com/0059/20
JOY NAFULA WANYONYI.

STA 205

Q1 Mean (x) $= \dfrac{8.6+9.4+7.9+6.8+8.3+7.3+9.2+9.6+8.7+11.4+10.35+5.4+8.14+5.5+6.9}{15}$

$\bar{x} = \dfrac{123.4}{15} = 8.23$

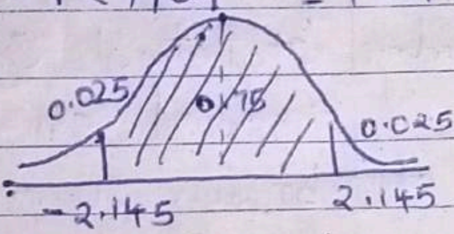
deviation $= 0.37, 1.17, -0.33, -1.43, 0.07, -0.93, 0.97, 1.37, 0.47, 3.17, 2.07, -2.83, -0.13, -2.75, -1.33.$

$d^2 = 0.0369, 1.3689, 0.1089, 2.0449, 0.6049, 0.8649, 0.9409, 1.8769, 0.2209, 10.0489, 4.2849, 8.4089, 0.0169, 7.4529$

$\Sigma d^2 = 39.1495$ $\qquad \Pi = 8.23$

Variance $= \dfrac{\Sigma d^2}{n-1} = \dfrac{39.1495}{15-1} = 2.7964$

$\sigma = \sqrt{2.7964} = 1.67$



Margin Error (E) $= Z\dfrac{\sigma}{\sqrt{n}} = 2.145 \times \dfrac{1.67}{\sqrt{15}}$

$= E = 0.92$

lower limit $= \Pi - E = 8.23 - 0.92$
$= 7.31$

Upper Limit $= \Pi + E = 8.23 + 0.92$
$= 9.15$

Confidence interval $= 7.31 < \Pi < 9.15$

Q2

Central Limit Theorem	Law of Large Numbers
States that for multiple samples taken from a population, of known mean and variance, If the sample size n is greater than or equal to 30, this will be Assumed to be a normal distribution even though the random variable X may be a non-normal.	States that as sample size grows the sample mean gets closer to the population mean irrespective whether the data set is normal or non-normal.

ii)

Level of Significance	Probability Value
This is a probability of a Type 1 error, set by researchers in advance which is usually the complement of confidence Level. Denoted by α	Used to Express level of statistical Significance often expressed as a p-value btween 0 and 1, the smaller the P-value the stonger the evidence.

iii)

t-statistics	z-statistics
Used when the sample size is less than 30	Used when the Sample size is greater than 30 $n \geq 30$
Used when population variance is unknown.	Used when population variance is known.

iv)

Type I	Type II
This is the mistake of rejecting the null hypothesis when it is true devoted as (α)	Mistake of failing to reject the null hypothesis when it is false devoted by (β)

Q3) Statistician has not Completed enough trials for the Experiment

Q4

a) $= 50(0.2) = 10$

$\mu = 10$

b) $\sigma^2 = 50 \times 0.2 \times 0.8$

$\sqrt{\sigma^2} = \sigma$

$\sigma = \sqrt{50 \times 0.2 \times 0.8}$

$\sigma = 2.8$

Q5 Any hypothesis tasking is done under Assumption that the null hypothesis is True

for example

Test at 0.5% level of claim that the True mean of a laptop is equal to 3. Suppose the sample results are $n = 100$ $\bar{x} = 2.84$ $\sigma = 0.8$

Step 1: State ē appropriate null (H_0) and Alternative (H_a) Hypothesis

$H_0 = 3$

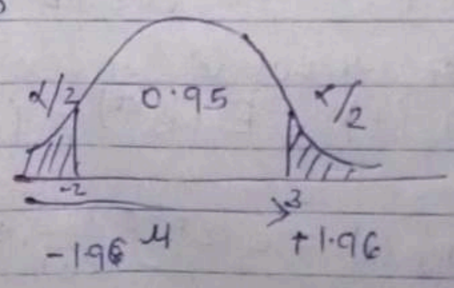
$H_1 \neq 3$ (

Step 2: Determine appropriate Techniques since we have σ then we can use a z-test

$n > 30$ hence we assume a normal distribution and we use ~~a~~ z-test.

Step 3: Determine the critical values

$\alpha = 0.05$



$CL = 1 - 0.25$
$= 0.95 - 95\%$

$0.975 \longrightarrow \pm 1.96$

Step 4: Compute the test statistic z-statistic

$$Z_c = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.84 - 3}{0.8/\sqrt{100}} = -2.0$$

Step 5:

Make a decision and conclusion

$z - {}^{-}2.0 < -1.96$ Under rejection region hence reject the Null hypothesis.

Conclusion: There is evidence that the mean is not equal to 3.

Q6 Mgf Moment generating Function

Is the expectation of a function of the random variable and is used to generate moments of random variables about the origin

$E(x), E(x^2), E(x^3) \dots, E(x^n)$

$$M(t) = E[e^{tx}] = \begin{cases} \sum_x e^{tx} \cdot P(x) & x : \text{discrete} \\ \int_x e^{tx} \cdot f(x) & x : \text{continous} \end{cases}$$

i) Normal distribution.

The probability density Function (pdf) of a random variable x with a Mean of $E(x) = \mu$ and variance of $var(x) = \sigma^2$ is

$$N(x : \mu, \sigma^2) = \frac{1}{\sqrt{(2\pi\sigma^2}} e^{-1/2 (x-\mu)^2/\sigma^2}$$

consider $\mu = 0$ $\sigma^2 = 1$ $x \sim N(0,1)$.

$$M_x(t) = E(e^{xt}) = \int e^{xt} \frac{1}{\sqrt{2\pi}} e^{-1/2 x^2} dt$$

$$m_x(t) = e^{1/2 t^2} \int \frac{1}{\sqrt{2\pi}} e^{1/2 (x-t)^2} dt$$

When the final equality follows from the fact that the expression under the integral is the

$N(x : \mu = t \;\; \sigma^2 = 1)$

$$M_x(t) = E(e^{xt}) = \int e^{xt} \frac{1}{\sqrt{2\pi\sigma^2)}} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2} \, dx$$

define $z = \frac{x-\mu}{\sigma} \approx x = \mu + z\sigma$

ii) Binomial Distribution

$$X \sim B(n, p).$$

$$P_x(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, 2, 3 \dots n$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

from Binomial theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{x} a^x b^{n-x}$$

$$M_x(t) = E(e^{tx}) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum \binom{n}{x} (pe^t)^x (1-p)^{n-k}$$

let $a = pe^t$

$b = 1 - p$

$$\sum_{x=0}^{n} \binom{n}{x} a^x b^{n-x}$$

$$= (a+b)^n = [pe^t + (1-p]^n$$

$$M_x(t) = [pe^t + (1-p)]^n$$

Q7 $M_{\bar{x}}(t) = [M_x(t/n)]^n$

$= M_{\bar{x}}(t) = E(e^{t\bar{x}})$

$= E(e^{t(x/n)})$

$= E(e^{t/n} \cdot e^{tx/n})$

$= e^{tx/n} E(e^{tx/n})$

but $E(e^{tx/n}) = M_x(t/n)$

$= e^{tx/n} M_x(t/n)$

hence

$M_{\bar{x}}(t) = [M_x(t/n)]^n$

Q8 Normal:

We use a 2 normal distribution you know or give the population standard deviation (σ)

If the sample provided is there

for t-distribution:

The sample size should be below 30

When we have an unknown standard deviation

Q9 Let $X \sim B(n, p)$ with $E(x) = 2$ and $var(x) = 4/3$

find $P(X < 4)$

$P(X < 4) = P(1) + P(2) + P(3)$

$E(x) = np = 2 \quad \ldots (1)$

$\sigma^2 = npq = 4/3 \quad \ldots (2)$

$\frac{npq}{np} = 4/3 \times 1/2 = 2/3 = q$

but $p + q = 1$

$p + \frac{2}{3} = 1$

$p = 1 - \frac{2}{3}$

$= \frac{1}{3}$

find n.

$\mu = np$

$2 = n \cdot \frac{1}{3}$ $\quad$ $2 = \frac{1}{3} n$

$n = 6$

$\therefore X \sim B(6, \frac{1}{3})$

but $P(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$P(1) = \binom{6}{1} \frac{1}{3} (1 - \frac{1}{3})^{(6-1)}$

$P(2) = \binom{6}{2} \frac{1}{3} (1 - \frac{1}{3})^{(6-2)}$

$P(3) = \binom{6}{3} \frac{1}{3} (1 - \frac{1}{3})^{(6-3)}$

$P(1) = 0.2633745$

$P(2) = 0.987664$

$P(3) = 1.1975209$

$\underset{P(1)}{0.2633745} + \underset{P(2)}{0.987664} + \underset{P(2)}{1.197509} = 3.2263$

$P(x < 4) = 3.2263$