Rutgers Math250 Intro to Linear Algebra **Homework - 1**
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Problem 1.1.3
Find the point (x_1, x_2) that lies on the line $x_1 + 5x_2 = 7$ and on the line $x_1 - 2x_2 = -2$.

Solution: Firstly we get augmented matrix from them:

$$\begin{bmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{bmatrix}$$

Then process it:

$$\begin{bmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 0 & 7 & 9 \\ 1 & -2 & -2 \end{bmatrix} \xrightarrow{1/7R_1} \begin{bmatrix} 0 & 1 & 9/7 \\ 1 & -2 & -2 \end{bmatrix} \xrightarrow{R_2 + 2R_1} \begin{bmatrix} 0 & 1 & 9/7 \\ 1 & 0 & 4/7 \end{bmatrix}$$

where:

$$x_1 = 4/7 \ and \ x_2 = 9/7$$

So the point $(4/7, 9/7)$ is such a point lies on the both two lines

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Problem 1.1.7
The following augmented matrix of a linear system has been reduced by row operations to the form shown. In each case, continue the appropriate row operations and describe the solution set of the original systems.

$$\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Solution: From provided augmented matrix we can notice one thing:
the R_3 means this system has a built-in contradiction. There is no values of variables could satisfy the R_3 equation because $0 = 1$ is never true. So this system is inconsistent, it has no solution.

Problem 1.1.9
The same problem with *Problem 1.1.7* but the matrix is:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$

Solution: We continue to process the provided augmented matrix:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix} \xrightarrow{1/2R_4} \begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_3 + 3R_4} \begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2+3R_3} \begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

So the solution set of the original system can be describe as:

$$x_1 = 4,\ x_2 = 8,\ x_3 = 5,\ x_4 = 2$$

Problem 1.1.13
Solve the systems:

$$x_1 - 3x_3 = 8$$

$$2x_1 + 2x_2 + 9x_3 = 7$$

$$x_2 + 5x_3 = -2$$

Solution: We get the augmented matrix from the provided equations:

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

Then we process it:

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix} \xrightarrow{R_2-2R_1} \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{bmatrix} \xrightarrow{R_2-2R_3} \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -5 \\ 0 & 1 & 5 & -2 \end{bmatrix} \xrightarrow{R_3-R_2} \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -5 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -5 \\ 0 & 1 & 0 & 3 \end{bmatrix} \xrightarrow{1/5R_2} \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 3 \end{bmatrix} \xrightarrow{R_1+3R_2} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$

So the solution set of the system is:

$$x_1 = 5,\ x_2 = 3,\ x_3 = -1$$

Problem 1.1.19
Determine if the system is consistent. Do not completely solve this system.

$$x_1 + 3x_3 = 2$$

$$x_2 - 3x_4 = 3$$

$$-2x_2 + 3x_3 + 2x_4 = 1$$

$$3x_1 + 7x_4 = -5$$

Solution: First we can get the augmented matrix from these equations:

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{bmatrix}$$

Then we process it:

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{bmatrix} \xrightarrow{3R_3} \begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -6 & 9 & 6 & 3 \\ 3 & 0 & 0 & 7 & -5 \end{bmatrix}$$

We get the equations from R_2 and R_3, we find:

$$\begin{gathered} x_2 - 3x_4 = -6x_2 + 9x_3 + 6x_4 = 3 \\ x_3 = 7/9x_2 - 4/3x_4 \\ x_2 = 6x_4 + 3 \\ 3x_4 + 3 = 3 \\ x_4 = 0 \end{gathered}$$

At this point, we know x_4. Were we to substitute the value of x_2 and x_1 into equation 2 and 4 separately, then we could compute x_3 from equation 1. So a solution exits, the system is consistent.

Problem 1.1.23
Determine the values(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

Solution: First we process the matrix:

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \xrightarrow{3R_1} \begin{bmatrix} 3 & 3h & 12 \\ 3 & 6 & 8 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 0 & 3h-6 & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

So from matrix, we get equation:

$$(3h-6)x_2 = 4$$

The matrix is the augmented matrix of a consistent linear system, so we have:

$$\begin{gathered} 3h - 6 \neq 0 \\ h \neq 3 \end{gathered}$$

So the values(s) of h is $h \in (-\infty, 3) \cup (3, +\infty)$

Problem 1.1.25
The same problem with *Problem 1.1.23* but the matrix is:

$$\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix}$$

Solution: First we process the matrix:

$$\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix} \xrightarrow{-4R_1} \begin{bmatrix} -4 & -12 & 8 \\ -4 & h & 8 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} -4 & -12 & 8 \\ 0 & h+12 & 0 \end{bmatrix}$$

So from matrix, we get equation:

$$(h+12)x_2 = 0$$

The matrix is the augmented matrix of a consistent linear system, so we have:

$$h + 12 \neq 0$$

$$h \neq -12$$

So the values(s) of h is $h \in (-\infty, -12) \cup (-12, +\infty)$

Problem 1.2.1
Determine which matrices are in reduced echelon form and which others are only in echelon form.

$$a. \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} b. \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} c. \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} d. \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

Solution:
For a:
The leading entries in R_1, R_2, R_3 are all 1. And also each leading 1 is the only nonzero entry in the column. So the matrix a is in reduced echelon form.

For b:
The leading entries in R_1, R_2, R_3 are all 1.And each leading entry of a row is in a column to the right of the leading entry of the row above it. All entries in a column below a leading entry are zeros. But in the C_3, there are two leading 1.
So the matrix b is only in echelon form.

For c:
The R_3 is a all zeros row, but R_3 is above on R_4 which is nonzero row.
So the matrix c is not in echelon form.

For d:
Each leading entry of a row is in a column to the right of the leading entry of the row above it. And all entries in a column below a leading entry are zeros. But the leading entries of R_2, R_3, R_4 is not 1.
So the matrix d is only in echelon form.

Problem 1.2.3
Row reduce the matrices to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

Solution: We process it:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \xrightarrow{-1/5(R_3-6R_1)} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 0 & 1 & 2 & 3 \end{bmatrix} \xrightarrow{-1/3(R_2-4R_1)} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix} \xrightarrow{R_3-R_2, R_1-2R_2} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The pivot position in final matrix and original matrix is $(R_2, C_2), (R_1, C_1)$ separately.
The columns 1 and 2 are pivot columns.