MODULATION

AMPLITUDE MODULATION

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What is modulation

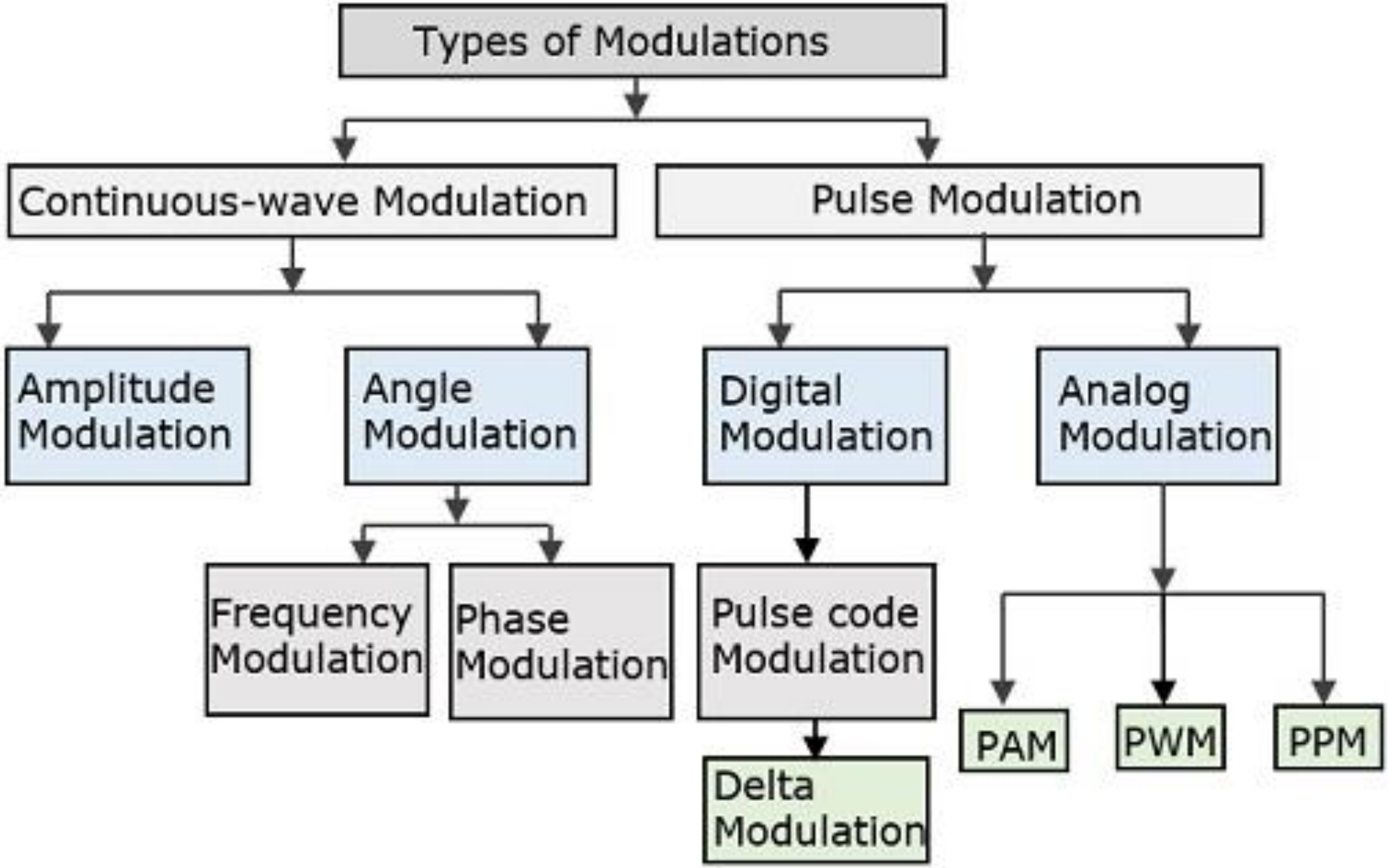
- Is the process of altering (modulating) the carrier signal (a very high frequency signal) with a modulating (baseband) signal.
- Modulation process can be classified into two broad classes:
  i. Continuous wave (CW) modulation
  ii. Pulse modulation (PM) modulation
- A sinusoidal wave is used as a carrier.

i) Continuous-wave (CW) Modulation

- In CW, a high frequency sine wave is used as a carrier wave. CW is divided into:
  i. **Amplitude Modulation:** the amplitude of the high frequency carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal.
  ii. **Angle Modulation:** The angle of the carrier wave is varied, in accordance with the instantaneous value of the modulating signal. Angle modulation is further divided into:
      i. **Frequency Modulation:** The frequency of the carrier wave is varied, in accordance with the instantaneous value of the modulating signal.
      ii. **Phase Modulation: T**he phase of the high frequency carrier wave is varied in accordance with the instantaneous value of the modulating signal.

ii) Pulse Modulation techniques

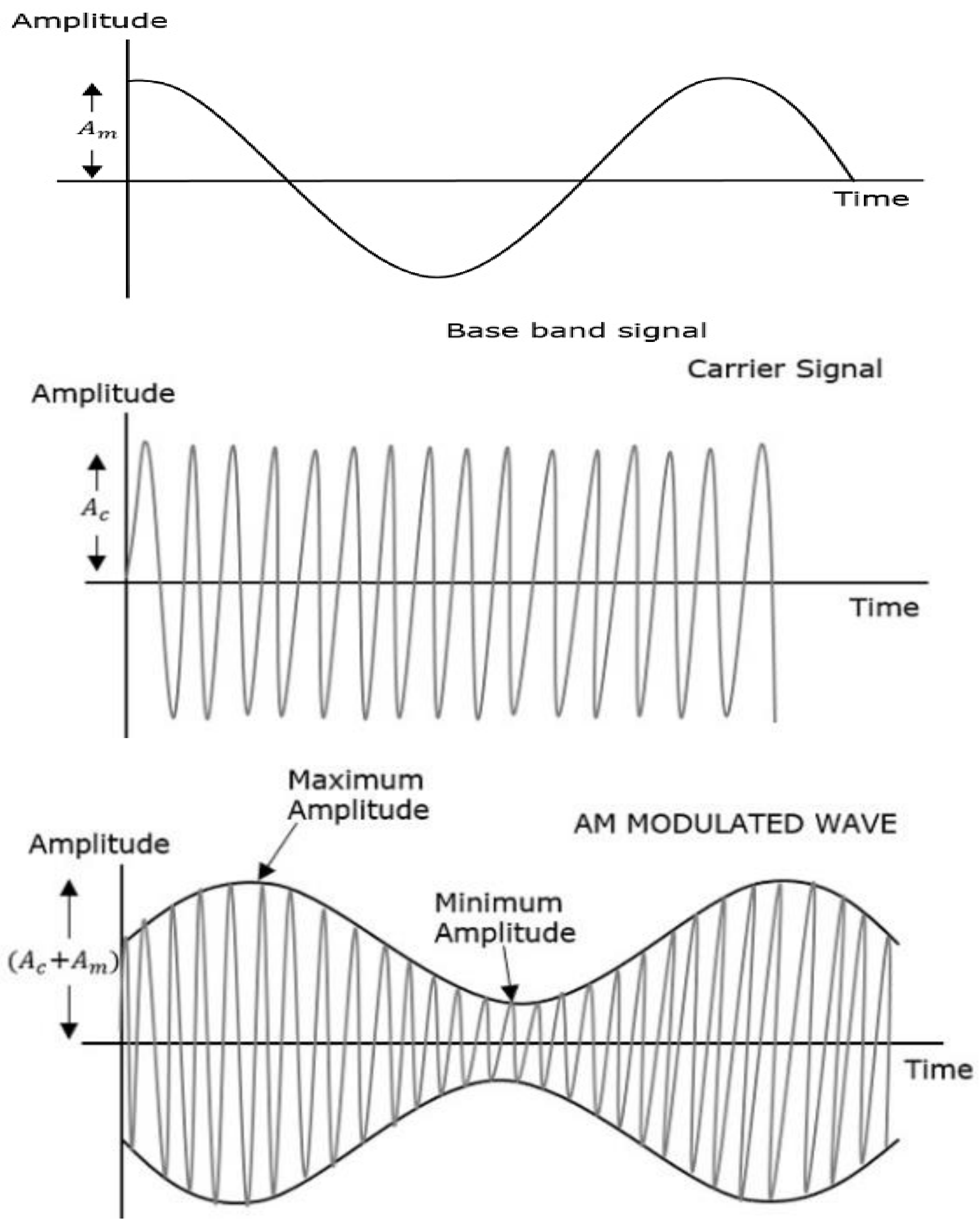
- A periodic sequence of rectangular pulses, is used as a carrier wave. This is further divided into:
- **Analog modulation** technique: The amplitude, duration or position of a pulse is varied in accordance with the instantaneous values of the baseband modulating signal then we call it **Pulse Amplitude Modulation (PAM)** or **Pulse Duration/Width Modulation (PDM/PWM)**, or **Pulse Position Modulation (PPM**) respectively.
- **Digital modulation**, uses **Pulse Code Modulation (PCM)** where the analog signal is converted into binary form. Resulting into a coded pulse train. This is further developed as **Delta Modulation (DM)**.

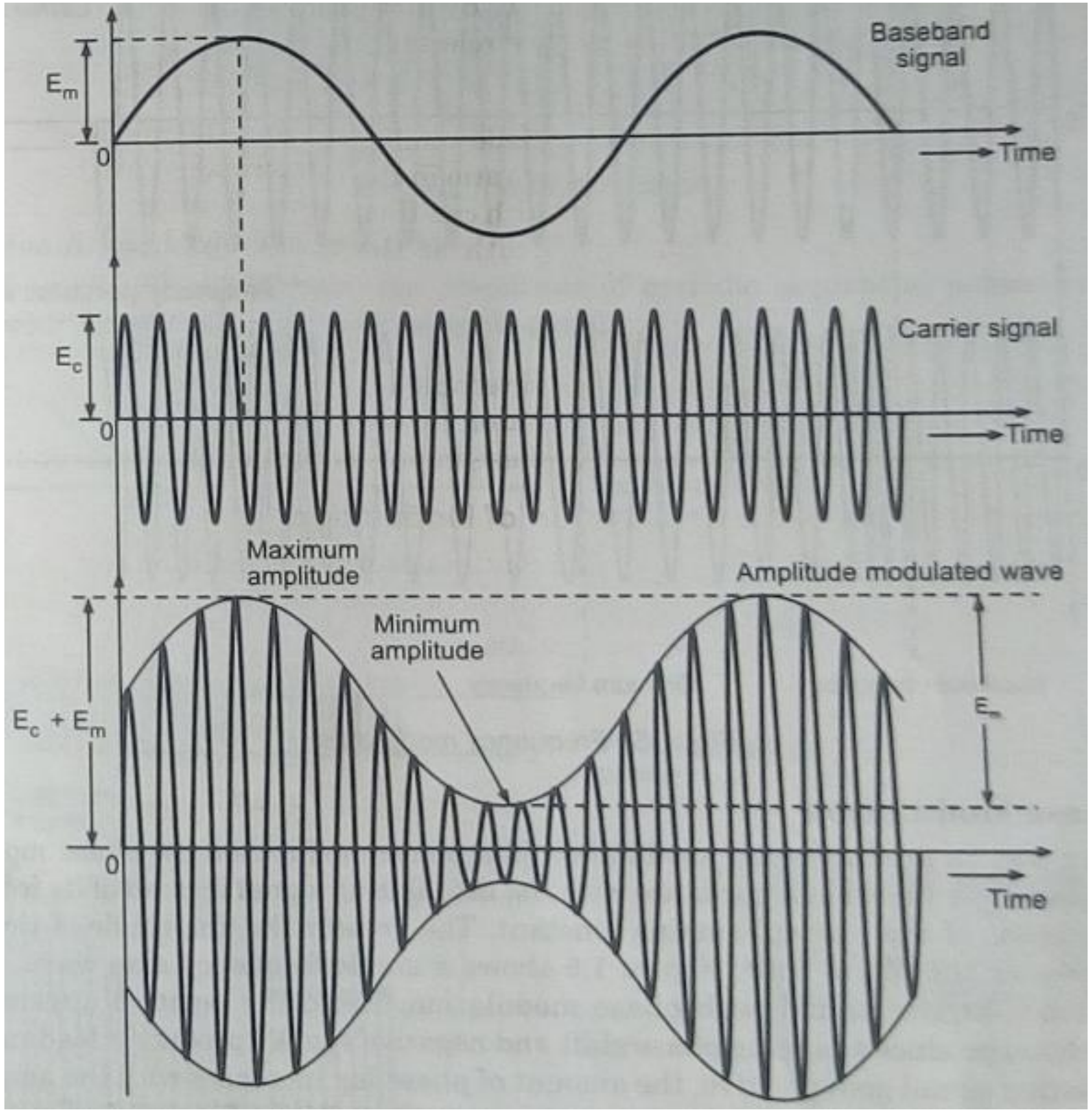

Types of Modulations
Continuous-wave Modulation
Pulse Modulation
Amplitude Modulation
Angle Modulation
Digital Modulation
Analog Modulation
Frequency Modulation
Phase Modulation
Pulse code Modulation
PAM
PWM
PPM
Delta Modulation

AMPLITUDE MODULATION

AMPLITUDE MODULATION

- In AM, the amplitude of a carrier signal is varied by the modulating signal.
- The instantaneous value of the carrier’s amplitude changes in accordance with the amplitude of the modulating signal.
- The **carrier signal** is a constant high frequency signal.
- A **modulating signal** is the message or information signal.
- The resulting signal is the **modulated signal**.


Amplitude
A_m
Time
Base band signal
Carrier Signal
Amplitude
A_c
Time
Maximum Amplitude
AM MODULATED WAVE
Amplitude
Minimum Amplitude
(A_c+A_m)
Time


Baseband signal
E_m
0
Time
Carrier signal
E_c
0
Time
Maximum amplitude
Amplitude modulated wave
Minimum amplitude
$E_c + E_m$
E_m
0
Time

- The imaginary line on the carrier wave is called **Envelope**.
- The envelope has the same shape as the message signal.

Modulation index, m or μ

- Modulation, m=$\frac{E_m}{E_c}$
- And is an important ratio. Why?
- As seen from the wave graphs of AM wave in slide 8 it can be seen that if E_m is greater than E_c then distortion will occur.
- Therefore E_m must be less than E_c for proper Amplitude Modulation. Thus $0 < m \leq 1$.
- Can also be expressed as percentage modulation.

- Modulating (thus baseband) signal is only preserved in the envelope of AM signal only if the modulating index is $0 < m \leq 1$.
- If $m > 1$ thus the amplitude of the message signal is greater than the amplitude of the carrier signal, distortion will occur.
- When $m > 1$, is referred to as overmodulated signal

Under-modulation

- If this value is less than 1, e.g. the modulation index is 0.5, then the modulated output is shown below.
- It is called as Under-modulation. Such a wave is called as an **under-modulated wave**.
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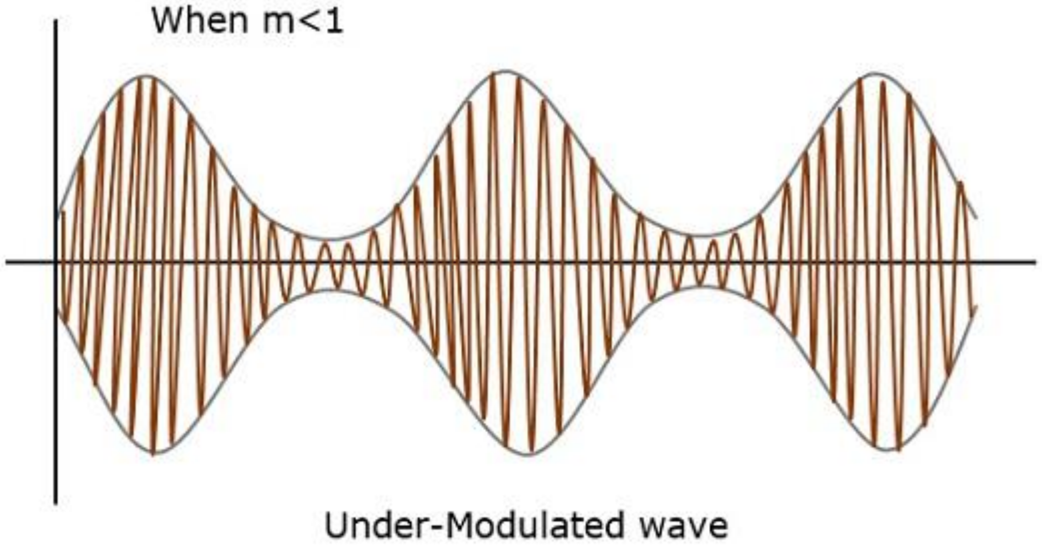
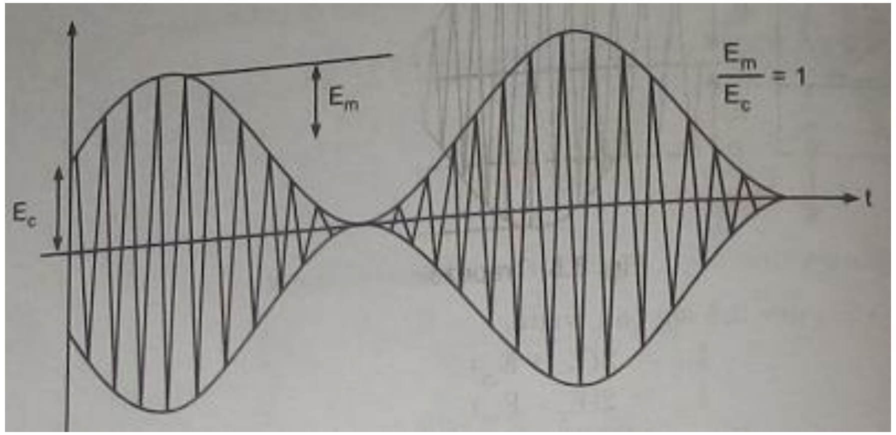
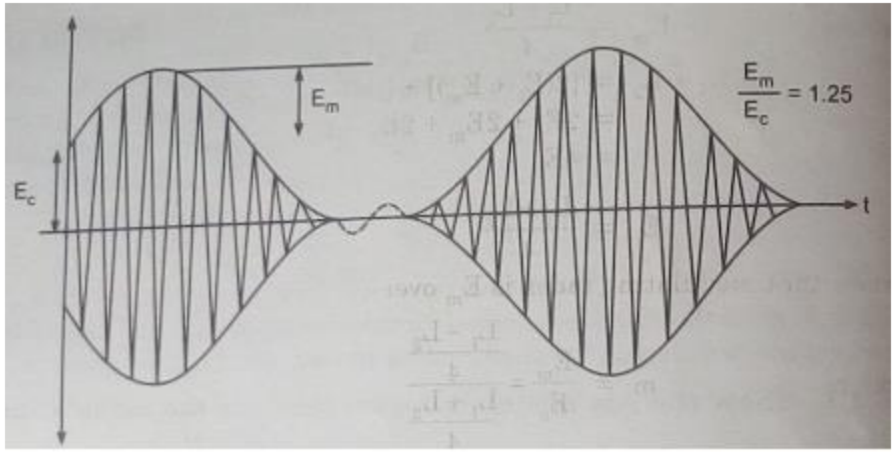


- Graph one for m=1

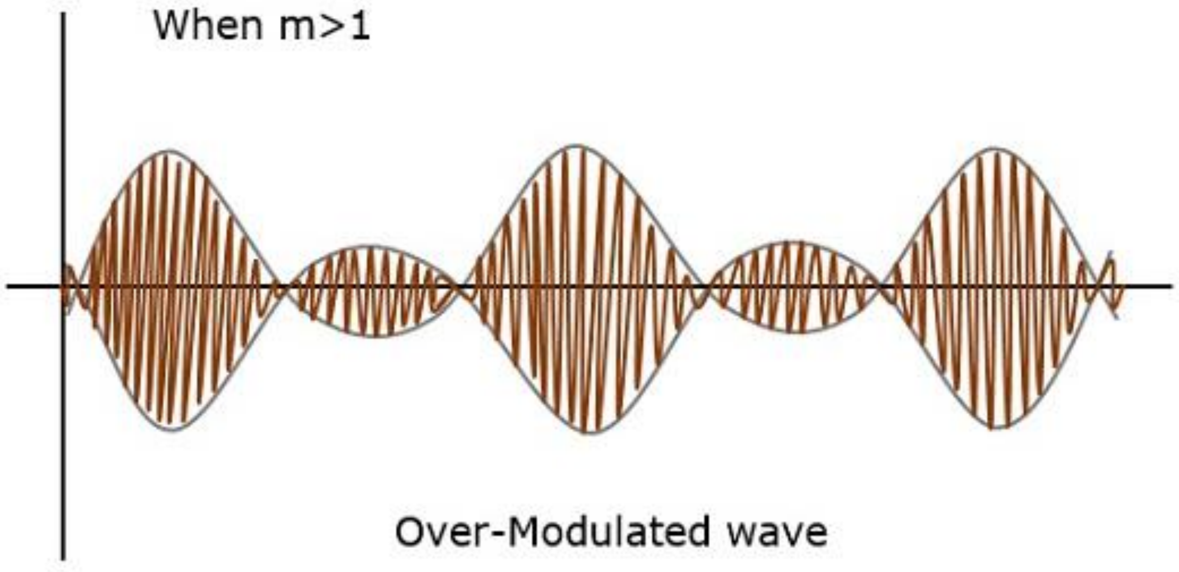
- Graph two for m>1
- There is phase reversal in the regions where the amplitude of the baseband signal exceeds the amplitude of the carrier signal.





over-modulation

- If the value of the modulation index is greater than 1, i.e., 1.5 or so, then the wave will be an **over-modulated wave**. It would look like the following figure:



Instantaneous value of the modulating signal

The magnitude of the modulating signal at any instant in time is given by the equation:

$$e_m = E_m \sin \omega_m t$$

where $e_m =$ instaneous amplitude

$E_m =$ maximum amplitude

$\omega_m = 2\pi f_m =$ angular frequency

$f_m =$ frequency of modulating signal

Instantaneous value of the carrier signal

The magnitude of the carrier signal at any instant in time is given by the equation:

$$e_c = E_c \sin \omega_c t$$

Where $e_c =$ instaneous amplitude

$E_c =$ maximum amplitude

$\omega_c = 2\pi f_c =$ angular frequency

$f_c =$ frequency of carrier signal

Instantaneous value of the amplitude modulated signal

The value of the modulated signal is given by:

$$E_{AM} = E_c + e_m$$
$$= E_c + E_m \sin \omega_m t$$

the instantaneous value of the AM wave can be given as:

$$e_{AM} = E_{AM} \sin \omega_c t$$
$$=(E_c + E_m \sin \omega_m t) \sin \omega_c t$$

Frequency spectrum of AM wave

We have seen that:

$e_{AM} = (E_c + E_m \sin \omega_m t) \sin \omega_c t$

Modulation index, m=$\frac{E_m}{E_c}$ therefore $E_m = mE_c$

Substituting $e_{AM} = (E_c + mE_c \sin \omega_m t) \sin \omega_c t$

$= E_c \sin \omega_c t + m\, E_c \sin \omega_m t \sin \omega_c t$

From trigonometry identity below:

$\sin a \sin b =$ ½ $[\cos(a - b)$ - cos(a+b)]

We write the equation as in the next slide:

Frequency spectrum of AM wave continued

$$e_{AM} = E_c \sin \omega_c t + \frac{mE_c}{2} \cos(\omega_c - \omega_m) t - \frac{mE_c}{2} \cos(\omega_c + \omega_m) t$$

We can see that:

$E_c \sin \omega_c t$ represents the unmodulated carrier

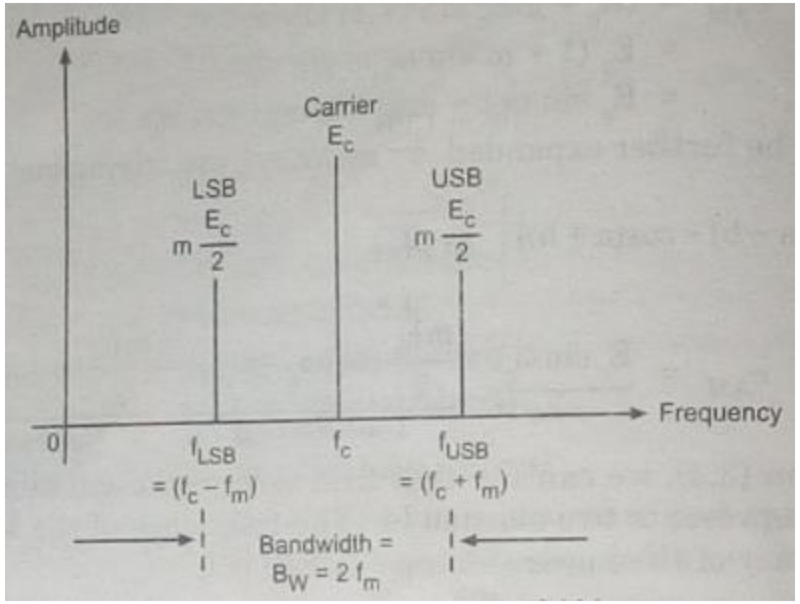
$\frac{mE_c}{2} \cos(\omega_c - \omega_m) t$ represents the lower sideband

$\frac{mE_c}{2} \cos(\omega_c + \omega_m) t$ represents the upper sideband

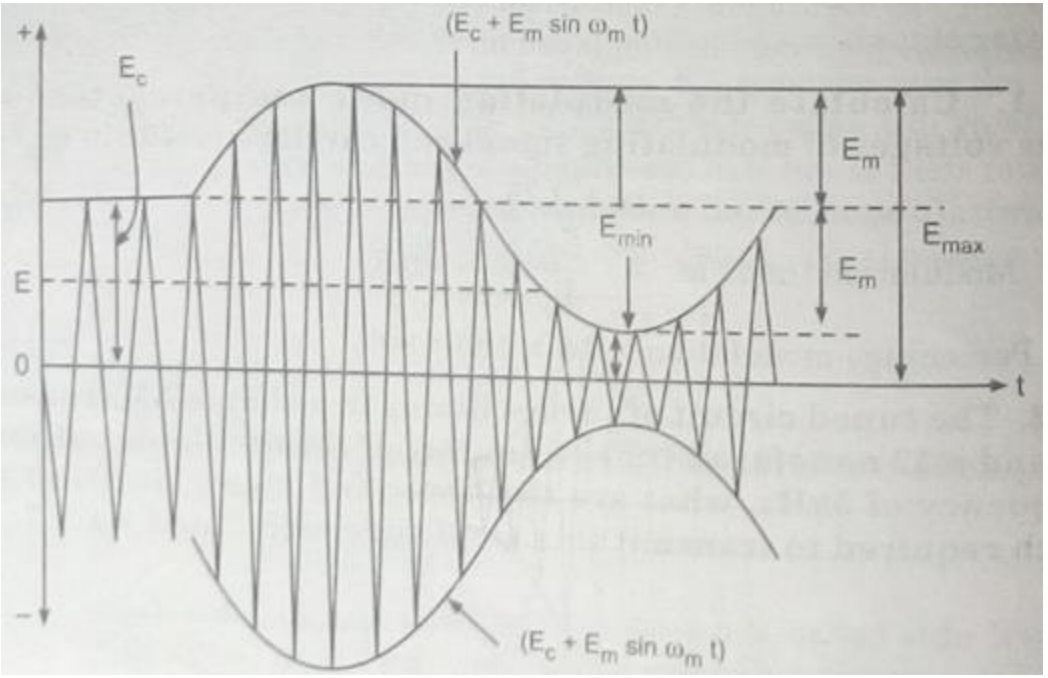
We note: the power of an AM wave is distributed in these three components

Representation of AM Wave

- An AM can be represented in a time domain or a frequency domain.
- In a frequency domain, signal amplitude versus frequency.



- In the time domain, amplitude versus time.



- Therefore: The modulated signal has:
  - The carrier frequency, f_c
  - new different frequencies called sidebands
    - Upper sideband, $f_{USB} = f_c + f_m$
    - Lower sideband, $f_{LSB} = f_c - f_m$

The bandwidth of an AM wave

- Substract the lowest frequency from the highest frequency
  - $B_w = f_{USB} - f_{LSB} = (f_c + f_m) – (f_c – f_m) = 2f_m$

- This tells us that the bandwidth requirement of an AM signal is twice the frequency of the modulating signal

- Note that $f_c \geq B_W$, this avoids overlap of the modulated spectra centered at *fc* and *–fc*
- If f_c < *B,* then the two copies of message spectra overlap and the information of *m(t)* is lost during modulation, which makes it impossible to get back *m(t)* from the modulated signal *m(t)* cos $\omega_c t$.
- Also for practical broadcast applications, a transmit antenna can radiate only a narrow band without distortion. Hence to avoid distortion caused by the transmit antenna, $\frac{f_c}{B_W}$ >> 1.

Example 1

Calculate the modulation index and percentage modulation, if instantenous voltages of modulating signal and carrier is $40\cos\omega_m t$ and $50\cos\omega_c t$ respectively.

i. $m = \frac{E_m}{E_c} = \frac{40}{50} = 0.8$

ii. % modulation = 80%

Example 2

For a certain system, the carrier and modulating signals are $60\,sin\,5000\pi\,t$ and $45\,\sin 800\pi\,t$. Determine the:

i. Modulating index
ii. Modulation percentage
iii. Minimum bandwidth required for the modulated signal

Solution:

From $E_c \sin \omega_c t$, we get amplitude of carrier is $E_c = 60$, $2\pi f_c = 5000\pi$

From $E_m \sin \omega_m t$, we get amplitude of modulating signal $E_m = 45$ and $2\pi f_m = 800\pi$

i. Modulating index = 45/60 = 0.75
ii. Percentage modulation = 0.75 x 100% = 75%
iii. Minimum bandwidth = $2f_m$ but $f_m = 800\pi/2\pi = 400$Hz

$= 2 \times 400$

$= 800$ Hz

Power of AM: Sidebands and carrier power

As noted power in an AM signal is distributed in the three components: the sidebands and the carrier power.

$$e_{AM} = E_c \cos \omega_c t + \frac{\mu E_c}{2} \cos(\omega_c + \omega_m) t + \frac{\mu E_c}{2} \cos(\omega_c - \omega_m) t$$

where μ is the modulation index and E_c is maximum amplitude of carrier signal

Power of AM wave is equal to the sum of powers of carrier, upper sideband and lower sideband frequency components

$total\ power$: $P_{total} = P_c + P_{USB} + P_{LSB}$

Since $P_{ave} = \frac{v_{rms}^2}{R} = \frac{\left(\frac{v_m}{\sqrt{2}}\right)^2}{2}$ Where v_{rms} is the rms value of cos signal

v_m is the peak value of cos signal

So interms of average power:

- The three components are:
- power of the carrier, $P_c = \frac{\left(\frac{E_C}{\sqrt{2}}\right)^2}{R} = \frac{E_C^2}{2R}$
- power of the upper sideband

$$P_{USB} = \frac{\left(\frac{E_C \mu}{2\sqrt{2}}\right)^2}{R} = \frac{E_C^2 \mu^2}{8R}$$

- power of the lower sideband = $\frac{E_C^2 \mu^2}{8R}$
- It can be seen that as modulation index is increased, there is more information power (thus P_{USB} and P_{LSB} hence a strong signal at the receiver. But also modulating index should be less than 1

- adding these three powers to get the power of AM wave

$$P_t = \frac{E_c^2}{2R} + \frac{E_c^2\mu^2}{8R} + \frac{E_c^2\mu^2}{8R}$$

$$P_t = \frac{E_c^2}{2R}\left(1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right)$$

$$P_t = P_c\left(1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right) = P_c\left(1 + \frac{\mu^2}{2}\right)$$

- This equation relates carrier power of AM wave to modulation index.

Now from $P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$, if we let modulating index u=1, then the power of AM wave is equal to 1.5 times the carrier power.

- So, the power required for transmitting an AM wave is 1.5 times the carrier power for a perfect modulation.

Transmission efficiency of an AM signal

- Is the ratio of transmitted information power to total transmitted power
- Transmission efficiency, $\eta = \frac{useful\ power}{total\ power} = \frac{P_{LSB}+P_{USB}}{P_{total}}$

$$= \frac{\left(\frac{\mu^2}{4}P_c+\frac{\mu^2}{4}P_c\right)}{P_C\left(1+\frac{\mu^2}{2}\right)} = \frac{\mu^2}{2+\mu^2}$$

- % Transmission efficiency $\eta = \frac{\mu^2}{2+\mu^2}$ x 100% provided condition $0 \leq \mu \leq 1$
- It can be seen that η increases monotonically with μ, and η_{max} occurs at μ **= 1** , for which η_{max} **=** 33.3%

Determine η and the percentage of the total power carried by the sidebands of the AM wave for tone modulation when $\mu = 0.5$ and when $\mu = 0.3$.

For $\mu = 0.5$,

$$\eta = \frac{\mu^2}{2 + \mu^2} 100\% = \frac{(0.5)^2}{2 + (0.5)^2} 100\% = 11.11\%$$

Hence, only about 11% of the total power is in the sidebands. For $\mu = 0.3$,

$$\eta = \frac{(0.3)^2}{2 + (0.3)^2} 100\% = 4.3\%$$

Hence, only 4.3% of the total power is in the sidebands that contain the message signal.

Example 1

- A modulating signal $m(t)$=10cos(2π x 10^3t) is amplitude modulated with a carrier signal c(t)=50cos(2π x 10^5t). Find the modulation index, the carrier power, and the power required for transmitting AM wave.
- Solution:
  - frequency of modulating signal is 10^3kHz
  - frequency of carrier signal is 10^5kHz
  - modulation index $\mathrm{u} = \frac{10}{50} = 0.2$
  - Carrier power $P_c = \frac{A_c^2}{2R} = \frac{50^2}{2(1)}$ = 1250W
  - Transmitting power = $P_t = P_c\left(1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right)$=1250(1+$\frac{0.2^2}{4}$ +$\frac{0.2^2}{4}$) = 1275W

Practice:

1. An audio frequency signal 10 sin(2π x 500t) is used to amplitude modulate a carrier of 50 sin(2π x 10^5t). Calculate:

i. Percentage modulation index (answer: 20%)

ii. Sideband frequencies (100.5kHz, 99.5kHz)

iii. Amplitude of each sideband frequencies (5V)

iv. Bandwidth required (1kHz)

v. Total power delivered to the load of 600Ω (2.125W)

vi. Transmission efficiency (1.96%)

Practice:

1. A 400W carrier is modulated to a depth of 80%. Calculate the total power in the modulated wave. (answer: 528W)
2. A broadcast transmitter radiates 20kW when the modulation percentage is 75. Calculate:
   i. the carrier power (15.6kW)
   ii. The power of each sideband (2.2kW)