STA 112: Lecture 1

SET THEORY

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Overview

Definition of probability measure

sample spaces

set notation and arithmetic

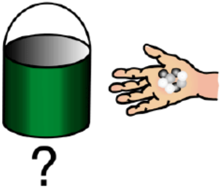
Probability measures

Important properties

Complements

Addition law

Difference between Statistics and Probability



Statistics: Given the information in your hand, what is in the box?

Probability: Given the information in the box, what is in your hand?

Based on: *Statistics*, Norma Gilbert, W.B. Saunders Co., 1976.

Definition of probaility

Likelihood, chance, tendency, trend

$$\text{Probability of an event} = \frac{\text{number of ways an event can occur}}{\text{number of possible outcomes}}$$

Probability has broad relevance in many areas

- An individual tests positive for HIV on a rapid screening test. What is the probability that the person actually is HIV-positive?
- What is the likelihood of contracting HIV in one act of sexual intercourse?
- What is the probability that someone with early stage breast cancer will survive for 10 years? Does the probability differ if she is 40 versus if she is 50?
- If a graduate program makes 10 offers of admission, what is the expected number of students that will enroll? What is the probability that 10 will enroll?
- In a room with 25 people, what is the probability that two share the same birthday?

Sample spaces I

Probability theory is a useful model for events that occur at random.

Sample space The set of all possible outcomes of an event

Example 1 . Driving to work, you go through 3 intersections. At each intersection you either stop (S) or continue (C). Sample space is set of all possible outcomes $\Omega = \{SSS, SSC, SCS, CSS, SCC, CSC, CCS, CCC\}$

Example 2 . A woman gives birth to twins. Sample space of possible gender combinations is $\Omega = \{FF, MF, FM, MM\}$

Sample space II

Example 3 . A person attempts to quit smoking. The time from the beginning of his quit attempt until potential relapse is all times greater than zero. $\Omega = \{t : t > 0\}$

Example 4 . During the next year, you will have a certain number of visits to the doctor. The sample space is $\Omega = \{0, 1, 2, ...\}$

Special events

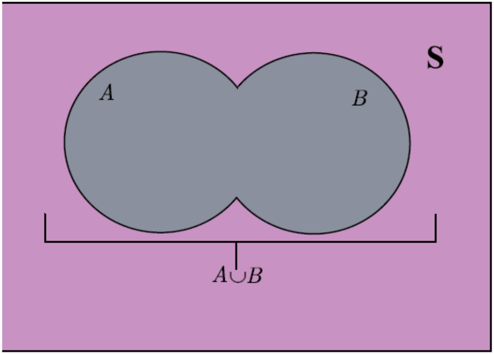
Null event An event that cannot happen .

Mutually exclusive events Two events that cannot both happen. For example event $A = \textit{Male}$ and $B = \textit{Pregnant}$ are two mutually exclusive events (as no males can be pregnant).

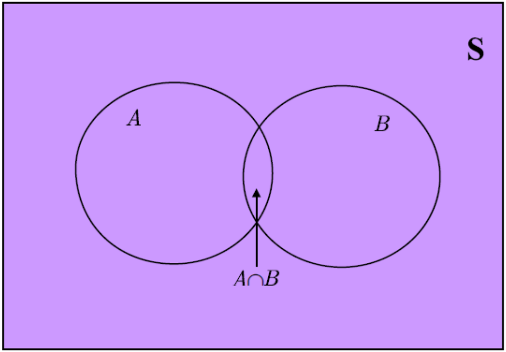
Operations on events

There are three main operation

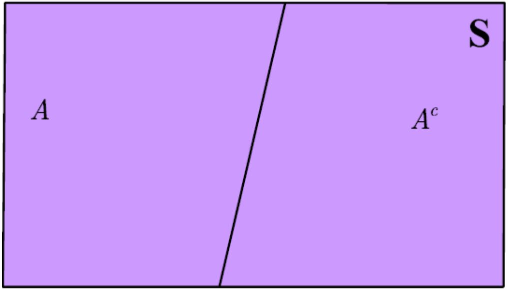
Union . $A \cup B$ is all the events in A and B .



Intersection $: A \cap B$ is the event that occurs both in A and B .



Complement . A^c or $\bar{A}$ is the event that A does not occur.



Properties of complements:

$A \cup A^c = \Omega$

$A \cap A^c = \emptyset$;

Example 1 (twins) A = one boy, B = no boys. Then

$$A = \{FM, MF\}$$
$$B = \{FF\}$$
$$A \cup B = \{FM, MF, FF\}$$
$$A \cap B = \emptyset;$$

Example 2 (twins) A = first is a boy. B = at least one boy.

$$A = \{MM, MF\}$$
$$B = \{MM, MF, FM\}$$
$$A \cup B = \{MM, MF, FM\}$$
$$A \cap B = \{MM, MF\}$$

Example 3 (visits to the doctor) A = exactly 3 visits to the doctor, B = 2 or more visits to the doctor.

$A = \{3\}$

$B = \{2, 3, 4, ...\}$

$A \cup B = \{2, 3, 4, ...\}$

$A \cap B = \{3\}$

Probability measure

Formally, it is a function labeled by P that maps subsets of the sample space to the $[0, 1]$ interval.
Satisfies the following axioms

1. If an event A is in the sample space, then $P(A) \geq 0$
2. The probability of all possible events is one; i.e.,$P(\Omega) = 1$
3. If A and B are disjoint (cannot happen simultaneously), then $P(A \cup B) = P(A) + P(B)$. This also holds for more than two disjoint events.

Complements : $P(A^c) = 1 - P(A)$

Example : Suppose each combination of twins is equally likely (probability 0.25). A = two boys. Then

$$A = \{MM\}$$

$$P(A) = 0.25$$

$$A^c = \{MF, FM, FF\}$$

$$P(A^c) = 1 - 0.25 = 0.75$$

Addition Law : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example : Compute the probability that the first twin is a boy or the second is a boy.

$$A = \{MF, MM\}$$

$$B = \{FM, MM\}$$

$$A \cap B = \{MM\}$$

$$\begin{aligned} P(A \cup B) &= P(\{MF, MM\}) + P(\{FM, MM\}) - P(\{MM\}) \\ &= 0.5 + 0.5 - 0.25 \\ &= 0.75 \end{aligned}$$