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# Assignment 3: Part 2

1. Consider a relation schema $R$ with attributes $ACBDEFGHIJ$ with functional dependencies $S$.

$$S = \{A \to E, BC \to AE, C \to ACF, DE \to A, EFG \to AB, I \to J, J \to AI\}$$

**(a)** State which of the given FDs violate BCNF.

Performing the closure test for each of the FDs yields the following results:

- $A^+ = AE$
- $BC^+ = ABCEF$
- $C^+ = ACEF$
- $DE^+ = ADE$
- $EFG^+ = ABEFG$
- $I^+ = AIJ$
- $J^+ = AIJ$

Since none of the functional dependencies are trivial, and none of the LHS are superkeys, all of them violate BCNF.

**(b)** Decompose R into BCNF using a lossless-join decomposition into a set of relations in BCNF using the BCNF algorithm given in class.

Decomposing $R$ based on the FD $A \to E$ yields $A^+ = AE$, which gives the following two relations:

- $R_1 = \{AE\}$
- $R_2 = \{ABCDFGHIJ\}$

Projecting $S$ onto these relations gives $S_1 = \{A \to E\}, S_2 = \{C \to ACF, I \to J, J \to AI\}$.

$A$ is now a superkey for $R_1$, so $R_1$ is now in BCNF. $R_2$ must be decomposed further, and decomposing it on the functional dependency $J \to AI$ yields $J^+ = AIJ$, which gives the following two relations:

- $R_3 = \{AIJ\}$
- $R_4 = \{BCDFGHJ\}$

Projecting $S_2$ onto these relations gives $S_3 = \{J \to AI, I \to J\}, S_4 = \{\}$.

By the closure test, $I^+ = J^+ = AIJ$, so $I$ and $J$ are both superkeys for $R_3$ and $R_3$ is in BCNF. $S_4$ is also in BCNF as it has no functional dependencies.

This leaves the following set of relations: $R' = \{\{AE\}, \{AIJ\}, \{BCDFGHJ\}\}$.

**(c)** State whether your decomposition is dependency preserving. If not, state which FDs are lost.

The decomposition is not dependency preserving as the following FDs are not preserved:

$$\{BC \to AE, C \to ACF, DE \to A, EFG \to AB\}$$

2. Consider a relation P with attributes ABCDEFGH and functional dependencies T.

$$T = \{A \rightarrow B, BC \rightarrow ACE, C \rightarrow B, EF \rightarrow CG, EFG \rightarrow ABCD, GH \rightarrow ABCD\}$$

**(a)** Compute all minimal keys for P.

| Attribute | Appears on | | Conclusion |
|---|---|---|---|
| | LHS | RHS | |
| D | | ✓ | not in any key |
| F, H | ✓ | | in every key |
| A, B, C, E, G | ✓ | ✓ | must check |

Compute closures for all combinations of A, B, C, E, G. F and H must be in every combination and D must never be in a combination.

- $FHA^+ = ABFH$
- $FHB^+ = BFH$
- $FHC^+ = ABCDEFHG$
- $FHE^+ = EFH$
- $FHG^+ = ABCDEFHG$

So, the minimal keys are $FHC$ and $FHG$.

**(b)** Compute a minimal basis for T. In your final answer, put the FDs into alphabetical order.

Simplify to singleton right-hand, number resulting FDs, let this be set S1:

1 $A \rightarrow B$
2 $BC \rightarrow A$
3 $BC \rightarrow C$
4 $BC \rightarrow E$
5 $C \rightarrow B$
6 $EF \rightarrow C$
7 $EF \rightarrow G$
8 $EFG \rightarrow A$
9 $EFG \rightarrow B$
10 $EFG \rightarrow C$
11 $EFG \rightarrow D$
12 $GH \rightarrow A$
13 $GH \rightarrow B$
14 $GH \rightarrow C$
15 $GH \rightarrow D$

Eliminate redundant FDs from S1.

| FD | Exclude | Closure | Decision |
|---|---|---|---|
| 1 | 1 | $A^+ = A$ | keep |
| 2 | 2 | $BC^+ = BCE$ | keep |
| 3 | 3 | $BC^+ = BCAE$ | discard |
| 4 | 3,4 | $BC^+ = BCA$ | keep |
| 5 | 3,5 | $C^+ = C$ | keep |
| 6 | 3,6 | $EF^+ = EFGC\ldots$ | discard |
| 7 | 3,6,7 | $EF^+ = A$ | keep |
| 8 | 3,6,8 | $EFG^+ = EFGBCDA\ldots$ | discard |
| 9 | 3,6,8,9 | $EFG^+ = EFGCDB\ldots$ | discard |
| 10 | 3,6,8,9,10 | $EFG^+ = EFGD\ldots$ | keep |
| 11 | 3,6,8,9,11 | $EFG^+ = EFGCBA\ldots$ | keep |
| 12 | 3,6,8,9,12 | $GH^+ = GHBCDA\ldots$ | discard |
| 13 | 3,6,8,9,12,13 | $GH^+ = GHCDBAE$ | discard |
| 14 | 3,6,8,9,12,13,14 | $GH^+ = GHD$ | keep |
| 15 | 3,6,8,9,12,13,15 | $GH^+ = GHCBAE$ | keep |

Remaining FDs, let this be set S2:

**1** $A \to B$

**2** $BC \to A$

**4** $BC \to E$

**5** $C \to B$

**7** $EF \to G$

**10** $EFG \to C$

**11** $EFG \to D$

**14** $GH \to C$

**15** $GH \to D$

Reduce LHS of any FDs with multiple attributes in S2.

**2** $BC \to A$
$C^+ = CBAE$ so we can reduce the LHS to C

**4** $BC \to E$
$C^+ = CBAE$ so we can reduce the LHS to C

**7** $EF \to G$
$E^+ = E$ so we can't reduce the LHS to E
$F^+ = F$ so we can't reduce the LHS to F

**10** $EFG \to C$
$EF^+ = EFGCD$ so we can reduce the LHS to EF

**11** $EFG \to D$
$EF^+ = EFGCD$ so we can reduce the LHS to EF

**14** $GH \to C$
$G^+ = G$ so we can't reduce the LHS to G
$H^+ = H$ so we can't reduce the LHS to H

**15** $GH \to D$
$G^+ = G$ so we can't reduce the LHS to G
$H^+ = H$ so we can't reduce the LHS to H

Remaining FDs after reduction, let this be set S3:

**1** $A \to B$

**2** $C \to A$

**3** $C \to B$

**4** $C \to E$

**5** $EF \to C$

**6** $EF \to D$

**7** $EF \to G$

**8** $GH \to C$

**9** $GH \to D$

Eliminate redundant FDs from S3.

| FD | Exclude | Closure | Decision |
|---|---|---|---|
| 1 | 1 | $A^+ = A$ | keep |
| 2 | 2 | $C^+ = CBE$ | keep |
| 3 | 3 | $C^+ = CABE$ | discard |
| 4 | 3,4 | $C^+ = CAB$ | keep |
| 5 | 3,5 | $EF^+ = EFDG$ | keep |
| 6 | 3,6 | $EF^+ = EFCGAB$ | keep |
| 7 | 3,7 | $EF^+ = EFCDAB$ | keep |
| 8 | 3,8 | $GH^+ = GHD$ | keep |
| 9 | 3,9 | $GH^+ = GHCAEB$ | keep |

The following set S4 of FDs with merged RHSs is a minimal basis:

1 $A \to B$

2 $C \to AE$

3 $EF \to CDG$

4 $GH \to CD$

**(c)** Using your minimal basis from the last subquestion, employ the 3NF synthesis algorithm to obtain a lossless and dependency-preserving decomposition of relation R into a collection of relations that are in 3NF.

The set of relations created using S4 is:

$$R1(A, B), \quad R2(C, A, E), \quad R3(E, F, C, D, G), \quad R4(G, H, C, D)$$

**(d)** Does your schema allow redundancy? Explain why or why not.

No, it does not because every relation in the schema satisfies BCNF.