GRAPH THEORY

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1 Graph

1.1 Basic Concepts

Graphs is a set of **Objects** and **Relations** between pairs of objects. A **Graph** $G = (V, E)$. Where: V is **Vertices/Notes**, and E is **Edges**.

If we have two nodes u and v. And between u and v, there is a edge e. We can say that e **Connects** u and v. u and v are **End Points** of e. u (v) and e are **Incident**. u and v are **Adjacent** and **Neighbors**.

And it is often convenient to consider **Directed Edges(Arcs)**. They describe **asymmetric** relations. For example, there is a flight from A to B, but not the other way around, such a graph is called **Directed**.

We should notice that if we want to describe a graph, we can write likes:
Objects:A, B, C, D, Relations:{{A, C}, {D, A}, {B, D}, {C, B}}
For directed graph:
Objects:A, B, C, D, Relations:{(A, C), (D, A), (B, D), (C, B)}

1.2 Vertex Degree

The **Degree** of a vertex is the number of its incident edges/neighbors. And the degree of a vertex v is denoted by $deg(v)$.
The **degree of a graph** is the maximum degree of its vertices. If a vertex's degree is 0, we call it as **Isolated Vertex**.
And a **Regular** graph is a graph where each vertex has the same degree(A regular graph of degree k is also called k-Regular).
The **Complement** of a graph $G = (V, E)$ is a graph $\overline{G} = (V, \overline{E})$.

1.3 Paths

A **Walk** in a graph is sequence of edges, such that each edge(except for the first one) starts with a vertex where the previous edge ended. The **Length** of a walk is the number of edges in it.
A **Path** is a walk where all edges are distinct. And a **Simple Path** is a walk where all vertices are distinct.
A **Cycle** in a graph is a path whose first vertex is the same as the last one. In particular, all the edges in a Cycle are distinct. A **Simple Cycle** is a cycle where all vertices expect for the first one are distinct(And there first vertex is taken twice).

1.4 Connectivity

A graph is called **Connected** if there is a path between every pair of its vertices. And a **Connected Component** of a graph G is a maximal connected sub-graph of G.