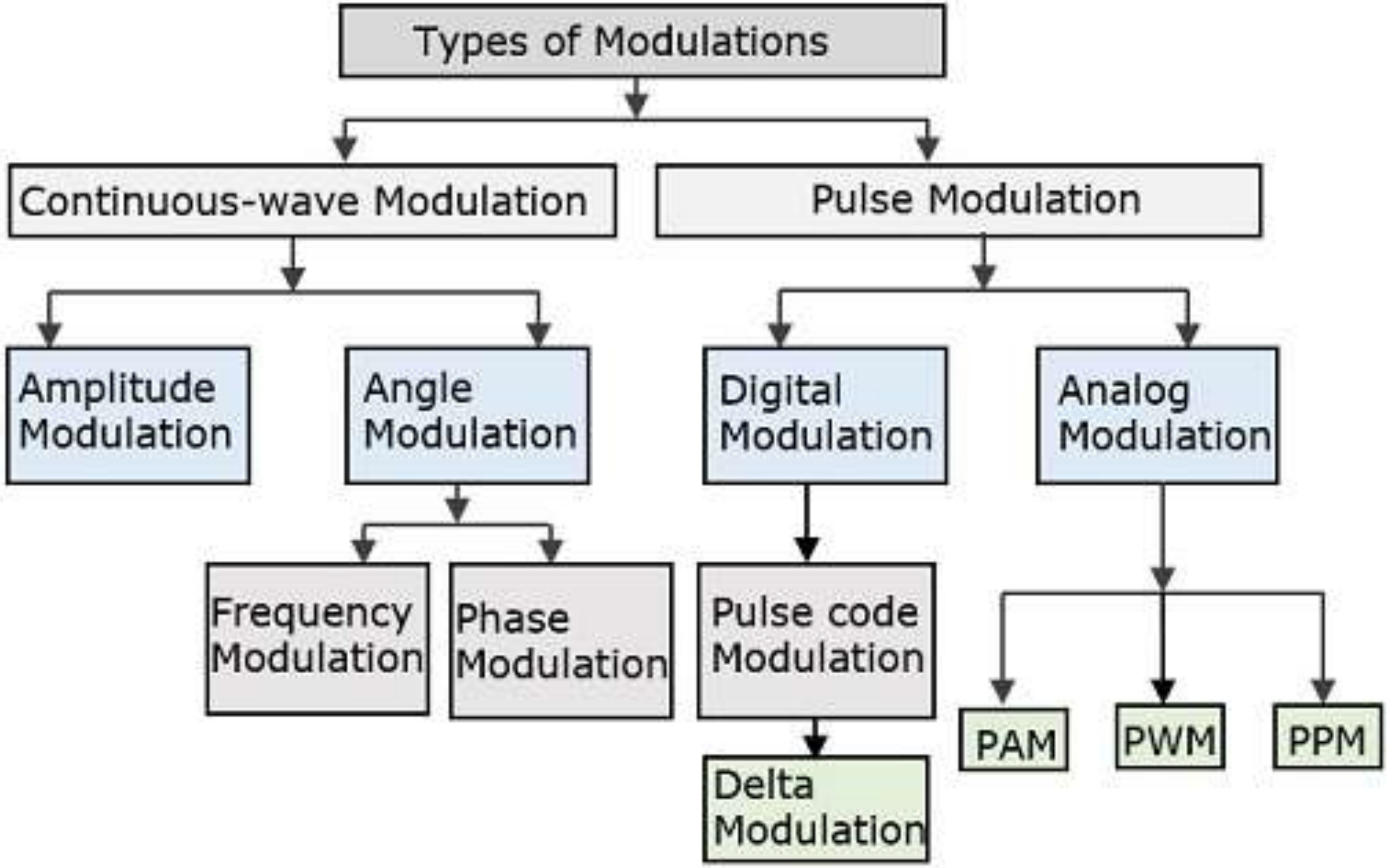
# ANGLE MODULATION

# Overview: Categories of modulation Where is angle modulation?



# ANGLE MODULATION

- The phase angle (θ) of a sinusoidal wave is varied with respect to time. While keeping the amplitude of the carrier frequency constant.
- So the frequency or phase of the carrier is varied according to the amplitude of the modulating (information) signal.
- They are two forms:
  i. frequency modulation: When amplitude of modulating signal is used to modulate frequency of carrier signal..
  ii. phase modulation: When amplitude of modulating signal is used to modulate phase (angle) of carrier signal.

# Mathematically expression of angle modulation

The angle modulated wave is given by:

$s(t)=V_c\cos[\omega_c t+\theta(t)]$

Where: $s(t)$= angle modulated wave

$V_c$ = peak carrier amplitude

$\omega_c = 2\pi f_c$ = carrier radian frequency

$\theta(t)$ = instantaneous phase deviation in radians

- The angle modulation is expressed mathematically as

$\theta(t) = F[m(t)]$ where $m(t)$ = the modulated signal

- Note that: varying frequency inherently causes phase to vary and vice versa.

- Therefore:
  - FM is generated when the frequency of the carrier is varied directly in accordance with the modulating signal.

  The change in frequency ($\Delta f$) is frequency deviation, and is the relative displacement of the carrier frequency in Hz.

  - PM is generated when the phase of the carrier is varied directly in accordance with the modulating signal.

  The change in phase ($\Delta\theta$) is phase deviation, and is the relative angular displacement in radians of the carrier in respect to a reference phase. Δ

# Mathematical analysis of PM

- Instanteous phase deviation = $\theta(t)$ radians
- Instantaneous phase = $\omega_c t + \theta(t)$ ............(i)
- For a modulating signal m(t),

Phase modulation = $\theta(t) = k_p\, m(t)$ in radians

where $k_p$ is phase sensitivity constant expressed in radian per volt

- Substituting $\theta(t)$ in eq (i) above we get:

instantaneous phase = $\omega_c t + k_p\, m(t)$

# Mathematical analysis of PM

- Also the general equation becomes:

$$s(t) = V_c \cos [\omega_c t + k_p\, m(t)]$$

- Substituting modulating signal, $m(t) = V_m \cos (\omega_m t)$ we get

$$s(t) = V_c \cos [\omega_c t + k_p\, V_m \cos (\omega_m t)]$$

# Mathematical analysis of FM

- Instantaneous frequency deviation = $\frac{d\,\theta(t)}{d\,t}$ rad/s
- Instantaneous frequency ($f_i$) = $\frac{d}{d\,t}[\omega_c t + \theta(t)]$ rad/s

$$= \omega_c + \frac{d\theta(t)}{d\,t} \quad \text{rad/s}$$

$$= 2\pi f_c + \frac{d\theta(t)}{d\,t}$$

$$= f_c + \frac{d\theta(t)}{2\pi\,d\,t} \quad \text{Hz}$$

# Mathematical analysis of FM continued

- Frequency modulation for a modulating signal m(t) can be given as

$$\text{frequency modulation} = \frac{d\,\theta(t)}{d\,t} = k_f\, m(t) \qquad \text{rad/s}$$

where $k_f$ is the frequency sensitivity of the modulator in rad/volt second

- From eq () , $\theta(t) = \int k_f\, m(t)\, d(t)$
- Substituting $\theta(t)$ in general equation we get:

$$s(t) = V_c \cos\left[\omega_c t + \int k_f\, m(t)\, d(t)\right]$$

Substitution $m(t) = V_m \cos(\omega_m t)$ we get

$$V_c \cos\left[\omega_c t + \int k_f\, V_m \cos(\omega_m t)\right]$$

# Mathematical analysis of FM continued

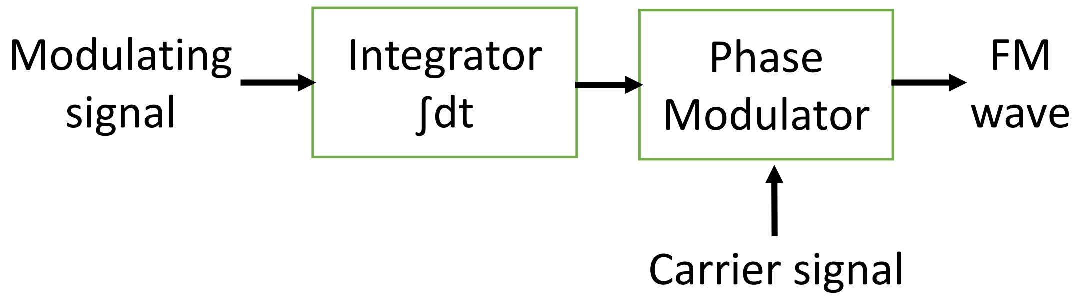
- Integrating we get: $V_c \cos [\omega_c t + k_f V_m \dfrac{\sin \omega_m t}{\omega_m}]$
- Substituting θ(t) we get the instantaneous frequency as:

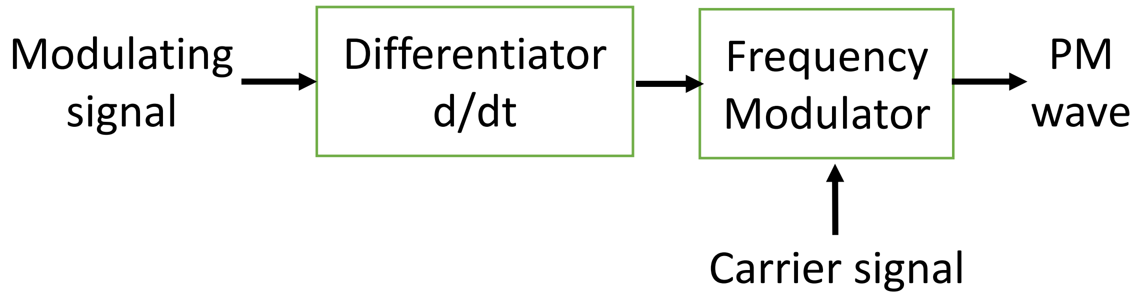
$$f_i = f_c + \frac{d}{2\pi\, d\, t}\left( \int k_f\, m(t)\, d(t)\right)$$

$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

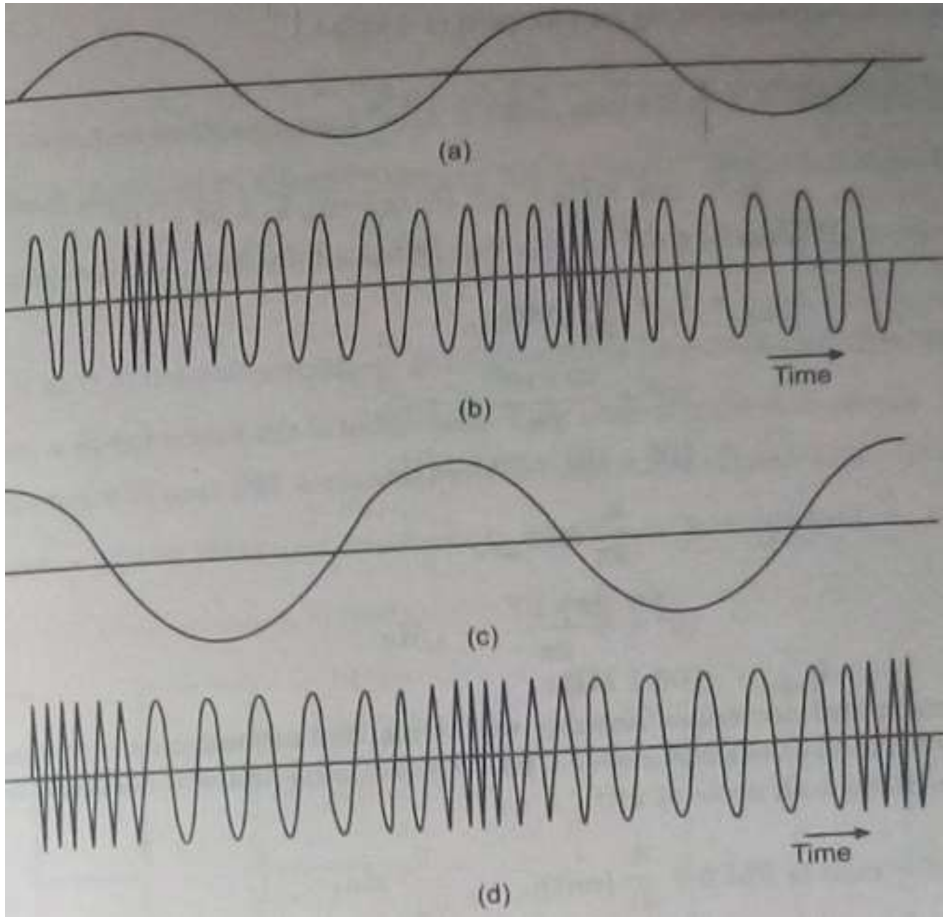
- From the above discussion we can realize that phase modulation and frequency modulation are not only very similar but are inseparable.
- Replacing m(t) in equating for PM with ∫ m(t) dt changes PM into FM.
- Thus a signal which is an FM wave corresponding to m(t) is also the PM wave corresponding to ∫ m(t) dt.
- Similarly, a PM wave corresponding to m(t) is the FM wave corresponding to d/dt m(t).

- In both PM and FM, the angle of the carrier is varied according to some measurement modulating signal m(t).
- In PM it is directly proportional to m(t) while in FM it is proportional to the integral of m(t)



- Fig a, indicates two complete cycles of a signal.
- Fig b, indicates the FM wave produced by m(t).
- To determine the phase modulated wave for m(t), we have seen that it is the same as the FM produced by d/dt [m(t)]
- Fig. c, shows the derivative of m(t). Note it is a cosine wave.
- Fig d, indicates the desired PM wave.

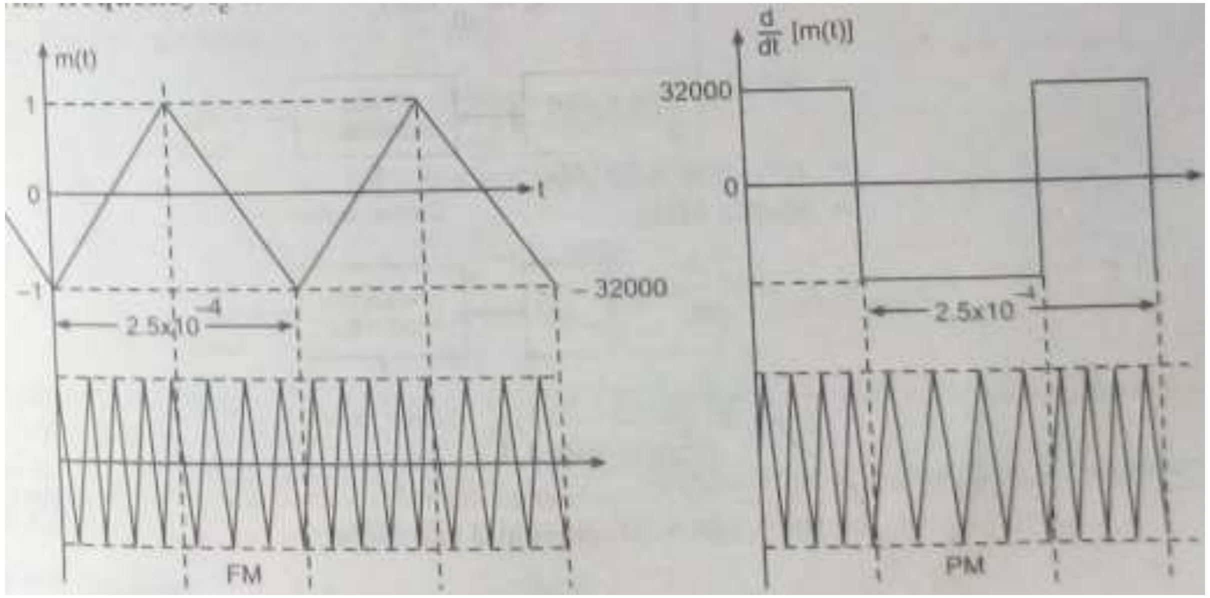


# Example:

- Determine the minimum and maximum frequencies in the modulated FM given m(t) is 4kHz with $V_{PP}$ = -1V to 1V, constants $k_f=2\pi x10^5$ and $k_p$= 8π and $f_c$= 100MHz.

Solution: if we synthesize the problem we see than the diagrams of FM and PM look as below:

- T=Period = 1/4kHz=$2.5x10^{-4}$s



For FM:

$$f_{min} = f_c + \frac{k_f}{2\pi} m(t)_{min} = 100\text{MHz} + \frac{2\pi x105}{2\pi} \text{ x } (-1) \text{ Hz} = 99.9\text{MHz}$$

$$f_{max} = f_c + \frac{k_f}{2\pi} m(t)_{max} = 100\text{MHz} + \frac{2\pi x105}{2\pi} \text{ x } (+1) \text{ Hz} = 100.1\text{MHz}$$

The FM carrier swings from 99.9MHz to 100.1MHz at 4kHz

For PM: note PM for m(t) is FM for d/dt [m(t)]

Note that the time derivative of triangular m(t) shown is the square wave as shown. So we find the FM for the derived m(t) as:

$$f_i = f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t) = f_c + \frac{8\pi}{2\pi}\frac{d}{dt}m(t) = f_c + 4\frac{d}{dt}m(t)$$

So the minimum frequency of phase modulated signal is:

$$f_{i\,min} = f_c + 4[\frac{d}{dt}m(t)]_{min} = 100\text{MHz} + 4[\frac{-2}{\frac{1}{2}(2.5x10-4)}]Hz = 99.872\text{MHz}$$

The maximum frequency of the phase modulated signal is:

$$f_{\mathrm{i\,max}} = f_{\mathrm{c}} + 4[\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{m(t)}]_{\mathrm{max}} = 100\mathrm{MHz} + 4[\frac{2}{\frac{1}{2}\,(2.5\mathrm{x}10{-}4)}]Hz$$

$$= 100.128\mathrm{MHz}$$

This indirect method of finding PM has been used here because the wave is a continuous wave, otherwise just use a direct method.

Since d/dt [m(t)] switches back and forth from a value of  +3200 to 3200 . The carrier swings from 99.9MHz to 100.1MHz.

# The Carrier swing of an FM signal

- The total variation in frequency from lowest to highest is known as carrier swing.

  carrier swing=2 x $\Delta f$

  where $\Delta f$ = frequency deviation

- Note that the frequency variation depends on loudness (amplitude) of the modulating signal. $k_f$ =

# Modulation index of FM

- Is the ratio of frequency deviation to the modulating frequency

$$\text{modulation index, } m_f = \frac{\text{frequency deviation}}{\text{modulation frequency}} = \frac{\Delta f}{f_m}$$

- Modulating index in FM may be greater than 1

# Mathematical Expression for single-tone frequency modulation

- Let carrier signal be: $c(t) = A \cos \omega_c t$
- Let the modulating signal be: $x(t) = V_m \cos \omega_m t$
- Let the FM wave be: $c(t) = A \cos \phi_i$

  where $\phi_i$ is instantaneous phase angle of the modulated wave
- We know instantaneous freq. of modulated signal is given by:

  $\omega_i = \omega_c + k_f \cdot x(t)$
- Substituting value of x(t), we get: $\omega_i = \omega_c + k_f \cdot V_m \cos \omega_m t$

# Mathematical Expression for single-tone frequency modulation ……continued

- Substituting value of x(t), we get: $\omega_i = \omega_c + k_f \,.\, V_m \cos \omega_m t$
- But we know frequency deviation is given as:

$$\Delta\omega = |k_f \,.\, x(t)|_{max} = k_f |x(t)|_{max} = k_f \,.\, V_m$$

- Therefore: $\omega_i = \omega_c + \Delta\omega .\cos \omega_m t$
- The total phase angle $\phi_i$ of the modulated wave is given as:
- $\phi_i = \int \omega_i \, dt$
- Putting the value of $\omega_i$ from equation above, we get:

$$\phi_i = \int [\omega_c + \Delta\omega .\cos \omega_m t\,]\, dt = = \omega_c\, t + \frac{\Delta\omega}{\omega_m} \sin \omega_m t$$

# Mathematical Expression for single-tone frequency modulation ......continued

- Putting the value of $\omega_i$ from equation above, we get:

$$\phi_i = \int [\omega_c + \Delta\omega.\cos\omega_m t\,]\,dt = \omega_c\,t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t$$

- Putting the value of modulation index, $m_f$ we get:

$$\phi_i = \int [\omega_c + \Delta\omega.\cos\omega_m t\,]\,dt = \omega_c\,t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t$$

since $\frac{\Delta\omega}{\omega_m} = m_f$ we now have: $\phi_i = \omega_c\,t + m_f \sin\omega_m t$

# Mathematical Expression for single-tone frequency modulation ……continued

- Substituting $\phi_f$ in equation $s(t) = A \cos \phi_i$ we get:

$$s(t) = A \cos [\omega_c t + m_f \sin \omega_m t]$$

And this is the mathematical expression for a single-tone FM wave.

# Exercise

1. A singletone FM is represented by the voltage equation

   $$v(t) = 12 \cos (6x10^8\, t + 5 \sin 1250t)$$

   Determine:

   i. Carrier frequency (Answer: 95.5 MHz)
   ii. Modulating frequency (199Hz)
   iii. Modulating index (5)
   iv. Maximum deviation (995Hz)
   v. Power that this FM wave will dissipate in 10Ω resistor. (7.2W)

# Exercise

- A 107.6MHz carrier signal is frequency modulated by a 7kHz sine wave. The resultant FM signal has a frequency deviation of 50kHz. Determine the following:
    i. The carrier swing of the FM signal (Answer: 100kHz)
    ii. The highest frequency attained by the modulated signal (107.65MHz)
    iii. The lowest frequency attained by the modulated signal (107.55MHz)
    iv. the modulation index of the FM wave. (7.143)

- Determine the frequency deviation and carrier swing for a frequency modulated (FM) signal which has a resting frequency of 105MHz and whose upper frequency is 105.007 MHz when modulated by a particular wave. Find the lowest frequency reached by the wave.

    (Answers: 7kHz, 14kHz, 104.993MHz)

- What is the modulation index of an FM signal having a carrier swing of 100 kHz when the modulating signal has a frequency of 8kHz?

    (50kHz)

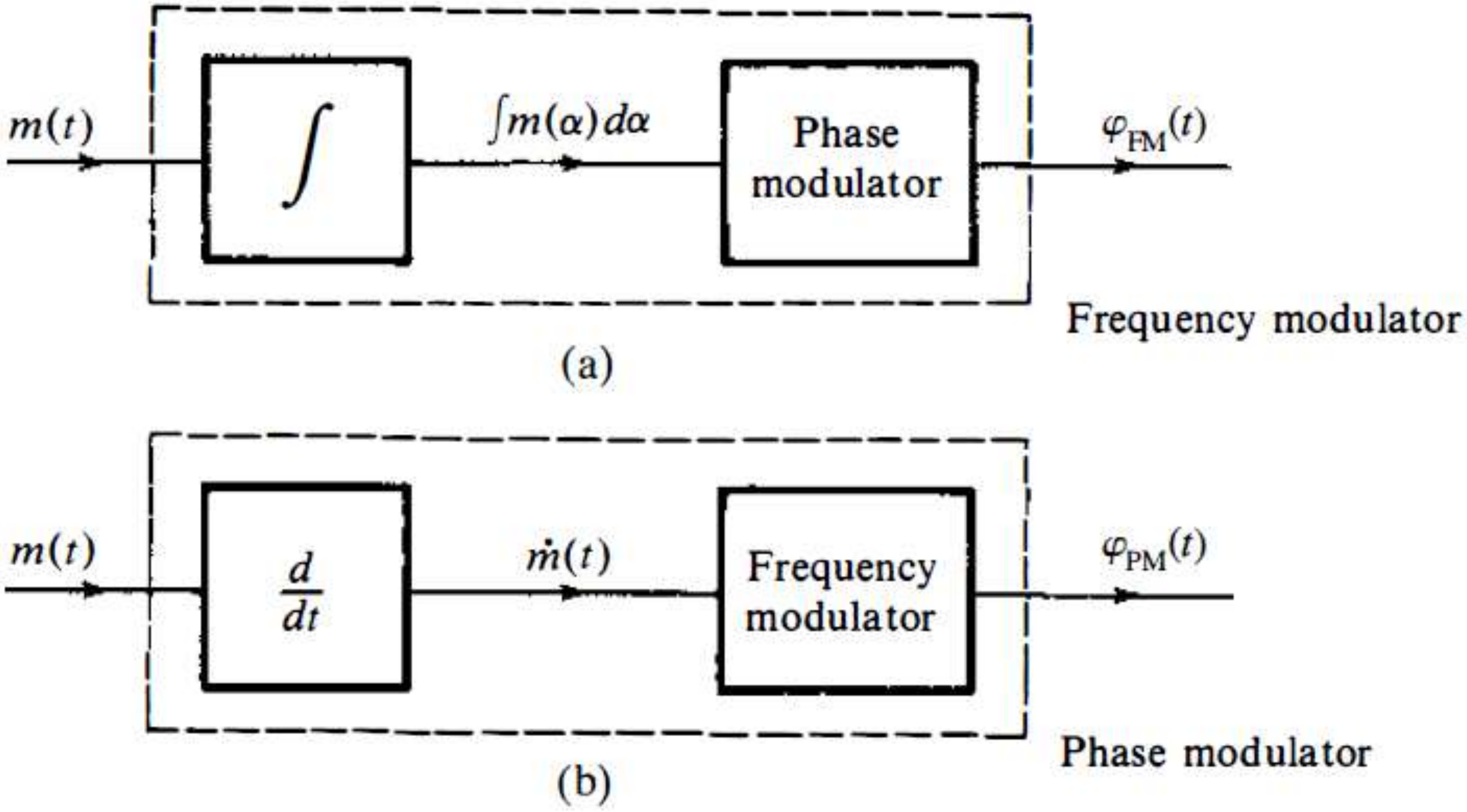
# Exercise

- An FM transmission has a frequency deviation of 20kHz. Determine:
  i. the percentage modulation of this signal if it is broadcasted in the 88-108MHz band. (26.67%)
  ii. The percentage modulation if this signal is broadcasted as the audio portion of a television broadcast. (80%)
- A certain country regulates the requirement for FM broadcast as follows: the maximum allowable frequency deviation from the carrier is 75kHz, the guard band on either side is 25kHz. Determine the number of FM radio channels that could fit in a 88MHz-108MHz available spectrum. (Answer: 100 stations)

# Relationship between FM and PM

- Replacing *m(t)* in Eq. (5.3b) with $\int_{-\infty}^{t} \mathrm{m}\,(\propto)\mathrm{d}\,(\propto)$ changes PM into FM.
- Thus, a signal that is an FM wave corresponding to *m(t)* is also the PM wave corresponding to $\int_{-\infty}^{t} \mathrm{m}\,(\propto)\mathrm{d}\,(\propto)$ as in (Fig. a)
- Similarly, a PM wave corresponding to *m(t)* is the FM wave corresponding to $\dot{m}$(t) as in (Fig. b).
- Therefore, methods of generation and demodulation of FM and PM use the same principles.
- In both PM and FM, the angle of a carrier is varied in proportion to some measure of *m(t).* In PM, it is directly proportional to *m(t),* whereas in FM, it is proportional to the integral of *m(t).*

# Phase and frequency modulation are equivalent and interchangeable.



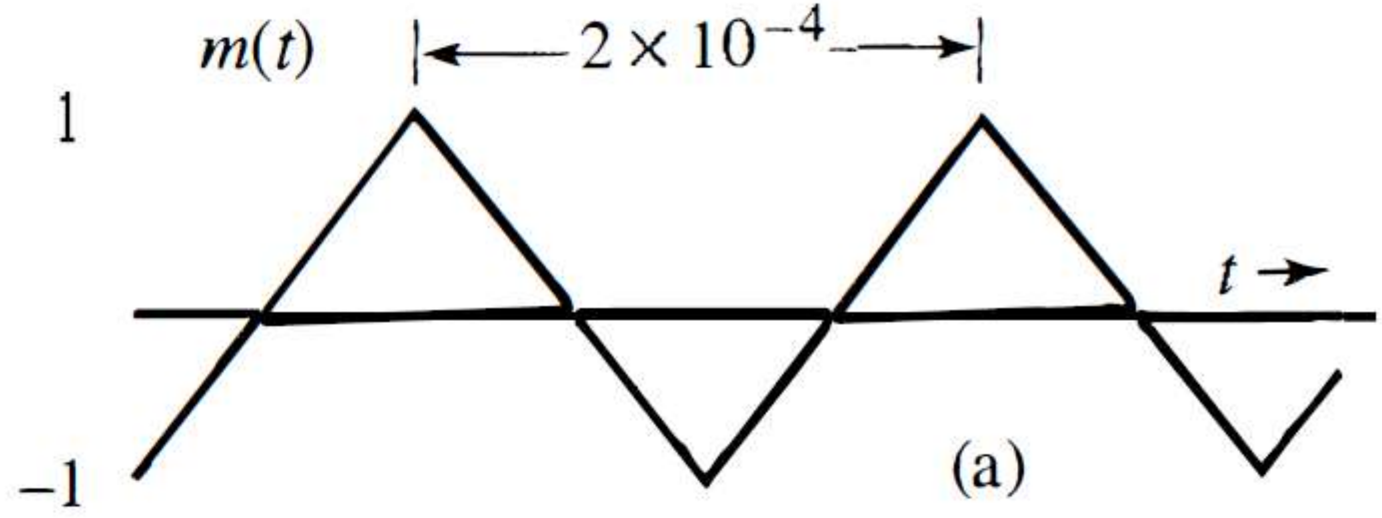
- In the previous fig.b:
  - a frequency modulator can be directly used to generate an FM signal
  - or the message input *m(t)* can be processed by a filter (differentiator) with transfer function *H(s)* = *s* to generate PM signals.
- (see more)

# Power of an Angle Modulated Wave

- Although the instantaneous frequency and phase of an angle-modulated wave can vary with time, the amplitude A remains constant.
- Hence, the power of an angle-modulated wave (thus PM or FM) is always $\frac{A^2}{2}$, regardless of the value of $k_p$ or $k_t$ .
- (see more)

# Example:

- Sketch FM and PM waves for the modulating signal m(t) shown in Fig. below. The constants $k_t$ and $k_p$ are $2\pi$x$10^5$ and $10\pi$, respectively, and the carrier frequency $f_c$ is 100 MHz.



# solution

- For FM:

  $\omega_i = \omega_c + k_f\, m(t)$

- Dividing throughout by $2\pi$, we get instantaneous frequency as:

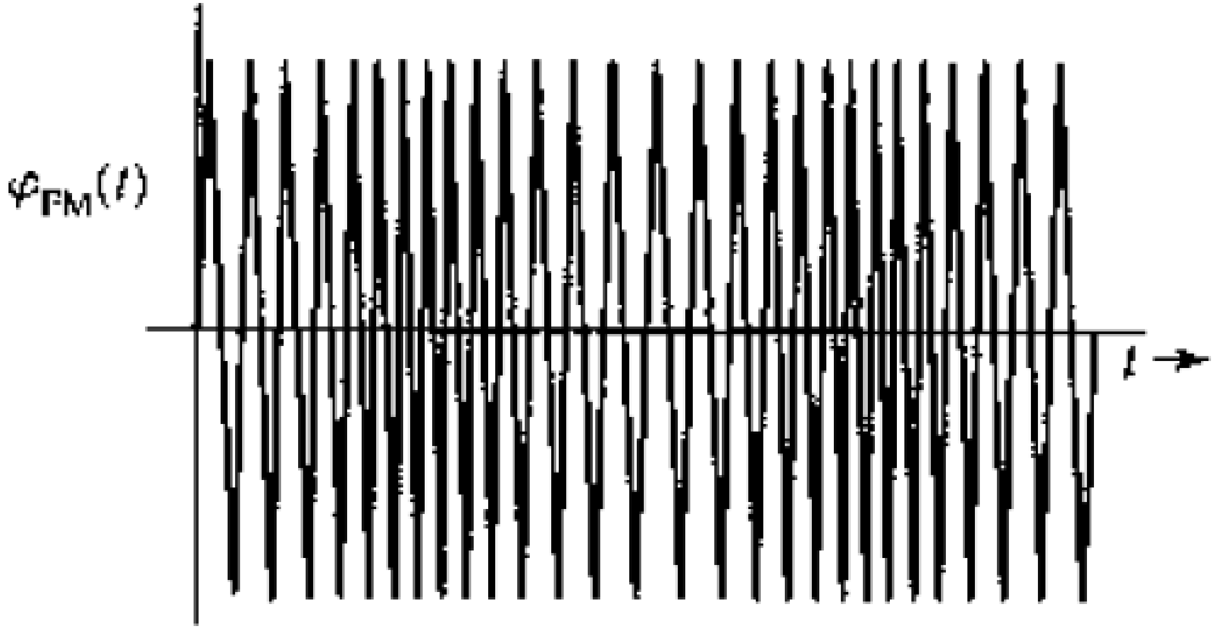
  $f_i = f_c + \dfrac{k_f}{2\pi}\, m(t)$

  $= 10^8 + 10^5 m(t)$

  $(f_i)_{min} = 10^8 + 10^5\, [m(t)]_{min} = 99.9\text{MHz}$

  $(f_i)_{max} = 10^8 + 10^5 [m(t)]_{max} = 100.1\text{MHz}$

- Because *m(t)* increases and decreases linearly with time, the instantaneous frequency increases linearly from 99.9 to 100.1 MHz over a half-cycle and decreases linearly from 100. 1 to 99.9 MHz over the remaining half-cycle of the modulating signal. As shown below




(b)

# •For PM:

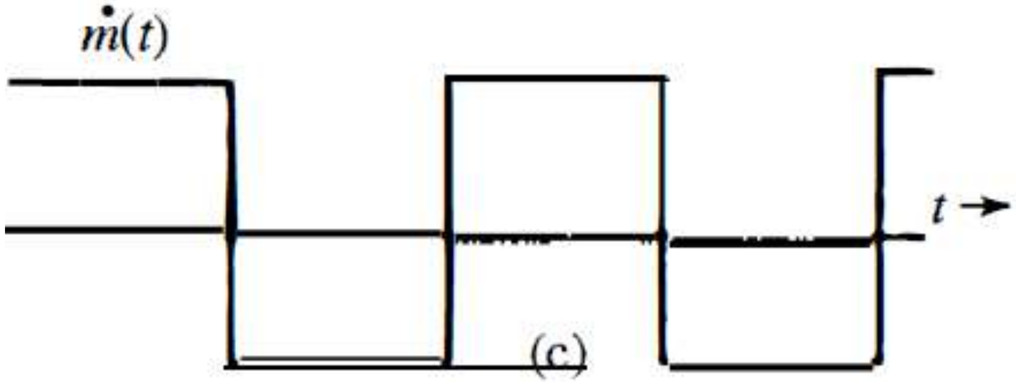
- note that PM for m(t) is FM for $\dot{m}(t)$

$$f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t)$$

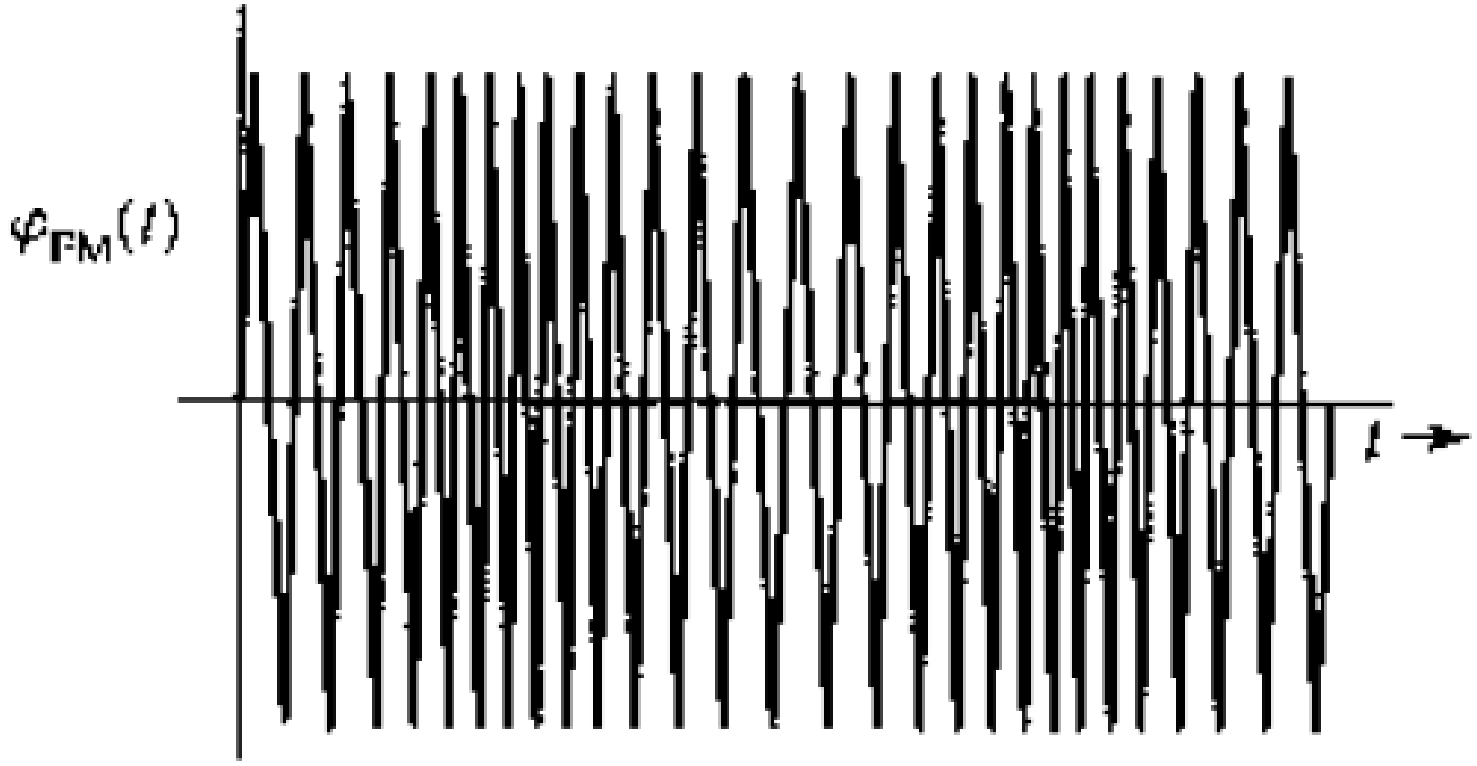
$$= 10^8 + 5\,\dot{m}(t)$$

$$(f_i)_{min} = 10^8 + 5\,[\dot{m}(t)]_{min} = 10^8 - 10^5 = 99.9\text{MHz}$$

$$(f_i)_{max} = 10^8 + 5\,[\dot{m}(t)]_{max} = 100.1\text{MHz}$$



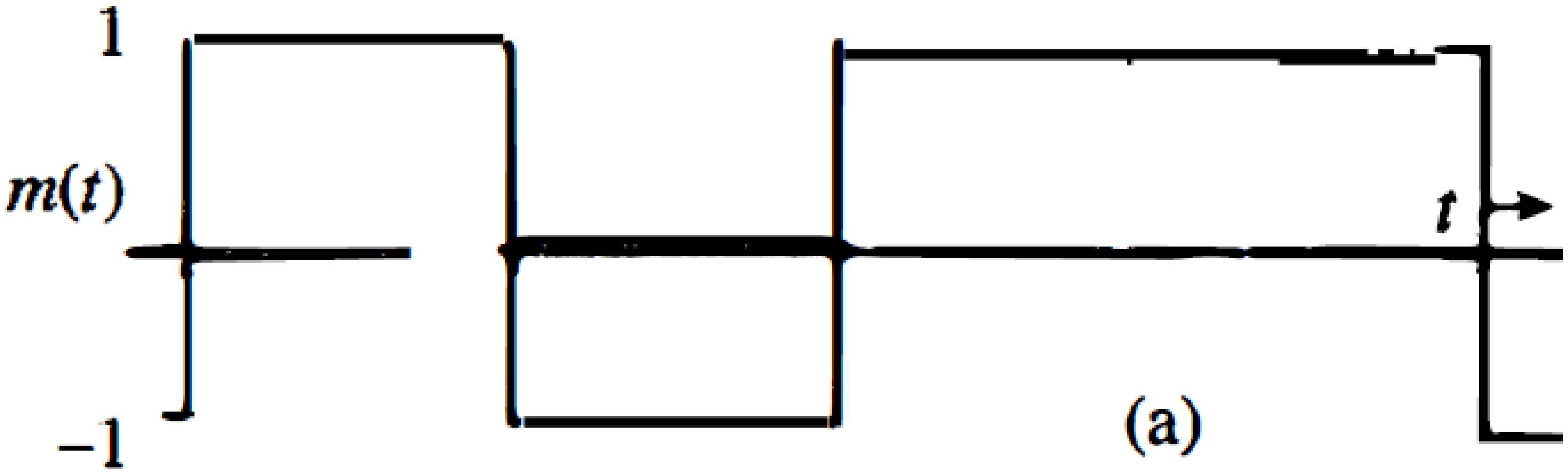
• PM sketch with frequency 99.9 to 100.1MHz




(b)

# Example

- Sketch FM and PM waves for the digital modulating signal *m (t)* shown below. The constants $k_t$ and $k_p$ are $2\pi$x $10^5$ and $\pi/2$, respectively, and $f_c$ =100 MHz.

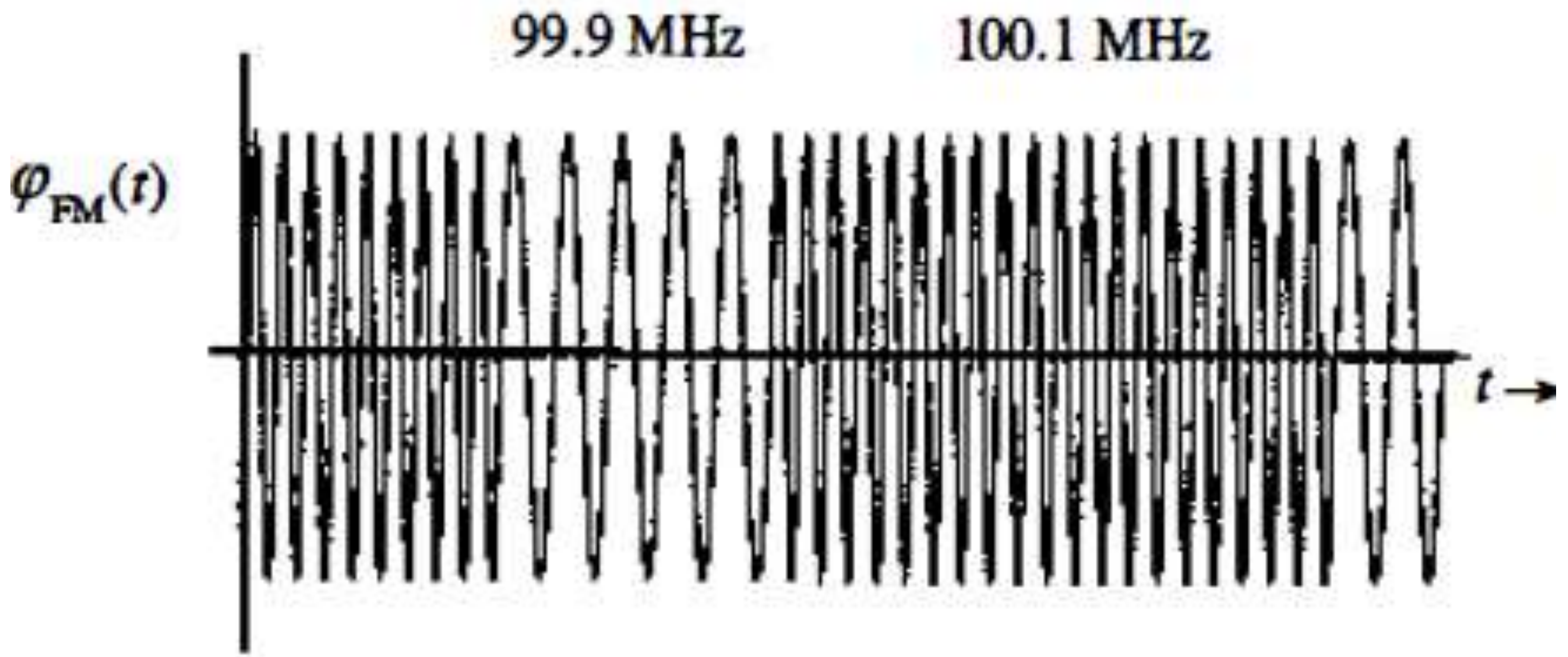



(a)

- For FM

$$f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$$

- Because *m(t)* switches from 1 to - 1 and vice versa, the FM wave frequency switches back and forth between 99.9 and 100.1MHz, as shown below.

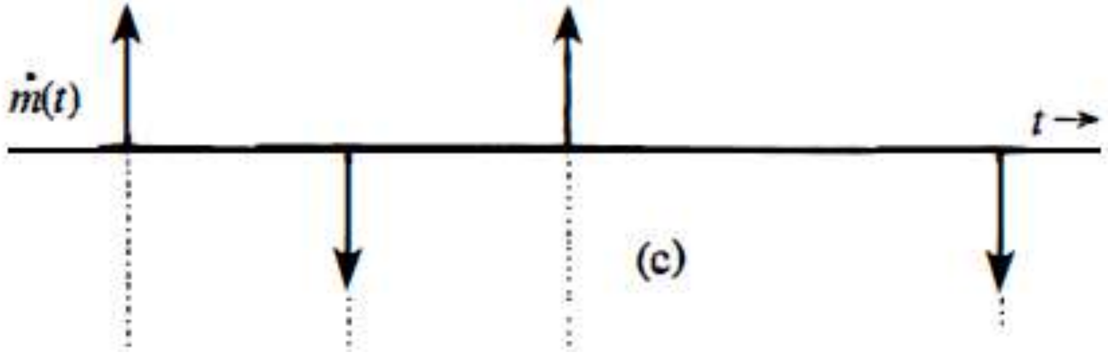


- Also note that:
- This scheme of carrier frequency modulation by a digital signal is called **frequency shift keying (FSK)** because information digits are transmitted by keying different frequencies.
- (to be discussed later)

- For PM:

$$f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t) \quad = 10^8 + \frac{1}{4}\dot{m}(t)$$

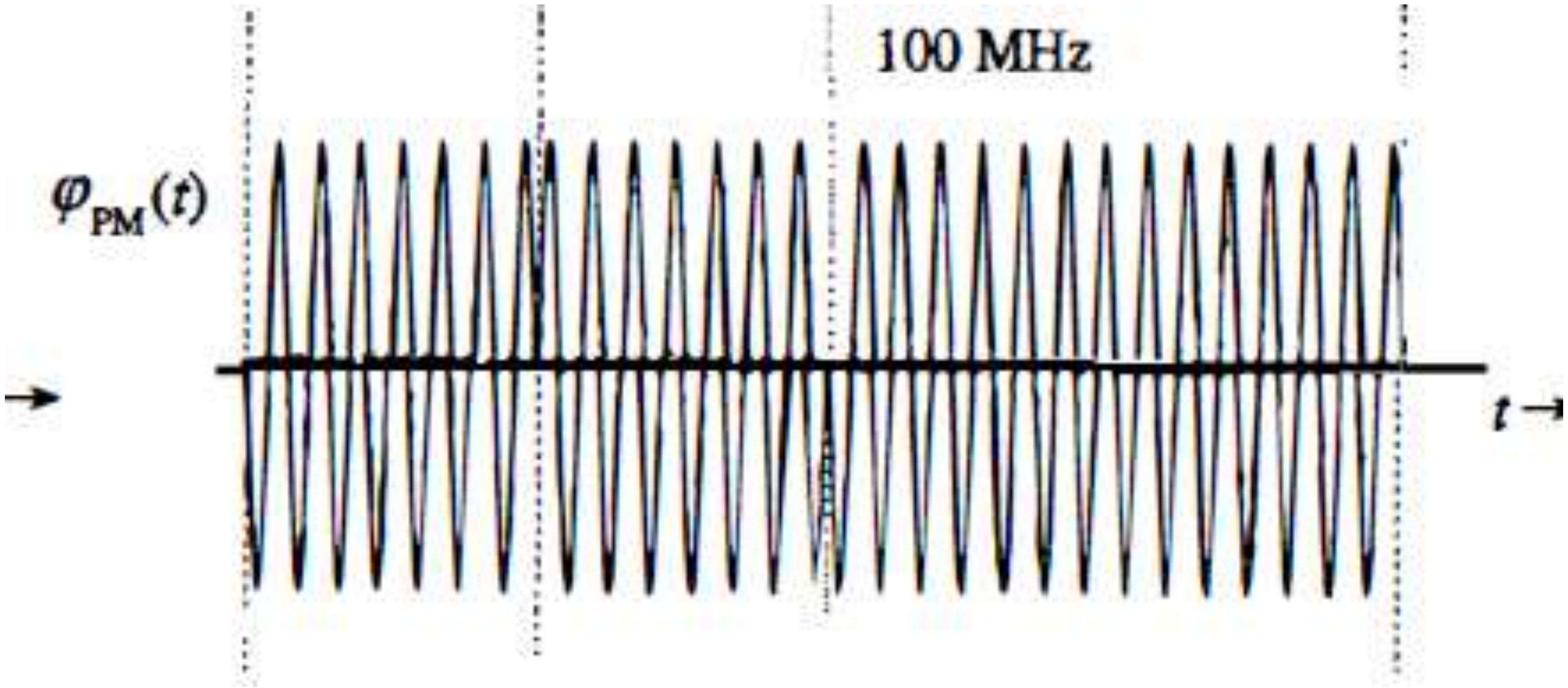
- The derivative $\dot{m}(t)$ see figure below is zero except at points of discontinuity of m(t) where impulses of strength ±2 are present.
- This means that the frequency of the PM signal stays the same except at these isolated points of time !



- It is not immediately apparent how an instantaneous frequency can be changed by an infinite amount and then changed back to the original frequency in zero time.
- Let us consider the direct approach

$$\varphi_{\mathrm{PM}}(t) = A\cos[\omega_c t + k_p m(t)]$$
$$= A\cos\left[\omega_c t + \frac{\pi}{2} m(t)\right]$$
$$= \begin{cases} A\sin\omega_c t & \text{when } m(t) = -1 \\ -A\sin\omega_c t & \text{when } m(t) = 1 \end{cases}$$

- This PM wave is shown in Fig. below. This scheme of carrier PM by a digital signal is called **phase shift keying (PSK)** because information digits are transmitted by shifting the carrier phase.
- Note that PSK may also be viewed as a DSB-SC modulation by m(t).



- The PM wave $\varphi_{PM}(t)$ in this case has phase discontinuities at instants where impulses of *m(t)* are located.
- At these instants, the carrier phase shifts by $\pi$ instantaneously. A finite phase shift in zero time implies infinite instantaneous frequency at these instants.
- This agrees with our observation about *m(t).*