

CSC 324 PRINCIPLES OF PROGRAMMING LANGUAGES

QUESTION ONE (COMPULSORY) [30 MARKS]

a. Why is it necessary for one to understand the history of programming languages before learning the principles of programming languages? (12 marks)

1. Evolution of concepts: Studying the history of programming languages provides insights into the evolution of programming concepts and paradigms over time. It helps learners understand the rationale behind the development of different language features and constructs.

2. Contextual understanding: Historical knowledge enables learners to understand the context in which programming languages were developed, including the technological advancements, design goals, and problem domains that influenced their creation.

3. Influence on modern languages: Many modern programming languages borrow ideas, syntax, and paradigms from earlier languages. Understanding the history of programming languages helps learners recognize and appreciate these influences in contemporary languages.

4. Learning from past mistakes: Examining the successes and failures of past programming languages can inform better design decisions in modern language development. It allows learners to identify common pitfalls, design deficiencies, and best practices in language design.

5. Appreciation of diversity: The history of programming languages showcases the diversity of approaches and paradigms in language design. It fosters an appreciation for the range of programming languages available and their suitability for different problem domains and programming paradigms.

6. Intellectual curiosity: Learning about the history of programming languages can be intellectually stimulating and enriching. It provides a broader perspective on the field of computer science and the role of programming languages in shaping computational thinking and problem-solving.

b. Consider the following program

Program P2

procedure W(var x: int)

begin

print x,

and

procedure D

begin

var n: int:

Win)

end

begin begini 2

10

D

end

If the language has dynamic scoping and parameters are passed by reference, simulate the program

Outputs [2 marks]

Output:

2 10

c. How does the execution of a machine code program on a von Neumann architecture computer occur? [3 marks]

1. Fetch: The CPU retrieves the next instruction from memory, using the program counter (PC) to determine the memory address.

2. Decode: The CPU decodes the instruction, determining its operation code (opcode) and operands.

3. Execute: The CPU performs the operation specified by the instruction, which may involve arithmetic calculations, data manipulation, or control flow operations.

4. Store: If necessary, the CPU stores the result of the operation back to memory or updates internal registers.

This process continues sequentially, with the CPU fetching, decoding, executing, and storing instructions until the program terminates.

d. Explain TWO programming language deficiencies that were discovered as a result of the research in software development in the 1970s [2 marks]

1. Lack of portability: Many programming languages of the 1970s were highly platform-dependent, meaning programs written in these languages could only run on specific hardware or operating systems. This lack of portability limited the reusability and interoperability of software components across different environments.

2. Poor support for abstraction: Early programming languages had limited support for abstraction mechanisms, such as data abstraction and procedural abstraction. This made it difficult to modularize and organize complex software systems, leading to code duplication, maintenance challenges, and reduced software quality and scalability.

e. Explain briefly the THREE fundamental features of an object-oriented programming language? [3 mark]

1. Encapsulation: Encapsulation is the bundling of data (attributes) and methods (functions or procedures) that operate on the data into a single unit called an object. It allows data hiding, where the internal details of an object's implementation are hidden from the outside world, and access to the object's data is controlled through well-defined interfaces.

2. Inheritance: Inheritance allows objects to inherit attributes and methods from parent classes (superclasses). It facilitates code reuse and promotes the creation of hierarchies of related classes, where subclasses (derived classes) inherit and extend the functionality of their parent classes. This promotes modularity, extensibility, and maintainability of code.

3. Polymorphism: Polymorphism enables objects of different classes to be treated uniformly through a common interface. It allows methods to be invoked on objects without knowing their specific types at compile time, promoting flexibility and code reuse. Polymorphism is typically achieved through method overriding (subclass provides a specific implementation of a method inherited from a superclass) and method overloading (multiple methods with the same name but different parameter lists).

f.) What arguments can you make against the idea of a single language for all programming domains? (2 marks)

1. Divergent requirements: Different programming domains have diverse requirements and constraints, such as performance, scalability, security, and domain-specific features. A single language may not adequately address all these requirements, leading to compromises in functionality or performance.

2. Specialization and optimization: Languages are often designed and optimized for specific use cases or domains, such as systems programming, web development, scientific computing, or artificial intelligence. Specialized languages can offer tailored features, libraries, and tools optimized for the specific needs of each domain, improving productivity and performance.

3. Learning curve and familiarity: Developers may already have expertise and familiarity with certain languages or paradigms that are well-suited to their domains. Introducing a new, unfamiliar language for all domains may require significant retraining, adaptation, and migration efforts, potentially impacting productivity and morale.

4. Language diversity and innovation: The diversity of programming languages fosters innovation, experimentation, and evolution in language design, enabling the exploration of new paradigms, features, and programming techniques. A single language may stifle diversity and inhibit the emergence of new ideas and approaches in software development.

g.) Some programming languages for example, Pascal have used the semicolon to separate statements, while Java uses it to terminate statements. Which of these, in your opinion, is most natural and least likely to result in syntax errors? Support your answer. [4 marks]

In my opinion, using semicolons to terminate statements, as in Java, is more natural and less likely to result in syntax errors. This is because semicolons serve as explicit terminators, clearly marking the end of a statement. In contrast, using semicolons to separate statements, as in Pascal, may lead to ambiguity or errors if a semicolon is accidentally omitted between statements. The explicit termination of statements with semicolons in Java makes the code structure clearer and less prone to misinterpretation or errors by both programmers and compilers.

h. Many contemporary languages allow two kinds of comments: one in which delimiters are used on both ends (multiple-line comments), and one in which a delimiter marks only the beginning of the current (one-line comments). Discuss the advantages and disadvantages of each of these with respect to design criteria. [4 marks]

Multiple-line comments:

Advantages:

1. Readability: Multiple-line comments allow programmers to write longer explanations, descriptions, or documentation for complex code segments, improving readability and understanding.

2. Organization: By enclosing multiple lines of code or text within comment delimiters, multiple-line comments help organize and structure the code, making it easier to navigate and maintain.

3. Documentation: Multiple-line comments are often used to generate documentation automatically using documentation generation tools or integrated development environments (IDEs), aiding in software documentation and knowledge transfer.

Disadvantages:

1. Verbosity: Writing lengthy comments for every code segment can make the code verbose and cluttered, detracting from the clarity and conciseness of the code.

2. Maintenance overhead: Multiple-line comments may require frequent updates and synchronization with the corresponding code segments, leading to maintenance overhead and potential inconsistencies between comments and code.

3. Risk of redundancy: Excessive or redundant comments may duplicate information already present in the code, increasing the risk of inconsistencies or discrepancies between comments and code.

Single-line comments:

Advantages:

1. Conciseness: Single-line comments are brief and concise, providing quick annotations or explanations directly adjacent to the relevant code segments, without cluttering the code.

2. Localization: Single-line comments are ideal for annotating specific lines or segments of code, allowing programmers to provide context or clarification without interrupting the flow of the code.

3. Inline documentation: Single-line comments can serve as inline documentation, providing brief explanations, reminders, or warnings directly within the code, facilitating understanding and maintenance.

Disadvantages:

1. Limited scope: Single-line comments are typically confined to a single line of code, restricting the amount of information that can be conveyed compared to multiple-line comments.

2. Lack of readability: In cases where longer explanations or documentation are necessary, single-line comments may become unwieldy, leading to reduced readability and comprehension.

3. Maintenance challenges: Single-line comments scattered throughout the codebase may be more difficult to manage and maintain compared to centralized multiple-line comments, increasing the risk of inconsistencies or outdated comments.

Languages continually evolve. What sort of restrictions do you think are appropriate for changes in programming languages? Compare your answers with the evolution of Java [4 marks]

Restrictions appropriate for changes in programming languages:

1. Backward compatibility: Changes to programming languages should strive to maintain backward compatibility with existing codebases and libraries to minimize disruption and facilitate migration.

2. Consistency and coherence: Changes should be guided by principles of consistency and coherence, ensuring that language features and syntax are intuitive, logical, and follow established conventions.

3. Performance considerations: Changes should be evaluated for their impact on runtime performance, memory usage, and compilation efficiency to avoid introducing unnecessary overhead or degradation in performance.

4. Community feedback: Changes should be informed by feedback from the programming community, including developers, users, and language designers, to address real-world needs, preferences, and concerns.

Comparison with the evolution of Java:

Java has evolved over time through a series of updates and releases, with each version introducing new language features, enhancements, and improvements while maintaining backward compatibility with previous versions. Java's evolution has been guided by principles of compatibility, consistency, performance, and community feedback. For example, Java introduced lambdas and streams in Java 8 to support functional programming paradigms, improving expressiveness and conciseness. However, these features were designed to seamlessly integrate with existing codebases and libraries, preserving backward compatibility and minimizing disruption for existing Java developers. Similarly, Java continues to evolve with each release, balancing the introduction of new features with the need to maintain stability and consistency across versions.

Why is it useful for a programmer to have some background in language design, even though he/she may never actually design a programming language (4 marks)

1. Understanding language principles: Knowledge of language design principles helps programmers understand the underlying concepts, paradigms, and trade-offs inherent in different programming languages. This enables them to make informed decisions when selecting or using programming languages in their projects.

2. Improved language proficiency: Familiarity with language design principles enhances a programmer's proficiency in using programming languages effectively, enabling them to leverage language features, idioms, and best practices to write clear, concise, and maintainable code.

3. Debugging and troubleshooting: Understanding language design enables programmers to diagnose and troubleshoot issues more effectively by analyzing language semantics, syntax, and behavior. It helps them identify potential pitfalls, ambiguities, or unintended consequences in code.

4. Language extension and customization: Knowledge of language design empowers programmers to extend or customize existing programming languages through domain-specific languages (DSLs), libraries, or frameworks tailored to specific problem domains or application requirements.

5. Collaboration and communication: Language design principles provide a common vocabulary and framework for communication among programmers, enabling effective collaboration, knowledge sharing, and exchange of ideas within the programming community.

QUESTION TWO [20 MARKS]

a. What is the meaning of lexeme and token as used in programming" [2 marks]

- Lexeme: In programming, a lexeme refers to the smallest unit of language recognized by the compiler or interpreter. It represents a sequence of characters in the source code that has a specific meaning in the programming language. Lexemes include identifiers, keywords, constants, operators, and punctuation symbols.

- Token: A token is a categorized group of lexemes identified by the lexical analyzer during the lexical analysis phase of compilation. Tokens represent meaningful units of language, such as keywords, identifiers, literals, and punctuation symbols. Tokens serve as input to the syntax analyzer for further processing in the compilation process.

b. Conventionally, all language developments require mastery of grammar or syntax. In which form is the programming language syntax commonly described? 14 marks]

Programming language syntax is commonly described in the form of a formal grammar. Formal grammars provide a systematic way to define the structure and syntax rules of a programming language using a set of production rules. The most commonly used form of formal grammar for describing programming language syntax is the Backus-Naur Form (BNF) or its variants, such as Extended Backus-Naur Form (EBNF). BNF provides a notation for specifying context-free grammars, which define the syntactic structure of programming languages in terms of production rules, non-terminal symbols, terminal symbols, and syntactic categories.

c.) Explain the meaning of a left-recursive grammar and an ambiguous grammar, [4 marks]

- **Left-recursive grammar:** A left-recursive grammar is a type of grammar where a non-terminal symbol can directly or indirectly derive itself as the leftmost symbol in one or more of its production rules. Left recursion occurs when a non-terminal A can produce a string that starts

with A itself. Left-recursive grammars can pose challenges for parsers, leading to inefficiency or ambiguity in parsing algorithms.

- **Ambiguous grammar:** An ambiguous grammar is a type of grammar where one or more strings in the language can be derived by more than one parse tree. Ambiguity in a grammar arises when there are multiple valid interpretations or meanings for the same input string. Ambiguous grammars can result in parsing conflicts and difficulties in determining the intended syntactic structure of a program.

d. Describe the two levels of use of operational semantics. Why can machine languages not be used to define statements in operational semantics? [4 marks]

1. Small-step semantics: Small-step semantics defines the operational semantics of programming language constructs by specifying individual reduction steps that transform the program state from one configuration to another. Each reduction step represents a small atomic computation or transition in the program execution.

2. Big-step semantics: Big-step semantics defines the operational semantics of programming language constructs by specifying the overall evaluation behavior of expressions or statements in terms of their final results or values. Big-step semantics directly evaluates the entire program or expression in a single step, without decomposing it into smaller reduction steps.

Machine languages cannot be used to define statements in operational semantics because machine languages operate at a lower level of abstraction, dealing with concrete hardware instructions and processor behavior rather than high-level language constructs. Operational semantics aims to provide a formal and abstract description of language behavior that is independent of specific machine architectures and execution environments, making it more suitable for reasoning about language semantics and program behavior at a higher level of abstraction.

e. i) In denotational semantics, what are the syntactic and semantic domains? [2 marks]

- **Syntactic domain:** The syntactic domain consists of abstract syntax trees (ASTs) or syntactic structures representing the syntactic structure of programming language constructs. It defines the structure of programs in terms of language entities such as expressions, statements, declarations, and control flow constructs.

- **Semantic domain:** The semantic domain consists of mathematical objects or values that represent the meaning or interpretation of language constructs. It defines the behavior, effects, and semantics of language constructs in terms of mathematical functions, relations, or mappings from syntactic structures to semantic values.

ii) What is stored in the state of a program for denotational semantics? [2 marks]

The state of a program for denotational semantics typically includes the **values of variables**, **data structures**, and **program counters** that represent the current state or configuration of the program during its execution. The state encapsulates the dynamic aspects of program execution, including the values of program variables, memory contents, and control flow information. In denotational semantics, the state is used to define the operational semantics of language constructs by mapping syntactic structures to semantic values based on the current program state.

f.) What TWO things must be defined for each language entity in order to construct a denotational description of the language? (4 marks)

Semantic Domain

- The semantic domain specifies the set of possible meanings or values that a language entity can have. It defines the mathematical structure that represents the semantics of the entity. For example, for a variable in a programming language, the semantic domain might be a set of values such as integers, floating-point numbers, or strings.

- The semantic domain provides the foundation for interpreting the language entity in mathematical terms, allowing us to map syntactic constructs to meaningful mathematical objects.

Semantic Function:

- The semantic function defines the mapping between the syntactic representation of a language entity and its corresponding meaning in the semantic domain. It specifies how the syntax of the language is interpreted and evaluated to produce semantic values.

- For example, the semantic function for an arithmetic expression might define how to evaluate the expression to obtain a numerical value. Similarly, the semantic function for a statement might describe its effect on the program state.

- The semantic function bridges the gap between the syntax of the language and its semantics, providing a formal mechanism for assigning meaning to language constructs.

QUESTION THREE (20 MARKS]

a. What is the difference between total correctness and partial correctness with regard to loop termination? [4 marks]

Total correctness: A loop is considered totally correct if it not only terminates but also produces the correct output for all possible inputs. Total correctness includes both termination and correctness of the output.

- **Partial correctness:** A loop is considered partially correct if it terminates and produces the correct output for the inputs for which termination is guaranteed. Partial correctness only focuses on the correctness of the output under the assumption that the loop terminates.

b. Explain the preconditions and post-conditions of a given statement mean in axiomatic semantics. [4 marks]

- **Preconditions:** Preconditions specify the conditions that must be satisfied before executing a given statement or program segment. They describe the initial state or context in which the statement is expected to execute successfully. Preconditions ensure that the statement's execution is meaningful and well-defined.

- **Post-conditions:** Post-conditions specify the conditions that must hold true after executing a given statement or program segment. They describe the expected outcome or effect of the statement's execution on the program state. Post-conditions verify that the statement has achieved its intended purpose or behavior.

c. In what fundamental way do operational semantics and denotational semantics differ? [2 marks]

- **Operational semantics** defines the meaning of language constructs in terms of their execution behavior on abstract machines or operational models. It focuses on describing the step-by-step execution of programs and the effects of language constructs on the program state.

- **Denotational semantics** defines the meaning of language constructs in terms of mathematical objects or values that represent their semantics. It focuses on providing a mathematical model of language behavior, where language constructs are mapped to semantic values or functions that capture their meaning and behavior.

d. The two mathematical models of language description are generation and recognition.

- **Generation:** In generation, the syntax of a programming language is defined by specifying rules or productions that generate valid syntactic structures or strings in the language. These rules describe the hierarchical structure of language constructs using a formal notation such as Backus-Naur Form (BNF) or its variants. By applying these rules recursively, valid program syntax can be generated systematically.

- **Recognition:** In recognition, the syntax of a programming language is defined by specifying a formal grammar or automaton that recognizes valid syntactic structures or strings in the language. The grammar or automaton defines a set of rules or patterns that describe the valid syntactic patterns or structures allowed in the language. By analyzing input strings according to these rules, valid program syntax can be recognized or validated.

Describe how each can define the syntax of a programming language.

[4 marks]

Using the grammar, show a parse tree and a leftmost derivation for each of the following statements: $AA(B+(CA))$

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14 marks)

f. What are the reasons why using BNF is advantageous over using an informal syntax description? [2 marks]

1. Formalism:

- BNF provides a formal and precise notation for describing the syntax of a programming language. It defines clear and unambiguous rules for constructing valid syntactic structures, leaving no room for interpretation or ambiguity.
- In contrast, informal syntax descriptions may lack precision and consistency, leading to misunderstandings and errors in language specification and implementation.

2. Automated Processing:

- BNF can be easily processed by parsing tools and compilers to automatically generate parsers for the language. Parsing tools can efficiently analyze BNF grammars to recognize and validate syntactic structures in input programs.
- Informal syntax descriptions, on the other hand, are not amenable to automated processing, requiring manual interpretation and implementation of syntax rules, which can be error-prone and time-consuming.

e. Parsing tree and leftmost derivation:

Grammar: $S \rightarrow AA \mid (B+(CA))$

Parsing tree:

...

S
 $/ \quad \backslash$
 $A \quad A$
 $| \quad / \quad \backslash$
 $B \quad + \quad CA$
 $| \quad |$
 $C \quad A$
 $|$
 ϵ
 \dots

Leftmost derivation:

\dots

$S \Rightarrow AA \Rightarrow (B+(CA))$

\dots

f. Advantages of using BNF over using an informal syntax description:

- Formality: BNF provides a precise and unambiguous notation for specifying the syntax of a programming language, facilitating rigorous analysis and interpretation of language constructs. It eliminates ambiguity and inconsistency inherent in informal descriptions, ensuring clarity and accuracy in defining language syntax.
- Machine readability: BNF can be easily parsed and processed by automated tools and parsers, enabling automated syntax checking, code generation, and program analysis. BNF serves as a standard notation for defining language syntax, promoting interoperability and tool compatibility across different programming environments and development tools.

QUESTION FOUR [20 MARKS]

a. What are the different grammar symbols for formal languages?

12 marks]

b. How does a lexical analyzer serve as the front end of a syntax analyzer?

14 marks]

e. Describe briefly the three approaches to building a lexical analyzer.

14 marks

d. Why are character classes used, rather than individual characters, for the letter and digit transitions of a state diagram for a lexical analyzer

14 marks)

e. Why are named constants used, rather than numbers, for token codes?

[2 marks]

f. Describe the complexity of parsing algorithms and explain the two distinct goals of syntax analysis?

14 marks]

a. The different grammar symbols for formal languages include:

1. Terminal symbols: Terminal symbols, also known as tokens or terminal symbols, are the basic units of the language's syntax. They represent the smallest meaningful elements of the language, such as keywords, identifiers, constants, and punctuation symbols. Terminal symbols appear in the input strings and cannot be further decomposed into smaller constituents.

2. Non-terminal symbols: Non-terminal symbols represent syntactic categories or abstract language constructs composed of other symbols, including both terminal and non-terminal symbols. Non-terminal symbols serve as placeholders for sequences of symbols that can be expanded into larger syntactic structures according to production rules.

3. Start symbol: The start symbol is a special non-terminal symbol that represents the top-level syntactic category or entry point of the language's syntax. It serves as the starting point for deriving valid sentences or program constructs in the language using production rules.

b. A lexical analyzer serves as the front end of a syntax analyzer by performing the following tasks:

1. Tokenization: The lexical analyzer scans the input source code character by character and groups characters into meaningful units called tokens. Tokens represent syntactic elements such as keywords, identifiers, literals, and operators, identified based on predefined lexical rules and patterns.

2. Lexical analysis: The lexical analyzer applies lexical rules and regular expressions to recognize and classify tokens according to their types. It identifies keywords, operators, identifiers, constants, and other language constructs, generating a stream of tokens as output.

3. Error handling: The lexical analyzer detects lexical errors, such as invalid characters or malformed tokens, and reports them to the syntax analyzer for error recovery and reporting.
4. Interface with syntax analyzer: The lexical analyzer interfaces with the syntax analyzer by providing the stream of tokens as input. The syntax analyzer uses this token stream to perform further syntactic analysis and parsing of the input source code.

c. Three approaches to building a lexical analyzer are:

1. Manual construction: In this approach, the lexical analyzer is implemented manually using programming languages such as C, C++, or Java. The developer writes code to scan the input source code character by character, apply lexical rules and regular expressions to recognize tokens, and generate a stream of tokens as output. Manual construction offers flexibility and control over the design and implementation of the lexical analyzer but requires significant effort and expertise.
2. Lexer generator tools: Lexer generator tools automate the generation of lexical analyzers from high-level specifications or formal descriptions of lexical rules and patterns. These tools take input in the form of regular expressions or lexical specifications and automatically generate efficient lexical analyzer code in programming languages such as C, C++, or Java. Popular lexer generator tools include Flex (for C/C++) and JFlex (for Java). Lexer generator tools simplify the development process and ensure consistency and correctness but may require learning curve and may limit flexibility compared to manual construction.
3. Integrated development environments (IDEs): Some integrated development environments (IDEs) provide built-in support for lexical analysis as part of their compiler or language processing toolchains. IDEs offer features such as syntax highlighting, code completion, and error checking, powered by built-in lexical analyzers that tokenize and analyze the input source code in real-time. IDE-based lexical analyzers provide convenience and seamless integration with the development workflow but may lack customization and extensibility compared to dedicated lexical analyzer implementations.

d. Character classes are used, rather than individual characters, for the letter and digit transitions of a state diagram for a lexical analyzer because:

1. Efficiency: Using character classes allows the lexical analyzer to recognize multiple characters with similar properties or patterns using a single transition in the state diagram. For example, a character class [a-zA-Z] represents all uppercase and lowercase letters, reducing the number of transitions and states in the finite automaton, thus improving efficiency and performance.
2. Compactness: Character classes help simplify the state diagram by reducing the number of transitions and states needed to represent lexical rules and patterns. By grouping similar characters into classes, the state diagram becomes more compact and easier to manage, enhancing readability and maintainability.

3. Flexibility: Character classes provide flexibility in defining lexical rules and patterns by allowing ranges or sets of characters to be specified using shorthand notation. This simplifies the specification of regular expressions and lexical rules, making it easier to define and update the lexical analyzer's behavior as needed.

e. Named constants are used, rather than numbers, for token codes for the following reasons:

1. Readability: Named constants provide descriptive names for token codes, making the token definitions more readable and understandable for developers and maintainers. Descriptive names convey the semantic meaning or purpose of each token, facilitating comprehension and debugging of the lexical analyzer code.

2. Maintainability: Named constants enhance code maintainability by decoupling token codes from specific numerical values or representations. If token codes need to be modified or extended, changes can be made to the named constants without affecting the rest of the lexical analyzer code. This promotes modularity and ease of maintenance, reducing the risk of errors or inconsistencies.

3. Portability: Named constants improve code portability by making the lexical analyzer code more platform-independent and language-agnostic. Developers can use consistent token definitions across different programming languages and environments, ensuring compatibility and ease of integration with other components or systems.

f. Parsing algorithms complexity and goals:

1. Complexity: Parsing algorithms exhibit different levels of complexity, depending on the parsing technique and the nature of the input language grammar. The complexity of parsing algorithms is typically characterized in terms of time complexity and space complexity, representing the computational resources required for parsing input strings of varying lengths.

2. Goals of syntax analysis:

- Correctness: The primary goal of syntax analysis is to ensure that the input program adheres to the syntactic rules and grammar of the programming language. Syntax analysis verifies the syntactic structure and organization of the program, detecting and reporting syntax errors or violations that may cause compilation or execution failures.

- Efficiency: Another goal of syntax analysis is to perform parsing and syntactic analysis efficiently, minimizing the computational resources and time required to process input programs. Efficient parsing algorithms optimize parsing speed and memory usage, enabling rapid development and compilation of software projects.

f. ****Parse Tree and Leftmost Derivation****:

The grammar for the statement `A=A*(B+(CA))` could be represented as:

...

$S \rightarrow A = A * (B + (C * A))$

...

****Parse Tree****:

...

```

      S
    / | \
  A  =  *
    / | \
  A ( )
    / \
  B  +
    / \
  C  A

```

...

****Leftmost Derivation****:

...

```

S → A = A * (B + (C * A))
  → A = A * (B + (C * A))
  → A = A * (B + (C * A))
  → A = A * (B + (C * A))

```


$$\rightarrow A = A * (B + (C * A))$$