INTRODUCTION TO PROBABILITY

LECTURE 4

BY MR THUO

Conditional Probability

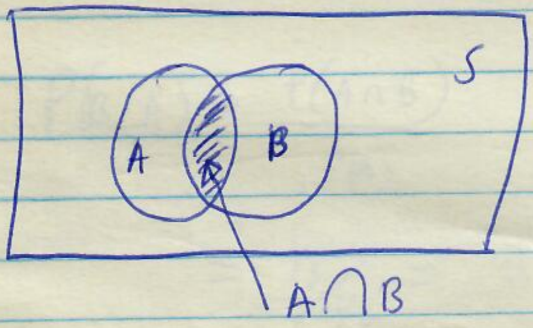
Let A and B be two events such that $P(A) > 0$. Denote by $P(B|A)$ the prob. of B given that A has occurred. Since A is known to have occurred, it becomes the new sample space replacing the original S.

From this we are led to the definition

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad \text{---- eq}_n\ (1)$$

$$P(A \cap B) = P(A)\, P(B|A) \quad \text{--- eq}_n\ (2)$$

or



~~the~~ In words, eq^tn 2 above we say that the prob that both A & B occur is equal to the prob that A occurs times the prob that B occurs given that A has occurred. We call $P(B/A)$ the conditional prob of B given A. ie// prob that B will occur given that A has occurred

Ex

Find the prob that a single toss of a die will result in a number less than 4 if

(a) no other information is given.

b It is given that the toss resulted in an odd number

Soln

a Let B denote the event {less than 4}

Since B is the union of the events 1, 2, or 3 turning up we write

$$P(B) = P(1) + P(2) + P(3) = \tfrac{1}{6} + \tfrac{1}{6} + \tfrac{1}{6} = \tfrac{1}{2}$$

b Letting A be the event {odd number} we see that

$$P(A) = \tfrac{3}{6} = \tfrac{1}{2}$$

NB (1, 2, 3, 4, 5, 6) Prob(odd) = 1/2
1/6 1/6 1/6 1/6 1/6 1/6

Also

$$P(A \cap B) = \tfrac{2}{6} = \tfrac{1}{3}$$

NB ie it is odd number & it is less than 4
(1, 2, 3, 4)
(1/6 1/6 1/6 1/6 = 2/6)

$$\therefore P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/3}{1/2} = = \underline{\underline{2/3}}$$

Independent Events

15

If $P(B|A) = P(B)$ i.e. the prob of B occuring is not affected by the occurrence or non-occurrence of A, then we say that A and B are independent events. This is equivalent to.

$$P(A \cap B) = P(A) \cdot P(B)$$

We say that three events A_1, A_2, A_3 are independent if they are pairwise independent:

$$P(A_j \cap A_k) = P(A_j)\, P(A_k) \quad j \neq k \quad \text{where}$$

$$j, k = 1, 2, 3$$

and

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2)\, P(A_3)$$

Ex 12

A fair die is tossed twice. Find the prob of getting a 4, 5, or 6 on the first toss and 1, 2, 3 or 4 on the second toss.

Soln

Let A_1 be the event "4, 5, or 6 on first toss" and A_2 be the event "1, 2, 3, or 4" on second toss."

Then we are looking for $P(A_1 \cap A_2)$

$$P(A_1 \cap A_2) = P(A_1)\, P(A_2|A_1) = = P(A_1) \cdot P(A_2) = \left(\tfrac{3}{6}\right)\left(\tfrac{4}{6}\right) = \underline{\underline{\tfrac{1}{3}}}$$

H/W

1 Find the prob of not getting a 7 or 11 total on either of two tosses of a pair of fair dice $\left(\frac{49}{81}\right)$

2 Two cards are drawn from a well-shuffled ordinary deck of 52 cards. Find the prob that they are both aces if the card is

a replaced $\left(\frac{1}{169}\right)$ ie $\left(\frac{4}{52}\right)\left(\frac{4}{52}\right)$

b not replaced $\frac{1}{221}$ ie $\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)$

3 Find the prob of a 4 turning up at least once in two tosses of a fair die. $= \frac{11}{36}$

4 Box 1 contains 3 red and 2 blue marbles while Box 2 contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box 1; if it turns up tails, a marble is chosen from Box 2. Find the prob that a red marble is chosen. $\left(\frac{2}{5}\right)$