices

1. They indicate general (aggregate) rather than specific changes.
2. They may be misinterpreted i.e CPI in Nigeria may not show differences in cost of living of rural/urban dwellers.
3. The weighting factor may become out of date.
4. Where sampling is involved in the collection of the required data, they may not be accurate.
5. Organisations entrusted with the publication of certain indices may not be reliable i.e NBS & CBN usual conflict on inflation rate.

Problems of index number construction

1. Different indices may have to be constructed for different purposes as an index that is useful for one purpose may not be appropriate for another.
2. The required data may not be available or they may be very expensive to collect.
3. There may be difficulty choosing items to be included in an index e.g CPI
4. There is problem of choosing appropriate weighting factor.
5. The choice of the base period may be arbitrary.

Real Vs Nominal changes in value

One important use of index number is for deflating monetary values of important economic variables such as wages, national income, export, import, e.t.c. in such a way as to enable us show real changes as against nominal changes in values assumed over time.

The formular for determing the real value at a given base period prices is follows:

$$RV_{n.o} = NV_n \times I_o / I_n$$

RV_n = current time's real value at base periods prices.

NV_n = current time point nominal value

I_o = base period's price index

I_n = current time points price index

Example

The table shows the average monthly wages received by registered nurse in a west African country and the consumer price index (1980 = 100):

Wages

Year	(#)	CPI
1980	1100	100.0
1985	3900	103
1990	7500	122
1995	18000	125
2000	32000	130
2005	65,000	136
2008	80,000	131

Determine the real wages in 2000, 2005 and 2008.

$$R_v \text{ 2000} = 32,000 \times 100/130 = 24,615.4$$

$$R_v \text{ 2005} = 65,000 \times 100/136 = 47,794.1$$

$$R_v \text{ 2008} = 80000 \times 100/131 = 61,068.7$$

Exercises

1. The table below shows the average prices and quantities of some household commodities demanded daily in a town for 2006 and 2008:

Commodity	Prices		quantities	
	2006	2008	2006	2008
Rice (kg)	50	60	800	700
Yam (1tubar)	30	32	600	630
Gari (1kg)	25	30	1000	1200
Beans (1kg)	60	80	600	400

Determine (2006 = 100).

- (a) The simple aggregate index for 2008
- (b) Mean of price relatives for 2008
- (c) Weighted average of price relatives for 2008
- (d) Laspeyres and Paasche's price indices for 2008

(e) Marshal Edgeworth's quantity index for 2008.

2. The table below shows Nigeria GDP (in ₦b) at current factor cost and price indices (1985=100) between 1980 and 1989.

Year	GDP	Index
1981	50.5	51.2
1982	51.6	55.1
1983	56.8	67.9
1984	63.0	94.8
1985	71.4	100
1986	72.1	105.4
1987	106.9	116.1
1988	142.7	181.2
1989	222.5	272.7

Determine the real GDP at 1985 constant factor cost.